A

SYSTEM OF CRYSTALLOGRAPHY,

WITH ITS

APPLICATION

TO

MINERALOGY.

BY

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PART I.

PRINCIPLES OF CRYSTALLOGRAPHY.

PREFACE.

There are many systems of crystallography in print, but none in general use. The different systems hitherto published have failed to satisfy the wants of the public. They are either too difficult to learn, or when learnt, too troublesome for service. Hence crystallography is little studied, either by chemists or mineralogists; and the consequence is, that, for want of a language in which observations can be recorded, the study of crystallisation also is shunned. How trifling is the annual addition made to our knowledge of the crystalline forms of minerals, and how vague and inaccurate are the descriptions which chemists give us of the forms of nearly all the crystallised products of the laboratory! Yet no one questions the importance of studying the productions of crystallisation, or the necessity of employing crystallography to record the facts which the examination of crystals brings to light. Indeed, since the discovery of "Isomorphism," everybody has become desirous of knowing how to describe the crystals that fall in his way; the mineralogist, the chemical analyst, and the manufacturing chemist, are alike solicitous to record their observations; but all have been so disheartened by the failure of former attempts to learn crystallography, that when they now recur to this science, it is only to demand *how it can be learnt* EASILY.

The present publication is an attempt to answer this favourite question; to show the way, not merely how to describe a crystal, but how to do it easily. Whether the attempt is successful or not, every reader will determine according to his peculiar standard of easiness. Persons accustomed to the investigations peculiar to abstract science, and those who have a distaste for calculations, will not agree on what constitutes an easy system of crystallography, and therefore will not coincide in their estimate of the value of this production. I trust, however, that in forming his opinion, the reader will not omit to take into consideration, how difficult a thing it is to give a popular character to *any* abstract science, and how especially difficult when that science is one which has to do with so vast a multitude of complex and troublesome details as those which constitute crystallography. To describe these de-

tails briefly and intelligibly, to bring them under general laws, and to show in plain language the practical use of these laws in investigating the forms of minerals, is a task of such extreme difficulty, that success, however desirable, is of problematical attainment.

In expressing my opinion that a popular system of crystallography was a desideratum, I by no means intend to undervalue the existing works on this science. When I state that they are not in use because they are too difficult to learn, or too troublesome for service, I allude in these expressions, less to the qualities of the publications, or the qualifications of their authors, than to the attainments of the major part of the students by whom crystallography ought to be learned. The students who enter our schools of chemistry, mineralogy, metallurgy, and mining, possess almost universally but a slender stock of mathematical knowledge; while the existing books on crystallography, or at least all that have any pretensions to science or system, are remarkable for containing investigations which require very high mathematical attainments. "Crystallography," says Whewell, "is essentially a mathematical subject. The striking mixture of simplicity and complexity which here, as in other parts of nature,—but yet more here than in any other part of nature,—offers itself to our notice, depends upon the combination of the primary forms belonging to the above systems [of crystallisation] with the geometrical and numerical laws by which other forms are derived from these. To trace the properties of such derived forms, and of their combinations, necessarily requires some *considerable portion of mathematical calculation*, which may, however, be of several kinds. Spherical trigonometry, solid geometry, and analytical geometry of three dimensions, may, any of them, be made to answer the purposes of the crystallographer. Haüy and Mohs, proceeding in the manner which, of the three, *implied the least extended acquaintance with mathematics*, employed in most instances particular constructions and calculations founded on solid geometry; and though they thus want the conciseness, beauty, and generality of other methods, they are perhaps, in consequence of this, *intelligible to a wider circle of students*." *Report on the Recent Progress and Present State of Mineralogy, addressed to the British Association*, 1832.

In illustration of these crystallographic calculations, which imply "the least extended acquaintance with mathematics," I beg to lay before the reader a few extracts from Haüy's "Traité de Cristallographie," Paris, 1822. 2 tome 8vo, pp. 1340.

Tome I. page 452.

$$\omega\mu = \frac{1}{2}a \frac{\left(\frac{2nxy-2y}{nxy+x} \cdot \frac{x+nxy+y}{nxy-x+y}\right)\sqrt{\frac{1}{4}g^2}}{\sqrt{\cdot\left(\frac{nxy+2x+y}{3nxy+3y-3x}\right)^2 a^2+\frac{1}{4}g^2}}$$

Tome I. page 452.

$$B\omega : \omega\mu :: 1 : \tfrac{1}{2}a . \frac{\dfrac{2nxy-2y}{nxy-x+y}\sqrt{\tfrac{1}{3}}}{\sqrt{\left(\dfrac{nxy+2x+y}{3nxy+3y-3x}\right)^2 a^2+\tfrac{4}{3}g^2}}$$

$$:: \sqrt{\left(\frac{nxy+2x+y}{3nxy+3y-3x}\right)^2 a^2+\tfrac{4}{3}g^2} : \tfrac{1}{2}a . \frac{2nxy-2y}{nxy-x+y}\sqrt{\tfrac{1}{3}}$$

$$:: \sqrt{(nxy+2x+y)^2\tfrac{1}{3}a+(nxy+y-x)^2 4g^2}$$

$$: (nxy-y)\sqrt{a^2}$$

Tome II. page 112.

$$d\vartheta = \frac{\sqrt{\dfrac{x^2(4g^2+4p^2+h^2)}{4}}\times gy\sqrt{\dfrac{16p^2+4h^2}{4g^2+4p^2+h^2}}}{\sqrt{(x+y)^2\dfrac{4p^2+h^2}{4}+(x-y^2g)^2}}$$

$$= \sqrt{\frac{g^2x^2y^2(4p^2+h^2)}{(x+y)^2\dfrac{4p^2+h^2}{4}+(x-y)^2g^2}}$$

Tome II. page 113.

$$di = \frac{2\dfrac{h}{n}xy}{\sqrt{(x+y)^2\dfrac{4p^2+h^2}{4}+(x-y)^2g^2}}$$

$$\times \frac{gp}{\sqrt{g^2x^2y^2\left[\left(1+\dfrac{h^2(x-y)}{nxy(4p^2+h^2)}\right)^2\dfrac{4p^2+h^2}{(x+y)^2\dfrac{4p^2+h^2}{4}+(x-y)^2g^2}\right]+\dfrac{1}{n^2}\cdot\dfrac{4p^2h^2}{4p^2+h^2}}}$$

Tome II. page 114.

$$du : di :: \sqrt{\frac{\dfrac{75}{13}}{4\ .\ \dfrac{75}{13}+\ 1}}$$

$$: \sqrt{\frac{\dfrac{12\ .\ 36}{13}}{\left(4.\dfrac{36}{13}\cdot\dfrac{14^2}{13^2}\cdot\dfrac{13}{9.\dfrac{13}{4}+\dfrac{36}{13}}+4.\dfrac{12}{13}\right)\left(9.\dfrac{13}{4}+\dfrac{36}{13}\right)}},$$

By way of contrast to the foregoing specimens of calculations, assumed to be, from their facility, "intelligible to a wide circle of students," I take leave to quote, on the following page, a few scientific calculations, from one of the most original and popular writers among the crystallographers of Great Britain, I mean Professor WHEWELL, from whose Memoir on "*A General Method of Calculating the Angles made by any Planes of Crystals, and the Laws according to which they are formed,*" these equations are extracted. See *Philosophical* Transactions *of the Royal Society of London, for the year* 1825.

Phil. Trans. 1825, *page* 94.

$$-\cos.\theta=\frac{\left\{\begin{array}{c}\frac{AA'}{d^2}+\frac{BB'}{e^2}+\frac{CC'}{f^2}\\ -\frac{A'B+AB'}{de}\cos.\gamma-\frac{A'C+AC'}{df}\cos.\beta-\frac{B'C+BC'}{ef}\cos.\alpha\end{array}\right\}}{\sqrt{\left\{\begin{array}{c}\left(\frac{A^2}{d^2}+\frac{B^2}{e^2}+\frac{C^2}{f^2}-\frac{2AB}{de}\cos.\gamma-\frac{2AC}{df}\cos.\beta-\frac{2BC}{ef}\cos\alpha\right)\times\\ \times\left(\frac{A'^2}{d^2}+\frac{B'^2}{e^2}+\frac{C'^2}{f^2}-\frac{2A'B'}{de}\cos.\gamma-\frac{2A'C'}{df}\cos.\beta-\frac{2B'C'}{ef}\cos.\alpha.\right)\end{array}\right\}}}$$

Page 95.

$$-\cos.\theta=\frac{AA'+BB'+CC'-(A'B+AB'+A'C+AC'+B'C+BC')\cos.\alpha}{\sqrt{\left\{(A^2+B^2+C^2-2(AB+AC+BC)\cos.\alpha)\,(A'^2+B'^2+C'^2-2(A'B'+A'C'+B'C')\cos.\alpha.)\right\}}}$$

Page 125.

$$\cos.\theta=\frac{1+AA'+BB'+(A+A')\cos.\varphi+(B+B')\cos.\psi+(A'B+AB')\cos\omega}{\sqrt{(1+A^2+B^2+2A\cos.\varphi+2B\cos.\psi+2AB\cos.\omega)\,(1+A'^2+B'^2+2A'\cos.\varphi+2B'\cos.\psi+2A'B'\cos.\omega)}}$$

Page 126.

$$\cos.\theta=\frac{1+1+\frac{(p+q)\,(q+r)}{pr}+2\cos.\varphi-\left(\frac{p+q}{r}+\frac{q+r}{p}\right)\cos.\varphi-\left(\frac{p+q}{r}+\frac{q+r}{p}\right)\cos.\varphi}{\sqrt{(1+1+\frac{(p+q)^2}{r^2}+2\cos.\varphi-2\frac{p+q}{r}\cos.\varphi-2\frac{p+q}{r}\cos.\varphi)(1+1+\frac{(q+r)^2}{p^2}+2\cos.\varphi-2\frac{q+r}{p}\cos.\varphi-2\frac{q+r}{p}\cos.\varphi)}}$$

$$=\frac{3pr+pq+q^2+qr-2\,(p^2+r^2+pq+qr-pr)\cos.\varphi}{\sqrt{((p+q)^2+2r^2-2\,(p+q-r)\,r\cos.\varphi))\,(q+r)^2+2\,p^2-2\,(q+r-p)\,p\cos.\varphi)}}$$

This "considerable portion of mathematical calculation," and these illustrations, show the discordant principle which keeps the existing works on crystallography out of general use. The doctrines they develop are based on a higher philosophy than comes within the reach of the ordinary student of crystallography. The books are written in an unknown tongue and cannot be read.

Reflecting on these matters, it occurred to me, that the mathematical difficulties of crystallography were not inherent in the science, but resulted from the manner in which it was commonly treated, and that it was only necessary to adopt a different method of treating it, to be enabled to remove many of these difficulties, and to present the science in a much simpler and more attractive form than had been hitherto accomplished. It were unnecessary to do this if the mathematical attainments of students of crystallography could be easily and extensively increased; but since experience shows this to be impossible, the next best proceeding is to simplify and remodel the science so as to adapt it to the circle of knowledge possessed by the popular student. What matters it, if we lose a little of the elegance and precision, by avoiding the splendid difficulties, of abstruse mathematics? Is it not better that our science should be even imperfectly mastered by the multitude, than be entirely restricted to the service of a few accomplished mathematicians? Led by this idea, I began to examine the mutual relations of the forms of crystals, and the methods by which their geometrical properties could be investigated by a student possessing a *minimum of mathematical knowledge*; and having done this, I contrived a series of symbols, by which the results of these investigations could be easily, intelligibly, and accurately recorded. The principle of notation upon which these symbols are founded, occurred to me at an early period of my inquiry into this subject; and though it is a principle which, from its fertility in symbols, proved susceptible of extensive application in the science, it is, at the same time, extremely simple in its nature. I cannot perhaps introduce the few remarks which I have to make in this Preface, better than by giving a preliminary notice of the principle of the proposed notation, and a sketch of the crystallographic machinery by the employment of which the general laws of the science are subsequently developed.

We begin, then, to lay the foundation of crystallography by assuming the existence of what is termed *a system of axes*. These axes are the three lines which indicate the length, the breadth, and the thickness of a crystal. They are respectively denoted by the signs p^a m^a and t^a. These axes necessarily cross one another in the centre of the crystal at right angles, and the length of each of them depends upon the length, the breadth, or the thickness of the crystal. To promote perspicuity, p^a is assumed to be the principal or perpendicular axis, m^a to be the middle or minor axis, or that which passes from the front to the

back of a crystal, and t^a to be the transverse axis, or that which passes from the left to the right side. When a crystal is held up in front of an observer, its longest diameter is put in the place of p^a, and its shortest in the place of m^a. To indicate differences in the length of the three axes, indices are put under the small letter ^{a}, thus: p^a_+ m^a_- t^a, which indices intimate that p^a is longer than t^a and m^a shorter than t^a. These differences are more specifically shown by figures, as p^a_3 m^a_1 t^a_2, or more indefinitely by letters, as p^a_x m^a_y t^a_z.

In the next place, we consider *the relation of the external planes of the crystal to these three axes.* A careful investigation shows that the planes of all crystals are reducible to seven different kinds. Of these, three kinds are characterised by the property of cutting one axis, three other kinds by that of cutting two axes, and one kind by that of cutting three axes. These peculiarities give us the power *to indicate the seven different kinds of planes by merely naming the axes which they cut.* The notation thus afforded is short, simple, and precise. For

P	signifies	2	planes that cut	axis p^a.
M	—	2	do.	— m^a.
T	—	2	do.	— t^a.
MT	—	4	do.	axes m^a and t^a.
PM	—	4	do.	— p^a and m^a.
PT	—	4	do.	— p^a and t^a.
PMT	—	8	do.	— p^a m^a and t^a.

No crystal can possibly present planes that differ from all the above seven kinds. This remarkable fact can be easily demonstrated. The planes marked P are horizontal and form the top and bottom of a crystal. Those marked M are vertical and form the front and back of a crystal. Those marked T are vertical and form the left and right sides of a crystal. These planes have fixed positions, from which they cannot swerve without instantly losing their identity. If the plane P, for example, were to incline ever so little from its absolutely horizontal position, it must assume such a position that, if sufficiently extended, it would cut either one or two of the horizontal axes, and thereby become equal to plane PM, or PT, or PMT, according to its particular inclination towards m^a or t^a or both. In the same manner it can be shown that if the plane M changes its situation, it must become MT, PM, or PMT, and that if the plane T changes its situation, it must become MT, PT, or PMT. From these and other considerations which are stated at length in the fifth section of the following work, it can be proved that the planes which are denoted by the seven symbols P, M, T, MT, PM, PT, PMT, are all the kinds that can possibly occur upon crystals. These seven symbols, therefore, constitute the whole alphabet of crystallography, but brief as this alphabet is, the powers of the characters are such as to enable it distinctly to name the innumerable crystals with which nature and art present us.

I proceed to notice the few contrivances, by the adoption of which we are enabled to bestow upon these seven symbols the degree of descriptive energy proper to qualify them for this extensive duty.

The planes differ in respect to the *number that is proper to each kind*, which number is 2, 4, or 8. All the planes of each kind constitute a "Form," and according as the planes cut one, two, or three axes, the forms are termed uniaxial, biaxial, or triaxial. The uniaxial forms, P, M, T, contain each *two* planes to the set; the biaxial forms, MT, PM, PT, contain each *four* planes to the set; and the triaxial form, PMT, contains *eight* planes to the set. On certain minerals these forms occur with *all* their planes, on others with only *half* their proper number, and on a third kind with only a *fourth part* of their number. These differences are denoted by marking the half-form with the prefix $\frac{1}{2}$, and the quarter-form with the prefix $\frac{1}{4}$; as $\frac{1}{2}$MT, $\frac{1}{4}$PMT.

Although the planes or Forms of crystals are limited to these seven kinds, there are innumerable *varieties* of each kind. Thus, of the biaxial form MT, there is a multitude of varieties, which differ from one another in the relation which the length of axis m^a bears to that of t^a, that is to say, in the relative distances from the centre of the crystal at which the two axes, m^a and t^a, are cut by the set of four planes which constitute the form MT. The same may be said of the biaxial forms PM and PT, and of the triaxial form PMT; the differences in the last of which relate to two particulars, namely, the comparative length of p^a to t^a, and of m^a to t^a. In consequence of this *variation in the quality of each of the seven kinds of forms*, it is necessary to provide a discriminating index for the symbol of each variety. This discriminating index is a vulgar fraction placed between the letters which compose the symbol of the form, and of which fraction the upper figure shows the length of the axis named by the left hand letter, and the lower figure the length of the axis named by the right hand figure. Thus, M$\frac{2}{3}$T denotes a variety of the form MT, in which axis m^a bears to axis t^a, the ratio of 2 to 3. The cross section of this form is a rhombus, whose two diagonals have the relation of 2 to 3. This method of notation is extremely simple; it is capable of universal application to rhombic forms; and it has a very important mathematical use, since the index of every form indicates the angle at which the external planes of the forms incline upon one another. This inclination is, in all cases, *twice the angle whose cotangent is the fraction contained in the symbol.* Thus, $\frac{2}{3} = .6667$ is the cotangent of 56° 18$\frac{1}{2}$′, and the obtuse external angle of the form M$\frac{2}{3}$T is 112° 37′.

In like manner, the obtuse external angles of the forms P$\frac{2}{3}$M and P$\frac{2}{3}$T are all 112° 37′, and their *acute* external angles are 180° — 112° 37′ = 67° 23.′ Hence, all the planes which surround a crystal in three directions, at right angles to one another, can be denoted by some combination of the symbols of the first six Forms, P, M, T, M$_x$T, P$_x$M, P$_x$T,

and all the planes of crystals which do not belong to this series, are examples of the seventh or triaxial form $P_xM_yT_z$.

None of the forms but PMT denote a complete crystal, and the reason is very evident. The length, breadth, and thickness of a crystal can be denoted by not less than three rectangular axes. If we, therefore, take forms which relate to two axes only, as P, M, PM, or P, T, PT, or M, T, MT, it is clear that these forms can show the boundaries of the crystal only in two directions, and therefore cannot describe a closed figure or complete crystal. But if we take a combination of forms which cut all the three axes, no matter how, those forms will make up a complete crystal, because they will indicate dimension in three directions, at right angles to one another. The form PMT is the simplest example of planes that cut three axes; but, P with MT, P with M and T, M with PT, T with PM, MT with PM, or PM with PT, are all combinations which produce complete crystals, because every one of them cuts all the three axes. This principle shows, in a very simple manner, how great a diversity of crystals may be described by the combination with one another of the seven fundamental forms which are denoted by the before-mentioned symbols.

Without going into farther details, which would be inconsistent with the limits of a Preface, and serve merely to anticipate what is given in the text, I may observe, that the Forms of all crystals, natural or factitious, can be described by thus naming them partly after the *particular axes* that they cut, and partly after the *particular lengths* of the cut axes. This is a clear and intelligible principle of notation, and one that forms a sound basis for mathematical investigations.

The *Forms*, or sets of planes, thus discriminated, combine together and produce complex forms or crystals, which, in crystallographic language, are termed *Combinations*. Thus, the three Forms or octahedrons $P\frac{1}{2}MT$, $PM\frac{1}{2}T$, $PMT\frac{1}{2}$, combine together and produce the combination commonly called the Icositessarahedron. The three rhombic forms $M\frac{1}{2}T$, $P\frac{1}{2}M$, $P\frac{2}{1}T$, combine together and produce the combination called the Pentagonal Dodecahedron. The three uniaxial forms P, M, T, produce the Cube. The three biaxial forms MT, PM, PT, produce the Rhombic Dodecahedron. And so on. All natural crystals, with the single exception of the octahedron, are *Combinations*, containing two or more different *Forms*, sometimes in equilibrium as to the relative magnitude of their planes, as in P, M, T. PMT, and sometimes having one form greatly predominant over the other, as in p, m, t. PMT, or P, M, T. pmt, where the large letters show which are the predominant planes, and the small letters the subordinate planes. The business of the crystallographer is to analyse these Combinations, to discover what Forms they are composed of, to prove the separate identity of each of these forms, and to describe the whole in accurate and intelligible symbols. The methods of effecting these analyses, of conducting these investigations, and of

constructing the symbols proper to denote every different kind of combination, are subjects which undergo a full discussion in the following pages.

But the analysis of crystals and the description of them in symbols, one by one, is not the whole business of the crystallographer. The scientific arrangement of crystals is fully as important as their individual examination; indeed, the latter operation should be considered merely as a prelude to the former. "By one," if I may use the language of WHEWELL, "by one we form a *mob* of species, by the other we brigade them into a well-ordered *army*."

A ready classification of crystals can be founded on the distinction of their planes or Forms into prismatic and pyramidal. We can thus provide six classes, dependent for their characters on the *inclination* and *number* of the planes of the crystal. The first class, Complete Prisms, contains two horizontal planes, with three or more vertical planes. The second class, Complete Pyramids, contains six or more inclined planes, in two equal sets, and having two solid angles on axis p^a: one at pole Z, and another at pole N. The third class contains Complete Prisms, combined with Incomplete Pyramids. The fourth class, Incomplete Prisms, combined with Complete Pyramids. The fifth class, Incomplete Prisms, combined with Incomplete Pyramids. And the sixth class, Incomplete Pyramids, which are combinations or forms that contain inclined planes only, but have no solid angles on axis p^a.

Each of these six Classes of crystals can be sub-divided into five Orders, whose characters depend upon the form of the horizontal cross section or *equator* of the crystal. The first order comprises those whose equator is square, the second those which are rectangular, the third rhombic, the fourth rhombo-quadratic, and the fifth rhombo-rectangular.

By these sub-divisions, all crystals are separated into 30 Orders, *the characters of which are easily discriminated at sight.* And the Orders can be farther separated into Genera, according to the ratios of the axes of the combination, as $p^a\, m^a\, t^a$, $p^a_x\, m^a\, t^a$, $p^a_x\, m^a_{15}\, t^a_{15}$, $p^a_x\, m^a_{14}\, t^a_{13}$, or $p^a_x\, m^a_y\, t^a_x$ which characters are also very easy of recognition.

Pursuant to this classification of crystals, their seven fundamental Forms are separated into two classes, as follows:—

Prismatic Forms: P, M, T, MT.

Pyramidal Forms: PM, PT, PMT.

Those of the first kind are either horizontal or vertical, and are considered to be the components of *prisms*. Those of the second kind are all oblique, or inclined, and are considered to be the components of *pyramids*. This discrimination of the Crystallographic Forms into two classes, leads, as I have shown, to a classification of crystals which is of great practical utility. It infers, however, the necessity of restricting or modifying the use of such terms as *oblique* and *doubly-oblique prisms*, because the oblique planes of such forms belong, in the present crys-

tallometrical method, not to prisms, but to pyramids. But the separation of the planes of crystals into *horizontal*, *vertical*, and *oblique*, leads also to the necessity of establishing a fixed *point of view* for crystals, in order that terms so strictly *relative* as horizontal, vertical, and oblique, may acquire a definite or positive acceptation. I propose, therefore, that the point of view of a crystal shall be in the prolongation of the minor axis m^a; that the crystallographer, when examining a crystal, shall always be assumed to look towards the south; and that a crystal, while under examination, shall be considered to be polarised. Then, the uppermost part of the crystal will be the zenith pole = Z, the lowermost part, the nadir pole = N, the part directly facing the spectator, the north pole = n, the part opposite, the south pole = s, the side opposite the spectator's right hand, the west pole = w, the side opposite, the east pole = e. The acceptation of these arbitrary terms, enables us to describe particular positions or points on a crystal, with a most convenient degree of accuracy. This advantage is not limited to the above six poles, for the compound terms Zn, Zs, Ze, Zw, Nn, Ns, Ne, Nw, and nw, ne, se, sw, enable us to refer, with equal perspicuity, to twelve polar positions, each intermediate between two of the six primary poles, and the terms Znw, Zne, Zsw, Zse, and Nnw, Nne, Nsw, Nse, enable us farther to refer to eight other polar positions, each respectively equidistant from three of the six primary poles.

It is in many cases convenient to refer to lines which pass from the centre of the crystal, and terminate in one or other of the above-named twenty-six poles. These lines are termed *Normals*. The normals which terminate in poles denoted by one, two, or three letters, are termed respectively unipolar, bipolar or tripolar normals. By means of these poles and normals we can refer with great accuracy to twenty-six different points on the surface of a crystal, and to the angles which the normals make with one another at the centre of the crystal, and which they also make with other lines that connect their poles. Thus, the inclination of a

$$\text{Tripolar Normal to any adjacent}\begin{cases}\text{Unipolar Normal} = 54^\circ\ 44'\\ \text{Bipolar Normal} = 35^\circ\ 16'\\ \text{Tripolar Normal} = 70^\circ\ 32'\end{cases}$$

Great use is made of this principle in investigating the forms of the crystals which belong to that class whose length, breadth, and thickness are alike.

For other practical purposes, I assume every crystal to be capable of division by sections, as follows:—By a horizontal section passing through the poles n, e, w, s, and parallel to the axes m^a and t^a. I call this section the *Equator*. It separates the Zenith half of the crystal from the Nadir half. There are four other sections, all vertical, and which are called *Meridians*. The two first of these are of most importance. The north meridian passes through the poles n, Z, s, N, and parallel to the axes p^a and m^a, and separates the east from the west half of the

crystal. The east meridian passes through the poles e, Z, w, N, and parallel to the axes p^a and t^a, and separates the north from the south half of the crystal. The north-east meridian passes through the poles Z, ne, N, sw. The north-west meridian through the poles Z, nw, N, se. The two first and two last meridians are situated at right angles to one another, but the common intersection of the four meridians is an angle of 45°, and all the meridians make an angle of 90° with the equator. By means of any two of these sections, situated at right angles to one another, a crystal is divided into equal quarters, and there are six possible combinations of this kind. By means of the four meridians a crystal is divided into long octants or eighths. By means of two meridians and the equator, the crystal is divided into octants of another kind. By means of the five sections together, a crystal is divided into sixteenths. If we want to distinguish one of these quarters, eighths, or sixteenths, from all the rest that can be produced, we can do it easily by naming its polar situation. Thus, the sixteenth Zn^2w, is that which is on the zenith side of the equator, the north side of the east meridian, the west side of the north meridian, and the north side of the north-west meridian. The outer angles of this sixteenth are at the poles Z, n, and nw, and the edges formed by the intersection of its meridians and equator, have the positions of the Z, n, and nw normals. The plane angles of the internal faces, and the interfacial angles of the internal edges of this sixteenth can be readily determined from these data.

The surface planes of a crystal, which are tangents to the above five sections, constitute collectively the *Zones* of those sections. Thus the planes M, T, MT, produce the prismatic or equatorial zone ; the planes P, M, PM, the north zone; the planes P, T, PT, the east zone; and the planes PMT, the north-east and north-west zone. Each of these zones is a belt of planes surrounding the edge of the corresponding section, and the axis of the zone is at right angles to the given section. The form PMT produces two zones, whose axes cross at a right angle.

Such are the foundations of the System of Crystallography which is communicated in the following work, respecting the execution of which I will now take the liberty to make a few explanatory observations.

The First Part is devoted to an account of the " Principles of Crystallography." This may be considered partly in the light of a Popular Introduction to this science, and partly as a series of original essays on the leading topics of Crystallographic research. The work was at first intended to contain only a brief account of the new system of notation, but being sent to the press as it was written, bit by bit, it gradually came to embrace a much wider range of subjects than the author originally proposed, until it finally comprehends a tolerably complete sketch of modern Crystallography.

In consequence of the composite character of the work, it presents a

greater amount of elementary matter, especially on mathematical topics, than it is usual to intermix with original investigations; while on the other hand, it contains much more argument and discussion than is perhaps adapted to an elementary treatise. Hence the critical reader may notice a want of unity in the several parts of the work, and the occurrence of some apparently needless repetitions. But it is hoped that faults of manner of this kind will not be found very detrimental, and will be considered to be compensated by the substantial merits of the associated information.

The first six sections of the work relate to topics already spoken of in this Preface, namely, to the Axes, Poles, Normals, Planes, and Sections of Crystals; to Prisms and Pyramids and their combinations; to the Classification of Crystals; to the possible limit to the variety of Planes that can occur upon Crystals; to the demonstration of the sufficiency of three rectangular axes for all useful references in Crystallography; and to the description of crystallographic Notation. This scientific machinery may appear cumbrous, but it is useful. The equator and meridians are as indispensable to the crystallographer as to the geographer, while axes and poles are no less necessary in the analysis of crystals, than are pistils and stamens in the discrimination of plants.

The seventh section relates to Cleavage, and to the doctrine of Primitive Forms; of these I need only say, that I have proposed an easy and exact method of denoting Cleavage by symbols, and argued for the total relinquishment of the doctrine of primitive forms, as one highly injurious to the science. Section eighth contains explanations relative to Forms and Combinations. Section ninth treats of the five Zones. Section tenth of the law of Symmetry, and of the distinction between Homohedral, Hemihedral, and Tetartohedral Forms. Section eleventh contains a new theory of crystallisation, founded on an opinion that electricity is the mainspring of crystallisation.

The reader will perceive that the first six sections are entirely confined to descriptions and explanations, and that all theoretical and general views,—all inferences drawn from foregone statements of facts,—are confined to the five last sections. By this separation and arrangement of particular and general truths, I have endeavoured to lead the popular student by the least difficult path to a knowledge of the science. I considered it, however, to be unnecessary to enter into much detail on theoretical matters, and I have therefore devoted but forty pages to the whole five last sections. But the speculative reader will not, I trust, hold these sections to be deficient in interest because they are limited in extent. He will find them to contain a variety of new views on some of the most curious topics of this branch of natural science; and although a thorough investigation of the objects discussed in them was unnecessary for the mere illustration of "the art of describing crystals," yet their farther pursuit, in reference to the physical theory of crystallisa-

tion, would neither be without interest to the student, nor advantage to science.

Section twelfth contains a popular account of the use of Spherical Trigonometry in Crystallography. The object of this section is to prepare and explain a collection of trigonometrical formulæ to be used empirically in crystallographic calculations, by persons who possess too little mathematical knowledge to prepare the formulæ, and yet sufficient to use them in the investigations proper to Crystallography. The method here followed of presenting the mathematical part of the subject in a state ready for the technical use of the crystallographer, appears to me to remove many of the difficulties which have heretofore impeded the popular study of this science.

The thirteenth section contains an inquiry into the variety of forms and combinations which occur upon the crystals of minerals. There are three objects proposed in this section. One is, to show the agreement between the natural crystals of minerals, and the mathematical forms and combinations described in the preceding twelve sections. Another object is, to give a partial explanation of the system of Crystallography proposed by Weiss, and modified by Rose, Naumann, Mohs, Miller, and others, and to prove that the new notation is adapted to describe all the forms and combinations belonging to that system, and represented to be all that occur in nature. The third object is, to show the methods by which the mathematical calculations requisite for these several purposes, can be made empirically by means of the prepared Formulæ and general Rules communicated in the twelfth section.

This last object is one of great importance to the student, and the solution of the problem that it presents, has received my best attention. It appeared to me at the beginning of my inquiry, that if the mathematical calculations could not be rendered short and easy, it would be needless to entertain any hope of ever bringing Crystallography among the number of the popular sciences. I set myself therefore to try what could be done towards simplifying and organising the calculations that were indispensably requisite to discriminate and identify the co-existing forms of complex combinations. This research produced a body of Crystallographic Analyses, which is, I believe, the most comprehensive and systematic that has ever been published. All the forms and fundamental combinations of each of Rose's six systems of Crystallisation, namely, the Octahedral, the Pyramidal, the Rhombohedral, the Prismatic, the Oblique Prismatic, and the Doubly Oblique Prismatic, have been fully investigated, and their relations are pointed out, on the one hand to crystallised minerals, and on the other hand, to the symbols employed to denote them. One peculiarity of the mathematical processes given in this part of the work is, that they are never *arbitrary*, but are always founded upon the simple principle of finding the relation of certain unknown to certain known sides and angles of the solid triangles

produced by the ideal analysis of the crystals subjected to examination. By a systematic adherence to this principle, the calculations are brought into *a regular train*, the *reason* of each of them is distinctly shown, and the *method of operating* is, by frequent repetition of the same thing, made habitual and easy.

It will be noticed, that many of the equations given in this section afford results that are not rigidly but only approximately accurate. This arises in some cases from the circumstance that angles are reckoned only to minutes, not to seconds. In other cases, from the adoption of logarithmic quantities having but four instead of seven figures after the point. The errors thus produced are of very little consequence. I have used short numbers, because the wearisome calculations introduced by the affectation of extreme accuracy, interrupt the train of reasoning, and disgust the reader entirely, instead of communicating to him a more exact amount of information.

Independent of any merit which this section may possess on the score of its mathematical simplifications, it presents a variety of new methods of crystallographic research, of which some may probably prove interesting to the more accomplished crystallographer, for whom the popular parts of the work can have no attraction. Dispensing, as I have done, with all notation, except that which refers to *a single system of three rectangular axes;* dispensing with primitive forms, with primary forms, with fundamental forms; with oblique prisms, and with doubly oblique prisms; assuming new views in relation to the difference between homohedral and hemihedral forms; subdividing the forms of the "oblique prismatic system" on grounds entirely new; and making other important changes in different departments of the science; it became necessary to make many new arrangements, and introduce a little new phraseology, to compensate for the loss of so many of the creations, and so much of the machinery, of former crystallographers. Whether the suggested alterations are amendments on the science, or merely examples of retrograde steps, time and experience will determine.

The fourteenth section of the work contains an account of Mr. Brooke's popular method of describing crystals by means of primary forms and tables of modifications. This section is intended to show that the doctrine of primary or primitive forms is useless and mischievous, being equally unadapted for popular and for scientific nomenclature.

The fifteenth section contains an inquiry into the means by which crystallographic notation may be made to attain the utmost limit of brevity. I have endeavoured to establish certain principles which ought always to govern the contrivance of notation, and to show how far extreme brevity of expression is practicable, and how far it is desirable.

The sixteenth section contains a table of sines and tangents adapted for working the equations described in the preceding sections; which, although too brief to replace the ordinary Tables of Logarithms, will

often prove a useful substitute, especially from its containing *the* INDICES *of the Symbols of all commonly occurring varieties of the seven crystallographic Forms.*

The Second Part of the work contains an account of the application of Crystallography to the investigation of the forms of crystallized Minerals.

The first section presents a tabular arrangement of minerals according to the six systems of crystallisation, described in the thirteenth section of Part I. The catalogue of minerals is translated from the one published in GUSTAV ROSE's *Elemente der Krystallographie.* It shows the crystallographic system, and, approximately, the chemical composition, of every particular Mineral or Isomorphous group of minerals.

The second section contains a catalogue of all the crystals or natural combinations peculiar to the minerals named in Rose's catalogue. The minerals are arranged in six classes, on the principles described in section thirteenth. Each class opens with an account of its peculiar axes, forms, and fundamental combinations. Under each mineral, its different natural crystals, or secondary forms, are described in symbols, each in a different line, with an abridged reference to several well known mineralogical works, which contain figures or descriptions of the crystals thus particularized.

The compilation of this catalogue cost me a great deal of labour, yet, after all, it is but an imperfect production, and gives only a provisional and approximate view of what I wished it to represent. When the calculations, upon which these symbols are founded, were made, I was not acquainted with the methods of investigation which are now printed in the thirteenth section. Indeed, the analytical formulæ of that section were contrived during, and subsequently to, the writing of this catalogue of minerals. They are the fruits of my attempt to reduce into precise notation and regular order, the vague and often discordant statements of the mineralogists whose works I have quoted in the text. Had I possessed this body of mathematical aid when I *began* my task, it would have been better performed; but even the best provision as to method, would not supply that want of material which is found by an author who attempts to study such a science as Crystallography in such a town as Glasgow, where there is no public collection of crystallised minerals to refer to, and where no public library contains the literature of a science so "dry" as Crystallography. The crystallographers of Berlin, and Freiberg, and Paris, would need to study in this city a little while, to be able to feel the weight of this difficulty. But for the kindness of Dr. THOMAS THOMSON, to whom I am indebted for the loan of several scarce books, my catalogue of minerals would have been even more inaccurate than it appears at present.

But although the table is not to be depended upon as absolutely

correct, it nevertheless presents a great amount of information respecting the natural crystals of minerals, and it affords a convenient method of comparing the recorded crystallographic combinations with those which may be presented by the crystals of any mineralogical collection to which the crystallographer may happen to have access. The use of such a table is to show the mineralogist what varieties of each mineral have been observed and described, and so lead him to the discovery of new and unrecorded combinations.

It is scarcely necessary to add, that the thirteenth section of the first part of this work is at once an introduction to, and a commentary on, the second section of the second part. The former contains instructions *how to do*, what the latter presents *done;* imperfectly done, indeed, but still presenting the basis of a work which will be highly useful when rendered complete.

The third section of this part contains a systematic arrangement of natural crystals, with a list of the minerals which are common to each crystal. This section is a counterpart to the last. One presents an account of all the crystals proper to each mineral. The other presents an account of all the minerals which are common to each crystal. The object of the former is to aid in completing the natural history of each mineral. The object of the latter is to direct the mineralogist how to discover the name of a mineral from an examination of its form. The classification of crystals adopted in this section, is necessarily different from the classification of minerals followed in the last section; the principles of classification being those which are described at page xv. The classes, orders, genera, and groups in which the minerals are here arranged, are sharply defined and strongly contrasted, and it appears to me, that the classification in question, with the subordinate contrivances which accompany it, are well qualified to constitute a useful GUIDE TO THE DISCRIMINATION OF MINERALS, the contrivance of which is the principal object of this section.

I shall endeavour to show the practical use to the Mineralogist, of the analytical method presented in this section, by contrasting it with MOHS's instructions for the use of his celebrated "*Characteristic.*" I quote from *Haidinger's Translation of Mohs's Treatise on Mineralogy*, vol. i, pp. 383—387.

"It will be useful to give a short explanation of the process used in the determination of minerals.

"If a mineral is to be determined, first its *Form*, if this be regular, must be ascertained, at least as far as to know the system to which it belongs. Then *Hardness* and *Specific Gravity* must be tried with proper accuracy, and expressed in numbers. It is sufficient however, to know the latter to one or two decimals. The specific character *requires* these data; they are also of use in the characters of the classes, orders, and genera. After this examination, the Characteristic may be applied, and it will at the same time point out what other characters are still wanting; so that a mere inspection of the mineral, or a very easy experiment, as for instance, to try the streak upon a file, or still better, upon a plate of porcelain biscuit, will very often be sufficient. The given individual is now carried through the subordinate characters of the classes, orders, genera, and species, one

after the other, comparing its properties with the characteristic marks contained in the characters of these systematic unities. From their agreement with some, and their difference from other characters, we infer, that the individual belongs to one of the classes, to one of the orders, to one of the genera, and to one of the species. Having advanced in this manner to the character of the species, it will in some instances be necessary, and in all cases advisable, for the sake of certainty, to have recourse to the dimensions of the forms. This is particularly necessary, if the genus to which the mineral belongs, contain several species having forms of the same system, as is the case in the genus Augite-spar. The common goniometer in most cases will suffice for determining the dimensions of the forms, the differences in the angles being in general so great, that they cannot easily be missed, even by the application of this instrument. If the differences be small, and their distinction require on that account a higher degree of accuracy, it will be necessary to recur to the reflective goniometer.

"It will seldom be necessary to read over the whole of any character of a class, order, genus, or species, excepting those which comprise the individual; one term that does not agree sufficing for its exclusion. Thus even the characters of the orders, though the longest, will not be found troublesome.

"The application of the Characteristic has been facilitated in a great measure by separating the absolute characteristic marks from the conditioned ones. It becomes still more easy and expeditious, by taking particular notice of some characters, which might be termed *prominent*. Such are a metallic appearance; a high degree of specific gravity, particularly if the appearance be not metallic; and a high degree of hardness. The observation of these will immediately decide whether an individual can belong to any particular class, order, genus, or species. It is understood, that if it be not thereby excluded, the other characters must next be examined, till either an excluding one be found, or if not, the individual may be considered as belonging to that class, order, &c., with which it has been compared and found to agree.

"An individual, which has been carried through the characters of the classes, orders, genera, and species, and whose systematic denomination has thus been found, is said to have been *determined*.

"In illustration of this, let us take the following example. *Let the form of the mineral which is to be determined, be a combination of a scalene eight-sided pyramid, of an isosceles four-sided pyramid, and of a rectangular four-sided prism; the cleavage parallel to the faces of two rectangular four-sided prisms, in diagonal position to each other; form and cleavage therefore pyramidal, or belonging to the pyramidal system. Let Hardness be* = 6.5; *Specific Gravity* = 6.9.

"In this case, both hardness and specific gravity are prominent characters, and exclude the individual at once from the first and third, but not from the second class; with the characters of this class, its other properties also perfectly agree. Hence the individual belongs to the second class.

"Comparing the properties of the individual with the characters of the orders in the second class; hardness and specific gravity will be found too great for the order Haloide; hardness too great for the orders Baryte and Kerate; both of them too great for the order Malachite and Mica; and specific gravity too great for the order Spar and Gem. But in the character of the order Ore, both hardness and specific gravity fall between the fixed limits, and cannot exclude the individual from this order. The other parts of this character are now to be taken into consideration. If the appearance of the individual be metallic, its colour must be black, otherwise it cannot belong to the order Ore. But *the appearance is not metallic*; therefore the colour of the individual is quite indifferent; that is, this conditional characteristic mark does not affect the individual, and consequently cannot decide. Since the appearance is not metallic, the individual must exhibit *adamantine* or imperfect metallic *lustre*. The first will be found, particularly in the fracture. The following characteristic marks refer to minerals of a red, yellow, brown, or black streak; and as the individual gives none of these, *its streak being uncoloured*, these characteristic marks do not come into consideration. The next mark requires, that if hardness be = 4.5 and less, the streak should be yellow, red, or black; but *hardness* is = 6.5, therefore the colour of the streak indifferent. If hardness be = 6.5 and more, and streak uncoloured; then specific gravity must be = 6.5 and more. Now this condition takes place; *hardness is* = 6.5, *streak is uncoloured*. But also the conditioned character takes place, *specific gravity being* = 6.9, which is greater than 6.5.

"In regard to the individual, which is to be determined, all the characteristic marks con-

stituting the Character of the order Ore, may be divided into two parts. The first part contains those which refer to the individual; the second those which do not; the last evidently cannot be decisive. But with the first, all the properties of the individual concur. These properties agree consequently with the whole character of the order, as far as it is applicable to the individual, and determine it to belong to the order Ore, or, in shorter terms, to be an *Ore*.

"It will be advisable to beginners, who do not yet possess a sufficient practice in the use of the Characteristic, also to compare the characters of the remaining orders, which will enable them to find out any error they might have committed in the comparison of the individual with the characters of the preceding orders. In the present case, the non-metallic appearance excludes the individual from the orders Metal, Pyrites and Glance; hardness from the order Blende; and both hardness and specific gravity from the order Sulphur. This fully confirms the above determination, and we must now return to the order Ore for comparing the properties of the individual with the generic characters which the order contains.

"Considering again hardness and specific gravity as prominent, the individual will be immediately excluded from the genera Titanium-ore, Zinc-ore, and Copper-ore, but not from the genus *Tin-ore*. The forms of the pyramidal system, and the uncoloured streak, show that it belongs to this genus. If we compare the individual with the remaining generic characters, we find that it is excluded from the genus Scheelium-ore by its too great hardness, and too little specific gravity; from the genera Tantalum-ore, Uranium-ore, Cerium-ore, Chrome-ore, Iron-ore, and Manganese-ore, by hardness and specific gravity, both of them being too great; as also by its uncoloured streak, which only agrees with that genus from which the individual differs most by its hardness and specific gravity. From all this we infer that the individual cannot belong to any other than to the fourth genus, and that we are therefore entitled to give it the name of *Tin-ore*.

"This genus contains but one species. The conclusion that the individual must belong to this species, might nevertheless be erroneous. There could exist a second species of this genus. Hence we must accurately consider the dimensions of the forms. If these coincide with the angles given in the character, the highest degree of certainty, that the individual belongs to or is *pyramidal Tin-ore*, will be obtained."

The determination of a Mineral, after the method described in Section III. Part II. of this work, requires the following particulars:—

1. Crystallographic Forms of the combination, described in symbols.
2. Axes of the combination.
3. Lustre or transparency.
4. Hardness.
5. Streak.
6. Specific gravity.
7. Cleavage.

These particulars are given by MOHS, as follows:—

1. CRYSTALLOGRAPHIC FORMS: *a*) A Scalene eight-sided pyramid. This is a dioctahedron of the pyramidal system $= p_x m_y t_z,\ p_x m_z t_y$. *b*) An isosceles four-sided pyramid. This may be $P_x MT$, or $P_x M$, $P_x T$. *c*) A rectangular four-sided prism (meaning a square prism). This may be either M, T, or MT. Hence the combination is one of the four following:—

$M, T. P_x MT,\ p_x m_y t_z,\ p_x m_z t_y.$ $M, T. P_x M,\ P_x T,\ p_x m_y t_z,\ p_x m_z t_y.$

$MT. P_x M,\ P_x T,\ p_x m_y t_z,\ p_x m_z t_y.$ $MT. P_x MT,\ p_x m_y t_z,\ p_x m_z t_y.$

A sight of the crystal would enable us to discard two of these symbols, which we cannot do from MOHS's description of the combination, because he does not tell us whether or not the four-sided pyramid and

four-sided prism belong to the same or to different zones. We have also to assume, since he does not condescend upon that point, that the planes P do not form part of the rectangular (quadratic) prism.

2. Axes = $p_x^a m^a t^a$.
3. Lustre = non-metallic; adamantine.
4. Hardness = 6.5.
5. Streak = uncoloured.
6. Specific gravity = 6.9.
7. Cleavage = m, t, mt.

This combination is immediately referred to Class 4, page 110, because the forms $M,_{,}^{'}T$, or MT, without P, constitute an *Incomplete Prism*, and because the forms $P_x MT$ or $P_x M$, $P_x T$, either with or without $p_x m_y t_z$, $p_x m_z t_y$, constitute a *Complete Pyramid.*

The combination is referred to Order 1, because the equator of M, T, or MT, is square.

It is referred to Genus 2, because the axes are $p_x^a m^a t^a$.

It is referred to Group *a*, because the prism M, T, or MT, has four vertical planes. See Part II. page 111.

The group thus referred to, contains fifteen Minerals, among which the twelfth in order *agrees in hardness* with our specimen. It also agrees in *lustre*, as tl signifies *non metallic* and *translucent.* It disagrees in *streak*, as br signifies *brown*, and not *uncoloured;* but in point of fact, MOHS, in another place, states the streak of the given Mineral to be *uncoloured to pale brown.* Finally, it agrees in *specific gravity*, as 7 is very close to 6.9; and there is no other Mineral in the same group which has any thing near the same hardness, without differing very greatly in the specific gravity.

The Mineral which thus agrees in its characters with the given specimen is Oxide of Tin. By means of the Index, we find the place of Oxide of Tin in the second section, page 34, and examine its cleavage, which is m,t,mt. $P_3^s M, P_3^s T$, which partly agrees with MOHS's statement, although it contains something more. We then look down the list of the natural crystals of Oxide of Tin, to find if any of them agree with the given combination, and among the varieties marked 4, 1, which mean Class 4, Order 1, from which Order we have just been referred, we find two crystals which agree very nearly with the given characters. These are

MT. $P_3^s MT$, $p_3 m_2 t$, $p_3 m t_r$.

MT. $P_3^s M$, $P_3^s T$, $p_3 m_2 t$, $p_3 m t_r$.

In the first of these combinations, the prism and quadratic pyramid belong to the same zone; in the second, they belong to different zones. Our determination of the specimen can go no farther, because MOHS has neglected to declare in his data, which of these two varieties he meant to refer to.

This single investigation shows that the method of analysis which is

here proposed, is preferable to that of MOHS's, not only on the score of simplicity and facility, but also in point of precision.

The fourth and last section of the work contains a description of the Models of Crystals employed to illustrate, not merely the general principles of this science, but its practical, mathematical, and mineralogical details. To the descriptions given in this section, I need only add, that the possession of models of this kind is indispensable for the comprehension of the science, by every one who does not study it according to the strict rules of mathematics. It is, perhaps, possible for an accomplished geometer to learn the principles of Crystallography without seeing either crystals or models; but the popular student need not attempt to learn this science without the aid to be derived from models of crystals, which afford that constant tangible correction of his erroneous ideas, which is indispensable to his progress and success.

I have purposely avoided, in this work, the discussion of two subjects of considerable interest to the crystallographer. One of these is Isomorphism; the other is the Optical Properties of Minerals.

In Rose's catalogue of Minerals, the species are arranged in Isomorphous groups, and as I have followed this arrangement, the reader has the opportunity of comparing with one another, many groups of natural crystals of the so-called Isomorphous minerals. The section also in which I have brought together the different minerals which crystallise in the same form, presents other Isomorphous groups which are deserving of notice, while at the same time it affords groups of dimorphous, isodimorphous, and plesiomorphous crystals, which illustrate other relations of the forms of minerals in an interesting and striking manner. Unfortunately, these catalogues are neither sufficiently correct nor sufficiently extensive, to warrant the drawing of conclusions exact enough either to confirm or overturn the Law of Isomorphism, such as it was announced by Mitscherlich:—"The same number of atoms combined in the same way, produces the same crystalline form, and the same crystalline form is independent of the chemical nature of the atoms, and is determined only by their number and relative position." If a hundred examples can be drawn from the following tables to confirm this law, we can also point out a hundred other examples that are repugnant to it. It would be unwise to reject or neglect a principle which is so important as Isomorphism, if it prove to be true, but it is unsafe to adopt as a law of nature, a rule which seems liable to so many exceptions. I am aware that Isomorphism is considered, by a large body of philosophers, to be firmly established, but I doubt whether they have allowed due weight to the evidence which is arrayed against it. The hope that it might be true, has probably induced many to exercise a biased judgment on its merits. It is much to be desired that the salts and minerals, commonly assumed to be isomorphous, should be thoroughly investigated, and the

acknowledgment of their absolute isomorphism be conceded only upon satisfactory crystallographic evidence.

The optical characters of crystals are passed over with a very slight notice, in consequence of my inability to afford the time that would be required to investigate the subject so carefully as to be enabled to write a popular account of it. Not that the mere description of the principal facts known respecting double refraction and the polarization of light would be attended with much difficulty; for in fact, the Notation and Crystallographic machinery employed in this work are adapted for readily conveying information on such points as those. But the consideration of the optical properties of minerals, in relation to the theory of crystallisation, which I have partially developed in the eleventh section of this work, would lead to a field of speculation so wide, that I cannot at present venture to enter upon it. I therefore leave this interesting subject for future investigation.

I once intended to add to this Treatise a third part, on the application of Crystallography to Chemistry. But when I came to examine the descriptions which chemical writers give of the forms of the crystallised products of the laboratory, I found them to be universally so vague, that I could not attempt to translate them into symbols. The crystallographer who would write a catalogue of crystallised chemical products, has a long task before him, for he must *make* as well as *measure* the substances that it would be necessary to describe. And not only must he *make the crystals*, but he must do so with a close examination of the circumstances of the crystallisation of each of them. He must accurately determine their chemical composition, and that also of the mother liquor from which they are taken. He should notice the effects produced by changes of temperature, by light, air, and mechanical obstructions; by chemical, electrical, and magnetical action, and so on. This is a task not likely to be soon undertaken; for although the result would be highly useful to science, the labour would be miserably unprofitable to the philosopher. The task, however, must be done, and well done, before we can derive substantial advantages from the doctrine of Isomorphism.

GLASGOW, *November 2d*, 1840.

PART I.

PRINCIPLES OF CRYSTALLOGRAPHY.

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SECTION I.—OF THE AXES OF CRYSTALS.

1. CRYSTALLOGRAPHY is the art of describing crystals.

2. A crystal is a homogeneous inorganic solid body, which possesses length, breadth, and thickness, and is bounded by plane faces.

3. The measures of the length, breadth, and thickness of a crystal are three imaginary lines which pass through its centre, cross one another there at right angles, and terminate at its surface.

4. These imaginary lines are called *Axes*.

5. The longest axis is the principal or perpendicular axis. Its symbol is p^a. Its position is vertical.

6. The next longest axis is the transverse axis. Its symbol is t^a. Its position is horizontal, and it passes from left to right.

7. The shortest axis is the minor axis. Its symbol is m^a. Its position is horizontal, and it passes from the front to the back of the crystal.

8. The point of view of a crystal is in the prolongation of the axes m^a. Hence the observer of a crystal has to hold it before him with the longest axis in a perpendicular position, and the broadest surface exposed to his eye. Variations from this rule will be explained hereafter.

9. The crystallographer is farther assumed to be always looking towards the SOUTH when examining a crystal. The front of the crystal, or the face that is turned towards him, is then exposed to the *north;* the parallel face, or that which is farthest from him, is exposed to the *south;* the face opposite to his left hand, to the *east,* and the parallel face, opposite to his right hand, to the *west.* The upper portion of the crystal is the *Zenith* portion, and the lower portion is the *Nadir* portion. The horizontal plane which separates these two portions, is the *Equator.* The symbols for these *terms of position* are as follow:

North, n.	East, e.	Zenith, Z.
South, s.	West, w.	Nadir, N.

Z (for Zenith) and N (for Nadir), but none of the other four letters, are written in capitals, in order that N = Nadir may never be mistaken for

n = north. The terminal points of the axis p^a are in Z and N. Those of the axis m^a are in n and s. Those of the axis t^a are in e and w.

p^a_Z is the zenith pole of the axis p^a.	m^a_s is the south pole of the axis m^a.
p^a_N is the nadir pole of the axis p^a.	t^a_e is the east pole of the axis t^a.
m^a_n is the north pole of the axis m^a.	t^a_w is the west pole of the axis t^a.

10. Any point on the surface of a crystal may be a *pole*, and is named in reference to its proximity to any two or three of the above six cardinal points; as nw = north west, Ze = Zenith east (midway between these two poles), &c. A direct line between a pole and the centre of the crystal is a NORMAL, every variety of which is named after the *pole* to which it is perpendicular. Thus a line from the centre of the crystal to the pole Znw is the Znw normal.

This arbitrary attribution of *polarity* to the axis of crystals, is intended to facilitate reference to the different faces, edges, and angles of crystals.

11. The axes of crystals vary in length, as the crystals that they belong to vary in length, in breadth, or in thickness. The symbols p^a m^a t^a employed without addition, denote the axes to which they relate to be of equal length. Crystals whose axes are all equal, are called *equiaxed* crystals. When *long* axes are to be denoted, the symbols are written p^a_+ m^a_+ t^a_+; and when *short* axes are to be denoted, the symbols are written p^a_- m^a_- t^a_-. The sign *plus* (+) employed in this manner, signifies *more than unity;* the sign *minus* (–) signifies *less than unity.*

12. When an exact value is to be given to the symbols, the signs + and – are replaced by figures that indicate the precise lengths of the respective axes. Thus, when the principal, minor, and transverse axes have the relation of 3, 1, and 2, the symbol is written p^a_3 m^a_1 t^a_2.

13. When the length of an axis is *unknown*, or when it is *variable*, that is to say, liable to be more or less than unity, its symbol is subscribed $_x$. When the three axes, p^a, m^a, t^a, are all of different, but of unknown lengths, they are written p^a_x m^a_y t^a_z.

14. ILLUSTRATIONS BY MEANS OF THE MODELS OF CRYSTALS.

[The reader will observe, that each of the Models of Crystals is marked with the letters P, M, T, and it is proper in this place, once for all, to inform him, that these letters have the following significations:

P signifies the zenith pole of the axis p^a.
M signifies the north pole of the axis m^a.
T signifies the west pole of the axis t^a.

In examining a model, it may be held in the left hand, or placed upon a support, with the face M on a level with, and exactly opposite to, the observer's eye, § 8. The several positions of top and bottom, front and back, left and right, are then easily discriminated.

On a few of the models that exemplify complicated forms, the Nadir, south, and east poles are marked, as well as the Zenith, north and west. See Models 95, 69, 117. This is done to prevent any misconception that might take place respecting the direction of any of the axes of such forms. The Zenith pole of any model is readily found by holding the solid in such a manner that the letters M and T stand the right way uppermost: p^a_Z will then be at the top of the model.]

Model 1. *The cube.* The axes are $p^{a}\ m^{a}\ t^{a}$.
Model 2. *The short quadratic prism.* The axes are $p^{a}_{-}\ m^{a}\ t^{a}$.
Model 3. *The long quadratic prism.* The axes are $p^{a}_{+}\ m^{a}\ t^{a}$.
Model 6. *The rhombic prism.* The axes are $p^{a}_{-}\ m^{a}\ t^{a}_{+}$.
Model 12. *The obtuse quadratic octahedron.* The axes are $p^{a}_{-}\ m^{a}\ t^{a}$.
Model 13. *The acute quadratic octahedron.* The axes are $p^{a}_{+}\ m^{a}\ t^{a}$.
Model 15. *The regular octahedron.* The axes are $p^{a}\ m^{a}\ t^{a}$.
Model 21. *The rhombic octahedron.* The axes are $p^{a}_{+}\ m^{a}_{-}\ t^{a}$.

The absence of any sign under the letter ^{a} signifies that the axis to which the letter ^{a} belongs is to be considered as unity = 1.

Section II.—OF THE PLANES OF CRYSTALS.

15. Crystals are bounded by *planes.* Where two planes meet they form an *edge.* Where more than two planes meet, they form a *solid angle.*

16. The Planes of crystals are flat faces bounded by straight lines. As respects their form, they are of three kinds,—trilateral, quadrilateral, and multilateral.

Trilateral Figures, or Triangles, are bounded by three straight lines, and have three angles. Considered in reference to their *sides*, they are of three kinds: *a*, *Equilateral*, or equal-sided, when the figure has three equal sides. See the planes of Models 15 and 117.—*b*, *Isosceles*, or equal-legged, when the figure has two sides equal. See the planes of Models 12 and 13.—*c*, *Scalene*, or unequal-legged, when the figure has three unequal sides. See the planes of Models 116 and 21.—Triangles are also of three kinds, when described according to the nature of their *angles: d*, *Right-angled*, when the triangle has one right angle. A square divided into two halves by a diagonal line, produces two right-angled triangles.—*e*, *Obtuse-angled*, when the triangle has one obtuse angle. See the small triangular planes on Model 40. These are obtuse isosceles triangles, as are also the planes of Model 119.—*f*, *Acute-angled*, when the triangle has three acute angles. See the triangular planes on Model 73.

g, The three interior angles of every triangle are equal to two right angles.—*h*, If two angles of a triangle be given, the third is equal to the difference between their sum and two right angles, or 180°.—*i*, Each of the angles in an equilateral triangle is $\frac{1}{3}$ of two right angles, or $\frac{2}{3}$ of one right angle, and therefore contains 60°.—*j*, In every right-angled triangle, the sum of the two acute angles is equal to one right angle, and therefore contains 90°.—*k*, In every isosceles right-angled triangle, each of the acute angles is equal to half a right angle, and therefore contains 45°.

l, Quadrilateral Figures are those contained by four straight lines—*quadrangles* or four-angled figures. All their angles together are equal to four right angles, or 360°. Quadrilateral figures are of six kinds, according to the nature of their sides: *m*, a *square*, has all its sides equal, and all its angles right angles. See the planes of Model 1.—*n*, a *rectangle* or oblong, has all its angles right angles, and its opposite sides, but not all its sides, equal. See the planes M and T of Model 2, and the six vertical planes of Model 7.—*o*, a *rhombus* or lozenge, has all its sides equal, but its angles are not right angles. See the planes P of Models 6, 84, 87. But the four angles of a rhombus are together equal to four right angles, or 360°; that is to say, one of its acute and one of its obtuse angles are together equal to 180°, the acute angle being exactly as much less than 90°, as the obtuse angle is greater.—*p*, a *rhomboid*, has its opposite sides equal to one another, but all its sides are not equal, and its angles are not right angles. See the planes P of Model 11.—*q*, a *trapezium*, is a four-sided figure, whose opposite sides are not parallel. See the planes of Model 22.—*r*, a *trapezoid*, has two sides parallel to each other, and two not so. See the eight similar planes on Models 76 and 115.

s, MULTILATERAL FIGURES are those contained by more than four straight lines—also called *Polygons*, or many-angled figures. A polygon is regular when all its sides are equal, and irregular when its sides are unequal. A 5-sided figure is a pentagon, a 6-sided, a hexagon, a 7-sided, a heptagon, an 8-sided, an octagon, a 10-sided, a decagon. The crystallographic polygons have not the same regularity as the geometrical polygons. Thus the pentagonal dodecahedron, Model 91, and the isosahedron, Model 92, both differ in the form of their faces from the geometrical solids which have the same names.

t, All the angles of a multilateral figure are together equal to twice as many right angles as it has sides, minus four right angles or 360°. That is to say, if you count the sides of a polygon, multiply the number by 180, and deduct 360 from the product, the result is equal to that obtained by adding together all the angles of the polygon. See § 79.

u, Wherever two of the lines which bound a plane meet, they form a *plane angle*. Of any two such angles, that one is the greater which has the greater opening or divergence, whatever may be the comparative lengths of the lines by which the angles are formed. If a circle be described from the vertex of any angle, the arc intercepted between the legs of the angle is called the *measure* of the angle, and the number of degrees which the arc contains, is said to be the number of degrees in the angle. A circle is commonly divided into 360 degrees, marked 360°. A right angle contains 90°, for 2 straight lines dividing a circle into four parts, produce at the centre 4 right angles, and $\frac{360}{4} = 90°$. Each degree of the circle is subdivided into 60 minutes, marked 60′, and each minute into 60 seconds, marked 60″. An angle of 43 degrees, 15 minutes, and 25 seconds, is therefore marked 43° 15′ 25″. An angle greater than a right angle is called an *obtuse* angle. See the plane angles of the plane P of Model 7. An angle less than a right angle is called an *acute* angle. See the plane angles of Model 117.

17. The EDGES of different crystals have different degrees of sharpness, which differences are estimated by measurement with the goniometer, and are expressed in degrees, minutes, and seconds. An edge which is formed by two planes that meet at a right angle, is denoted by 90°. A sharper edge is denoted by any degree from 89° 59′ to 0°. A blunter edge by any degree from 90° 1′ to 180°.

I do not describe the goniometer, because it is figured and described in almost every elementary work on Mineralogy. (See PHILLIPS's *Introduction*, page xxxi.) It consists of a jointed pair of straight edges which can be applied to a crystal where two planes meet to form an edge, and afterwards to a semicircular scale, whereby the divergence of the planes is measured, and the value of the angle determined. Those who do not possess a goniometer, may measure the angles of the Models of crystals by means of a flat rule and a semicircular brass sector, such as commonly form part of a case of drawing instruments; but small natural crystals cannot be measured in that way, nor is it indeed either a convenient or accurate mode of measuring.

18. The SOLID ANGLES of crystals are named according to the number of planes which meet together to produce them,—as, three-faced angles, six-faced angles, eight-faced angles, and so forth. They constitute the *points* or *corners* of crystals.

Discrimination of different kinds of Planes.

19. ALL THE PLANES THAT BOUND A CRYSTAL, CUT ONE, TWO, OR THREE OF ITS AXES.

The *Denominations* of the Planes are the initial letters of the names of the Axes which they cut. Thus—

P is the denomination of the planes that cut the axis p^a.
M is the denomination of the planes that cut the axis m^a.
T is the denomination of the planes that cut the axis t^a.
MT is the denomination of the planes that cut both m^a and t^a.
PM is the denomination of the planes that cut both p^a and m^a.
PT is the denomination of the planes that cut both p^a and t^a.
PMT is the denomination of the planes that cut p^a m^a and t^a.

These planes occur in *sets*, which differ in their number as follows:

Of P, M, T,	there are 2 to each set.
Of MT, PM, PT,	there are 4 to each set.
Of PMT,	there are 8 to the set.

Sometimes the planes occur in *portions* of sets, and such a portion is generally the *half* or the *quarter* of a complete set. All the varieties are subject to occur in half sets, but only the set PMT is commonly liable to be quartered. The symbol which denotes a *half set*, is the vulgar fraction $\frac{1}{2}$, and a *quarter set*, the vulgar fraction $\frac{1}{4}$, prefixed to the symbol or denomination which denotes the whole set of planes—as, $\frac{1}{2}$ PM, $\frac{1}{4}$ PMT, &c. The dividing power of the fraction extends only over the symbol which stands betwixt itself and the next following comma, unless several symbols which are all to be divided by the same fraction, are placed within parentheses, as $\frac{1}{2}$ (PM, PMT), in which case the fraction divides the whole of the enclosed symbols.

20. The following diagram is intended to show the relative positions of these sets of planes in reference to the system of three axes. The principal axis is denoted therein by p^a, the transverse axis by t^a, and the minor axis by m^a. The crystallographer is assumed to be in front of the diagram, and his eye to be situated in the prolongation of the axis m^a. The classification of planes into vertical, horizontal, and inclined, which follows hereafter, is made in reference to this assumption.

There are several terms used in relation to Crystallographic Sections, which may be conveniently explained here.

A Crystallographic Section is the outline of a plane produced by the imaginary cutting of a crystal through the centre into two equal and similar halves. Such sections can be made in a multiplicity of directions, but there are only five which are of practical importance in the present system of crystallography. Of these five sections, one is horizontal and four are vertical, as follow:

The *Equator*, or horizontal section, which divides the zenith from the nadir portion of the crystal, see § 9, is shown in the diagram by the lines 6 12 9 3. It bisects the poles m_n^a m_s^a t_e^a t_w^a.

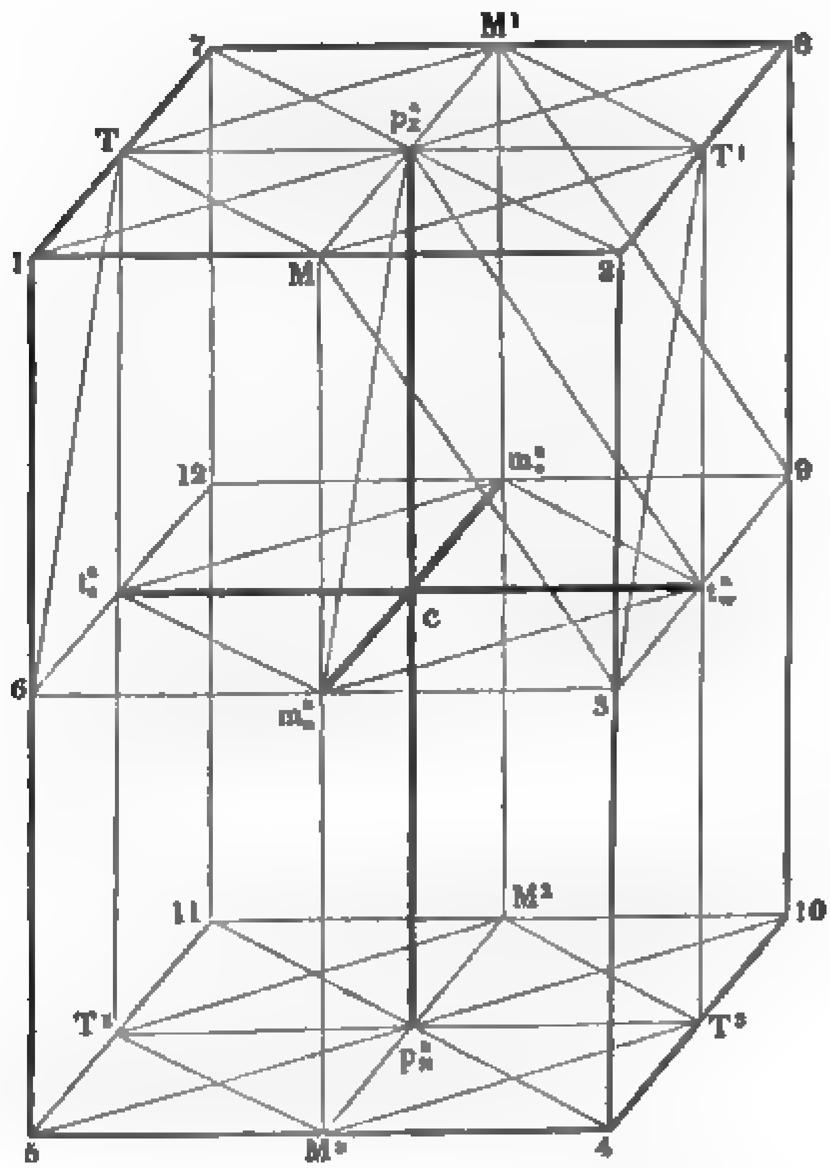
The *North Meridian*, or first vertical section, is shown by the lines M M_2 M_3 M_1. It bisects the poles p_Z^a p_N^a m_n^a m_s^a.

The *East Meridian*, or second vertical section, is shown by the lines T T_2 T_3 T_1. It bisects the poles p_Z^a p_N^a t_e^a t_w^a.

The *North-east Meridian*, or third vertical section, is shown by the lines 1 6 5 8 9 10. It bisects the poles p_Z^a p_N^a, and cuts the equator at the ne (north-east) and sw (south-west).

The *North-west Meridian*, or fourth vertical section, is shown by the lines 2 3 4 7 12 11. It bisects the poles p^a_z p^a_n, and cuts the equator at the nw and se.

The positions of these sections are fixed and invariable, and have no dependence upon the form, or the number of planes, belonging to any single crystal.



21. The application of the *terms of position*, quoted in § 9, will be easily understood in reference to the planes, edges, and angles of the imaginary crystal which is represented in the above diagram. The point c being the centre of the crystal, and the plane 6 12 9 3 being the equator, all the lines that proceed from point c, or from any other part of the equator, *upwards*, must strike planes, edges, or angles that are contained in the *zenith* portion of the crystal;—all the lines that proceed from the same points *downwards*, must strike planes, edges, and angles that belong to the *nadir* portion;—and all the lines that proceed from c horizontally, affect neither zenith nor nadir, but are directed towards the north, east, south, or west poles, or to some situation between those cardinal points. Consequently, the line c p^a_z points to the Zenith, c p^a_n to the Nadir, c m^a_n to the north, c m^a_s to the south, c t^a_e to the east, c t^a_w to the west. And any line pointing from c *diagonally as respects the axes,*

must point to Zn, Ze, Zs, Zw, or to Nn, Ne, Ns, Nw, or to some intermediate situation, as Znw or Nse.

In examining these relations, the reader is requested to recollect the explanation of the polaric positions given in § 9.

Fixed Positions shown in the Diagram.

Planes:				*Positions.*
1	2	8	7	Z
5	4	10	11	N
1	5	11	7	e
2	4	10	8	w
1	2	4	5	n
7	8	10	11	s
T	6	3	T^1	Zn
M	3	9	M^1	Zw
M	3	T^1		Znw
M	M^2	T^3	T^1	nw
M	M^2	T^2	T	ne
T^1	T^3	M^3	M^1	sw
M^1	M^3	T^2	T	se
Edges:				
1	5			ne
2	4			nw
8	10			sw
7	11			se
1	2			Zn
4	5			Nn
7	8			Zs

Edges:		*Positions.*
10	11	Ns
1	7	Ze
5	11	Ne
2	8	Zw
4	10	Nw
Solid Angles:		
	1	Zne
	2	Znw
	7	Zse
	8	Zsw
	4	Nnw
	5	Nne
	10	Nsw
	11	Nse

Lines:

All lines that are drawn upon a plane have the same position as that plane; thus, T T^1—M M^1—1 8—2 7— are all Z

ILLUSTRATIONS.—A. Place Model 1, the cube, in a position for examination, and observe the situations of its planes, edges, and solid angles: (§ 14),—

The uppermost plane is Z, the lowermost, N.
The front plane is n, the back, s.
The plane opposite to your left hand is e.
That opposite to your right hand is w.
The two front vertical edges are ne and nw.
The two back vertical edges are se and sw.
The front upper edge is Zn, the lower, Nn.
The back upper edge is Zs, the lower, Ns.
The upper side edges are Ze and Zw.
The lower side edges are Ne and Nw.
The two upper front solid angles are Znw and Zne.
The two upper back solid angles are Zsw and Zse.
The two lower front solid angles are Nnw and Nne.
The two lower back solid angles are Nsw and Nse.

B. Place Model 63 in position before you. What is to be noticed respecting this form, the rhombic dodecahedron, is, that its 12 planes have

exactly the same polaric positions as the 12 edges of the cube, Model 1; that 6 of its solid angles, being those that are formed by the meeting of four planes in each, have the positions of the 6 faces of the cube; and that 8 of its solid angles, being those that are formed by the meeting of three planes in each, have the positions of the 8 solid angles of the cube.

C. Place Model 15 in position. Observe that its 8 planes have the polaric positions of the 8 solid angles of Model 1, and of the 8 three-faced angles of Model 63; that its 6 solid angles have the positions of the 6 planes of Model 1, and of the 6 four-faced angles of Model 63; and that its 12 edges have the positions of the 12 edges of Model 1, and of the 12 planes of Model 63. Model 15 is called the octahedron.

D. Model 32. The 6 octangular planes of this model have the polaric situations of the 6 square planes of Model 1. The 12 rectangular planes have the positions of the 12 rhombic planes of Model 63. The 8 hexangular planes have the positions of the 8 triangular planes of Model 15.

E. Model 33. The planes of this model are the same in number and in polaric position, as those of Model 32, but they differ in their relative sizes, and, as a consequence, in their forms.

F. Model 34. The planes of this form are also the same in number and polaric position as those of Model 32, but they differ in relative size and in form from the planes of both the preceding Models.

G. It follows from the foregoing observations, that each of the three Models 32, 33, and 34 contains all the planes of the three Models 1, 63, and 15; but that they all differ among themselves in the relative sizes of the planes; so that in

Model 32, the *cube* predominates, and the dodecahedron and octahedron are subordinate:

In Model 33, the *octahedron* predominates, and the cube and the dodecahedron are subordinate:

In Model 34, the *dodecahedron* predominates, and the cube and the octahedron are subordinate.

H. But although the planes peculiar to the three simple crystals alter their size and shape when they form part of the three complex crystals, *they never change their polaric positions.* The angles across their edges are, therefore, always the same. Any one plane of the cube inclines upon any other plane of that form, at an angle of 90°. Any one plane of the rhombic dodecahedron makes upon any other plane an angle of 120°. Any one plane of the regular octahedron makes upon any other plane an angle of 109° 28′. It matters not whether you measure these angles by applying the goniometer to Models 1, 63, or 15, or to Models 32, 33, 34; the results are the same. *Polaric positions are fixed and invariable,* and the planes of crystals are said to belong to one form or to another, according as they are found to occupy one or other polaric position.

Particular Description of each set of Planes.

22. *Of the Planes* P.—There are two of them, and their positions are shown by the lines 1 2 8 7 and 5 4 10 11 in the diagram in § 20. One of these planes forms the top and the other the bottom of the crystal. They are HORIZONTAL and parallel to one another. They cut the axis p^a, and are parallel to the axes m^a and t^a. The symbol P signifies two planes, which are all that belong to this *set*, or *complement*, of planes. The symbols which denote these two planes separately are PZ for the one that cuts the pole p^a_z, and PN for that which cuts the pole p^a_N.

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23. *Of the Planes* M.—There are two of them, and their positions are shown by the lines 1 2 3 4 5 6 and 7 8 9 10 11 12 in the diagram § 20. One forms the front and the other the back of the crystal. They are VERTICAL and parallel to one another. They cut the axis m^a, and are parallel to the axes p^a and t^a. The symbol M signifies two planes. The front plane by itself is denoted by Mn, the back plane by Ms. The former cuts the pole m^a_n, the latter the pole m^a_s.

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24. *Of the Planes* T.—There are two of them, and their positions are shown by the lines 2 3 4 10 9 8 and 1 6 5 11 12 7. One forms the left and the other the right side of the crystal. They are VERTICAL and parallel to one another. They cut the axis t^a, and are parallel to the axes p^a and m^a. The symbol T signifies two planes. Separately, the plane 2 3 4 10 9 8 is marked Tw, and the plane 1 6 5 11 12 7 is marked Te. The former cuts the pole t^a_w; the latter cuts the pole t^a_e.

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25. When the symbols P, M, T, denote the planes of an equiaxed crystal (§ 10), the symbols are written without addition. But when one of the axes of the crystal which the symbols relate to, is longer or shorter than the other two axes, then the signs + or −, or a number, as the case may be, is subscribed below that symbol which is the representative of the planes that cut the longer or shorter axis (§ 11, 12). Thus:

P,M,T, intimate that the Planes P, M, T cut the Axes p^a m^a t^a at equal distances from the centre of the crystal.

P_-,M,T intimate that the Axis p^a is cut by the Planes P nearer to the centre of the crystal than the Axes m^a and t^a are cut by the Planes M and T.

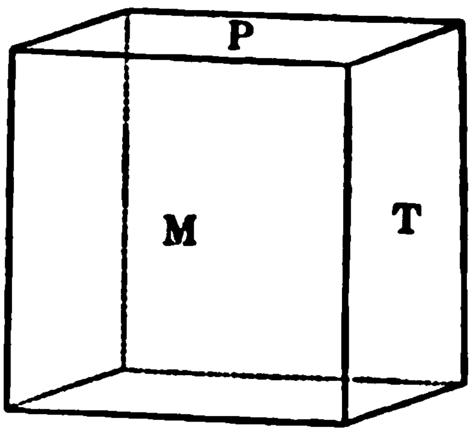
P_+,M_-,T intimate that the Axes p^a m^a t^a are cut by the Planes P, M, T *unequally*,—p^a being cut at the *greatest*, and m^a at the *least* distance from the centre of the crystal.

P_3,M_1,T_2 intimate the same general fact as P_+,M_-,T, but give a more precise account of the three different dimensions of the axes.

The lengths of the Axes are found by measuring the lengths of those Planes of the crystal to which the axes are parallel. Thus, taking Model 1 for an example, the length of the axis p^a is equal to the height of the planes M or T, the length of the axis m^a is equal to the width of the planes P or T, and the length of the axis t^a is equal to the width of the planes P or M.

EXAMPLES OF THE FORMS P, M, T.

Model 1. *The Cube.* P,M,T.



The annexed figure, and Model 1, both represent the cube. This form has six planes, namely, the set P, the set M, and the set T. The axes are p^a m^a t^a. The planes are consequently square, for the prolongation of any one axis would cause four of the planes to be rectangles. There are necessarily twelve edges and eight corners.

The equator is a square with the angles situated ne, se, sw, nw. The north meridian is a square with the angles Zn, Zs, Nn, Ns. The east meridian is a square with the angles Ze, Zw, Ne, Nw. The north-east meridian is a rectangle with the angles Zne, Zsw, Nne, Nsw. The north-west meridian is a rectangle with the angles Znw, Zse, Nnw, Nse.

The symbol to express this form is P,M,T.

Model 2. *Short Quadratic Prism.* $P\frac{1}{2}$,M,T.

This form, like the cube, has six planes, namely, the set P, the set M, and the set T. But its axes are p^a_1 m^a_2 t^a_2, in which it differs from the cube, which is equiaxed. The planes P are squares, like the same planes of the cube, because, with the planes M and T, and with m^a and t^a equal, the planes P cannot be any thing else than squares. The planes M and T are rectangles, which form is the necessary consequence of the shortness of the axis p^a in relation to the axes m^a and t^a.

The equator is a square, and all the meridians are rectangles, the angles of which have the same positions as the angles of the equator and meridians of the cube.

The symbol to express this form is P_-,M,T ; or $P\frac{1}{2}$,M,T.

Model 3. *Long Quadratic Prism.* $P_{1\frac{1}{4}}$,M,T.

The difference between this form and the two preceding, results solely from the comparative length of the axis p^a. The planes are the same, but the axes are p^a_5 m^a_4 t^a_4. The equator is a square. The meridians are all rectangles. The angles are the same as those of Models 1 and 2.

The symbol to express this form is P_+,M,T; or $P_{1\frac{1}{4}}$,M,T.

Model 5. *Rectangular Prism.* P_+,M_-,T.

The rectangular prism agrees with the cube in having 6 planes, in

three pairs, each pair cutting the two ends of one axis at right angles. It differs from the cube in having the axes m^a and t^a unequal, be the length of the axis p^a what it may. It differs from the quadratic prisms in the same particular. The planes of Model 5 are all rectangles. The equator, and the four meridians, are rectangles, the angles of which have the same positions as the angles of the equator and meridians of the cube and the quadratic prisms. The axes of the model measure p^a_{15} m^a_{12} t^a_{13}.

The symbol is P_{+},M_{-},T; or P_{15},M_{12},T_{13}.

Model 80. P.$P_{+}$$M_{-}$T.
Example of a combination which has the planes P without M and T.

Model 70. M.$P_{+}$$M_{-}$T.
Example of a combination which has the planes M without P and T.

Model 111. T,MT_{+}.PT_{+}.
Example of a combination which has the planes T without P and M.

Model 61. M,T.PM_{+},PT_{+}.
Example of a combination which has the planes M and T without P.

A great many other combinations of this kind may be seen upon glancing over the set of models, but the above are sufficient to illustrate the preceding illustration of the symbols P, M, T, and to show that the planes which they represent never change their polaric positions when they appear upon a solid, either separately or together, in combination with various other planes. Consequently, if you want to know whether a crystal possesses the planes P, M, T, you have only to assume one of its planes to be = PZ, and then to hold that plane in the position of PZ and look for the other 5 planes of P, M, T, at the poles N, n, e, s, w.

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26. *Of the Planes* MT.—There are four of them, and their positions are shown by the lines T M M^2 T^2, M T^1 T^3 M^2, T^1 M^1 M^3 T^3, M^1 T T^2 M^3. Every single plane cuts the two axes m^a and t^a and is parallel to the axis p^a. They constitute together the four sides of a VERTICAL PRISM, whose edges bisect the axes m^a and t^a, and whose axis coincides with the axis p^a. The four planes are parallel two and two, and all of them are equivalent to one another. The symbol MT signifies the whole four planes, which in this case constitute the complement, whereas in all the foregoing cases, *two* planes constitute a complement.

The positions and symbols of the individual planes of this set are as follow:

That between the poles m^a_n and t^a_w is MTnw.
That between the poles m^a_n and t^a_e is MTne.
That between the poles m^a_s and t^a_w is MTsw.
That between the poles m^a_s and t^a_e is MTse.

27. The half of this set of planes is denoted by $\frac{1}{2}$MT, which signifies two vertical planes whose side edges bisect the two axes m^a and t^a.

28. When the symbol MT denotes the planes of a form whose axes m^a and t^a are *equal*, the symbol is used without addition. The alternating angles on m^a and t^a of the vertical prism which is formed by the intersection of the four planes MT, are in that case all right angles (90°), and the form of the equator of the crystal is a square, with its angles due n, e, s, w.

29. But when the axes m^a and t^a are unequal, the symbol of the planes requires subscription with the sign + below the representative of the longer axis. Thus:

> MT_{+} denotes the vertical planes of a prism, the equator of which is a rhombus, having its longer diagonal parallel with the axis t^a, and its shorter diagonal parallel with the axis m^a.

In this case, the angles formed by the intersection of the four vertical planes are not right angles, but alternately *obtuse* at m^a_n and m^a_s, and *acute* at t^a_e and t^a_w; the one angle being as much *more* than 90° as the other angle is *less* than 90°, so that in every instance, the angle at m^a_n added to the angle at t^a_w or the angle at m^a_s added to the angle at t^a_e, is equal to 180°.

In like manner, the symbol $M_{+}T$ denotes the vertical planes of a rhombic prism, having its obtuse angles on the axis t^a and its acute angles on the axis m^a. In this case, as in that before mentioned, the angles of two different prismatic edges taken together are equal to 180°. And this is to be generally understood of the angles formed by the intersecting planes of rhombic prisms, so that if one angle of such a prism be known, the discovery of the other angle is made by subtracting the sum of the known angle from 180°. See § 16, *o*.

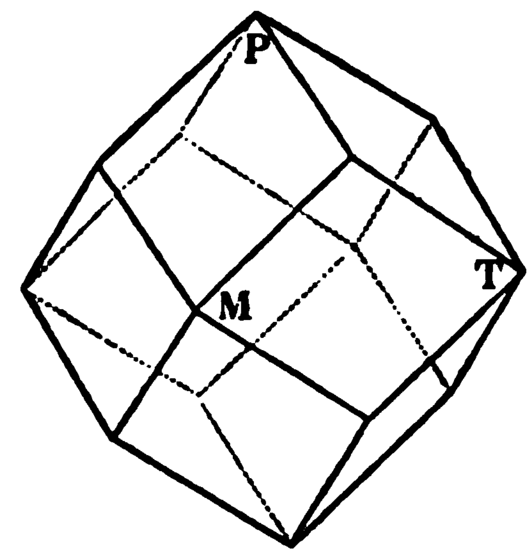
When the subscribed signs + and − do not denote the relative lengths of axes with sufficient precision, it is necessary to follow the method of numerical notation which has been described in paragraph 12.

30. It sometimes happens that we find upon the same crystal the planes MT_{+}, MT_{+}, where one + signifies a greater quantity than the other +. In this case, it is best to replace + by a number in both symbols. But if it is impossible to find the lengths of the axes, so as to be able to describe them numerically, the next best method is to double the symbol $\ddagger$ for the longer axis. Thus, MT_{+}, $MT_{\ddagger}$. By this means the general relations of the axes, as respects length, are indicated as well as they can be without numbers. A similar expedient may be adopted to indicate an *extremely short* in comparison with a *short* axis, in cases where the exact comparative lengths of the two axes are unknown. Thus, in $M_{-}T$, $M_{=}T$, the sign $=$ signifies *very short*, or shorter than the measure indicated by the sign −. See § 73.

EXAMPLES OF THE FORMS $MT, MT_{+}, M_{+}T$.

31. Model 63. *Rhombic Dodecahedron.* MT.PM,PT.

The four vertical planes of this form, that is to say, those that are directed to the polaric points ne, nw, se, sw, are the planes MT. See § 21, Illustration B, or the annexed figure, in which one of the planes of this set is marked with M and T at the n and w poles. The equator of this form, as observed in § 28, is a square. There are no meridians, because the prism MT runs to infinity upon the axis p^a, and we are not now, in the explanation of this or any other variety of the form MT_x, regarding the nature of the terminations of the prism.

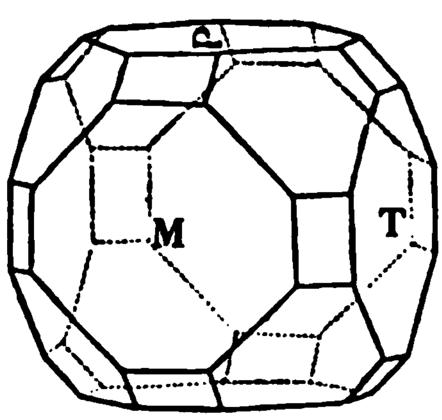


32. Model 4. *Eight-sided Prism.* Quadratic prism with the lateral edges replaced. P_+,M,T,MT.

This model has all the planes of the long quadratic prism, Model 3, with the addition of the four planes that constitute the set MT. The planes MT are distinguished on the model from the planes M and T by being narrower than those planes. They are all at right angles to the planes P. The axes of this crystal are p^+_+ m^a t^a. The equator is an octagon, having the sides M, T, exposed to n, e, s, w, and the sides MT exposed to ne, nw, se, sw.

The symbol which expresses this form is P_+,M,T,MT.

33. Model 32. The *Cube* combined with the rhombic dodecahedron and the regular octahedron, as represented in the marginal figure.—Model 33. The *octahedron* combined with the cube and the rhombic dodecahedron.—Model 34. The *rhombic dodecahedron* combined with the cube and the octahedron. See Illustrations § 21. The planes MT are *marked* upon Model 32, and they can be easily discriminated from all different planes upon the other two models, if their polaric positions be attended to.



34. In each of these three forms, the equator is an octagon, as is the equator of Model 4. Whenever, indeed, a prism contains the planes MT in addition to the planes M and T, and has no other vertical planes, the equator is always an octagon, whatever may happen to be the form of its meridians. In other words, the two four-sided prisms which produce the planes M, T, and MT, occurring upon the same crystal, cut off each other's edges and produce a single vertical prism of eight sides.

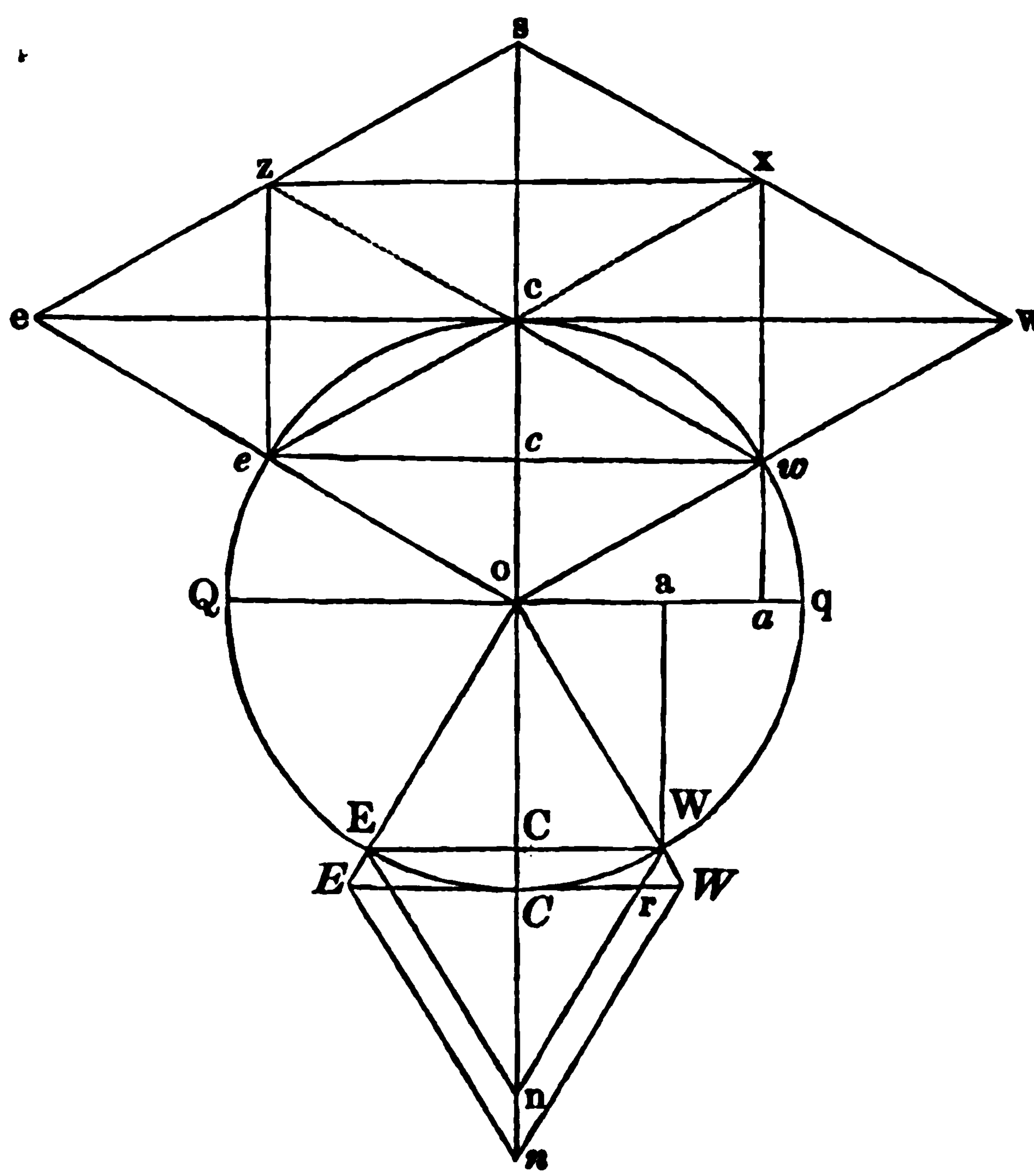
35. Model 6. *Rhombic Prism.* P_-,MT_+; or $P\frac{4}{10}$,$MT\frac{13}{10}$.

The set of planes MT_+ constitutes the four vertical planes of this model. To ascertain the numerical value of the sign +, the axis m^a can be measured from n to s, and the axis t^a from e to w. By this means m^a

is found to bear to t^a the relation of 10 to 13, so that the exact symbol for this form is $M_{10}T_{13}$, or $MT\frac{13}{10}$. The equator of this form is a rhombus, with its obtuse angles at the poles n and s, and its acute angles at the poles e and w.

36. Model 7. ***Regular Six-sided Prism.*** P_x, T_-, MT_+; or $P_{0.4}, T_{0.866}, MT_{1.732}$.

The planes of this model are the set MT_+, in combination with the sets P and T. The axes of the model are p_-^a m_+^a t^a, but the horizontal axes of the set of planes MT_+ are not the same as the horizontal axes of the model. The length of the axis m^a is the same, both of the form MT_+ and of Model 7, and it is ascertained by measurement from n to s. The length of the axis t^a of the model is also ascertained by measurement from e to w. But the axis t^a of the form MT_+ is the longer diagonal of the rhombus that would have been produced had the crystal or combination been, like Model 6, destitute of the planes T. The occurrence of the latter planes has served to cut off the acute edges of the rhombus and to convert it into a regular hexagon. It is therefore impossible to ascertain the length of the axis t_+^a of the symbol MT_+ by lineal measurement of the distance between the two extreme points, and an evil is produced for which we have no remedy but trigonometrical calculation. We can, indeed, by measuring the angle at m_n^a, easily determine the value of the angle removed from t_w^a, (See § 16 *o*,) but having done that, we have still to determine the length of the axis or line from t_w^a to t_e^a, and this is what we can only do properly by means of trigonometry.



Trigonometrical Determination of the Comparative Lengths of the Axes of Rhombuses.

37. I shall begin by explaining several general terms that are employed in trigonometry, and then show how the information furnished in that explanation is to be applied to the solution of problems in crystallography.

38. A straight line drawn from the centre to the circumference of a circle is called its *radius;* as oQ, oc, oq, o*C* in the diagram in page 14. These lines are all equal to one another.

39. A straight line drawn through the centre of a circle and terminated both ways by the circumference, its called is *diameter;* as the diameters c*C* and Qq.

40. A *quadrant* is one of the four parts into which a circle is divided by two diameters intersecting each other at right angles. The circle being divided into 360 degrees, the quadrant contains 90°. The circle in the diagram is divided by the diameters Qq and *C*c into the four quadrants Qoc, coq, qo*C*, *C*oQ.

41. An *arc* is part of a circle separated from the rest of it by the two legs of an angle, or otherwise. The following arcs are shown in the diagram:

Arcs of 30°.	Arcs of 60°.	Arcs of 90°.	Arcs of 120°.
Q*e*	*e*c	Qc	*ew*
*w*q	c*w*	cq	*wC*
W*C*	qW	q*C*	*Ce*
*C*E	EQ	*C*Q	Qq is 180°.

42. The difference between an arc and a quadrant, is called the *complement* of that arc. The difference between an angle and a right angle is called the *complement* of that angle.—Thus, the arc Q*e* is the complement of the arc *e*c, and conversely *e*c is the complement of the arc *e*Q. Again, the angle *w*oq is the complement of the angle *w*oc, and conversely, *w*oc is the complement of *w*oq.

43. Two arcs whose sum is a semicircle, or two angles which are together equal to two right angles, are called *supplements* of each other. Thus, the arc Q*e* is the supplement of the arc *e*q, and the angle Qo*w* is the supplement of the angle *w*oq.

44. The straight line that joins the extremities of an arc is called its *chord*.—Thus, the line *ew* is the chord of the arc *ecw*. The line Qoq is the chord of the arc (the semicircle) Qcq; and the line EW is the chord of the arc E*C*W.

45. A straight line drawn from one extremity of an arc, perpendicular to the diameter that passes through the other extremity, is called the *sine of that arc*, or the *sine of the angle which the arc measures;* and the part of the diameter that is intercepted between the sine and the arc is called the *versed sine* of the arc or the angle. Hence, the sine of an arc is half the chord of its double.—Thus, the line *wa* is the sine of the arc *w*q, or

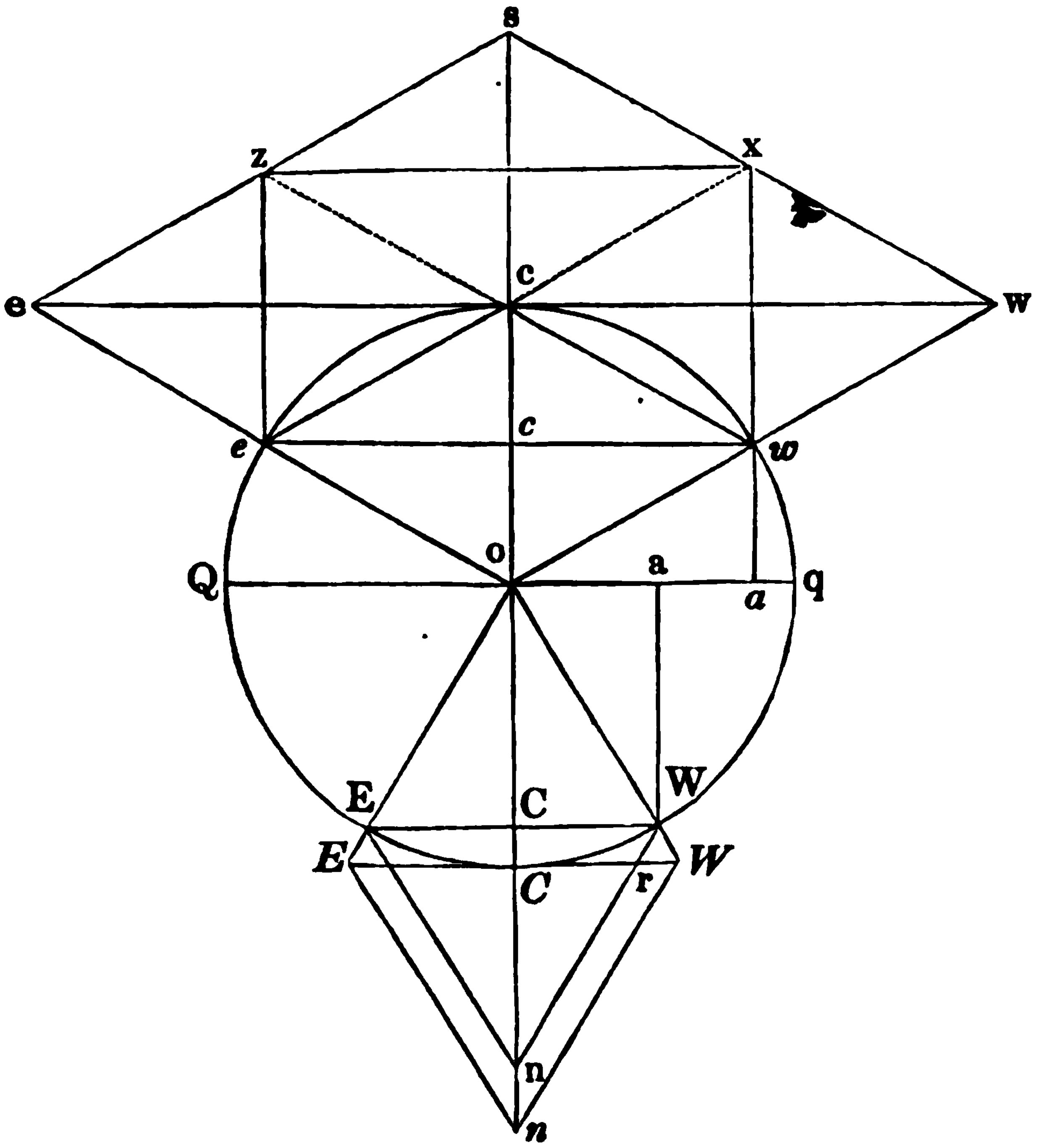
of the angle *w*oq, and the line *aq*, intercepted between the sine *wa* and the arc *w*q, is the versed sine of the same angle. Again, the line WC is the sine of the arc W*C*, or of the angle Wo*C*, and the line C*C*, intercepted between the sine WC and the arc W*C*, is its versed sine. Lastly, the line EW is the chord of the arc E*C*W, which is the double of the arc *C*W, and the portion CW is the half of that chord.

46. The *cosine* of an arc is the sine of its complement.—Thus, *wa* is the sine of the arc *w*q, and *wc* is its cosine, or the sine of its complement *w*c. Conversely, *wc* is the sine of the angle co*w*, and *wa* is its cosine, being the sine of its complement *w*oq. And in the under quadrant, WC is the sine of the arc W*C*, and Wa is its cosine; or Wa is the sine of the arc Wq, and WC is its cosine.

47. When one of the legs of an angle is prolonged beyond one of the extremities of an arc, and is cut there by a straight line that touches the other extremity of the arc, and is at the same time perpendicular to the other leg of the angle, the straight line so intercepted between the two legs of the angle, and beyond the arc, is called the *tangent* of that angle or that arc. The shorter of the two legs of the angle is called the *radius* of the arc, and the longer leg is called the *secant*. The form produced by the three straight lines in contact is a right-angled triangle, and the right angle of the triangle is situated where the tangent touches the radius and the extremity of the arc.—Thus, ocw is a right-angled triangle; and c is its right angle; *w*c is the arc; o*w*w the secant; and cw the tangent. The triangle coe has the same properties as cow, and the tangent is ce. Again, o*CW* is another right-angled triangle; *C* is the right angle; *C*W is the arc; o*C* the radius, o*W* the secant, and *CW* the tangent. The triangle o*CE* is similar to o*CW*, and the tangent is *CE*.—The tangent of an arc of 45° (the half of a right angle) is equal to the radius of the arc = 1. The tangent of an arc of more than 45° is greater than the radius. The tangent of an arc of less than 45° is less than the radius.

48. The *coversed sine*, *cotangent*, and *cosecant* of an arc, are respectively the versed sine, tangent, and secant of its complement—as, the cotangent of an arc of 60° is the tangent of an arc of 30°, and so on.

49. In a book entitled "MATHEMATICAL TABLES, by Charles Hutton," and in many other works relating to Logarithms, there is to be found a table entitled a "Table of Natural Sines, Versed Sines, Tangents, Secants, Cosines, Coversed Sines, Cotangents, and Cosecants," in which the comparative lengths of the lines indicated by these terms is *expressed in numbers*, for every degree and every minute of a quadrant; and the table is so arranged, that if the value of the angle measured by any arc be given, the length of its sine, cosine, tangent, secant, &c., can be seen on reference to the page of the table where that degree is printed; or, if any sine and its cosine, or any tangent and its radius, be given, the value of the corresponding arc and angle can also be determined therefrom. In a subsequent section of this work, I shall give a short table of the same kind, sufficient to show its nature and use.



PROBLEMS.

50. *The rhombus* e s w o, *depicted in the diagram, is the equator of a crystal, and the angle* e o w *is found, by measurement with the goniometer, to be* 120°. *Required, the length of the axes* os *and* ew.

The angle of 120° is equal to the angle of an edge of Model 7 at m_n^s.

51. We see at once that the rhombus is divided by its two axes into 4 equal and similar right-angled triangles, and we take it for granted that any thing that is true of one of these triangles is true of all the others. We begin, therefore, by taking the triangle cow, which measures at o the half of 120°, or 60°. In this triangle, co is the radius, which is to be always taken as unity = 1; cw is the tangent; and ow is the secant. We turn to the Table of Natural Sines, &c, and at 60° we find the following numbers; (Radius being = 1):—

Angle	Radius	Tangent	Secant
60°	1	1.7320508	2.0000000

Consequently, the line co is to the line cw, as 1 is to 1.7320508, and the axis so bears to the axis ew the same relation. If therefore the axis so be equal to m^s and the axis ew to t_+^s, and if the lines es, sw, wo, oe be the four planes of the set MT_+, the exact numerical substitute for the sign + is 1.7320508, and the symbol for this variety of the rhombic prism is $MT_{1.732}$. We see at the same time that the breadth of the planes $MT_{1.732}$ from o to w is equal to the length of the axis m^s from o

to s, because sow is an equilateral triangle, which has three sides of equal length.

52. Let us again examine the triangle cow. If the angle at o is = 60° and *cw* is the arc that measures that angle, *cw* is its sine, and its cosine, or the sine of its complement, is *wa*, which is evidently equal to *co*. If now, we turn to the Table of Sines, &c., and seek there for the sine and cosine of an angle of 60°, instead of seeking for its tangent and secant, we find them to be as follows:—

	Sine.	Cosine.
	8660254	5000000
which × 2 =	1.7320508	1.0000000

Now *co* bears the same relation to *cw* that co does to *cw*, or so to ew: this result is therefore the same as the former, and gives for the rhombus *ecwo*, the axes m^{s}_{1} $\mathrm{t}^{a}_{1.732}$.

53. *The rhombus* o*EnW is the equator of a crystal, and the angle E*o*W is found by measurement with the goniometer to be* 60°. *Required, the length of the axes* o*n and EW.*

54. We proceed as in § 51, to ascertain the comparative lengths of the three sides of the right-angled triangle o*CW*, in which the angle *C*o*W* is $\frac{60°}{2}$ = 30°. The line o*C* is the radius = 1, *CW* the tangent, and o*W* the secant. Referring to the Table, we find the following numbers:—

Angle	Tangent	Secant
30°	.5773503	1.1547005.

Consequently, the line *CW* is to the line *C*o, and the axis *EW* is to the axis o*n* as .5773503 is to 1.0000000. Now

.5773503 : 1.0 :: 1.0 : 1.7320508

which result is the converse of that obtained in § 51. Hence, if o*n* be = m^{s}_{1} and *EW* be equal to t^{a}, the symbol for the set of planes that have the equator o*En*W is $\mathrm{M}_{1.732}\mathrm{T}$.

We see also in this case, as in the former, that the breadth of the planes M+T is equal to the shorter axis, which here is t^{a}; for, as 2.0 is the double of 1.0, so is 1.1547005 the double of 0.5773503.

55. Let us again examine the triangle o*CW*, or rather let us begin with the given angle of 30°, and refer it to the arc *C*W and the angle *C*oW. In this case, CW is the sine of that angle, and Wa is its cosine, which is evidently equal to the line Co. Referring to the Table of Sines, we find

Angle	Sine	Cosine
30°	5000000	8660254
which × 2 =	1.0000000	1.7320508

This is the same result that we arrived at in § 52, save that the numbers apply to different axes. The reason of this is rendered obvious by a

slight examination of the two equators drawn upon the diagram. In the upper equator, the axis t^a is long, and the axis m^a is short, whereas in the lower equator, the axis t^a is short, and the axis m^a is long.

56. The inference to be drawn from the foregoing examinations is, that when the equator of a crystal is a rhombus, which has its shorter diagonal on the axis m^a and its longer diagonal on the axis t^a, the length of the axis t^a is to that of the axis m^a as the tangent of the arc that measures half the obtuse angle of the rhombus is to unity.

Longer Axis		Shorter Axis		Tangent of 60°		Radius = 1
t^a	:	m^a	::	1.7320508	:	1.0

57. *There is a rhombic equator, whose axes* m^a *and* t^a *have the relation of* 10 *to* 13. *Required, the measure of the obtuse angle at* m^a_n, that is to say, *the measure of the angle of incidence of the plane* MT_+ne *upon the plane* MT_+nw.

58. 10 to 13 is the same ratio as 1 to 1.3. 10 or 1 is the radius of the arc that measures the required angle, and 13 or 1.3 is its tangent. We seek in the Table of Sines, &c., and in the column of tangents, for 1.3, which number is easily found, as all the numbers run in regular order. The nearest number to 1.3 that can be found is 1.3000904 which is the tangent of an arc of 52° 26′. This is of course the half only of the obtuse angle at m^a_n which consequently measures twice 52° 26′ or 104° 52′.

It will be found by measurement with the goniometer that this is very nearly the value of the obtuse angle at m^a_n of the rhombic prism represented by Model 6, and described in § 35.

59. *The acute angles of a rhombic equator are cut off by two straight lines which separate portions that are isosceles triangles. Reguired, the value of the angles formed by the straight lines with the residual portions of the sides of the rhombus.*

The angles of the rhombus are given at 120° and 60°.

This case is represented by the rhombus eswo in the last diagram: *ez* and x*w*, are the two straight lines; *e*z*e* and x*w*w are the two triangles cut off; *ezs*, sx*w*, x*wo*, o*ez*, are the four new angles whose value is required.

The problem is solved in the same manner as the geometrical proposition, that "the angles made by one line meeting another, between its extremities, are together equal to two right angles."

Let the line *wx* be supposed to meet the line sw, then the angles *w*xw and wxs are together equal to two right angles = 180°. Now the angle esw is given at 120°, consequently the angle cxw is 120°, and the angle *w*xw, which is the half of cxw, is 60°. Further, the angle *w*wx is given at 60°, the angle cwx, which is the half of *w*wx, is 30°, wherefore the angle *w*xw must be 60°. Hence, the angle *w*xs is 180° — 60° = 120°. But if the angle *w*xs is 120°, so also are the angles x*w*o, o*ez*, and *ezs*. Therefore all the new angles are 120°, and the resulting figure is a regular hexagon, similar to the equator of Model 7.

60. *The same problem as § 59 is given, but is now to be considered in reference to the separation of the obtuse angles of the rhombus.*

Diagram and measurements of the rhombus as before; but the straight lines now in question, are those marked zx and *ew*; the triangles cut off are zsx and *eow*; and the new angles produced are ezx, zxw, w*we*, and *we*e. Principle of solution, as already explained.

Let the line zx be supposed to meet the line sw, then the angles zxs and zxw are together equal to two right angles, or 180°. Now the angle xw*w* is given at 60° and the angle xwc is consequently = 30°, and as xwc is equal to sxz, the angle wxz is equal to two right angles, minus the angle sxz; or to 180° — 30° = 150°. But the angle zxw is equal to the angles w*we*, *we*e, ezx, and consequently each of the four new angles is = 150°.

61. It will be observed that the value of the new angles produced by the replacement of the angles of a rhombus by straight lines is not altered by the approximation of the straight lines either to the middle point of the rhombus or the vertex of the replaced angles. It will also be observed, that each new angle that is produced when a straight line cuts off the angle of a rhombus, is *equal to a right angle added to half the value of the angle that is cut off;* and the reason is very obvious. The angle sx*w*, for example, has been found to be = 120°, and it contains the right angle zx*w*, which is 90°, and the angle sxz, which is 30°, or the half of the acute angle xw*w*, which is 60°. Again, the angle zxw has been found to be =150°, and it contains the right angle zx*w*, which is 90°, and the angle *w*xw, which is 60°, or equal to the angle csx, which is the half of the obtuse angle of 120°, zsx.

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62. From this digression into trigonometry, I return to the consideration of the forms that are produced by the complement of planes MT_x.

63. Model 8. *Six-sided Prism.* P_x, T_-, MT_+, or $P_{0.65}$, $T_{0.71}$, $MT_{1.732}$. This model contains the planes MT_+ in combination with P and T. The angle at m^a_n is 120°, consequently the axes of the planes are m^a_1, $t^a_{1.732}$. The planes T are parallel to the axis m^a, and consequently the angles formed by T upon $MT_{1.732}$ ought to be 120°, which the goniometer proves them to be. The axis t^a of the model is found by measurement across the plane P, to be 0.7 of the length of the axis m^a, and the axis p^a is found to be 0.65 of the axis m^a. Of course, the angles formed by T and MT_+ upon P, are all angles of 90°.

The difference between the forms represented in Models 7 and 8 depends entirely upon the dissimilar lengths of the axis t^a.

64. Model 9. *Six-sided Prism; a macle, double, or twin crystal.* P_x, T_+, MT_+, or $P_{2.2}$, $T_{1.2}$, $MT_{2.12}$.

Each prismatic half of this model contains the planes MT_+, in combination with the planes P and T. The angle at m^a_n is 129° 30′. The half of this angle is 64° 45′, the tangent of which is 2.12. By lineal measurement the axes of the model are found to be $p^a_{2.2}$ m^a_1 $t^a_{1.2}$.

The angles formed by the planes $MT_{2.12}$ upon T, are found by adding 90° to half the acute angle of $MT_{2.12}$. Now, as the obtuse angle is 129° 30,′ the acute angle must be 180°—129° 30′ = 50° 30′. The half of this is 25° 15′ which added to 90° gives 115° 15′, and this is in fact the angles that are formed by T upon MT_{+}, as may be determined by the goniometer. The method of denoting the *macle* I shall describe hereafter.

65. The angle formed by each of the planes MT, upon M or T, Model 4, can be readily calculated upon the principle explained in § 59—61. The angle formed by M upon T is 90°, and that formed by MT upon MT is the same. Therefore, in the combination, the angle of 90° between M and T is cut off by a straight line, and $\frac{90}{2} + 90° = 135°$, which will be found, by measurement with the goniometer, to be the value of the angle of every vertical edge of Model 4.

66. Model 10. *Twelve-sided Prism.* P_{-}, M, T, MT_{+}, $M_{+}T$.

Besides the two terminal planes of this model, which are the set P, there are upon it the sets M,T, MT_{+}, and $M_{+}T$. Of these twelve vertical planes, six are large and six small. One of the latter, the north plane, is marked M, and one of the large planes, the west plane, is marked T. Parallel to these may be found the corresponding planes Ms and Te. Upon putting the crystal into position for examination, it will be seen that the large planes are similar in number and position to the six vertical planes of Model 7, and that the small planes are equal to planes which may be supposed to have cut off the six vertical edges of Model 7. These edges we have found to measure 120°, and according to § 61, if they were cut off by a plane equally, the resulting edges should be $\frac{120°}{2} + 90° = 150°$. Now upon measuring the angle formed by a large upon a small side plane of Model 10, it is found to be 150°; so that this form would seem to consist of two hexagonal or six-sided prisms cutting one another, which is really the case, for the model contains, first, the set of planes $MT_{1.732}$, which gives an angle of 120° at m^{a}_{n}. This angle may be found by measuring the incidence upon one another of the two large planes on each side of the north plane, applying the goniometer horizontally across the plane M. The situations of the four planes of the set $MT_{1.732}$ are n^2w, n^2e, s^2w, s^2e; and their acute edges are cut by the two large planes $T_{0.866}$, situated e and w. Next, there is a rhombic prism of precisely the same relative dimensions but having m^a long, which produces the four smaller side planes that are situated nw^2, ne^2, sw^2, se^2. The symbol for these four planes is $M_{1.732}T$. The acute edges of this set of planes are cut by the two small planes $M_{0.866}$, situated n and s. The proof that all this is correct, lies in the fact that all the vertical edges of Model 10 have angles of 150°, which could not be the case if $M_{+}T$ was different in the relative lengths of its two axes from MT_{+}.

67. But, it may be asked, how happens it that the planes of the form

M, $M_{+}T$ are *small*, and those of the form T, MT_{+}, *large?* The answer is, that the entire form T,MT_{+}, although it is most visible upon the combination, is in fact of smaller dimensions than the form M,$M_{+}T$. Its planes therefore approach nearer to the centre of the compound, and leave less room for the planes of the other form. It often happens, indeed, in the combination of SIMILAR *forms*, that they are not EQUAL, and whenever this is the case, the *largest form* that enters into the combination, is that of which *least is to be seen* upon the solid. The planes that approach nearest to the central point of a compound form necessarily cut off the planes, or part of planes, that project to a distance from the centre. Thus, it will be seen that the planes MT on Model 4 are narrower than the planes M and T. Now the equators of MT and M, T, are both square, but though *similar*, they are not *equal*, for if the diameters of MT and M, T, be measured across the plane P, that of MT will be found to be longer than that of M, T. The planes MT are consequently farther from the centre of the combination than the planes M, T, and on that account they are smaller.

68. Models 32, 33, 34, which have been several times referred to, all contain the same planes, and in the same positions, and they are denoted by the same symbols. Yet the three forms are quite different from one another, and the difference arises entirely from this circumstance, that the planes upon each form though *similar* to those upon the others, are not *equal* to them.—Model 32 consists of a small cube in combination with a large dodecahedron and a large octahedron; Model 33 of a small octahedron in combination with a large dodecahedron and a large cube; Model 34 of a small dodecahedron in combination with a large cube and a large octahedron. In all these cases, the form which is *most visible* is the smallest of all the forms that belong to the combination.

69. It is evident from these considerations, that it is of great importance to us to have a method of distinctly and conveniently describing the variations in the relative size of planes which arise from the causes that I have just investigated. The method which it has occurred to me is best adapted for this purpose, is to write the symbols that express the *large planes*, by which I mean those that are *most visible*, in capital letters, as P, M, T, MT. PM, PT, PMT; and the symbols that express the *small planes*, by which I mean those that are *least visible*, in small letters, as p, m, t, mt. pm, pt, pmt. Upon this plan—

Model 4, is P_{+},M,T, mt.
Model 10, is P_{x}, $m_{.866}$, $T_{.866}$, $MT_{1.732}$, $m_{1.732}t$.
Model 32, is P,M,T, mt. pm, pt, pmt.
Model 33, is p, m, t, mt. pm, pt, PMT.
Model 34, is p, m, t, MT. PM,PT, pmt.

It sometimes happens that the planes upon a crystal are of three kinds as respects their comparative size: namely, 1, very large; 2, very small; 3, intermediate. I may instance Model 35, in which the square planes are very large, the triangular planes very small, and all the others inter-

mediate. The first kind may be expressed in print by capitals, P, M, T; the second kind by small letters, p, m, t; the third kind by small capitals, ᴘ, ᴍ, ᴛ. In manuscript, these varieties are distinguished by writing the first in capitals, the second in small letters, and the third in small letters with two lines scored below them.

EXAMPLE.

Model 35. P,M,T, ᴍᴛ. ᴘᴍ, ᴘᴛ, ½ ᴘᴍᴛ, ½ pmt.

70. *Polaric Positions round the Equator.* In the description of Model 10, I have used the signs n^2w, nw^2, and some others, not yet explained, to indicate certain polaric positions. It will be proper to explain these terms fully, in order to prevent ambiguity.

Suppose the equator to be a circle, and suppose radii proceeding from the centre of the circle, (see o in the diagram in § 37, and the equivalent point c in the diagram in § 20,) in all directions towards the circumference. The extremities of these lines indicate polaric positions, and the planes which cross them occupy the particular positions that the lines point out. Let the circle *C*Qcq, in § 37, be this equator, and let o*C* be a radius pointing northward. Then *C* on the line *ECW* will be the polaric position n, Q the position e, c on the line ecw the position s, and q the position w. A radius striking the arc *C*q, midway between *C* and q would point out the position nw; exactly opposite would be se, and at right angles to these would be ne and sw. A radius striking the arc *C*q midway between the positions n and nw, would point out the position that I have marked n^2w, which means nearer to n than to w. The corresponding position, midway between the positions nw and w is marked nw^2, which means nearer to w than to n.

EXAMPLES.

71. A crystal that has the planes M,T, MT, MT_+, M_+T, exhibits one plane of each set in the north-west quarter of the crystal, arranged in the following order:

n	n^2w	nw	nw^2	w
M,	MT_+,	MT,	M_+T,	T

72. A crystal that has the planes M, T, MT, MT_+, $MT_‡$, M_+T, $M_‡T$, exhibits them in the following order:

n	n^3w	n^2w	nw	nw^2	nw^3	w
M,	$MT_‡$,	MT_+,	MT,	M_+T,	$M_‡T$,	T

Any other quadrant of the equator of the combination would exhibit the planes in a corresponding arrangement, as:

e	ne^3	ne^2	ne	n^2e	n^3e	n
T,	$M_‡T$,	M_+T,	MT,	MT_+,	$MT_‡$,	M

73. The twelve planes which form a zone round the equator of model 10, have the following positions:

	e	ne^2	n^2e	n	n^2w	nw^2	w
Northern:	T,	m_+t,	MT_+,	m	MT_+	m_+t,	T
	e	se^2	s^2e	s	s^2w	sw^2	w
Southern:	T,	m_+t,	MT_+	m	MT_+	m_+t,	T

In *Phillips's Mineralogy*, page 174, there is a figure of a crystal of Fluorspar, which has 32 vertical planes, the arrangement of which is as follows, taking the nw quadrant as an example:

n	n^4w	n^3w	n^2w	nw	nw^2	nw^3	nw^4	w
M,	$MT_{\overset{+}{++}}$,	$MT_{\ddagger}$,	MT_+,	MT,	M_+T,	$M_{\ddagger}T$,	$M_{\overset{+}{++}}T$,	T

The signs $_+$, $\ddagger$, $\overset{+}{++}$, in all these symbols indicate the great, greater, and greatest *distance* of the poles of the axes referred to, while the figures 2, 3, 4, indicate the great, greater, and greatest *proximity* of the poles to whose symbol they are added. The figures which indicate the lengths of axes are consequently in an inverse ratio to those that indicate the proximity of the planes to particular poles. The greater the figure that multiplies the length of any axis, the further are the residual portions of the planes that appear upon a combination, removed from the poles of that axis.

74. It will be recollected that the axis t^a of the form MT_+ is longer than the axis t^a of the form MT, and that the axis t^a of the form $MT_{\ddagger}$ is still longer than the axis t^a of the form MT_+: the axes m^a being taken at unity in each case. On the other hand, it is shown in the above series of positions, that when these sets of planes have cut one another in a combination, the planes of $MT_{\ddagger}$ appear nearer to the planes M than do the planes of MT_+, and that the planes of MT_+ appear nearer than the planes of MT. Hence the polaric positions of individual planes indicate the comparative lengths of the axes of the rhombic prisms to which they belong, so that when we see upon a crystal a plane situated betwixt two others, which we know to be M and MT, we may be sure that the symbol of the plane in question is MT_+, since all the planes of the form M_+T, must of necessity appear upon a crystal, if they appear at all, between MT and T, and cannot by any possibility occur between M and MT. The reason of this has been explained in § 67, where it is shown that when the planes of rhombic prisms are cut by other forms, it is generally the portions that are attached to the poles of the shorter axes of the rhombic prisms that remain upon the combination. The portions attached to the longer axes are cut off and lost.

75. Model 11. *Right Rhomboidal Prism*, P,$M_{\llcorner}$, $\frac{1}{2}M_+T$.

This model contains the planes P and M, in combination with the planes $\frac{1}{2}$MT, that is to say the half of the set MT. See § 27. Upon applying the goniometer to MT and M across the nw edge, we find the angle to be 120°. Again applying it to the sw edge, we find it to be 60°.

The two opposite angles in ne and se agree with these. The equator is a rhomboid. Deducting 90° from 120°, we have 30° as the value of the angle formed by a line passing from the south to the north pole, and another passing from the north to the west pole. This angle is exactly that formed by the rhombus of 120° and 60° as described in §§ 53—55:

for 30° × 2 = 60°, = angle of the acute edge at m^{a}_{n},
and 180° — 60° = 120°, = angle of the obtuse edge at t^{a}_{w}.

Hence the precise symbol for the planes of model 11 is P,M, $\frac{1}{2}M_{1.732}$T nw, se. The polaric positions are added to show which two planes of the set M+T are present.

The relation which this form bears to the large rhombic prism $M_{1.732}$T of which it may be supposed to be a segment, is shown in the diagram in p. 14, by the lines E*E*rW, which lines represent the form of the equator of model 11. Another method of explaining the form would be that of referring it to the rhomboid *e*xwo, which forms the half of the rhombus eswo, depicted in the diagram on page 14. Considered in this light, and having regard to its dimensions, Model 11 would be a *macle* consisting of 2½ or of 10 rhombic prisms of 120° and 60°, combined so as to form a single crystal, and the symbol for it would be P_x,M$T_{1.732}$ × 10.

76. *Designation of the Planes and Axes of Crystals by separate Symbols:*—

In many cases it is difficult to add to the symbols that denote the planes of crystals, the signs that denote the lengths of their axes, without endangering the perspicuity of the symbols that designate the planes. It is better therefore when crystals require a complex symbol, to denote the length of their axes by a separate symbol. Thus in the case of model 11, the best method of denoting the length of its axes appears to me to be this:—

P,M,$\frac{1}{2}M_{1.732}$T nw, se.p^{a}_{16} m^{a}_{13} t^{a}_{23}

These measurements are taken along the edges or across the planes of the crystal, and serve to give a general idea of its principal dimensions, independently of the information communicated by the other symbols in respect to the number and disposition of the planes of the crystal. See Section 1., §§ 1—14.

77. Model 91. *Pentagonal Dodecahedron*, MT_2.PM_2,P_2T.

When model 91 is put into position, it will be found to possess four vertical planes which form part of a rhombic prism that has obtuse angles of 126° 52′ at the poles m^{a}_{n} and m^{a}_{s}. Now the half of 126° 52′ is 63° 26′, and the tangent of this arc is 1.999859. Hence the formula for this set of 4 planes is $MT_{1.999859}$, or briefly MT_2.

The planes PM_2, P_2T, which are also possessed by the form exhibited in model 91, will be explained in subsequent sections. I am treating here only of the vertical planes of crystals.

78. Model 68. *Tetrakishexahedron*, $MT_2M_2T.PM_2P_2M,PT_2P_2T$.
Place this model in position, and observe that it has eight vertical planes, divided from one another by 4 vertical edges at ne, nw, se and sw, and meeting in 4 angles at n, e, s, and w. The two northern and two southern planes, make up the four sides of a rhombic prism = MT_+. The two eastern and two western planes make up the sides of a rhombic prism = M_+T. These two prisms intersect one another, and therefore mutually cut off their acute angles. Upon measuring the obtuse angle at m^{s}_{n}, it is found to be, or it ought to be, 126° 52′, but some of the models are incorrect, and measure only about 120°. The half of 126° 52′ is 63° 26′, the tangent of which arc is 1.999, as I have shown above. This gives the formula MT_2 for the southern and northern planes. The obtuse angle at t^{e}_{w} also measures 126° 52′, which gives the formula M_2T for the eastern and western planes.—The planes that are not vertical will be described hereafter.—The horizontal axes of this model are m^{s} t^{e}.

79. *Geometrical relations of the angles of an Equator, or other section of a crystal.*—The equator of Model 68 is an octagon or figure of 8 sides, and it may serve to afford us an illustration of the use to be derived from the geometrical proposition contained in § 16, *t*.—The mineral kingdom presents us with several varieties of the tetrakishexahedron, which differ among themselves in the relative lengths of the axes of the rhombic prisms whence their planes are derived, as $MT\frac{3}{2}$, MT_2, $MT\frac{5}{2}$, &c. In many works on mineralogy, these different forms are described by merely giving the angles which their planes form upon one another; such as, for example, the angle formed by the plane MT_2n^2w upon the adjoining plane $M_2T\,nw^2$, measured by the goniometer across the nw vertical edge. I will suppose this angle to be given at 143° 8′ and that it is required to learn from that datum what is the value of the sign + in the symbol MT_+, M_+T.

80. The equator has eight angles and 8 sides. According to the rule given at § 16, *t*, I multiply the 8 sides by 180°, which gives 1440°, and then I deduct 360°, which leaves 1080°. This should be equal to the sum of all the eight angles added together. But one of the angles is given at 143° 8′, and there are four similar angles at nw, ne, sw, and se. Now 143° 8′ × 4 is 572° 32′, which, deducted from 1080°, gives 507° 28′ for the value of the four intermediate angles at n, e, s, and w. Dividing 507° 28′ by 4, I have 126° 52′ as the value of a single obtuse angle, and this, divided by 2 is 63° 26′, which, as I have shown above, is the angle whose tangent gives 2 for the equivalent of the sign + in the formula MT_+,M_+T, producing the symbol MT_2, M_2T.

81. Several other useful problems in crystallography can be solved in a similar manner. Thus, suppose we had the form MT_+,M_+T, and knew the measurement of the obtuse angle at m^{s}_{n} to be 126° 52′, and that we were required to deduce from that measurement alone the value of the 8 angles of the equator of the form. Of course 4 of the angles, namely, those at n, e, s, w, are 126° 52′, but the other 4 are unknown.

To find them, I multiply 126° 52′ by 4, which gives 507° 28′. This sum deducted from 1080°, leaves 572° 32′, which is the aggregate value of the 4 other angles. I divide again by 4, which gives 143° 8′ for the value of the single intermediate angle: consequently the equator has alternate angles of 126° 52′ and 143° 8′.

82. We learn from the foregoing calculations, that 1080° is the constant aggregate value of the eight angles of an equator that has eight sides, being $8 \times 180 - 360° = 1080°$. From a similar calculation, we learn that 720° is the constant aggregate value of the six angles of an equator that has six sides, $6 \times 180 - 360° = 720°$; and that 360° is the constant aggregate value of the four angles of an equator that has four sides, $4 \times 180 - 360° = 360°$. Again, if we divide 1080°, the constant aggregate value of the 8 angles of an octagon, by 4, we have a product of 270°. This is the aggregate value of 2 of the angles. Now it very often happens, that 4 of the angles of an octagon are equal to one another, and the 4 others are also equal to one another, but different from the first four; an example of this is given in § 80. When this is the case, one angle of each kind added together is = 270°, so that, if we know the value of either of these angles, that of the other is ascertained by deducting the known angle from 270°.—Finally, the six angles of a hexagon, which are together always = 720°, very frequently consist of 4 angles of one kind and two of another kind. Then of course 2 of the first and 1 of the second kind of angles are together equal to 360°. If you know the sum of the odd angle, that of the two others will be equal to 360° minus the odd angle. If you know the value of one of the pair of angles, you have but to double its sum and deduct the product from 360° to find the value of the odd angle. The knowledge of these relations frequently enables one to understand descriptions of crystals so imperfectly given as to be otherwise unintelligible.

83. *Problem.* The vertical planes of Model 47, which represents a combination of the cube with the pentagonal dodecahedron, the cube being subordinate, are m, t, MT_2. Required, the value of the angles formed by m upon MT_2, and by MT_2 upon t.

The obtuse angle at m^a_n of the form MT_2, is found by seeking in the Table of Tangents for the arc of the tangent 2.000000. That arc is 63° 26′, and the double of it is equal to the angle at m^a_n, which is 126° 52′. Then $\frac{126° 52'}{2} + 90° = 153° 26'$. This is the value of the angle formed by the plane m upon the plane MT_2.

When the obtuse angle of a rhombus at m^a_n is 126° 52′, the acute angle at t^a_w is $180° - 126° 52' = 53° 8'$. And $\frac{53° 8'}{2} + 90° = 116° 34'$. This is the value of the angle formed by the plane t upon the plane MT_2. Upon measuring the angles of Model 47 with the goniometer, you will find that these results are correct.

84. *Control over the correctness of this reckoning.* The planes m, t, MT_2 give eight sides, and therefore eight angles, to the equator; and

these angles must be in sets of four and four alike; namely, 4 angles formed by m upon MT_n, and 4 formed by t upon MT_x. Now:

$$\left.\begin{array}{l} 4 \times 153^\circ\ 26' = 613^\circ\ 44' \\ 4 \times 116^\circ\ 34' = 466^\circ\ 16' \end{array}\right\} = 1080^\circ\ 00',$$

which is the constant aggregate value of the angles of an octagon. § 82.

85. This method of examining the geometrical relations of the angles of the equator, or of any other section of a crystal, affords the means of *controlling the correctness of the mechanical measurements of the external angles of crystals.* It is a principle of control similar to that which the chemist has over the results of an analysis in the knowledge which he possesses of the value of equivalent proportions. There are many published measurements of the angles of crystals, which stand greatly in need of the corrections that could be made by using this power of control.

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86. *Definition of the word* PRISM.—It will be evident to the examiner of the foregoing data, that the symbols M, T, MT, $M_{+}T$, MT_{+}, serve to denote every vertical plane that can possibly occur around the equator of any crystal; for all planes whose positions are n, e, s, w, are denoted by M or T, and all that have intermediate positions, as ne, nw, se, sw, n^2e, ne^2, &c., are denoted by some variety of MT_x. Consequently, if we arbitrarily assume these vertical planes to be, in conjunction with the horizontal planes P, the COMPONENTS OF PRISMS, we have in P, M, T, MT, MT_{+}, $M_{+}T$, a formula which comprehends every variety of prism which can possibly occur in nature or in art—that is to say, of prism considered independently of inclined or pyramidal terminations. I propose therefore to restrict the meaning of the word *prism* to forms which contain these planes and no others.

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87. *Forms of the Equators of Prisms.*—For a practical purpose that will be hereafter explained, we may here take a view of the different forms that can be assumed by the equators of prisms under every possible change of circumstance.—I believe that these forms may all be reduced to five principal varieties, as follow:—

	Axes.		Planes.		Equators.
1	$m^a\ t^a$	—	M,T	=	a square.
	$m^a\ t^a$	—	MT	=	a square.
	$m^a\ t^a$	—	M,T,MT	=	a square combined with a square.
2	$m^a\ t^a_{+}$	—	M,T_{+}	=	a rectangle.
	$m^a_{+}\ t^a$	—	M_{+},T	=	a rectangle.
3	$m^a\ t^a_{+}$	—	MT_{+}	=	a rhombus.
	$m^a_{+}\ t^a$	—	$M_{+}T$	=	a rhombus.
	$m^a\ t^a$	—	$MT_{+},M_{+}T$	=	a rhombus with a rhombus.
4	$m^a\ t^a$	—	$M,M_{+}T$	=	a square combined with a rhombus.
	$m^a\ t^a$	—	T,MT_{+}	=	a square combined with a rhombus.
5	$m^a\ t^a_{+}$	—	$M,M_{+}T$	=	a rectangle combined with a rhombus.
	$m^a_{+}\ t^a$	—	T,MT_{+}	=	a rectangle combined with a rhombus.

88. *Of the Planes* PM.—There are four of them. The first plane is shown by the lines T p^a_z T^1 3 m^a_n 6 in the diagram in § 20. The others are not shown on the diagram. The second plane passes from 6 m^a_n 3 to T^2 p^a_N T^3; the third from T^2 p^a_N T^3 to 9 m^a_s 12; and the fourth from 9 m^a_s 12 to T p^a_z T^1. The four planes have the following polaric positions: Zn, Zs, Nn, Ns. Every one of these planes cuts the two axes p^a and m^a, and is parallel to the axis t^a. They are all INCLINED. They are equivalent to one another and are parallel two and two. They form together a HORIZONTAL PRISM, whose axis coincides with the axis t^a, and whose edges are directed towards Z, n, N, s. The symbol for the complement of four planes is PM. The symbol for half the complement is $\frac{1}{2}$ PM. The planes are individually denoted by PM Zn, PM Zs, PM Nn, PM Ns.

The north meridian forms the cross section of the horizontal prism PM, and coincides with the plane p^a_z m^a_n p^a_N m^a_s. The form of this north meridian depends upon the relative lengths of the axes p^a and m^a.

a, If these axis are equal, the north meridian is a square.

b, If the axis p^a is longer than the axis m^a, the north meridian is a rhombus with the acute angles upon the axis p^a.

c, If the axis m^a is longer than the axis p^a, the north meridian is a rhombus with the acute angles upon the axis m^a.

d, In the first case, the symbol for the planes is PM. In the second case, it is P_+M. In the third case, it is PM_+.

The instructions given in §§ 50—61 for determining the relative lengths of the axes m^a and t^a situated in a rhombic equator, apply equally to the determination of the lengths of the axes p^a and m^a situated in a rhombic meridian.

EXAMPLES OF THE PLANES PM, PM_+, P_+M.

89. Model 63. *Rhombic Dodecahedron*, MT.PM,PT.

See § 21 illustration B, and § 31.

The Zn plane of the form PM is marked on this model with P at the Z pole and with M at the n pole. The whole four planes of PM meet at their acute points, and form a zone round the crystal in the direction Z,n, N,s. The north meridian, which is a square with its angles in Z, n, s, N, cuts across this set of planes. The planes PM incline upon the planes MT at an angle of 120°.

90. Model 12. *Obtuse Quadratic Octahedron*, PM_+,PT_+.

The Zn plane of this model is marked with P at the Z pole, and with M at the n pole. The angle across the edge at the n pole is found, by measurement with the goniometer, to be 83° 38′, and the angle formed by the plane Zn upon the plane Zs, measured over the apex, is found to be 96° 22′. Consequently, the axes are p^a m^a_+, and the form of the north meridian is a rhombus with its acute angles on m^a. To learn the comparative lengths of the two axes, we divide 96° 22′ by 2, which gives 48° 11′, and seek in the Logarithmic Tables for the tangent of this angle,

which is 1.1177846. This number is the length of the axis m^a when p^a is taken as unity. Now 1.1177846 is to 1, as 19 is to 17. We may therefore describe PM_+ as $P_{17}M_{19}$, or as $PM\frac{19}{17}$, or as $PM_{1.118}$. I think that the last method of notation is the best, because it gives the readiest reference to the Table of Tangents, &c., whenever we wish to learn the value of the edges of a crystal from the length of its axes.

If the lengths of the axes of this model were reversed, so as to make p^a the longer axis, the symbol for the form would be $P_{1.118}M$, and the north meridian would be a rhombus with its acute angles at Z and N.

91. Model 13. *Acute Quadratic Octahedron,* P_+M, P_+T.

The Zn plane of this model is marked with P at the Z pole and with M at the n pole. The angle formed by the plane Zn upon the plane Nn across the equator at n, is 137° 10′; and that formed by Zn upon Zs over the apex Z is 42° 50′; as will be found upon applying the goniometer to the model. The north meridian is therefore a rhombus, with its acute angles at Z and N. The half of the obtuse angle 137° 10′ is 68° 35′, the tangent of which is 2.5495160; so that the axes have the relation of $p^a_{5.1}$ m^a_2, and the symbol for the set of planes is $P_{5.1}M_2$ or $P_{2.55}M$.

If the obtuse angle at m^a_n, instead of being 137° 10′, had been 136° 24′, then the axes would have had the simple relation of p^a_5 m^a_2; because the half of 136° 24′ is 68° 12′, the tangent of which is 2.5001784.—The symbol for the planes would then have been $P_{2.5}M$ or $P\frac{5}{2}M$.—The difference between 137° 10′ and 136° 24′ is one that cannot be properly discriminated by measurement upon the model; but 137° 10′ is the measurement derived from the crystals of a mineral called Anatase, the general form of which is represented by the model.

92. Model 32., Model 33., Model 34. } Cube combined with the rhombic dodecahedron, and the octahedron = P,M,T,MT.PM,PT,PMT.

In these three models the planes PM, are distinguished by their equal inclination upon the planes P and M, with both of which they make angles of 135° or $\frac{90°}{2} + 90°$. On model 32 the four planes are inclined rectangles; on model 33, they are also inclined rectangles; and on model 34 they are octagons. On all the 3 models, the positions are Zn, Zs, Nn, Ns.

93. Model 14. *Quadratic Octahedron in combination with an Acute Quadratic Octahedron, the former subordinate.* pm, P_+M, pt, P_+T.

The plane marked P at the pole Z on model 14 is pm Z^2n. The plane marked M at the pole n is $P_+M\ Zn^2$.

The angle formed by the plane pmZ^2n upon the plane pmZ^2s, measured over the apex, is 90°. This is not the correct angle of the mineral which this model was intended to represent, but it has been made so by accident. 90° is the angle which denotes a square north meridian, so

that the planes pm belong to an equiaxed form, and the symbol is simply pm. The angle formed by the plane $P_+M\ Zn^2$ upon the plane $P_+M\ Nn^2$ across the edge marked M, is 136° 47′. The half of this angle is 68° 23½′, the tangent of which is 2.5246392. Hence the symbol for the planes is $P_{2.525}M$, or briefly but less exactly $P\frac{5}{2}M$.

The symbol for the model is therefore pm, $P\frac{5}{2}M$, pt, $P\frac{5}{2}T$.

94. The form of the north meridian of the combination represented by model 14 is an octagon. We will examine the value of its angles:

2 × 90°	at Z and N	= 180° 00′
2 × 136° 47′	at n and s	= 273° 34′
All the angles known are therefore		= 453° 34′

But the eight angles of the meridian are together equal to

1080° 00′. See § 82

Deduct 453° 34′ = Value of the four known angles.

Leaves 626° 26′ = Value of the four unknown angles.

And the fourth part of 626° 26′ is 156° 36½. This is the value of each of the four angles formed by the incidence of the planes pm upon the planes $P_{2.525}M$, as will be found by applying the goniometer to the model.

95. Model 91. *Pentagonal Dodecahedron.* $MT_2.PM_2,P_2T$. See § 77.

The letter P marked upon this model shows the edge formed by the meeting of the planes PM_2 Zn and PM_2 Zs. Upon applying the goniometer to these two planes, across this edge, the value of the angle of their incidence is found to be 126° 52′. The half of this is 63° 26′, the tangent of which is 1.999589. Hence the formula for the set of planes PM_+, two of which appear at the top of the model, and two at the bottom, is PM_2.

96. Model 47. *The Cube combined with the Pentagonal Dodecahedron, the former subordinate.* $p,m,t,MT_2.PM_2,P_2T$. See § 83.

The plane pZ is marked P upon model 47. The plane m n is marked M. The plane t w is marked T. The planes MT_2 are the vertical planes situated between the planes m and t. The planes PM_2 are the front and back inclined planes situated between the planes p and m. The following is the proof. The angle at Z of the planes PM_2 Zn and PM_2 Zs, is 126° 52′. See § 95. Now as the plane P cuts these two planes equally, it should make angles of 153° 26′ with both of them; for

$$\frac{126^\circ\ 52'}{2} + 90^\circ = 153^\circ\ 26'$$

and upon applying the goniometer to the model, this will be found to be the correct angle. See § 77, 95.

97. Model 68. *Tetrakishexahedron.* $MT_2,M_2T.PM_2,P_2M,PT_2,P_2T$. See § 78.

The planes PM_2, P_2M, form a zone of 8 planes round the edge of the north meridian. The planes PM_2 occupy the positions Z^2n, Z^2s, N_2n, N_2s. The planes P_2M occupy the positions Zn^2, Zs^2, Nn^2, Ns^2. The angles are given at § 78 to 82, and all that is said there respecting the means of investigating the properties of the equator of this form, applies equally to the investigation of its north meridian.

98. Model 79. *Oblique Rectangular Prism.* $M, T_+ . \frac{1}{2} PM_+$ Zn,Ns. p^a_x m^a t^a_+.

The position which this crystal is made to assume, seems, like the positions of Models 2, 6, 7, 10, and others, to militate against the general rule, § 8, that a crystal is to be held with its longest axis in a perpendicular position. It is to be remembered, however, that the length of a prism, or, what comes to the same thing, the length of its axis p^a, is an extremely variable quantity, so that crystals whose side planes are invariable as respects the relation of the axes m^a and t^a, often differ greatly in their length.

The plane marked M on Model 79 is the plane M n. The plane marked T is the plane T w. The plane marked P is the plane PM_+ Zn. The angle which this plane forms with M, across the edge Zn, is 113° 8′. Deducting 90° for the value of the right angle of the prismatic plane M n, we have 23° 8′ for the value of half the acute angle of the rhombus PM_+. The sine of this angle is 3928722 and the cosine is 9195931. Its cotangent is 2.3406928. The radius is 1. The axis p^a of the form PM bears therefore to the axis m^a the ratio of 1 to 2.34, and the symbol of the set of planes is $PM_{2.34}$. But only two planes of the set are present, and these have the positions Zn and Ns.

The three axes of the model are approximatively equal to p^a_2 m^a_5 t^a_8. Hence the exact symbol for the model is $M, T_+, \frac{1}{2} PM_{2.34}$ Zn,Zs. p^a_2 m^a_5 t^a_8.

The north meridian of Model 79 has the same geometrical properties as the equator of Model 11. See § 75.

99. Model 83. *Obtuse Rhombohedron.* $MT_+ . \frac{1}{2} PM_+$ Zn,Ns.

The angle formed by the plane MT_+ nw upon the plane MT_+ ne, measured across the edge marked M, is about 105° 5′. Some of the models are not quite exact in the angle. The half of 105° 5′ is 52° 32½′, and the tangent of the latter is 1.3051896. Hence the planes MT_+ are $MT_{1.3}$. See § 35.

The angle formed by the plane PM_+ Zn upon the north vertical edge is *about* 102°. Deducting 90° we have 12° for half the acute angle of the rhombus PM_+. The sine of 12° is 2078117, its cosine 9281476, its radius 1.0, its cotangent 4.7046301. Hence the symbol for PM_+ is $PM_{4.7}$. But there are only two of the set of planes PM_+ present on the combination, which two planes have the positions Zn,Ns.

In the foregoing calculations I have taken first the axis m^a and afterwards the axis p^a for unity; but it would be better to consider the axis m^a as unity in both cases, since that axis is common to both the forms

MT_+ and PM_+. The only correction which it is necessary to make in this view, is to take the tangent instead of the cotangent of the acute angle of 12°, and use that sum in expressing the relation of the axes. The tangent of 12° is 0.212556, so that the form PM_+ becomes P_-M, and must be termed $P_{0.213}M$, instead of $PM_{4.7}$. The combination exhibited by Model 83, will then be expressed by $MT_{1.3}$, $\frac{1}{2}P_{0.213}M_-Zn,Ns$, or, in general terms, MT_+, $\frac{1}{2}$ P_-M Zn,Ns. To this must be added, the relation of the axes of the crystal $=$ p^a_8 m^a_{10} t^a_{13}, because the dimensions of the crystal are not told by the symbols of the planes.

100. Model 85. *Obtuse Rhombohedron.* MT_+. $\frac{1}{2}PM_+$ Zn,Ns.

The angle formed by the vertical plane MT_+ nw upon the adjoining vertical plane MT_+ ne, measured across the vertical edge at M, is about 128°, but it ought, according to a measurement of Haüy's, to be 134° 26′. The half of this angle is 67° 13′, the tangent of which is 2.3808444; so that the symbol for the prismatic planes is $MT_{2.38}$.

The angle formed by the plane PM_+Zn upon the vertical edge at n, is about 130° 26′. Deducting 90°, we have 40° 26′ for half the acute angle of the rhombus PM_+. The tangent of this is 0.8520704, which gives the symbol $P_{0.852}M$, and which bears a very simple relation to the symbol $P_{0.213}M$ of Model 83; as 213 × 4 = 852.

Hence the symbol for Model 85 is $MT_{2.38}$.$\frac{1}{2}P_{0.852}M$ Zn,Ns. To which is to be added the dimensions of the axes of the crystal p^a_2 m^a_3 t^a_4 .

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101. *Of the Planes* PT.—There are four of them. First, the plane shown by the lines M p^a_Z M^1 9 t^a_W 3 in the diagram in page 6; secondly, a plane passing from 9 t^a_W 3 to M^2 p^a_N M^3; thirdly, a plane passing from M^2 p^a_N M^3 to 6 t^a_e 12; and fourthly, a plane passing from 6 t^a_e 12 to M p^a_Z M^1. Each of these planes cuts the two axes p^a and t^a in the same manner. They are all INCLINED. They are parallel to the axis m^a, they are parallel two and two among themselves, and they are all equivalent to one another. They constitute together a HORIZONTAL PRISM whose edges bisect the two axes p^a and t^a, and whose axis coincides with the axis m^a. The polaric position of the four planes is Ze, Zw, Ne, Nw. The symbol for the complement of four planes is PT. The symbol for half the complement is $\frac{1}{2}$ PT. The planes are denoted individually by the symbols PT Ze, PT Zw, PT Ne, PT Nw.

The cross section of the horizontal prism PT takes place upon the plane p^a_Z t^a_e p^a_N t^a_W. It consequently coincides with the east meridian. The form of it depends upon the relative lengths of the two axes p^a and t^a.

a, If both axes are equal, the east meridian is a square, and the symbol for the planes is PT.

b, If the axis p^a is longer than the axis t^a, the east meridian is a rhombus with its longer diagonal parallel to the axis p^a. The symbol for the planes is then P_+T.

c, If the axis t^a is longer than the axis p^a, the east meridian is a rhombus with its longer diagonal parallel to the axis t^a. The symbol for the planes is in that case PT_+.

d, The measurement of the two axes of the east meridian is effected, and its general properties are ascertained, by the methods described in §§ 50—61.

EXAMPLES OF THE PLANES PT, PT_+, P_+T.

102. The varieties of the form PT differ in no other respect from the varieties of the form PM, than that they are situated at right angles to PM, and have the same relation to the axes p^a and t^a that the form PM has to the axes p^a and m^a. Hence the same models afford most of the requisite examples.

103. Model 63. *Rhombic Dodecahedron.* MT.PM,PT.

The four planes PT are situated Ze, Zw, Ne, Nw. They touch one another at the acute points of the rhombuses, and form a zone round the crystal, which gives for the east meridian a square with its angles at Z, e,w,N. See §§ 21 B, 31, 89.

104. Model 12. *Obtuse Quadratic Octahedron.* PM_+,PT_+.

This form has been already explained at § 90. The exact symbol of the set of planes PT_+ is $PT_{1.118}$, and the model is $= PM_{1.118},PT_{1.118}$.

105. Model 13. *Acute Quadratic Octahedron.* P_+M,P_+T.

This form was explained at § 91. The exact symbol for the set of planes P_+T is $P_{2.55}M$, and the model is $= P_{2.55}M,P_{2.55}T$.

106. Models 32, 33, 34. *Combinations of the Cube, Rhombic Dodecahedron, and Octahedron.*

The planes PT differ in no respect from the planes PM, only that they are situated at right angles to the latter. See § 33. They form angles of 135° with the planes P and T, and, of course, angles of 90° with the planes M, as do all the planes that cut both p^a and t^a and not m^a. The symbols for the three combinations are given in § 69.

107. Model 14. *Combination of a Quadratic Octahedron with an Acute Quadratic Octahedron, the latter prevailing.* pm, $P\frac{5}{2}M$, pt, $P\frac{5}{2}T$.

The planes pt have the positions Z^2e, Z^2w, N^2e, N^2w.
The planes $P\frac{5}{2}T$ have the positions Ze^2, Zw^2, Ne^2, Nw^2.

The letter T at the pole t^a_w of the model marks the plane $P\frac{5}{2}T$ Zw^2. All these planes are parallel to the axis m^a.

The model has in other respects been so fully explained in §§ 93, 94, that it is only necessary to add that the east meridian of the present combination has the same properties as the north meridian of the combination pm, $P\frac{5}{2}M$, which has been described in § 94.

108. Model 91. *Pentagonal Dodecahedron.* $MT_2.PM_2,P_2T$.

The sets of planes MT_2 and PM_2 have been already fully described in §§ 77, 95. I have therefore to confine this notice to the planes P_2T, which are the four planes that meet in two pairs so as to form horizontal edges across the poles t^{a}_{e} and t^{a}_{w}. The angle formed by the plane P_2T Zw upon the plane P_2T Nw, measured by applying the goniometer across the edge at t^{a}_{w}, is 126° 52′, which, as already explained in the §§ referred to above, gives the relation of 2 to 1 for the axes p^{a} and t^{a}, and authorizes the symbol P_2T.

109. Hence we see that Model 91 represents a form produced by the intersection of three similar and equal rhombic prisms, each of them having one of its cross axes longer than the other, and each having its infinite axis coincident with one of the axes of the crystal or combination, so that the three prisms cross one another in the centre at right angles. There are several varieties of this form to be found in the mineral kingdom, the most important of which have the following proportional axes :—

1 : $\frac{4}{3}$, which gives the symbol $MT\frac{4}{3}.PM\frac{4}{3},P\frac{4}{3}T$.
1 : $\frac{3}{2}$, which gives the symbol $MT\frac{3}{2}.PM\frac{3}{2},P\frac{3}{2}T$.
1 : 2, which gives the symbol $MT_2.PM_2,P_2T$.

These are all easily discriminated by examining the angles formed by the inclination of their planes one upon another, as shown in the following table, or by their inclination upon the planes of P,M,T, or MT.PM,PT, or PMT, or of any form with which they may occur in combination; as will be hereafter explained circumstantially:

Combinations.	*Inclination of the plane* MT_+nw *upon :* MT_+ne.	PM_+ Zn.	P_+T Zw.
$MT\frac{4}{3}.PM\frac{4}{3},P\frac{4}{3}T$............	106° 16′	118° 41′	118° 41′
$MT\frac{3}{2}.PM\frac{3}{2},P\frac{3}{2}T$............	112° 37′	117° 29′	117° 29′
$MT_2.PM_2,P_2T$............	126° 52′	113° 35′	113° 35′

110. Model 47. *Combination of the Cube and the Pentagonal Dodecahedron, the latter predominating.* p,m,t, $MT_2.PM_2,P_2T$.

The planes of the form P_2T, or TP_2, are essentially the same as the planes of the forms MT_2 and PM_2, already fully described in §§ 83, 96. The 4 planes of P_2T are those situated between the planes marked P and T on the model. The east meridian of the form p,t,P_2T is an octagon, exactly similar in form and angles to the octangular north meridian of the planes p,m,PM_2, and to the octangular equator of the planes m,t,MT_2. The method of determining the value of the angles that are formed by the inclination of the plane p or t upon P_2T, has been described in § 83.

111. Model 68. *Tetrakishexahedron,* $MT_2,M_2T.PM_2,P_2M,PT_2,P_2T$.

This model has twenty-four planes, of which I have already described sixteen in §§ 78 and 97, where I have also explained the principles upon which we proceed in determining what are the properties of the combination that is exhibited by the model. It seems to be necessary only

to add, that the rest of the 24 planes, namely, the sets PT_2, P_2T, are those which form the bounds of the east meridian of the model and occupy the following positions:

$$PT_2 = Z^2e, Z^2w, N^2e, N^2w.$$
$$P_2T = Ze^2, Zw^2, Ne^2, Nw^2.$$

112. We perceive from the account that has been given of Model 68, that it represents a combination of six similar and equal rhombic prisms, each of them having one of its cross axes longer than the other, and consisting of three pairs of prisms, each pair cutting two axes unequally and inversely. This combination is therefore a sort of double of that represented by Model 91. See § 109. The mineral kingdom presents several varieties of the tetrakishexahedron, particularly the following:—

Axes.		Resulting Planes.
$1 : \frac{3}{2}$	=	$MT_{\frac{3}{2}}, M_{\frac{3}{2}}T . PM_{\frac{3}{2}}, P_{\frac{3}{2}}M, PT_{\frac{3}{2}}, P_{\frac{3}{2}}T$.
$1 : 2$	=	$MT_2, M_2T . PM_2, P_2M, PT_2, P_2T$.
$1 : \frac{5}{2}$	=	$MT_{\frac{5}{2}}, M_{\frac{5}{2}}T . PM_{\frac{5}{2}}, P_{\frac{5}{2}}M, PT_{\frac{5}{2}}, P_{\frac{5}{2}}T$.
$1 : 3$	=	$MT_3, M_3T . PM_3, P_3M, PT_3, P_3T$.
$1 : 5$	=	$MT_5, M_5T . PM_5, P_5M, PT_5, P_5T$.

113. These combinations may be all discriminated by the difference in the angles at which their planes incline upon one another, as shown in the annexed table, or upon the planes of any other forms with which it is possible for them to occur in combination. And we have in our knowledge of the geometrical properties of the equator, and the east and north meridians of the different varieties, an easy and efficient power of control over the accuracy of the mechanical measurements which may seem to distinguish any one of these combinations from the others. Suppose a form of this kind to be given and the angle at the pole n, formed by the incidence of the plane $MT_+ n^2w$ upon the plane $MT_+ n^2e$, to be stated to be $= 136°$. We test the accuracy of this measurement as follows:—The half of 136° is 68°, the tangent of which is 2.4750869, or, when doubled, 4.9501738. This is very nearly the same as the angle of the form $\frac{5}{2}$, but the angle formed by the form $\frac{5}{2}$ is not exactly 136° but 136° 24′, for the half of the last named angle $= 68°\ 12′$ has a tangent $=$ 2.5001784. Hence we conclude that the given form is in reality that known by the term $\frac{5}{2}$, and that the given angle of 136° was erroneous.

Combinations.	*Inclination of the plane* $MT_+ n^2w$ *upon:* $MT_+ n^2e$.	$M_+T\ nw^2$.	$P_+M\ Zn^2$.
$MT_{\frac{3}{2}}, M_{\frac{3}{2}}T . PM_{\frac{3}{2}}, P_{\frac{3}{2}}M, PT_{\frac{3}{2}}, P_{\frac{3}{2}}T$,	112° 37′	157° 23′	133° 49′.
$MT_2, M_2T . PM_2, P_2M, PT_2, P_2T$,	126° 52′	143° 8′	143° 8′.
$MT_{\frac{5}{2}}, M_{\frac{5}{2}}T . PM_{\frac{5}{2}}, P_{\frac{5}{2}}M, PT_{\frac{5}{2}}, P_{\frac{5}{2}}T$,	136° 24′	133° 36′	149° 33′.
$MT_3, M_3T . PM_3, P_3M, PT_3, P_3T$,	143° 8′	126° 52′	154° 9′.
$MT_5, M_5T . PM_5, P_5M, PT_5, P_5T$,	157° 23′	112° 37′	164° 3′.

114. Model 89. *Acute Rhombohedron.* $MT_+ . \frac{1}{2}PT_+ Zw, Ne$.

It will in this case, as in that described in § 99, be proper to consider as unity the axis that is common to both forms. At present, this axis is t^2.

The angle formed by the incidence of the plane MT_+ nw upon the plane MT_+ sw is 78° 28′, the half of which is 39° 14′, and the tangent of 39° 14′ is 0.8165493 = axis m^s when t^s is = 1. This gives the symbol $M_{0.817}T$ for the four vertical planes of the combination represented by the model.

The angle formed by the incidenee of the plane PT_+ Zw upon the w vertical edge is about 110°. I deduct 90° for the right angle, and have a remainder of 20°, the tangent of which, = p^s, is 0.3639702. This gives the symbol $P_{0.364}T$.

Hence the formula for Model 89 is $M_{0.817}T.\ \frac{1}{2}P_{0.364}T$ Zw,Ne.

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115. *Of Rhombohedrons in General.*—The method of describing rhombohedrons given in §§ 99, 100, 114, affords a constant mark of distinction between the two kinds that are commonly called *obtuse* and *acute* rhombohedrons. The general formula for all obtuse rhombohedrons is $MT_+.\ \frac{1}{2}P_-M$, Zn,Ns. The general formula for all acute rhombohedrons is $M_-T.\ \frac{1}{2}P_-T$ Zw,Ne. Both kinds are right rhombic prisms, having the planes MT_+, but the obtuse rhombohedrons are terminated by the incomplete complement of planes $\frac{1}{2}P_-M$ and the acute rhombohedrons by the incomplete complement of planes $\frac{1}{2}P_-T$.

116. It is however usual with crystallographers to hold rhombohedrons in such a position as to give formulæ very different from the above. Thus the obtuse rhombohedron, Model 85, is held with the two obtuse solid angles at the poles Z and N, and the planes turned towards Zn, Zse, Zsw, Ns, Nne, Nnw. This gives the formula

$$\tfrac{1}{2}PM_+ \text{ Zn,Ns, } \tfrac{1}{2}P_-M_+{}^{!}T \text{ Zse,Zsw,Nne,Nnw.}$$

On the other hand, the acute rhombohedron, Model 89, is held with its two acute solid angles at the poles Z and N, but with its six planes also in the positions Zn,Zse,Zsw,Ns,Nne,Nnw. This produces the formula

$$\tfrac{1}{2}P_+M \text{ Zn,Ns, } \tfrac{1}{2}P_+M_-T \text{ Zse,Zsw,Nne,Nnw.}$$

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117. *Of Oblique Rhombic Prisms.*—The two models numbered 84 and 87 exhibit examples of the forms commonly called oblique rhombic prisms. They have a certain resemblance to the rhombohedrons, but the resemblance does not hold in all points. If you suppose all the oblique rhombic prisms and the rhombohedrons to have three axes crossing one another in the centre, but situated in directions parallel to the planes of each form, then, in the rhombohedrons the three axes will be alike, but in the oblique rhombic prisms there will be always one axis shorter or longer than the other two. The rhombohedron is the point of unity between a short oblique rhombic prism and a long one, just as the cube is the point of unity between a short quadratic prism and a long one. See Models 1, 2, 3. The consequence of the inequality of the length of the axes of oblique rhombic prisms, is, that they have always two rhombic planes and four rhomboidal planes, whereas the whole six

planes of every rhombohedron are rhombuses. It is however the analogy betwixt the oblique rhombic prisms and the rhombohedrons, as compared with the analogy between quadratic prisms and the cube, that has induced me to reject the commonly received method of holding the rhombohedrons with two angles on the axis p^a and to consider them to be examples of oblique rhombic prisms.

118. In making use of the term *oblique rhombic prism*, I employ the commonly received language of crystallography; but in subjecting the solids known by that term to symbolic description according to the principles laid down in this work, I do not consider them to be OBLIQUE prisms. The definition that I have given of the word *axes*, in § 2, namely, that they are "three imaginary lines which pass through the centre of a crystal, cross one another there *at right angles*, and terminate at its surface," does not permit of the assumption of any other than *right* or *vertical* prisms; nor does it appear to me to be either necessary or advantageous to consider any of these prisms to be oblique, since the fact is that the *prisms* are really straight, and the *terminations* alone are inclined. All the *prismatic* planes of such forms can be easily designated by symbols derived from the general formula M,T,MT,MT_x, (§ 86); and all the *inclined, pyramidal*, or *terminal* planes, whether the terminations of each combination be monofacial or multifacial, can be equally well denoted by some term of the series PM_x, PT_x, $P_xM_xT_x$, as will be satisfactorily proved in a subsequent section. § 198.

119. Of the rhombic or rhombo-rectangular prisms that are terminated by single oblique planes, there are the following three varieties:

1. Those terminated by the planes $\frac{1}{2}PM_x$; as Model 84.
2. Those terminated by the planes $\frac{1}{2}PT_x$; as Model 87.
3. Those terminated by the planes $\frac{1}{4}$PMT; as Model 108.

120. Model 84. *Oblique Rhombic Prism.* MT_{+}. $\frac{1}{2}$P_M, Zn,Ns.
The form is to be held so as to place the four short edges situated between the rhomboidal planes, in a vertical position.

The angle formed by the incidence of the plane MT_{+}ne upon the plane MT_{+}nw, at the n pole, is 124° 34′. The half of this is 62° 17, the tangent of which is 1.9033738. This gives the formula $MT_{1.9}$.

The incidence of the planes P_M upon the n vertical edge is 104° 57′; or rather, this ought to be the measure, for some of the models are incorrect and measure nearly 10° more than this. 104° 57′ — 90° = 14° 57′. The tangent of this is 0.2670141, which gives the formula $P_{0.267}M$.

The lengths of the axes of the model are about p^a_3 $m^a_{5\frac{1}{2}}$ t^a_{11}.

The symbol for Model 84 is therefore $MT_{1.9}$. $\frac{1}{2}P_{0.267}M$ Zn,Ns, p^a_3 $m^a_{5\frac{1}{2}}$ t^a_{11}.

121. Model 87. *Oblique Rhombic Prism*, M_T. $\frac{1}{2}$P_T Zw,Ne.
As the last form was the counterpart of the obtuse rhombohedrons, so is this the counterpart of the acute rhombohedrons; possessing the same planes in the same polaric positions, and only differing from them in the comparative length of the axis p^a.

The angle formed by the incidence of the plane MT_{+} nw upon the plane MT_{+} sw is 87° 42′. The tangent of the half of this angle is 0.9606421. Hence the symbol for the prismatic planes, the axis t^{a} being taken for unity, is $M_{0.96}T$.

The angle formed by the incidence of the plane P_T Zw upon the w vertical edge is 106° 6′. Deducting 90° for the right angle, we have 16° 6′ for the value of the half of the acute angle of the set of planes P_T. The tangent of 16° 6′ is 0.2886352. Hence the symbol is $P_{0.289}T$.

The axes of the crystal are about p^{a}_{11} m^{a}_{33} t^{a}_{33}.

The symbol for Model 87 is therefore $M_{0.96}T.\frac{1}{2}P_{0.289}T$ Zw,Ne. p^{a}_{1} m^{a}_{33} t^{a}_{33}.

RETROSPECT.

122. We have now examined all the varieties of planes that cut either one axis or two axes, and we may here very properly consider what degree of power these planes possess of producing complete crystals by their combinations with one another.

The complements P,M,PM_{x}, produce a zone of planes round the north meridian, or an infinite prism upon the axis t^{a}, but they produce no complete form. The complements P,T,PT_{x} produce a zone of planes round the east meridian, or an infinite prism upon the axis m^{a}, but they produce no complete form. The complements M,T,MT_{x} produce a zone of planes round the equator, or an infinite prism upon the axis p^{a}, but they produce no complete form. And no alteration of the relative lengths of the three axes can make any one of these zones of planes produce closed or complete crystals.

The order in which the planes that belong to the zone P,M,PM_{x}, dispose themselves upon a complex combination, may be exemplified by reference to the figure at page 174 of PHILLIPS's *Mineralogy*, which I have already referred to in § 73. I shall take only the Zn quarter of the north meridian as an example. The arrangement of the planes is similar in the other three quarters of the crystal.

P	Z
PM^{+}_{++}	Z^4n
PM^{+}_{+}	Z^3n
PM_{+}	Z^2n
PM	Zn
$P_{+}M$	Zn^2
$P^{+}_{+}M$	Zn^3
$P^{+}_{++}M$	Zn^4
M	n

The order in which the planes that belong to the zone P,T,PT_{x}, dispose themselves upon a complex combination, may be exemplified by reference to the figure at page 174 of PHILLIPS's *Mineralogy*, which I have already referred to in § 73. I shall take only the Zw quarter of the east meridian as an example. The arrangement of the planes is similar in the other three quarters of the crystal.

P	Z
PT^{+}_{++}	Z^4w
PT^{+}_{+}	Z^3w
PT_{+}	Z^2w
PT	Zw
$P_{+}T$	Zw^2
$P^{+}_{+}T$	Zw^3
$P^{+}_{++}T$	Zw^4
T	w

The order in which the planes that belong to the zone M,T,$MT_{\prime\prime}$ arrange themselves upon such a combination, has been explained in § 73. Those who may not have an opportunity of referring to PHILLIPS's *Mineralogy*, may compare this tabular arrangement with the marked planes upon Model 32. The planes P,PM,M, and P,PT,T, have the same positions upon the figure and the model. The planes PM^{+}_{++}, PM^{+}_{+}, PM_{+}, lie upon the combination parallel with the plane PM, and between PM and P, the plane PM^{+}_{++} being nearest of them all to the plane P. And the rest are arranged correspondingly.

123. It is evident that no single zone, however numerous its planes may be, can ever make a complete crystal. On the other hand, many complete forms or crystals are produced by such complements of these planes as are so situated on the different axes as to *cross* or *cut one another*. It is of no consequence under what angle of inclination this takes place—the act of crossing is the main point; and any two endless prisms, of any dimensions, that cross one another at any angle, may produce a closed form or complete crystal. The only necessary condition to produce this end is that the two principal axes of the combining prisms cross one another at the same level.

Each of the following combinations of complements produces a closed form or crystal:

1,	P,M,T.	6,	PM,MT.
2,	P,MT.	7,	PM, ½MT.
3,	P,M, ½MT.	8,	PT,M.
4,	P,T, ½MT.	9,	PT,MT.
5,	PM,T.	10,	PT, ½MT.

And a great variety of different crystals may be produced by the combination of these forms with one another, or with other sets of planes.

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124. *Of the Planes* PMT.—There are eight planes denominated PMT. One of these is shown in the diagram in § 20 by the triangle p^a_z m^a_n t^a_w, which cuts the three axes p^a m^a t^a and is parallel to the inverse triangle MT'3. This latter triangle is produced by the ideal removal of the corner or solid angle marked 2 on the parallelopepidon depicted in the diagram. Now there are eight similar solid angles on this form, from which it follows that there can be placed upon the axes p^a m^a t^a precisely eight triangular planes similar to the one marked p^a_z m^a_n t^a_w. Namely,

	Poles cut by the Planes.	*Planes produced.*
Above the Equator.	p^a_z m^a_n t^a_w	PMT Znw.
	p^a_z m^a_s t^a_w	PMT Zsw.
	p^a_z m^a_s t^a_e	PMT Zse.
	p^a_z m^a_n t^a_e	PMT Zne.
Below the Equator.	p^a_N m^a_n t^a_w	PMT Nnw.
	p^a_N m^a_s t^a_w	PMT Nsw.
	p^a_N m^a_s t^a_e	PMT Nse.
	p^a_N m^a_n t^a_e	PMT Nne.

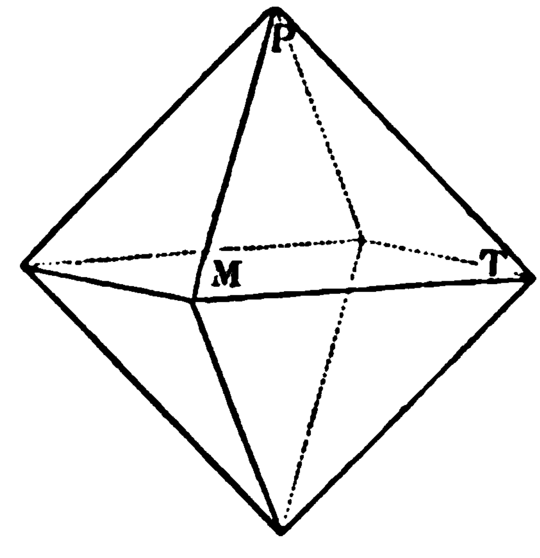
The polaric positions of the individual planes are denoted by the signs of the three poles which each plane cuts. The eight planes are all INCLINED. They are equivalent to one another. They all cut the three axes p^a m^a t^a and they are parallel to no axis. They consist of four pair of parallel planes so connected as to constitute TWO HORIZONTAL RHOMBIC PRISMS which cross one another *and produce a* COMPLETE FORM, or CRYSTAL. This is a property not possessed by any of the sets of planes described previously—for as the planes of every other set cut at most only *two* axes, and have always an axis of their own which coincides either with p^a m^a or t^a, every prism which these sets form runs to infinity upon the coinciding axis, and only produces a complete (closed) form when it is met and *crossed* by a prism that runs in a different direction. Thus, the complement M,T, or the complement MT, runs to infinity upon the axis p^a, producing the four vertical sides of a prism, but never forming a complete figure until it is crossed by the complement P, PM, or PT, or by something equivalent. See § 122.

125. The symbol for this complement of eight planes is PMT. The symbol for half the complement is $\frac{1}{2}$PMT. The symbol for the fourth part of the complement is $\frac{1}{4}$PMT. All the planes denoted by these symbols bisect the three axes p^a m^a t^a. They comprise four pair of parallel planes, but the planes which constitute the half and quarter complements very frequently do not occur in parallel pairs. It is easy however to indicate the particular planes which may be present in any fractional part of the complement PMT, by adding to the symbol PMT the polaric positions of those particular planes.

126. The two horizontal prisms which concur to produce the complement $P_xM_xT_x$ sometimes cross one another *at right angles*, and sometimes *obliquely*.

THE REGULAR OCTAHEDRON, PMT.

127. When the prisms cross one another at right angles, they cut the horizontal axes m^a and t^a equally and at an angle of 45°. If this occurs with an equiaxed crystal, or, to speak more precisely, with prisms of such dimensions as can produce an equiaxed crystal, the resulting form is the geometrical regular octahedron, which is figured in the margin, and represented by Model 15. This form has the following properties:—The two rhombic prisms by whose intersection it is produced, have angles of 70° 32′ and 109° 28′, and consequently have diagonals that are nearly equal to the numbers 10 and 7, for the tangent of 35° 16′, the half of 70° 32′, is 0.7071664. The longer of these diagonals coincides with the axis p^a of the resulting crystal. The incidence of any two planes measured over the pole Z is 70° 32′, and measured across the equator is 109° 28′. The axes m^a and t^a being cut at equal distances from the centre, the



form of the equator is a square, with the angles at n, e, s, w. The north meridian is a square, with the angles at Z, n, N, s; the east meridian a square with the angles at Z, w, N, e; while the north-east and north-west meridians, which cut through the planes of the crystal, are necessarily rhombuses of the same dimensions as the prisms by which the octahedron is produced. The shape of the planes is that of an equilateral triangle, and the angle of their incidence upon one another, measured across any edge, is 109° 28′. These characters are sufficient to distinguish the *regular octahedron* from all other varieties of the octahedron; and this is the form which is intended to be represented by the symbol PMT when written without addition.

Isosceles Octahedrons, P_xMT, PM_xT, PMT_x.

128. When the two rhombic prisms which produce an *octahedron* by crossing one another *at right angles*, have other dimensions than those which produce the *regular* octahedron, then the octahedron which is produced must necessarily have different dimensions from the dimensions of the regular octahedron. Only two different variations can however occur. The rhombic prisms may be either more obtuse or more acute at the angles which come upon the axis p^s than the prisms which produce the regular octahedron. In the former case the resulting octahedron will have the perpendicular axis *shorter* than the horizontal axes. In the latter case, the resulting octahedron will have the perpendicular axis *longer* than the horizontal axes. One of these forms would require the symbol P_-MT, the other, the symbol P_+MT.

129. But this explanation of the possible variations that can take place in this operation, rests upon the supposition that, in the three cases particularised, the rhombic prisms always cross one another at the level of the horizontal axes m^s and t^s, and this supposition is necessary so long as we confine our attention to the consideration of the production of octahedrons that have a *square equator*.

130. When however we extend our observation to octahedrons which can be produced by the crossing of two rhombic prisms, still at right angles to one another, and still in such a manner as to cut two axes at angles of 45°, but at other levels than that of the equator, or in different positions as regards the longer diagonals of the cross sections of the prisms, we arrive at the following results.

Obtuse Isosceles Octahedrons, P_-MT, PM_-T, PMT_-.

P_-MT is referred to in § 128.

131. Let a rhombic prism cut the system of three axes in the direction of a line passing from Zw to Ne. Let a similar and equal rhombic prism cut the three axes in the direction of a line passing from Ze to Nw. In both cases the shorter diagonal of the rhombic prism is to be situated parallel to the axis m^s. The two prisms will cross one another at right angles, and they will cut the axes p^s and t^s equally at an angle of 45°. The resulting combination will require the symbol PM_-T.

132. Let a rhombic prism cut the system of three axes in the direction of a line passing from Zn to Ns. Let a similar and equal rhombic prism cut the three axes in the direction of a line passing from Zs to Nn. Let the shorter diagonal of the rhombic prism be, in both cases, parallel to the axis t^s. The two prisms will cross one another at right angles, and and they will cut the axes p^s and m^s equally at an angle of 45°. The resulting combination will require the symbol PMT_.

EXAMPLES OF P_MT, PM_T, AND PMT_.

133. Hold Model 12 in such a position that its two obtuse solid angles shall be at Z and N, and its four acute solid angles at n, e, s, w. The equator will then be a square, and the north and east meridians will both be rhombuses with their acute angles on the four poles of the equator. This is the Octahedron P_MT described in § 128.

134. Hold the same model in such a position that its two obtuse solid angles shall be at n and s, and its four acute solid angles at Z,e,N,w. The equator will then be a rhombus with its acute angles at e and w. The north meridian will be a rhombus with its acute angles at Z and N. The east meridian will be a square, with its angles at Z,e,N,w. This is the octahedron PM_T described in § 131.

135. Hold the same Model in such a position that its two obtuse solid angles shall be at e and w, and its four acute solid angles at Z,n,N,s. The equator will then be a rhombus with its acute angles at n and s. The north meridian will be a square with the angles at Z,n,N,s. The east meridian will be a rhombus with its acute angles at Z and N. This is the octahedron PMT_, described in § 132.

136. I have shown in § 67, and in other places, that when the planes of a form that has a short axis and a long one, enter into combination with other planes, it is generally those parts of the planes which, in the uncombined form, project to the poles of the longer axis, that cannot be seen on the combined form; the most projecting portions of the unequiaxed form being replaced or cut off by the planes of combination which replace them.

137. If, with this consideration in mind, we examine the forms indicated by the symbols P_MT, PM_T, and PMT_, we shall perceive that when these forms enter into combination with other forms whose axes are shorter than the axis that is considered unity in the form P_MT, it is necessarily those portions of the planes which extend so as to form four acute solid angles, and yield the square section, that must be cut off by the planes of replacement, because these are the portions of the planes that are farthest removed from the centre of the crystal. It follows also of necessity, that when these forms have been cut in this manner by combining planes, the portions which remain upon the combinations must be the following:—

138. Of P_MT, there will remain four planes surrounding the pole Z,

and having the positions Z^2nw, Z^2ne, Z^2sw, Z^2se, and four planes surrounding the pole N, and having the positions N^2nw, N^2ne, N^2sw, N^2se.

139. Of PM_T, there will remain four planes surrounding the pole n, and having the positions Zn^2w, Zn^2e, Nn^2w, Nn^2e, and four planes surrounding the pole s, and having the positions Zs^2e, Zs^2w, Ns^2e, Ns^2w.

140. Of PMT_, there will remain four planes surrounding the pole e, and having the positions Zne^2, Zse^2, Nne^2, Nse^2, and four planes surrounding the pole w, and having the positions Znw^2, Zsw^2, Nnw^2, Nsw^2.

141. It is of no consequence by what means—by what description of planes—the parts of planes that disappear in such a case are removed. The positions of the *remaining* portions of the planes is in no respect altered by any accidental circumstance that may have attended the replacement of the absent portions, neither are their positions at all changed by the nature, number, or positions of the superinduced planes—the planes of replacement or combination. The portions of the planes of an unequiaxed form which touch the shorter of the two unequal axes of the form, are never replaced, never driven from their polaric positions, or altered in any respect, except that of being diminished in size, by the abstraction of those portions of the planes that extended towards the longer of the two axes of the form, excepting when the unequiaxed form is cut by another form whose three axes are all shorter than the shortest of the axes of the unequiaxed form. This happens, for example, when the form P$\frac{1}{2}$MT is cut by the form P$\frac{1}{4}$,M,T, or by P$\frac{1}{4}$,M$\frac{1}{4}$,T$\frac{1}{4}$, or by any form whose planes fall nearer to the centre of the combination than do any of the poles of the forms P_MT, PM_T, PMT_. From the above positions we may safely draw the inference, that if the three forms described as P_MT, PM_T, and PMT_, *were to cut one another*, and come together upon *one crystal*, that crystal must exhibit planes in all the positions described in §§ 138, 139, 140, and be a combination of no less than 24 equal and similar planes. It is indeed easy to perceive that the result cannot be otherwise. The uncombined planes of P_MT have four acute solid angles at the poles m^a_n, m^a_s, t^a_e, t^a_w. Let this form be taken as = $P_1M_2T_2$. If it combines with the form $P_2M_1T_2$, it is evident that the second form will cut off all those parts of the planes $P_1M_2T_2$ which extend along the axis m^a beyond the distance m^a_1 measured from the centre towards m^a_n and m^a_s, while simultaneously the form $P_1M_2T_2$, will cut off all those parts of the planes of $P_2M_1T_2$ which extend along the axis p^a beyond the distance p^a_1 measured from the centre towards p^a_z and p^a_N. If the combination be then cut by the third form $P_2M_2T_1$, the latter will cut off all those parts of both the preceding forms which extend along the axis t^a beyond the distance t^a_1 measured from the centre towards t^a_e and t^a_w, while it will itself suffer a deprivation of all those parts of its planes that extend along the axes p^a and m^a beyond the points p^a_1 and m^a_1. By this threefold operation, the six poles of the combination are fixed at the points p^a_z1, p^a_N1, m^a_n1, m^a1, t^a_e1, t^a_w1, and thus an equiaxed crystal is produced by the combination of three equal and similar unequiaxed forms.

142. Model 22. *Icositessarahedron.* P_MT, PM_T, PMT_.

This model represents the form alludud to in the last paragraph. It is in fact the result of the combination of the three sets of octahedral planes P_MT, PM_T, PMT_. The four planes round the pole p^{a}_{Z} and the four round the pole p^{a}_{N}, are those described in § 138. The four planes round the pole n, and the four round the pole s, are those described in § 139. The four planes round thê pole e, and the four round the pole w, are those described in § 140. Altogether there are 24 planes, in 6 sets of 4 each, if considered in relation to the planes of the cube, or in 8 sets of 3 each, if considered in relation to the planes of the octahedron. The form of the planes is that of a symmetrical trapezium. The positions of the planes have been already described.

143. Several varieties of this form have been found among minerals, the two principal of which have the following symbols:—

$$P\tfrac{1}{2}MT,\ PM\tfrac{1}{2}T,\ PMT\tfrac{1}{2},$$
$$P\tfrac{1}{3}MT,\ PM\tfrac{1}{3}T,\ PMT\tfrac{1}{3}.$$

The first of these forms is represented by Model 22. The other has nearly the same shape. Its planes are the same in number and have the same positions, and consequently its edges and solid angles are the same in number. But in general appearance the other form approaches more nearly to that of a cube, whereas Model 22 bears a resemblance to the octahedron. The reason of the difference is sufficiently obvious upon an examination of the relative lengths of the axes of the two forms, for as the three axes of one of the forms have to the three similar axes of the other, the relation of $\frac{1}{3}$ to $\frac{1}{2}$, it follows of the form $\frac{1}{3}$ that its 6 solid angles at Z, N,n,e,s,w, must be more obtuse, and the 8 solid angles at Znw, Zne, Zsw, Zse, Nnw, Nne, Nsw, Nse, must be more acute than those of model 22. Now the 6 angles in both forms represent the positions of the angles of the octahedron, and the 8 angles represent the positions of the angles of the cube; so that one form necessarily approximates to the octahedron and the other to the cube; both forms being consequent upon the difference in the primary dimensions of the rhombic prisms whence they are derived.

The two forms may be discriminated as follows:—

144. $P\frac{1}{2}MT, PM\frac{1}{2}T, PMT\frac{1}{2}$.—The angle formed by the two horizontal edges that meet at m^{a}_{n}, is 126° 52′. The half of this is 63° 26′, whose tangent is 1.999859 or 2.0. Equal to m^{a}_{1} t^{a}_{2}. The angle formed by the two inclined edges that meet at m^{a}_{n} is the same as that formed by the horizontal edges, and has the same tangent, and is therefore equal to m^{a}_{1} p^{a}_{2}. This gives the formula $PM\frac{1}{2}T$ for the eight planes that meet at the poles m^{a}_{n} and m^{a}_{s}. From similar measurements made across the poles p^{a}_{Z} and t^{a}_{w}, we derive the formula $P\frac{1}{2}MT$ for the 8 planes that meet at the poles p^{a}_{Z} and p^{a}_{N}, and the formula $PMT\frac{1}{2}$ for the 8 planes that meet at the poles t^{a}_{e} and t^{a}_{w}.

145. $P\frac{1}{3}MT, PM\frac{1}{3}T, PMT\frac{1}{3}$.—The angle formed by the meeting of

any two edges at any one of the 6 poles of the crystal is 143° 8′, from which is derived the tangent 3.000282, which gives the formula P_1M_3T, or $P\frac{1}{3}MT$, &c. The measurements are alike at all the six poles.

146. *Angles that characterise these two forms:*

$P\frac{1}{2}MT$ Z^2nw upon:			$P\frac{1}{3}MT$ Z^3nw upon:		
$P\frac{1}{2}MT$ Z^2se	=	109° 28′	$P\frac{1}{3}MT$ Z^3se	=	129° 31′
$P\frac{1}{2}MT$ Z^2ne	=	131° 49′	$P\frac{1}{3}MT$ Z^3ne	=	144° 54′
$PM\frac{1}{2}T$ Zn^2w	=	146° 27′	$PM\frac{1}{3}T$ Zn^3w	=	129° 31′
$PMT\frac{1}{2}$ Znw^2	=	146° 27′	$PMT\frac{1}{3}$ Znw^3	=	129° 31′

Acute Isosceles Octahedrons, P_+MT, PM_+T, PMT_+.

147. I now return to the position quitted in § 130, in order to explain a case different from that taken up in § 131, but already alluded to in § 128.

Let the two rhombic prisms which we suppose to cut the system of three axes when in the act of producing the octahedral forms, with the consideration of which we are now occupied, be *acute*, not, as tacitly admitted in § 131, *obtuse* prisms. Let them be, for example, prisms whose cross sections would somewhat resemble the form of the north meridian of Model 13. See § 91. With this understanding, we will now retrace the steps that were taken in § 128 and subsequently.

148. Let the two rhombic prisms be situated in a horizontal position with their acute edges at the points Z and N, or, what comes to the same thing, with their longer diagonals parallel to the axis p^a. Let them cut the axes m^a and t^a equally at an angle of 45°, and cross one another at right angles. The result of this process, as already stated in § 128, will be a combination whose symbol must be P_+MT.

149. Let one of the rhombic prisms cut the system of three axes in the direction of a line passing from Zw to Ne; and the other prism cut the axes in the direction of a line passing from Ze to Nw—both prisms having their longer diagonals situated parallel to the axis m^a, and both cutting the axes p^a and t^a equally at an angle of 45°. The resulting octahedron will be such as to require the symbol PM_+T.

150. Let one of the same prisms cut the three axes in the direction of a line passing from Zn to Ns, and let the other prism cut the axes in the direction of a line passing from Zs to Nn—both prisms having their longer diagonal parallel to the axis t^a, and both cutting the axes p^a and m^a equally at an angle of 45°. The resulting octahedron will be such as to require the symbol PMT_+.

Examples of P_+MT, PM_+T, and PMT_+.

151. Hold Model 13 in such a position that its two acute solid angles shall be at Z and N, and its four obtuse solid angles at n, e, s, w. The

equator will then be a square, and the east and west meridians will both be rhombuses, with their acute angles at Z and N. This is the octahedron $P_{+}MT$ described in § 148.

152. Hold Model 13 in such a position that its two acute solid angles shall be at n and s, and its four obtuse solid angles at Z, e, N, w. The equator will then be a rhombus with its acute angles at n and s. The north meridian will be a rhombus with its acute angles at n and s. The east meridian will be a square with its angles at Z, e, N, w. This is the octahedron $PM_{+}T$ described in § 149.

153. Hold the same model in such a position that its two acute solid angles shall be at e and w, and its four obtuse solid angles at Z, n, N, s. The equator will then be a rhombus with its acute angles at e and w. The north meridian will be a square with its angles at Z, n, N, s. The east meridian will be a rhombus with its acute angles at e and w. This is the octahedron PMT_{+} described in § 150.

154. Let us put Model 13 into the position described in § 151, and denoted by $P_{+}MT$, and let us examine the effects that must be consequent upon the combination of such a form with other planes. For the reasons that I have stated in §§ 67, 136, 141, I form the opinion that, when $P_{+}MT$ occurs in combination with *any form* whose axis p^{s} is less than the sum indicated by the term + in $P_{+}MT$, there must be a replacement of those parts of the planes $P_{+}MT$, that touch the poles p^{s}_{z} and p^{s}_{N}. The shape of the planes substituted for the portions of the planes that are removed, depends upon the particular form by which the replacement is effected. Thus, if the planes of combination are the complement P,MT, there will be only a single horizontal plane substituted for each solid angle removed, as shown by Model 80. If the planes of replacement are the complement $P_{-}M_{+}T_{+}$ (equivalent to Model 12), the two four-sided acute pyramids cut off from the apices of $P_{+}MT$, will be replaced by two four-sided obtuse pyramids, such as are exhibited by Model 14. In all cases of this sort, the planes of $P_{+}MT$ will remain untouched where they meet to form the equator; so that, however the form may be cut away about the poles p^{s}_{z} and p^{s}_{N}, we shall still find about the equator eight planes that have the following polaric positions:—

$$Zn^{2}w^{2},\ Zn^{2}e^{2},\ Zs^{2}e^{2},\ Zs^{2}w^{2}.$$
$$Nn^{2}w^{2},\ Nn^{2}e^{2},\ Ns^{2}e^{2},\ Ns^{2}w^{2}.$$

The multiplication by 2 of the poles around the equator, shows that the residual portions of the planes are equally near to any two poles at that level, but at a greater distance from the poles p^{s}_{z} and p^{s}_{N}, which is the necessary result of the equality of the equatorial axes and the comparative greater length of the vertical axis.

155. Let us next place Model 13 in the position prescribed in § 152, to make it represent the form $PM_{+}T$.—It is evident that in this case, the portions of the planes most liable to be removed from the form in the event of combination are those attached to the poles n and s, and that

the portions most likely to remain upon the combination are the following eight planes attached to the edges of the east meridian:—

$$Z^2ne^2,\ Z^2nw^2,\ Z^2se^2,\ Z^2sw^2.$$
$$N^2ne^2,\ N^2nw^2,\ N^2se^2,\ N^2sw^2.$$

156. We shall now place Model 13 in the position which is represented in § 153, as being peculiar to the form PMT_+.—When this form enters into combination, the portions of its planes that are most liable to be replaced are those that extend to the poles t^{n}_{e} and t^{s}_{w}, and the remnants of planes most likely to be found upon the combination, are those attached to the north meridian, which are the eight following:—

$$Z^2n^2e,\ Z^2n^2w,\ Z^2s^2e,\ Z^2s^2w.$$
$$N^2n^2e,\ N^2n^2w,\ N^2s^2e,\ N^2s^2w.$$

157. Upon comparing these three results we find that the 24 remnants of planes that may be exhibited by the forms P_+MT, PM_+T, and PMT_+, in combination, have all different positions; that every plane has a basis attached to two poles of the combined form; that there are twenty-four of these bases joined two and two together at the base; that the twelve edges between these twenty-four bases have the positions of the twelve edges of the octahedron; and that lines drawn perpendicular to the centres of these twelve bases, will all unite in the positions Zne, Znw, Zse, Zsw, Nne, Nnw, Nse, Nsw, which are coincident with the centres of the planes of the regular octahedron, or with the solid angles of the cube. It follows that if the three forms P_+MT, PM_+T, and PMT_+, were to combine to form one crystal, the combination must have all the twenty-four planes described in §§ 154, 155, 156; that these planes must meet in pairs at each of the four sides of the equator, of the north meridian and of the east meridian; that they must meet again in lines that run from the six poles of the crystal towards the points that coincide with the centres of the eight planes of the octahedron; and that, meeting in this manner, they make a complete form by cutting each other into twenty-four equal and similar obtuse isosceles triangles.

158. Model 17. *Triakisoctahedron.* P_+MT, PM_+T, PMT_+.

This model exhibits the result of the threefold combination described in the last paragraph, embracing all the planes of the forms P_+MT, PM_+T, PMT_+. There are consequently 24 planes arranged in 8 sets of 3 each as compared with the octahedron, or in 12 sets of 2 each as compared with the rhombic dodecahedron, with which this form has some analogy. All the planes are obtuse isosceles triangles, the bases and apices of which meet in the points described in § 157. It is easy to discriminate the planes belonging to each of the three complements. The 8 planes whose bases are attached to the equator, and which have the positions Zn^2w^2, &c., being those that are farthest removed from the axis p^2, are consequently those that have the formula P_+MT. The 8 planes whose bases are attached to the east meridian, and which are therefore

ırthest removed from the axis m^a, are the set $PM_{+}T$. The 8 planes 'hose bases are attached to the north meridian, and which are therefore ırthest removed from the axis t^a, are the set PMT_{+}.

159. It will be useful to compare the forms and positions of the planes ' Model 22 with those of Model 17. We perceive that on the former odel, *the Icositessarahedron*, every plane touches *one* of the six poles ' the crystal, and thence proceeds towards *two* other poles, which it ›es not reach; and that on the latter model, *the triakisoctahedron*, ery plane touches *two* poles equally, and thence stretches towards *one* her pole, which it does not reach. Hence we infer that the axes of the ndamental octahedrons of which these two combinations are constited, are essentially different from one another; and that the axes of e fundamental forms of Model 22 have the relation of –, +, +, and ose of Model 17, the relation of +, –, –. The observation of the ›laric positions of the planes that we find upon complex combinations, rves thus to guide us in forming an opinion of the comparative lengths of ıe axes of the simple forms to which the planes belong. The following wo rules are of special service to us when we are making observations f this nature:—

1. When the polaric position of a plane is such as to prove that it nust cut two axes if extended all ways till it meets the axes, the plane vill be found upon the combination nearest to that one of the two axes vhich it cuts nearest the centre of the crystal, and the more unequal the ength of the two axes, the nearer will the plane be found to the shorter ıxis.

2. When the polaric position of a plane is such as to show that it must ɔut three axes when extended in all directions till it meets the axes, then ;he comparative lengths of the three axes may be determined from the ɔosition which the plane occupies upon the combination. *a*, If it is placed :lose to one axis, and passes thence equally towards two other axes but vithout touching them, the relation of the axes will be p^{a}_{-}, m^a, t^a, or –, +, +. See Model 22.—*b*, If it is placed equally close to two axes, and hence proceeds equally towards a third axis but without touching it, the elation of the axes will be p^{a}_{+} m^a t^a, or +, –, –. See Model 17.—*c*, If the ›lane is placed close to one axis, proceeds thence chiefly towards the econd axis, and only slightly towards the third, thus having an unequal elation to all the three axes, then these axes must necessarily be denoted ›y p^{a}_{+} m^a t^{a}_{-}. See Model 25, which will be more fully described hereıfter. § 177.

160. The varieties of the Triakisoctahedron which have been found n the mineral kingdom, are the three following:—

1. $P_{\frac{3}{2}}MT$, $PM_{\frac{3}{2}}T$, $PMT_{\frac{3}{2}}$.
2. P_2MT, PM_2T, PMT_2.
3. P_3MT, PM_3T, PMT_3.

They are discriminated by attention to the following angles:—

$PMT_{+}\ Z^2n^2w$ upon		$P_{\frac{3}{2}}MT$, &c.	P_2MT, &c.	P_3MT, &c.
$PMT_{+}\ Z^2n^2e$	=	129° 31′	141° 3′	153° 28′
$P_{+}MT\ Zn^2w^2$	=	162° 39½′	152° 44′	142° 8′
$PM_{+}T\ Z^2nw^2$	=	162° 39½′	152° 44′	142° 8′

The combination represented by Model 17 is the second of these three, or that denoted by the symbol P_2MT, PM_2T, PMT_2.

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161. These six varieties of the isosceles octahedron, namely, the three obtuse varieties $P_{-}MT$, $PM_{-}T$, PMT_{-}, and the three acute varieties $P_{+}MT$, $PM_{+}T$, PMT_{+}, are all that can occur under the essential condition of having one short axis and two long axes. All other changes in the axes have relation to the variable value of the quantities represented by the signs − and +, and do not affect the unmarked axes. For this reason, I propose, when all the three permutations of one kind occur upon the same crystal, to abridge the symbols as follows:—

3 $P_{-}MT$ instead of $P_{-}MT, PM_{-}T, PMT_{-}$.
3 $P_{+}MT$ instead of $P_{+}MT, PM_{+}T, PMT_{+}$.

in which examples the term 3 signifies the three regular permutations of the three axes. When only one or two similar forms occur upon a combination, they must be particularized. The six forms never occur as single uncombined crystals, because all the obtuse forms presented in substance would, according to this system of crystallography, be described as $P_{-}M, P_{-}T$, and all the acute forms as $P_{+}M, P_{+}T$, in the idea of their being quadratic (square-based) octahedrons pertaining to the series P_xM, P_xT. The isosceles octahedrons formed by the permutations of P_xMT are to be regarded universally as forms peculiar to complex combinations.

Scalene Octahedrons. $P_xM_yT_z$.

162. I return now to § 126, in which it will be seen that we have still to investigate the nature of the octahedrons that are produced when the combining rhombic prisms cross one another *obliquely*.

163. I shall proceed to consider the nature of the different symmetrical scalene octahedrons that can be produced by the combination of two similar and equal prisms whose cross section is an acute rhombus having angles of 143° 24′ and 36° 36′, and whose diagonals are consequently nearly as the numbers 1 and 3, because the tangent of 71° 44′, the half of 143° 24′, is 3.029632. You will find the angle of 143° 24′ to be nearly that of the incidence of the Zenith planes of Model 21 upon the Nadir planes, the measurement being taken with the goniometer right across the horizontal edge that surrounds the equator of the model.

164. It may be useful, in the first place, to show the nature of the principal *isosceles octahedrons* which can be produced by the combina-

tion of two rhombic prisms of the above-named dimensions. These forms will serve as points of comparison.

a, If one of the prisms passed along the axis t^a, with the longer diagonal of its cross section parallel with the axis p^a, it would produce the planes P_3M. If the other prism passed along the axis m^a, with its longer diagonal parallel with p^a, it would produce the planes P_3T. This completes the acute quadratic octahedron P_3M, P_3T.

b, If one prism passed along t^a with its longer diagonal parallel with m^a, it would produce the planes PM_3. If the other prism passed along m^a with its longer diagonal parallel with t^a, it would produce the planes PT_3. This completes the obtuse quadratic octahedron PM_3, PT_3.

c, If one prism, having its longer diagonal parallel with the axis p^a, and striking the system of axes at the level of the equator, passed midway between m^a and t^a, cutting both axes equally at an angle of 45°, and if this prism was crossed at right angles by the other similar prism, similarly situated, the combination resulting from the operation would be the acute quadratic octahedron $P_3M_{1.414}T_{1.414}$.

d, If the two prisms crossed each other under the circumstances just recited, with the single difference of having the shorter instead of the longer diagonal of the prisms parallel with the axis p^a, the resulting form would be the obtuse quadratic octahedron $P_1M_{4.242}T_{4.242}$.

e, I have here to explain the cause of the apparent variations in the lengths of the axes represented in examples *a*, *b*, *c*, *d*. I shall give the explanation with reference to the lower part of the diagram in page 6.

The lines T^2 T^3 and M^2 M^3 show the course of the prisms as described in cases *a* and *b*, and the square 4 5 11 10 may be considered to be the base of the resulting octahedrons. But the course of the prisms described in cases *c* and *d* is represented by the lines 4 11 and 5 10, and the base of the resulting octahedrons is shown by the square M^2 T^2 M^3 T^3. In the first two cases the axes of the crystal coincide with the axes of the rhombic prisms and are of the same length, but in the last two cases the axes of the crystal are represented by the lines M^2 M^3 and T^2 T^3, while the axes of the prisms are represented by so much of the lines 4 11 and 5 10 as lies between the centre p^a_N and the lines M^2 T^2 M^3 T^3; one set of axes crossing the other set at an angle of 45°. The consequences are, that the axes of the crystal bear to the axes of the prisms, in cases *a* and *b* the ratio of 1 to 1, but in cases *c* and *d*, a ratio equivalent to that which the secant of an arc or angle of 45° bears to the radius of that arc; *i.e.* the ratio of 1.414 to 1.000. Hence the axes of these crystals are equal to the axes of the prisms × 1.414, which number being, in case *c*, multiplied by 1, gives 1.414, and in case *d*, multiplied by 3, gives 4.242.

If it is requisite to consider the axes m^a and t^a of these octahedrons to be unity, the above symbols may be converted into others, as follows:—

$$1.414 : 3.0 :: 1.0 : 2.121$$
$$4.242 : 1.0 :: 1.0 : 0.236$$

These proportions give the symbols $P_{2.121}MT$ for $P_3M_{1.414}T_{1.414}$; and $P_{.236}MT$ for $P_1M_{4.242}T_{4.242}$.

f, By certain processes of derivation which have been already fully explained, we can account for the production of the following isosceles octahedrons, which are equivalent in dimensions to those just described:

$PM_{.236}T$, as described in § 131.
$PMT_{.236}$, as described in § 132.
$PM_{2.121}T$, as described in § 149.
$PMT_{2.121}$, as described in § 150.

g, The relation which the axes of an isosceles octahedron $P_{-}MT$ or $P_{+}MT$, bear to the axes of the rhombic prisms that produce it, being necessarily a constant character, and one that is easily examined, is, in consequence, a character of great utility; for it enables us to determine the ratio of the axes of such octahedrons with very little trouble, either of measurement or calculation, as I shall show by a few examples.

h, The angle of incidence of the planes of the regular octahedron that meet at the equator is 109° 28′. The tangent of the half of this angle is 1.414, which is equal to the axis p^a, the corresponding radius being a line passing from the centre of the crystal towards the north-west, and cutting the equator at the point nw. This line is = 1.0. It forms with either of the axes m^a or t^a an angle of 45°, and on the principle laid down above (letter *e*), either m^a or t^a is equal to the secant of this angle; therefore = 1.414. Hence the three axes of this crystal p^a m^a t^a are each equal to 1.414.

i, It is stated in § 146, that the angle of incidence of the plane $P\frac{1}{2}MT$ Z^2nw upon the plane $P\frac{1}{2}MT$ Z^2se is = 109° 28′. The angle of incidence of two planes of the same form across the equator must therefore be equal to the supplement of this angle, or to 70° 32′. The tangent of the half of this angle is 0.707, which is equal to p^a. On comparing this with *the constant value of the axes* m^a or t^a, *namely*, 1.414, we see at once the relation of 0.707 to 1.414, = $\frac{1}{2}$ to 1, upon which relation the symbol $P\frac{1}{2}MT$ is founded.

j, It is also stated in § 146, that the incidence of two upper planes of the form $P\frac{1}{3}MT$ over the apex of the crystal, is 129° 31′. I again take the supplement of that angle as the measure of the incidence of two planes of the same form across the equator, and find the tangent of its half = 25° 14$\frac{1}{2}$′, to be 0.4714. On comparing this with the constant value of the axes m^a and t^a = 1.414, I see the foundation of the symbol $P\frac{1}{3}MT$, for 1.414 divided by 0.4714, gives 3.

k, You may have to solve a problem that is the converse of those above described. Suppose, for example, that you have the symbol $PMT_{2.121}$, and desire to know the value of the angle across the Zn edge, that is to say, the angle of incidence of the plane $PMT_{2.121}$ Z^2n^2w upon the plane $PMT_{2.121}$ Z^2n^2e. What you have to do in this case is to multiply all the axes of the crystal by 1.414, so as to make p^a and m^a each = 1.414, and to increase t^a in a corresponding ratio. This multiplication produces $P_{1.414}M_{1.414}T_{2.999}$. The angle which is desired lies in the direction t^a_w Zn t^a_e. Zn is the middle of this term, and Zn is situated midway

between p^a_z and m^a_n. A line drawn from Zn to the centre of the crystal, which line I will call the polaric line Zn, is equally inclined to the axes m^a and p^a, and forms an angle of 45° with both of them, because the axes p^a and m^a cross one another at an angle of 90° and Zn divides this angle into two equal angles. The length of the polaric line Zn is limited at one end by the centre of the crystal, and at the other by the straight line or edge that connects p^a_z with m^a_n. Hence the polaric line Zn is the radius of an angle of 45°, and the axes p^a and m^a are both secants of that angle. Consequently, the length of the polaric line Zn is 1.0, because p^a and m^a are each = 1.414. Finally, the polaric line Zn = 1.0, is the radius of an angle whose tangent is the axis t^a, already found to be = 2.999, and the angle that corresponds to this tangent, or to 3.00028, which is nearly the same, is 71° 34′, the double of which = 143° 8′ is the desired value of the angle across the Zn edge.

165. Let the aforesaid two rhombic prisms, § 163, cross one another at the level of the equator with their longer diagonals parallel with the axis p^a. Let the line of their direction be closer to the axis t^a than to the axis m^a, and let them cut both m^a and t^a so as to produce angles of 50° 58′ and 39° 2′ with those axes.

This process will produce a scalene octahedron similar to Model 21, the measurements of which are as follow:—

The equator is a rhombus with angles of 101° 56′ at m^a_n and m^a_s, and of 78° 4′ at t^a_e and t^a_w.—The north meridian is a rhombus with angles of 133° 53′ at m^a_n and m^a_s, and of 46° 7′ at p^a_z and p^a_N. The east meridian is a rhombus with angles of 124° 36′ at t^a_e and t^a_w, and 55° 24′ at p^a_z and p^a_N. —The model agrees pretty closely with these measurements, as will be found upon applying the goniometer to its edges, in the indicated directions.

166. The scalene octahedron is so called in consequence of its planes being scalene triangles. It has 8 planes that occupy the same polaric positions as the 8 planes of the regular octahedron. It has three unequal axes which are always p^a_+ m^a_- t^a. That is to say, it is to be made a rule that every simple scalene octahedron is to be held with the longest axis in the place of p^a, and the shortest axis in the place of m^a. I have called m^a the *minor* axis, with a view to the establishment of this general rule as to the polaric position of any simple scalene octahedron. *In all examples which follow, whenever a scalene octahedron, or a rhombic prism occurs, I shall make the shorter diagonal of the rhombic base of the octahedron, or the shorter diagonal of the rhombic prism, coincide with the axis* m^a. And, for the sake of producing uniformity in the mode of calculating formulæ for the simple scalene octahedrons, I shall always consider the axis t^a as unity, which will give them the general symbol P_+M_-T. The scalene octahedrons of other dimensions than P_+M_-T, which only occur upon combinations, and not as substantive uncombined crystals, are of course not embraced in this regulation.

167. Let us now determine the comparative length of the three axes

of the scalene octahedron Model 21. The obtuse angle of the equator at m^a_n is $= 101° 56'$. The half of this is $50° 58'$, and the tangent of $50° 58'$ is 1.2334, which gives the ratio of $m^a_1 t^a_{1.2334}$; but as I have resolved to make the axis t^a = unity, I take, with that purpose, not the *tangent* of the angle of $50° 58'$, but its *cotangent*, which is 0.8107, and this supplies the symbol $m^a_{0.81} t^a_{1.00}$.

I proceed next to ascertain the relation of the axis p^a to the axes m^a and t^a, and as I propose again to consider t^a = unity, I perceive that the investigation is one that relates to the form of the east meridian of the crystal, because the diagonals of that meridian are the axes p^a and t^a. The obtuse angle of the east meridian at t^a_w is $= 124° 36'$, the half of which is $62° 18'$. In this case I must take the *tangent* of the angle and not the *cotangent*, because I have to estimate the longer of the two axes, and not, as in the last paragraph, the shorter. The tangent of $62° 18'$ is 1.9047, which gives the symbol $p^a_{1.9} t^a_{1.0}$.

The three axes of the crystal have consequently the ratio of $p^a_{1.9} m^a_{0.81} t^a$, and the symbol which designates the complete form is $P_{1.9}M_{0.81}T$, which is a variety of the general symbol P_+M_-T, § 166. Every other variety of the simple scalene octahedron can be expressed by a similar numerical variation of the same general symbol; and the numerical value of the three axes of such an octahedron can be always ascertained in the above manner. The *cotangent* of half the north angle of the equator is equal to the axis m^a; and the *tangent* of half the west angle of the east meridian is equal to the axis p^a: the axis t^a being considered as unity in both cases.

Having thus fully explained the derivation of a scalene octahedron from two right rhombic prisms, by their intersection at a certain degree of obliquity, and having also explained the method that is to be employed to determine the properties of the resulting form, I shall content myself with giving a very brief description of the several other scalene octahedrons that may be derived from the same two rhombic prisms by processes so entirely analogous to the foregoing, that the mere description of the forms is of itself sufficient to point out the mode of derivation.

168. THE SIX VARIETIES OF THE SCALENE OCTAHEDRON.

a. P_-MT_+, or $P_{.81}MT_{1.9}$.—Hold model 21 in such a position that its three axes, which are held to terminate in the solid angles of the Model, become $p^a_- m^a t^a_+$; that is to say, place the two most acute solid angles at t^a_e and t^a_w, the two most obtuse solid angles at p^a_s and p^a_N, and the two intermediate solid angles at m^a_n and m^a_s. The Model then represents the form whose symbol is $P_{.81}MT_{1.9}$. This form differs in no respect from the form that has been designated, $P_{1.9}M_{0.81}T$, except in its position. If it were given to me to be described as a substantive form, I should term it $P_{1.9}M_{.81}T$, making its longest axis equal to p^a and its shortest equal to m^a, agreeably to the rule laid down in § 166. The form $P_{.81}MT_{1.9}$ is one that occurs only in combination with other sets of planes, and this remark applies to all the varieties of octahedral forms that follow. It is

h these scalene octahedrons, as with the isosceles octahedrons described §§ 128—161, they are described *individually* that they may be cor-tly descriminated when they are found in combination with one other, but not with a view to their being considered and described as many substantive forms or complete crystals. The three obtuse sceles octahedrons = P_-MT, PM_-T, PMT_-, § 144, must all, as separ-crystals, be denominated P_-M, P_-T, in which case they would be ee equal and similar obtuse quadratic octahedrons; and the three te isosceles octahedrons P_+MT, PM_+T, PMT_+, § 145, must all, as arate crystals, be denominated P_+M, P_+T, being considered as three al and similar acute quadratic octahedrons. In like manner, the six ieties of the scalene octahedron, which I have to describe here, and ich are all similar and equal to the one already described, must, as arate crystals, in comformity with the general rule, § 166, be denomin-d P_+M_-T, as six similar and equal scalene octahedrons. This ex-nation is given to show the importance of paying attention to the itions of these and of all varieties of the octahedron, since it is the itions, and not the shape, or the number of the planes, which consti-es that difference in the form which requires a difference in symbols convey the idea of it.

b. P_+M_-T, or $P_{1.9}M_{.81}T$.—Hold Model 21 in such a position that its es become $p^a_+m^a_-t^a$. This is the form already fully described in § 165.
c. PM_+T_-, or $PM_{1.9}T_{.81}$.—Hold Model 21 in such a position that its es become $p^am^a_+t^a_-$. It then represents the form PM_+T_-.
d. P_-M_+T, or $P_{.81}M_{1.9}T$.—Hold Model 21 in such a position that its es become $p^a_-m^a_+t^a$. It then represents the form P_-M_+T.
e. PM_-T_+, or $PM_{.81}T_{1.9}$.—Hold Model 21 in such a position that its es become $p^am^a_-t^a_+$. It then represents the form PM_-T_+.
f. P_+MT_-, or $P_{1.9}MT_{.81}$.—Hold Model 21 in such a position that its es become $p^a_+m^at^a_-$. It then represents the form P_+MT_-.

169. These six are all the symmetrical scalene octahedrons that can be oduced by any method of altering the position of the simple form repre-nted by Model 21. We can hold it in positions that will produce rious mixed or incomplete forms, but we cannot hold it so as to exhibit y other variety of symmetrical scalene octahedron. The reason is, that s is the greatest number of *permutations* to which three dissimilar axes n be subjected. But there may be a great many varieties of each of ese six *kinds* of the scalene octahedron, each variety depending for its culiarites, partly upon the original proportions of the rhombic prisms by hose intersection it is produced, and partly upon the degree of the obli-ity of their crossing. All of them, however, are capable of discrimination reference to the distances from the centre of the crystal at which the ree axes $p^am^at^a$ are cut by the planes which constitute each form. The ape of the planes of every scalene octahedron depends upon the distance each corner of the triangular face from the centre of the crystal. hese distances, however, are not different in every individual crystal,

but constant for all the crystals of the minerals of one species; so that the shape of the planes, or the lengths of the axes of a scalene octahedron, is a character which serves to discriminate one mineral species from another, among those that are in common subject to assume the form of a scalene octahedron.

170. *What befalls the planes of the scalene octahedrons when they combine with other planes.*

When the scalene octahedrons combine with other forms they suffer replacement at one or other of their poles according to the ratio which their axes bear to the axes of the forms which combine with them. They are in this matter subject to the same laws as the isosceles octahedrons, and much of what I have written in relation to the latter applies equally to the forms that are now under consideration, more especially what is contained in §§ 136—143, 154—159.

EXAMPLES.

171. Model 80. $p_{+}.\ P_{1.9}M_{.81}T.\ p^{a}_{+}m^{a}_{-}t^{a}.$

This is the scalene octahedron $P_{+}M_{-}T$, § 165, combined with the planes P, the latter subordinate. If, in such a combination, the planes P had a very short axis, the combination would have the appearance of a rhombic table with its terminal edges replaced. The symbol would then have to be $P_{-}.\ p_{1.9}m_{.81}t.\ p^{a}_{-}m^{a}t^{a}_{+}.$

172. Model 70. $\text{M}_{-}.\ P_{1.9}M_{.81}T.\ p^{a}_{+}m^{a}_{-}t^{a}.$

This is the scalene octahedron $P_{1.9}M_{.81}T$, combined with the planes M, the latter subordinate, but still not very small, and therefore to be represented by a small capital letter. If, in such a combination, the planes M were very large, or, what comes to the same thing, if the axis m^{a} was very short, the combination would be a vertical rhombic table having its edges replaced, and its symbol would be $M_{-}.\ p_{1.9}m_{.81}t.\ p^{a}_{+}m^{a}_{-}t^{a}.$

173. Model 66. $M_{.81}T.P_{1.9}M_{.81}T.\ p^{a}_{+}m^{a}_{-}t^{a}.$

This is the scalene octahedron $P_{1.9}M_{.81}T$ combined with the rhombic prism $M_{.81}T$, neither form predominating. If the vertical planes had been much smaller in proportion to the inclined planes, the symbol must have been $m_{.81}t.\ P_{1.9}M_{.81}T.\ p^{a}_{+}m^{a}_{-}t^{a}.$ If they had been much larger in proportion to the others, the symbol would have been $M_{.81}T.\ p_{1.9}m_{.81}t.\ p^{a}_{+}m^{a}_{-}t^{a}.$

174. Model 120. $p_{1.9}t.\ P_{1.9}M_{.81}T.\ p^{a}_{+}m^{a}t^{a}_{-}.$

This is the scalene octahedron $P_{1.9}M_{.81}T$, combined wlth the planes $P_{1.9}T$, the latter subordinate.

175. The planes of replacement on this form, and those on the form represented by Model 66, have the same proportional axes as the planes of the scalene octahedron, upon which they are superinduced. The equator of the prismatic planes $M_{-}T$, Model 66, is similar to the equator of the octahedral planes $P_{+}M_{-}T$, Model 21. And the east meridian which

bisects the planes P_+T, Model 120, is similar to the east meridian that passes through the edges of the octahedron P_+M_-T, Model 21. In these two cases, there would be no replacement if all the forms were *equal* as well as *similar*, but not being equal, there is a displacement of that portion of the form represented by Model 21 that is larger than the equator of M_-T in Model 66, or than the east meridian of P_+T in Model 120. And the forms M_-T and P_+T take the place of the portions of $P_{1.9}M_{81}T$ that are displaced in each example. We have in this series of Models, 21, 80, 70, 66, 120, a good illustration of the statements made in § 67, respecting the replacement by the planes of small crystals, of portions of planes cut off from large crystals.

176. *What befals the planes of the scalene octahedrons when they combine with one another.*

The case that is to be explained under this title, is of the same importance in respect to the scalene octahedrons as the cases explained in §§ 133—145, and §§ 151—160, were in respect to the isosceles octahedrons —for, just as the complex octahedrons which are represented by Models 22 and 17 are formed by the combination of certain groups of isosceles octahedrons, so there are other complex octahedrons which are formed by the combination of certain groups of scalene octahedrons, and it is to the consideration of these complex scalene octahedrons that I now proceed.

177. Model 25. *Right Hemihexakisoctahedron with parallel faces.*

$$P_-MT_+,P_+M_-T,PM_+T_-.$$

The reader is in the first place referred to what is said in § 159, 2 *c.* respecting the partial replacement of the planes of scalene octahedrons, or those whose axes are $p^a_x\, m^a_y\, t^a_z$, when they combine with one another.

Place Model 25 in an upright position for examination.

178. Hold Model 21 in the position described in § 168 *a*, which brings its axes to the ratio of $p^a_-\, m^a\, t^a_+$. Compare the four planes that touch the pole p^a_z of Model 25 with those parts of the planes of Model 21 that touch the same pole. You will see that their inclinations and positions are nearly the same. The two models have not the same axes, or they would be exactly alike at the pole referred to. They represent different minerals, and were not intended to be used for a comparison of this nature; yet they serve to show that if Model 21 was ground down at the four acute poles, the residual planes left about the pole p^a_z would resemble the four upper planes of Model 25. If you next examine the 4 planes that touch the pole p^a_N of Model 25, you will find that they agree, in a similar manner, with the planes that touch the same pole of Model 21. The inference to be drawn from the result of this examination is, that the eight planes that surround the poles p^a_z and p^a_N of Model 25 belong to a form such as is represented by Model 21, when held in the

position described in § 168 *a*, so as to require the symbol P_-MT_+ to designate it.

179. Hold Model 21 in the position described in § 186 *b*, so as to make its axes agree with p^a_+ m^a_- t^a. Then compare the four planes that touch the pole m^a_n of Model 25, with those parts of the planes of Model 21 that touch the same pole, and observe that the two models agree with one another. Make the same comparison between the planes that are situated about the pole m^a_s of both models. You will thus perceive that Model 25 contains also the eight planes that are denoted by the symbol P_+M_-T.

180. Hold Model 21 in the position described in § 168 *c*, so as to make its axes $=$ p^a m^a_+ t^a_-. Then compare the eight planes that touch the poles t^a_e and t^a_w of Model 25 with those portions of the planes of Model 21 that touch the same two poles. The similarity of their positions and inclinations will satisfy you that Model 25 contains the eight planes which are peculiar to the form denoted by the symbol PM_+T_-.

181. The twenty-four planes of Model 25 are consequently those which are produced when the three forms P_-MT_+, P_+M_-T, PM_+T_-, cut one another and come together upon one crystal. The combination is of the same nature as that which takes place when the three obtuse isosceles octahedrons P_-MT, PM_-T, PMT_-, combine to produce the form represented by 3 P_-MT, Model 22; or, as that which takes place when the three acute isosceles octahedrons P_+MT, PM_+T, PMT_+ combine to produce the form represented by 3 P_+MT, Model 17.

182. The comparison that I have instituted in § 159 between Models 22 and 17, may now be usefully extended to Model 25. I have shown, § 159: 1,2, *a*,*b*, that we can judge of the comparative lengths of the *axes* of the forms that are exhibited on Models 22 and 17, from the information that is to be gained by observing the polaric positions of their respective planes, and in § 159, 2, *c*, I have stated that a similar procedure in respect to such forms as are represented by Model 25, would give similar information regarding their axes. It is easy to test the accuracy and utility of this statement by an examination of the models.

183. With Models 22, 17, and 25 placed before us at the same level, and supported in upright position by the mouths of three wine glasses, we can, by directing our notice to the Znw octant of each form, readily make a comparative examination of all three.

184. Model 22. We observe that the plane marked P, which occupies the position Z^2nw, touches the pole p^a_z and thence proceeds *equally* towards the poles m^a_n and t^a_w which it does not touch. This is the character of a plane of the form P_-MT. The two planes that are between the plane Z^2nw and the equator must both have a longer perpendicular axis than the plane above them, else they would cut the axis p^a nearer the centre of the crystal than it is cut by the plane P_-MT; but as in that case the plane P_-MT would not appear upon the model at all, but would be entirely displaced by the other two planes, this is a consideration that demonstrates the truth of the assumption that the plane which

occupies the position $Z^{2}nw$ is a plane belonging to the form $P_{-}MT$. By a similar train of reasoning it may be proved that the plane which touches the pole m^{a}_{n} and is marked M, is a plane of the form $PM_{-}T$, and that the plane which touches the pole t^{a}_{w} and is marked T, is a plane of the form PMT_{-}. And that, as each of these three planes has seven counterparts in the other seven octants of the crystal, the model must necessarily represent the combination $P_{-}MT$, $PM_{-}T$, PMT_{-}, or 3 $P_{-}MT$.

185. Model 17. The plane marked P and M which touches the poles p^{a}_{z} and m^{a}_{n} equally, and proceeds thence towards the pole t^{a}_{w} which it does not touch, and which therefore occupies the position $Z^{2}n^{2}w$, is a plane of the form PMT_{+}. The plane which touches the poles p^{a}_{z} and t^{a}_{w} equally, and proceeds thence towards the pole m^{a}_{n} which it does not touch, and which consequently occupies the position $Z^{2}nw^{2}$, is a plane of the form $PM_{+}T$. It is evident from the direction of the line of combination between these two planes, which is from the point Z to the point Znw, that they both cut the axis p^{a} at the same distance from the centre of the crystal. If either of these planes had cut the axis p^{a} nearer to the centre than the other plane, it would have prevented that other plane from touching the pole p^{a}_{z}. The plane that touches the poles m^{a}_{n} and t^{a}_{w} equally, and proceeds thence towards the pole p^{a}_{z} which it does not touch, and which consequently occupies the position $Zn^{2}w^{2}$, is a plane of the form $P_{+}MT$. It is evident that this plane touches the axis p^{a} at a greater distance than either of the planes that occupy the positions $Z^{2}n^{2}w$ and $Z^{2}nw^{2}$, for it is entirely separated from the pole p^{a}_{z} by their intervention. It is also evident that the plane $P_{+}MT$ cuts the axis t^{a} shorter than that axis is cut by the plane PMT_{+}, because the latter is entirely separated by it from the pole t^{a}_{w}; and finally it is evident that $P_{+}MT$ cuts the axis m^{a} shorter than that axis is cut by the plane $PM_{+}T$, for it entirely separates by its intervention that plane from the pole m^{a}_{n}. —All these examples concur in proving that the planes found upon a complex equiaxed combination which is composed of unequiaxed forms, are those parts of the planes of the unequiaxed forms that are situated upon the poles of the shorter or shortest of their unequal axes.

186. Model 25. The above illustrations are intended to be preparatory to the examination of Model 25, and if I have succeeded in conveying my ideas, the reader will find little difficulty in comprehending the nature of the combinations of which this model is the type. The three planes situated in the Znw octant of Model 25, partly resemble those contained in the same octant of Model 22 and partly those of Model 17. That is to say, they resemble those of Model 22 in being each attached to one pole, while they resemble those of Model 17 in being each directed more upon two poles than upon the third. For example, the plane marked P touches the pole p^{a}_{z} and proceeds thence towards the poles m^{a}_{n} and t^{a}_{w}, but principally towards the pole m^{a}_{n}, although it touches neither of them. The plane marked M touches the pole m^{a}_{n}, and proceeds thence towards the poles t^{a}_{w} and p^{a}_{z}, but principally towards the pole t^{a}_{w}, yet touches neither. The plane marked T touches the pole t^{a}_{w},

proceeds thence towards the poles p^{a}_{z} and m^{a}_{n}, but principally towards the pole p^{a}_{z}, yet without touching either of them. This is the relation described in § 159, 2, *c*. It is evident from these positions, that each of the three planes that are situated in the Znw octant of Model 25, must, if extended till they cut all the three axes, cut them all unequally; that the axes of the plane P must have the ratio of $p^{a}_{-}\, m^{a}\, t^{a}_{+}$; that the axes of the plane M must have the ratio of $p^{a}_{+}\, m^{a}_{-}\, t^{a}$; that the axes of the plane T must have the ratio of $p^{a}\, m^{a}_{+}\, t^{a}_{-}$; and that consequently the combination must consist of the planes $P_{-}MT_{+}$, $P_{+}M_{-}T$, $PM_{+}T_{-}$, or be a combination of three equal and similar scalene octahedrons; for that the simple forms *must* be equal and similar is demonstrated by the exact *symmetry* of the sets of planes in every one of the eight octants of the combination, and by the precise *equality* of the three planes in each octant. It is also evident from the positions of the planes, that the three octahedral forms that are present on the combination, are those described in § 168 *a*, *b*, *c*, and not those described in § 168, *d*, *e*, *f*.

187. The foregoing observations all refer to an *octant*, or *one eighth part* of the combination; but this limited observation is quite sufficient for all practical purposes, since all the eight octants of an octahedron are equal and similar. It is worthy of remark, that the octants are the divisions into which an octahedron is divided by the equator, and the east and west meridians of the form. The equator separates the zenith from the nadir portion of the combination. The north meridian separates the east from the west portion. The east meridian separates the north from the south portion. The whole form is thus equally and very conveniently divided into eight portions, and we are enabled to simplify our observations and calculations by referring them to one of these octants, and assuming it as an axiom that what is true of one of them is true of the whole. I therefore always refer to the Znw octant of a combination, as that is the octant which is commonly depicted in figures of crystals, and as indeed it is that which it is generally most convenient to examine. For the same reason, the Models of Crystals have all been marked with P,M,T, at those poles of the axes which denote the limits of this particular octant, namely, at the poles p^{a}_{z}, m^{a}_{n}, and t^{a}_{w}.

Method of denoting the polaric positions of planes on the combinations of scalene octahedrons.

188. The unequal bearings of the planes of the Hemihexisoctahedron upon the poles of the crystal, renders it necessary to mark the polaric position of each of the twenty-four planes by symbols so distinct as not to be readily misunderstood. I propose to do this as follows:—The symbol Znw denotes, as a general term, the Zenith-north-west octant of an octahedral form, or any plane that bears equally upon the three poles which limit that octant. Z^2nw, Zn^2w, Znw^2 denote a plane in that octant which bears more upon one pole than upon the two others. Z^2n^2w, Zn^2w^2, Z^2nw^2, denote a plane in that octant which bears more upon any two poles than upon the third. Following out this system of notation, the symbol Z^3n^2w may be held to denote a plane that bears principally upon the pole

p^a_z, less upon the pole m^a_n, and still less upon the pole t^a_w; which bearings would indicate the symbol P_-MT_+ as the formula of the set to which the plane belongs. Secondly, the symbol Zn^3w^2 may indicate a plane that bears principally upon the pole m^a_n, less upon the pole t^a_w, and still less upon the pole p^a_z, which bearings serve to indicate a plane the symbol for whose complement would be P_+M_-T. Thirdly, the symbol Z^2nw^3 may indicate a plane that bears principally upon the pole t^a_w, less upon the pole p^a_z, and still less upon the pole m^a_n, which bearings point out the symbol PM_+T_- as characteristic of the set to which this plane belongs. These three planes are all that belong to the Znw octant of the combination, and being thus denoted by three different signs, the whole combination is in fact denoted, for every other octant can be denoted in a similar manner; retaining always the measures of proximity ^{2,3} and changing Z for N, n for s, w for e, and so on, as the different octants require. In this manner the following twenty-four symbols may be produced to distinguish the twenty-four planes of the combination represented by Model 25. For the convenience of subsequent reference, I have added the value of the axes of the component octahedrons of the combination which the model represents. The proofs of this value will be given afterwards.

Planes at p^a_z.	*Planes at* m^a_n.	*Planes at* t^a_w.
$P\frac{1}{3}M\frac{1}{2}T\ Z^3n^2w$	$PM\frac{1}{3}T\frac{1}{2}\ Zn^3w^2$	$P\frac{1}{2}MT\frac{1}{3}\ Z^2nw^3$
$P\frac{1}{3}M\frac{1}{2}T\ Z^3n^2e$	$PM\frac{1}{3}T\frac{1}{2}\ Zn^3e^2$	$P\frac{1}{2}MT\frac{1}{3}\ Z^2sw^3$
$P\frac{1}{3}M\frac{1}{2}T\ Z^3s^2w$	$PM\frac{1}{3}T\frac{1}{2}\ Nn^3w^2$	$P\frac{1}{2}MT\frac{1}{3}\ N^2nw^3$
$P\frac{1}{3}M\frac{1}{2}T\ Z^3s^2e$	$PM\frac{1}{3}T\frac{1}{2}\ Nn^3e^2$	$P\frac{1}{2}MT\frac{1}{3}\ N^2sw^3$
Planes at p^a_N.	*Planes at* m^a_s.	*Planes at* t^a_e.
$P\frac{1}{3}M\frac{1}{2}T\ N^3n^2w$	$PM\frac{1}{3}T\frac{1}{2}\ Zs^3w^2$	$P\frac{1}{2}MT\frac{1}{3}\ Z^2ne^3$
$P\frac{1}{3}M\frac{1}{2}T\ N^3n^2e$	$PM\frac{1}{3}T\frac{1}{2}\ Zs^3e^2$	$P\frac{1}{2}MT\frac{1}{3}\ Z^2se^3$
$P\frac{1}{3}M\frac{1}{2}T\ N^3s^2w$	$PM\frac{1}{3}T\frac{1}{2}\ Ns^3w^2$	$P\frac{1}{2}MT\frac{1}{3}\ N^2ne^3$
$P\frac{1}{3}M\frac{1}{2}T\ N^3s^2e$	$PM\frac{1}{3}T\frac{1}{2}\ Ns^3e^2$	$P\frac{1}{2}MT\frac{1}{3}\ N^2se^3$

189. The mineral kingdom has furnished three varieties of the Right Hemihexakisoctahedron with parallel faces: two of them as substantive crystals, and the third in combination with other planes. The symbols for these three varieties are as follows:

$$1,\ P\tfrac{1}{3}M\tfrac{1}{2}T,\ PM\tfrac{1}{3}T\tfrac{1}{2},\ P\tfrac{1}{2}MT\tfrac{1}{3}.$$
$$2,\ P\tfrac{1}{4}M\tfrac{1}{2}T,\ PM\tfrac{1}{4}T\tfrac{1}{2},\ P\tfrac{1}{2}MT\tfrac{1}{4}.$$
$$3,\ P\tfrac{1}{5}M\tfrac{1}{3}T,\ PM\tfrac{1}{5}T\tfrac{1}{3},\ P\tfrac{1}{3}MT\tfrac{1}{5}.$$

They may be discriminated from one another by the difference in the angles at which their planes incline upon one another, as shown in the following table:

COMBINATIONS.	*Inclination of the plane* $P_-MT_+\ Z^3n^2w$ *upon:* $P_-MT_+\ Z^3n^2e$.	$P_-MT_+\ Z^3s^2w$.	$P_+M_-T\ Zn^3w^2$.	$PM_+T_-\ Z^2nw^3$.
$P\frac{1}{3}M\frac{1}{2}T, PM\frac{1}{3}T\frac{1}{2}, P\frac{1}{2}MT\frac{1}{3}$	149° 0′.	115° 23′.	141° 47′.	141° 47′
$P\frac{1}{4}M\frac{1}{2}T, PM\frac{1}{4}T\frac{1}{2}, P\frac{1}{2}MT\frac{1}{4}$	154° 47′.	128° 15′.	131° 49′.	131° 49′
$P\frac{1}{5}M\frac{1}{3}T, PM\frac{1}{5}T\frac{1}{3}, P\frac{1}{3}MT\frac{1}{5}$	160° 32′.	118° 59′.	131° 5′.	131° 5′

It is the first of these three combinations that is represented by Model 25, as may be proved by measuring with the goniometer the angles of the incidence of its planes. The second form is distinguished from the other two by having trapezoids and not trapeziums for its planes, the three edges that radiate from the point Znw being parallel with the three edges that join the poles p^a_z, m^a_n, and t^a_w.

190. The method of proceeding to determine the value of the axes which belong to the component octahedrons of a combination of this kind, is as follows. I take Model 25 as an example.

a. As respects the planes that touch the poles m^a_n *and* m^a_s.—The goniometer is applied to the edges of the equator across the pole m^a_n. The angle is found to be 112° 38′. The half of this is 56° 19′, of which are the tangent is 1.5004. Hence the axes m^a and t^a have the ratio of $m^a_1\, t^a_{1.5004}$, or $m^a_2\, t^a_3$, or $m^a\frac{2}{6}\, t^a\frac{3}{6}$, or $m^a\frac{1}{3}\, t^a\frac{1}{2}$.

The goniometer is next applied to the edges of the north meridian, across the pole m^a_n. The angle is found to be 143° 8′. The half of this is 71° 34′, the tangent of which is 3.0003. Hence the axes m^a and p^a have the ratio of $m^a_1\, p^a_{3.0003}$, or $m^a_1\, p^a_3$, or $m^a\frac{1}{3}\, p^a 1$, or $m^a\frac{2}{6}\, p^a\frac{6}{6}$, or $m^a_2\, p^a_6$.

Hence the ratio of all the axes is $=$ $p^a_6\, m^a_2\, t^a_3$ or $p^a 1\; m^a\frac{1}{3}\; t^a\frac{1}{2}$, and the symbol for the planes is $PM\frac{1}{3}T\frac{1}{2}$, as quoted in the above tables. The measurement of the edges at the pole m^a_s gives the same results as the measurements at the pole m^a_n.

b. As respects the planes that touch the poles t^a_e *and* t^a_w.—The angle formed by the meeting of the two edges of the equator at the pole t^a_w is 143° 8′. That formed by the meeting of the two edges of the east meridian at the pole t^a_w is 112° 38′. The two angles formed by the edges at the pole t^a_e are precisely similar. These measurements give the ratio of $m^a_3\, t^a_1$ and $p^a_{1\frac{1}{2}}\, t^a_1$, or $p^a_{1\frac{1}{2}}\, m^a_3\, t^a_1$, which requires the symbol $P\frac{1}{2}MT\frac{1}{3}$.

c. As respects the planes that touch the poles p^a_z *and* p^a_N.—The value of the angle of the north meridian at the pole p^a_z is 112° 38′. This gives the ratio of $p^a_1\, m^a_{1\frac{1}{2}}$. The value of the angle of the east meridian at that pole is 143° 8′. This gives the ratio of $p^a_1\, t^a_3$. The planes therefore require the symbol $P\frac{1}{3}M\frac{1}{2}T$. The measurements at the pole p^a_N are exactly the same as those at the pole p^a_z.

d. Control over the accuracy of these measurements. The equator of the combination is an octagon, wherefore all its angles should be together equal to 1080°. See § 82.

$$\left.\begin{array}{lllr}
\text{The angle at } m^a_n & \text{has been found to be} & (a) = & 112^\circ\ 38' \\
\text{.........}\quad m^a_s & \text{...............} & (a) = & 112^\circ\ 38' \\
\text{.........}\quad t^a_w & \text{...... ....} & (b) = & 143^\circ\ \ 8' \\
\text{.........}\quad t^a_e & \text{...............} & (b) = & 143^\circ\ \ 8'
\end{array}\right\} = 511^\circ\ 32'.$$

511° 32′ deducted from 1080°, leaves 568° 28′ for the value of the four remaining angles of the equator, which angles are those produced by the incidence of the planes of the form $PM\frac{1}{3}T\frac{1}{2}$ upon the planes of the form $P\frac{1}{2}MT\frac{1}{3}$, and which are all equal to one another. This aggregate

sum divided by 4 gives 142° 7′ as the value of each of these four angles, and the application of the goniometer to any one of them shows this to be correct.

The north meridian and the east meridian have each eight angles of the same value as the angles of the equator, as can be easily proven, either by direct measurement, or by calculations similar to the foregoing.

191. These examples show how very useful it is to examine, in all cases, the geometrical relations of those three sections of a crystal, which I have named the north meridian, the east meridian, and the equator; for as the poles of every simple octahedron rest in these three sections, and as their axes lie always two in one section, and the third in another section, at right angles to that which contains the two axes, we have in general several easy methods of measuring two of the axes and calculating the length of the third axis of every simple octahedron that forms part of a combination. If, for example, we have access to the north pole of a crystal, the angle of the equator gives the ratio of the axes m^a to t^a, and the angle of the north meridian gives us the ratio of the axes m^a and p^a. If we have access to the west pole, the angle of the equator gives the ratio of m^a to t^a, and the angle of the east meridian gives the ratio of t^a to p^a. If we have access to the zenith pole, the angle of the north meridian gives the ratio of p^a to m^a, and the angle of the east meridian gives the ratio of p^a to t^a. If we cannot measure the angles at these three poles, but have access to the poles m^a_s, t^a_e, or p^a_N, we obtain the same results. If the apices of the forms are cut from all the poles, but replaced by planes that are perpendicular to the axes, we can still obtain exact results by measuring the edges of combination, and calculating the value of the displaced apices, according to the method that I have explained in treating of the properties of the form that is represented by Model 47. § 83.

192. *Left Hemihexakisoctahedron with parallel faces.*

$$P_-M_+T,\ PM_-T_+,\ P_+MT_-.$$

Place Model 25 in upright position, and then turn it round a quarter of a revolution on its principal axis, so as to bring the letter M to the pole t^a_w, and the letter T to the pole m^a_s. It will then exhibit the following forms:—

P_-M_+T, or $P\frac{1}{3}MT\frac{1}{2}$, described § 168 *d*. The planes touch the poles p^a_z and p^a_N.

PM_-T_+, or $P\frac{1}{2}M\frac{1}{3}T$, described § 168 *e*. The planes touch the poles m^a_n and m^a_s.

P_+MT_-, or $PM\frac{1}{2}T\frac{1}{3}$, described § 168 *f*. The planes touch the poles t^a_e and t^a_w.

The combination contains altogether 24 planes, which individually occupy the following positions:

Planes at p^{a}_{Z}	*Planes at* m^{a}_{n}	*Planes at* t^{a}_{w}
$P\frac{1}{3}MT\frac{1}{2}\ Z^{3}nw^{2}$	$P\frac{1}{2}M\frac{1}{3}T\ Z^{2}n^{3}w$	$PM\frac{1}{2}T\frac{1}{3}\ Zn^{2}w^{3}$
$P\frac{1}{3}MT\frac{1}{2}\ Z^{3}ne^{2}$	$P\frac{1}{2}M\frac{1}{3}T\ Z^{2}n^{3}e$	$PM\frac{1}{2}T\frac{1}{3}\ Zs^{2}w^{3}$
$P\frac{1}{3}MT\frac{1}{2}\ Z^{3}sw^{2}$	$P\frac{1}{2}M\frac{1}{3}T\ N^{2}n^{3}w$	$PM\frac{1}{2}T\frac{1}{3}\ Nn^{2}w^{3}$
$P\frac{1}{3}MT\frac{1}{2}\ Z^{3}se^{2}$	$P\frac{1}{2}M\frac{1}{3}T\ N^{2}n^{3}e$	$PM\frac{1}{2}T\frac{1}{3}\ Ns^{2}w^{3}$

Planes at p^{a}_{N}	*Planes at* m^{a}_{s}	*Planes at* t^{a}_{e}
$P\frac{1}{3}MT\frac{1}{2}\ N^{3}nw^{2}$	$P\frac{1}{2}M\frac{1}{3}T\ Z^{2}s^{3}w$	$PM\frac{1}{2}T\frac{1}{3}\ Zn^{2}e^{3}$
$P\frac{1}{3}MT\frac{1}{2}\ N^{3}ne^{2}$	$P\frac{1}{2}M\frac{1}{3}T\ Z^{2}s^{3}e$	$PM\frac{1}{2}T\frac{1}{3}\ Zs^{2}e^{3}$
$P\frac{1}{3}MT\frac{1}{2}\ N^{3}sw^{2}$	$P\frac{1}{2}M\frac{1}{3}T\ N^{2}s^{3}w$	$PM\frac{1}{2}T\frac{1}{3}\ Nn^{2}e^{3}$
$P\frac{1}{3}MT\frac{1}{2}\ N^{3}se^{2}$	$P\frac{1}{2}M\frac{1}{3}T\ N^{2}s^{3}e$	$PM\frac{1}{2}T\frac{1}{3}\ Ns^{2}e^{3}$

193. Of the six symmetrical scalene octahedrons that are described in § 168, the Right Hemihexakisoctahedron with parallel faces, comprehends the three first, and the Left Hemihexakisoctahedron with parallel faces, the three last. The two combinations are precisely alike if considered as substantive crystals, but they differ altogether in their positions, and consequently in the denominations of the planes that compose them. These differences are fully explained in the above table, and it does not appear to me to be necessary to dwell at all upon the examination of the Left Hemihexakisoctahedron, since what has been said of the means of investigating the properties of the *Right* form applies equally to the *Left*. The former combination sometimes occurs in the mineral kingdom as a substantive crystal, the latter never. This, with the differences of position, are the only grounds of distinction between them.

194. Model 23. *Hexakisoctahedron.* $P_{-}MT_{+}$, $P_{+}M_{-}T$, $PM_{+}T_{-}$, $P_{-}M_{+}T$, $PM_{-}T_{+}$, $P_{+}MT_{-}$.

This is a combination of the six symmetrical scalene octahedrons, which, when combined in two groups, form the Right Hemihexakisoctahedron with parallel faces, and the Left Hemihexakisoctahedron with parallel faces. The following description of the positions of the 48 planes upon Model 23 is intended to show that no other inference than the above can be legitimately drawn from the information afforded by the examination of these positions.

The model bears somewhat the appearance of a cube that has a low eight-sided pyramid upon each of its six planes, and $8 \times 6 = 48$ planes; or of a rhombic dodecahedron that has a four-sided pyramid upon each of its twelve planes, and $12 \times 4 = 48$ planes. It also has some resemblance to an octahedron that has a six-sided pyramid upon each of its 8 planes, and $6 \times 8 = 48$ planes. There exists in the mineral kingdom some forms of this kind, in which the octahedral form is more prominent than it is in the model, which represents a variety that has a tendency to the form of the cube. All these varieties agree in having 48 planes disposed in the same relative positions, and they differ only in the angles at which the planes incline one upon another, the cause of which difference may be ultimately traced to the difference in the dimensions of the

rhombic prisms which formed the simple scalene octahedrons that are the components of this complex combination.

Planes of the Znw *octant of the Hexakisoctahedron.*

195. It will be sufficient for our purpose to examine the relations of the six planes that constitute the Znw octant of the combination, because these six planes comprehend one belonging to each of the six combining octahedrons; and the knowledge of the position of one plane of each complement leads to a knowledge of the whole. These six planes are divided into three groups, namely, two planes that touch the pole p^a_z, two that touch the pole m^a_n, and two that touch the pole t^a_w. Then, of the two planes that touch the pole p^a_z, one is attached to the north meridian and the other to the east meridian of the crystal; of the two planes that touch the pole m^a_n, one is attached to the north meridian and the other to the equator; and of the two planes that are attached to the pole t^a_w, one is attached to the east meridian and the other to the equator.

a. The plane that touches the pole p^a_z and is attached to the north meridian is in the position Z^3n^2w, and therefore is a plane belonging to the complement P_-MT_+.

b. The plane that touches the pole p^a_z and is attached to the east meridian, is in the position Z^3nw^2, and therefore is a plane belonging to the complement P_-M_+T.

c. The plane that touches the pole m^a_n and is attached to the equator, is in the position Zn^3w^2, and therefore is a plane belonging to the complement P_+M_-T.

d. The plane that touches the pole m^a_n and is attached to the north meridian, is in the position Z^2n^3w, and therefore is a plane belonging to the complement PM_-T_+.

e. The plane that touches the pole t^a_w and is attached to the east meridian, is in the position Z^2nw^3, and therefore is a plane belonging to the complement PM_+T_-.

f. The plane that touches the pole t^a_w and is attached to the equator, is in the position Zn^2w^3, and therefore is a plane belonging to the complement P_+MT_-.

a, *c*, *e*, are the three complements of planes that produce the combination called the Right Hemihexakisoctahedron with parallel faces.

b, *d*, *f*, are the three complements of planes that produce the combination called the Left Hemihexakisoctahedron with parallel faces.

Consequently, the Hexakisoctahedron is a combination that contains all the six regular permutations of the symmetrical scalene octahedron.

The two tables given in §§ 188, 192, show the symbols and positions of all the forty-eight planes that belong to this combination, upon the assumption that P_-MT_+ signifies $P\frac{1}{3}M\frac{1}{2}T$; and although the numeral values of the symbols are changed when other varieties of the form are to be indicated, the signs of the positions are correct for all.

196. As the symbols for the three combinations of the scalene octahedrons are very long, I propose to abridge them as follows:—

6 P_-MT_+, instead of P_-MT_+, P_+M_-T, PM_+T_-, P_-M_+T, PM_-T_+, P_+MT_-, the symbol for the Hexakisoctahedron.

3 P_-MT_+ Z^3n^2w, instead of P_-MT_+, P_+M_-T, PM_+T_-, the symbol for the Right Hemihexakisoctahedron with parallel faces.

3 P_-M_+T Z^3nw^2, instead of P_-M_+T, PM_-T_+, P_+MT_-, the symbol for the Left Hemihexakisoctahedron with parallel faces.

197. *Varieties of the Hexakisoctahedron*, 6 P_-MT_+.

The mineral kingdom has afforded the following varieties of this complex combination:

6 $P_{\frac{1}{3}}M_{\frac{1}{2}}T$.
6 $P_{\frac{1}{4}}M_{\frac{1}{3}}T$.
6 $P_{\frac{1}{4}}M_{\frac{1}{2}}T$.
6 $P_{\frac{1}{11}}M_{\frac{1}{3}}T_{\frac{1}{3}}$.
6 $P_{\frac{1}{7}}M_{\frac{1}{3}}T$.

These five combinations can be discriminated by attention to the following angles of the incidence of their planes upon one another.

Combinations.	*Inclination of the plane* P_-MT_+ Z^3n^2w *upon:* P_-MT_+ Z^3n^2e.	P_-M_+T Z^3nw^2.	PM_-T_+ Z^2n^3w.
6 $P_{\frac{1}{3}}M_{\frac{1}{2}}T$.	149° 0′	158° 13′	158° 13′
6 $P_{\frac{1}{4}}M_{\frac{1}{3}}T$	157° 23′	147° 48′	164° 3′
6 $P_{\frac{1}{4}}M_{\frac{1}{2}}T$.	154° 47′	162° 15′	144° 3′
6 $P_{\frac{1}{11}}M_{\frac{1}{3}}T_{\frac{1}{3}}$.	152° 7′	166° 57′	140° 9′
6 $P_{\frac{1}{7}}M_{\frac{1}{3}}T$.	165° 2′	158° 47′	136° 47′

The combinations that are at the beginning of this list approximate to the form of the octahedron, those at the end rather to the form of the cube. By some crystallographers, the former are termed Hexakisoctahedrons and the latter Octakishexahedrons, a variation in nomenclature which is of little importance. Model 23 is intended to represent the combination 6 $P_{\frac{1}{7}}M_{\frac{1}{3}}T$, but its angles are not made sufficiently correct to afford measurements strictly justificatory of those contained in the above table. They are however exact enough to distinguish this variety from all the others, and to show the positions and general bearings of the different sets of planes. The figure of the Hexahisoctahedron that is commonly given in books on Mineralogy, as *the diamond form*, is the combination denoted by 6 $P_{\frac{1}{3}}M_{\frac{1}{2}}T$.

All the octahedral forms, simple and complex, occur in combination with the different prismatic forms. The suite of models presents numerous examples of such combinations, and this is perhaps the proper place for the description of them; but as I propose to add a Second Part to this work, treating of the application of Crystallography to Mineralogy, I shall reserve these details till I come to speak of the Minerals which afford examples of each particular combination.

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198. I have now described the properties of all the varieties of octahedrons which can possibly occur. They are as follow :—

$$\left.\begin{array}{l} PM_x\ \S\ 88 \\ PT_x\ \S\ 101 \end{array}\right\} = PM_x,\ PT_x.\ \S\S\ 104,\ 105.$$

$$PMT.\ \S\ 124.$$

$$\left.\begin{array}{l} \left.\begin{array}{l} P_-MT \\ PM_-T \\ PMT_- \end{array}\right\} = 3\ P_-MT.\ \S\ 142. \\ \left.\begin{array}{l} P_+MT \\ PM_+T \\ PMT_+ \end{array}\right\} = 3\ P_+MT.\ \S\ 158. \end{array}\right\} = 6\ P_xMT.$$

$$\left.\begin{array}{l} \left.\begin{array}{l} P_-MT_+ \\ P_+M_-T \\ PM_+T_- \end{array}\right\} = 3\ P_-MT_+.\ \S\ 177. \\ \left.\begin{array}{l} P_-M_+T \\ PM_-T_+ \\ P_+MT_- \end{array}\right\} = 3\ P_-M_+T.\ \S\ 192. \end{array}\right\} = 6\ P_-MT_+.\ \S\ 194.$$

Every other octahedral or pyramidal form is *a combination of two or more of these complements of planes or of their fractions.* Hence the following formula embraces every variety of pyramid that can occur, either alone or in combination with prismatic planes :

$$PM_x,\ PT_x,\ PMT,\ 6\ P_xMT,\ 6\ P_xM_yT_z.$$

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199. *Forms of the Equators of Pyramids.*—An examination of the forms of the equators of Pyramids, gives the same results as a similar examination of the equators of Prisms. What these results are, I have shown in the table contained in § 87. It is only necessary to explain here a few terms that will be employed hereafter to designate the different varieties of equator.

A *Square* Equator is one whose sides are parallel to the axes m^a and t^a, or cut these axes only at angles of 45°, and are all of equal length when extended till they meet. Models 1, 4, 12, 17. A *Rectangular* Equator is one whose sides are parallel to the axes m^a and t^a, and equal to one another two and two, but not all equal. Model 19. A *Rhombic* Equator is one whose sides, when extended till they intersect one another, form one or more rhombuses. Models 6, 21, 22. A *Rhombo-Quadratic* Equator is one whose sides, when extended till they meet, form both a square and a rhombus. Models 47, 91. A *Rhombo-Rectangular* Equator is one whose sides, when extended till they intersect one another, form both a rhombus and a rectangle. Models 50, 51, 8, 7. To complete the square in Model 91, and the rectangle in Models 8, 7, lines are supposed to be drawn parallel to the axis t^a, through the poles m^a_a and m^a_e.

Synopsis of Planes.

200. Before concluding the present section, it will be useful to take a general survey of the mutual relations of the different sets of Planes and their Axes.

SYNOPSIS OF PLANES.	Planes that produce PRISMS.				Planes that produce PYRAMIDS.		
Planes that cut Equal Axes,	P	M	T	MT	PM	PT	PMT
Planes that cut Unequal Axes,	P_- P_+	M_- M_+	T_- T_+	MT_+ M_+T	PM_+ P_+M	PT_+ P_+T	P_-MT PM_-T PMT_- P_+MT PM_+T PMT_+ P_-MT_+ P_+M_-T PM_+T_- P_-M_+T PM_-T_+ P_+MT_-
Number of Planes to each set,	2	2	[illegible]	[illegible]	4	4	8
Axes cut by the Planes of each set, …………………	p^2	m^2	t^2	m^2 t^2	p^2 m^2	p^2 t^2	p^2 m^2 t^2
Axes parallel to the Planes of each set, ………………	m^2 t^2	p^2 t^2	p^2 m^2	p^2	t^2	m^2	*none.*
Position of the Planes of each set, when the *point of view* of the crystal is in the prolongation of the axis m^2,…	Horizontal, *top and bottom*	Vertical, *front and back*	Vertical, *left and right.*	Vertical, *diagonal.*	Inclined, *front and back*	Inclined, *left and right.*	Inclined, *diagonal.*

Abridged Symbols for Octahedral Combinations. See §§ 161, 196.

3 P_-MT = P_-MT, PM_-T, PMT_-.

3 P_+MT = P_+MT, PM_+T, PMT_+.

3 P_-MT_+ Z^2n^2w = P_-MT_+, P_+M_-T, PM_+T_-.

3 P_-M_+T Z^2nw^2 = P_-M_+T, $PM\ T_+$, P_+MT_-.

6 P_-MT_+ = P_-MT_+, P_+M_-T, PM_+T_-, P_-M_+T, PM_-T_+, P_+MT_-.

Section III.—OF PRISMS AND PYRAMIDS AND THEIR COMBINATIONS WITH ONE ANOTHER.

201. A Prism is a solid contained by three or more *vertical planes* of the same altitude, and two equal, similar, and parallel *horizontal planes*. The vertical planes are those whose symbols are M, T, MT, MT_+, M_+T. The horizontal planes are the set P. These are the only planes that belong to a prism. See § 86.

202. A Complete Prism must have the two horizontal planes P, and at least three vertical planes, which number, and not less, will make up a complete form. A complete prism may have any number of vertical planes greater than three, but it cannot have less, because with less than three vertical planes we have not a complete form. It is seldom however that prisms have less than *four* vertical planes.

203. An Incomplete Prism may have any number and combination of the prismatic planes P, M, T, MT, MT_+, M_+T, other than suffices to produce a complete prism. For instance, it may have the two horizontal planes without any of the vertical planes, or it may have all the vertical planes without the horizontal planes, or finally, it may have the two horizontal planes with any two, but not more than two, of the vertical planes. It is obvious, from this description of the Incomplete Prism, that it cannot be a self-existing form, and that its planes can only appear upon a crystal in combination with the planes of some other form.

204. A Pyramid is a solid contained by eight, twelve, twenty-four, forty-eight, or more, *Inclined Planes*, generally triangular, but sometimes quadrangular, the symbols of which planes are

$$PM_x, PT_x, PMT, 6\,P_xMT, 6\,P_xM_yT_z.$$

These are all the planes that belong to a pyramid. There are no vertical and no horizontal planes. See § 198. The inclined planes must have such positions upon every crystal as to form a solid angle at the pole p^a_z and another at the pole p^a_N.

205. The Geometrical *Pyramid* is a plano-facial solid contained by three or more plane triangles which have a common vertex, and whose bases are the sides of a plane rectilineal figure which forms the base of the solid. The common vertex of the plane triangles is called the vertex of the pyramid, and the remaining face is called the base.

206. The Crystallographical *Pyramid* consists of *two* geometrical pyramids joined base to base. By some crystallographers this form is called the *double* pyramid. But as crystallographical pyramids are *always* double, it is better, because it simplifies notation, to consider the double pyramid as a *single form*. On this account I call the lower pyramid the complement of the upper, and consider the *double* pyramid to be *one* pyramid.

207. A Complete *Pyramid* is one that has at least six, but may

have any greater number of inclined planes, in two sets of equal number; one set situated above the equator, and the other below it. There must be a solid angle at the pole p^{*}_{z} and another at the pole p^{*}_{N}.

208. The Complete Pyramids of eight or twelve planes have commonly triangular planes that taper gradually from the equator to the poles p^{*}_{z} and p^{*}_{N}; as witness Models 15, 12, 13, 26. But there are other complete pyramids which do not taper gradually from the equator to the poles, and others of which the planes are not triangular. See Models 22, 17, 25, 23. These forms are produced by planes belonging to combinations that are described by a formula expressing several permutations of the complement P_xMT or $P_xM_yT_z$. But all Complete Pyramids agree with one another in three essential particulars:—They have none but *inclined* planes; they have the zenith portion similar and equal to the nadir portion of the form; and they are terminated by solid angles at the two poles p^{*}_{z} and p^{*}_{N}.

209. An Incomplete Pyramid is a form that has inclined planes, but is without solid angles at the poles p^{*}_{z} and p^{*}_{N}, where it is terminated by oblique planes, or by straight edges, or by the horizontal planes P. An Incomplete Pyramid may have any number of inclined planes.

THREE KINDS OF COMBINATIONS OF PRISMS WITH PYRAMIDS.

210. A Complete Prism combined with an Incomplete Pyramid. This is a crystal that contains the planes of the Complete Prism, § 202, combined with the planes of the Incomplete Pyramid, § 209.

A. It must have,

a. The two horizontal planes P.

b. At least three of the vertical planes M, T, MT_x.

c. Some, and it is of no consequence how many, of the inclined planes PM_x, PT_x, P_xMT, $P_xM_yT_z$.

And these are necessarily situated upon the crystal between the horizontal and vertical planes, in accordance with the polaric positions proper to each form that enters into the combination.

B. This combination must have no solid angles at the poles p^{*}_{z} and p^{*}_{N}.

211. An Incomplete Prism combined with a Complete Pyramid. This crystal exhibits the vertical planes of the Incomplete Prism, § 203, combined with the planes of the Complete Pyramid, § 207.

A. It must have,

a. Eight or more inclined planes forming two equal and similar pyramids, whose bases are parallel with the equator of the combination, and whose apices form solid angles at the poles p^{*}_{z} and p^{*}_{N}.

b. Some, and any number or combination, of the prismatic planes M, T, MT, MT_{+}, $M_{+}T$, which necessarily appear round the equator, replacing part of the base of the pyramid.

B. This combination must not have the horizontal planes P.

212. **An Incomplete Prism combined with an Incomplete Pyramid.** This crystal contains the planes of the Incomplete Prism, § 203, combined with the planes of the Incomplete Pyramid, § 209.

The characters of this combination are as follow:

a. It may have the horizontal planes P, but then it must not have more than two vertical planes.

b. It may have the horizontal planes P, without any vertical planes.

c. It may have any number of vertical planes, provided it is without the horizontal planes.

d. If it has the horizontal planes P, it may have any number and combination of the inclined planes to form the incomplete pyramid.

e. If it is without the horizontal planes P, the inclined planes of the incomplete pyramid must be such as do not form solid angles at the poles p^a_z and p^a_N.

Section IV.—OF THE CLASSIFICATION OF CRYSTALS.

213. The arbitrary definitions of Prisms, Pyramids, and their Combinations, which are given in the preceding Section, §§ 201—212, provide *six distinct terms* explanatory of different crystallographic combinations.

The Table of the Forms of the Equators of Prisms and Pyramids, explained in §§ 87 and 199, provides *five other characters* adapted to distinguish different crystallographic combinations.

I propose to employ these two sets of characters as the foundation of a Classification of Crystals, as follows:

CLASS I.—Complete Prisms.

Definition § 202.

Order	1. Square, . .	Examples.	Models	1 to 4
	2. Rectangular, .		—	5
	3. Rhombic, . .		—	6
	4. Rhombo-Quadratic, .		—	
	5. Rhombo-Rectangular, .		—	7 to 11

The title of each Order indicates the form of the Equators of the Crystals which it comprehends. § 199.

CLASS II.—Complete Pyramids.

Definition § 207.

Order	1. Square, . .	Examples.	Models	12 to 18
	2. Rectangular, . .		—	19 to 20
	3. Rhombic, . .		—	21 to 25
	4. Rhombo-Quadratic, .		—	
	5. Rhombo-Rectangular, .		—	26

CLASS III.—Complete Prisms combined with Incomplete Pyramids.

Definition § 210.

Order	1. Square, . .	Examples. Models	27	to	44
	2. Rectangular, . .	—			45
	3. Rhombic, . .	—			46
	4. Rhombo-Quadratic, .	—	47	to	49
	5. Rhombo-Rectangular, .	—	50	to	58

CLASS IV.—Incomplete Prisms combined with Complete Pyramids.

Definition § 211.

Order	1. Square, . .	Examples. Models	59	to	65
	2. Rectangular, . .	—			
	3. Rhombic, . .	—	66	to	68
	4. Rhombo-Quadratic, .	—			69
	5. Rhombo-Rectangular, .	—	70	to	75

CLASS V.—Incomplete Prisms combined with Incomplete Pyramids.

Definition § 212.

Order	1. Square, . .	Examples. Models	76	to	78
	2. Rectangular, . .	—			79
	3. Rhombic, . .	—	80	to	90
	4. Rhombo-Quadratic, .	—	91	to	95
	5. Rhombo-Rectangular, .	—	96	to	116

CLASS VI.—Incomplete Pyramids.

Definition § 209.

Order	1. Square, . .	Examples. Models	117, 118
	2. Rectangular, . .	—	
	3. Rhombic, . .	—	119
	4. Rhombo-Quadratic, .	—	
	5. Rhombo-Rectangular, .	—	120

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214. Directions for putting a Crystal into a Proper Position for Examination and Description.

As the foregoing Classification of Crystals is founded upon the differences that exist in the positions of the planes of crystals when they are held in upright position, it follows that one and the same crystal may be made to belong to different classes of crystals by merely holding it in different positions. Thus, Model 1, the cube, P,M,T, which belongs to Class I., Order 1, *Square Prisms*, may be held so as to exhibit the planes M. PT; or the planes T. PM; and in either of the latter cases,

he crystal belongs to Class V., Order 2, being " an Incomplete Prism combined with an Incomplete Pyramid, and having a Rectangular Equator." This single example proves the necessity of having a method agreed upon for putting crystals into a proper upright position, so as to establish uniformity in notation. I shall therefore give a few rules that may be observed on this point, but I give them with the confession that hey are too vague to be rigidly adhered to, and that I am at present unable to make them so exact as to be quite satisfactory.

General Rule.

a, The first and principal rule has been given in §§ 8 and 166, and may be here recited. The point of view of a crystal is in the prolongation of the minor axis m^s. Hence the observer of a crystal has to hold t before him with the longest axis in a perpendicular position, and the broadest side exposed to his eye. The axes then become p°_{+} m^{s}_{-} t^{e}.

Rules drawn from the distinction between Prisms and Pyramids.

b, When the crystal is a prism, the planes M,T,MT_x are to be held in a vertical position. A prism is known by the characters given in §§ 86, 201, 220.

c, When the crystal is a pyramid, it is to be held with the two principal solid angles, namely, the two sharpest or two bluntest where any two equal and opposite solid angles are different from the other solid angles, or at any rate, with two similar solid angles, upon the poles p°_{s} and p°_{N}. The characters by which a pyramid is known, are given in §§ 198, 204, 224.

d, Combinations of Prisms with Pyramids, such as are described in § 210—212, are to be held with the prismatic planes in a vertical position.

Rules drawn from the form of the Equator. See §§ 87, 199.

e, If the equator is a square, and the crystal is a prism, or an unequiaxed pyramid, the angles of the equator are to be placed at nw, ne, sw, se.

f, If the crystal is an equiaxed pyramid, as PMT, the angles of the equator are to be placed at n, s, e, w.

g, If the crystal is a combination of a prism with a pyramid, and the pyramid or pyramidal combination is such a one as forms an equiaxed crystal when alone, the crystal must be so placed as to bring the planes of the pyramid or pyramidal combination into the polaric positions which are proper to them as the planes of a self-existing combination.

This rule refers specially to the combinations of the cube with the different octahedral forms.

h, If the pyramid which is found upon such a combination is one that produces an unequiaxed pyramid when it is separate from the prism, he combination must be placed so as to make the planes of the pyramid agree *with the symbols PM, PT,* and not with PMT.

i, If the equator is a rectangle, its longer diameter is to be placed upon the axis t^a.

j, If the equator is a rhombus, its obtuse angles are to be placed at the poles m^a_n, m^a_s.

k, If the equator is a combination of a rhombus with a square or of a rhombus with a rectangle, it is to be placed with the obtuse angles of the rhombus at the poles m^a_n and m^a_s.

The last three rules are however merely repetitions of *a*, the first rule.

Section V.—OF THE POSSIBLE LIMIT TO THE VARIETY OF PLANES THAT CAN OCCUR UPON CRYSTALS.

215. I have shown that with the exception of the planes denoted by PMT, no single set contained in the synopsis § 200, can of itself produce a complete crystal. Yet all the planes which occur upon crystals belong to one or other of the varieties described in this synopsis, whence it follows, that *all crystals consist of combinations of these sets of planes, and may be denoted by combinations of their symbols.* As the letters of our alphabet, which are not words alone, serve to form all written words by combination, so these planes, few in number and simple in their relations, but incomplete of themselves, produce by combination all the immense variety of perfectly crystallised forms which is presented to our notice, not only in the mineral kingdom, but in the factitious productions of the chemist's laboratory.

Our Synopsis of Symbols, § 200, is, therefore, a Crystallographic Alphabet. It represents not simply a few of the planes which are found here and there upon particular crystals, but it shows all *the planes* that can occur upon crystals considered collectively. That this is a true proposition will be manifested by an examination of the general relations of the planes which these symbols serve to denote.

216. *Of the Plane* P.—Its essential characters are—to be *horizontal;* to be parallel to the axes m^a and t^a; and to cut the axis p^a. It cannot lose any of these characters without ceasing to be the plane P. If you imagine it to pass ever so little out of the perfectly horizontal position, it no longer remains parallel to the axes m^a and t^a. If one edge of it descends in the front towards the axis m^a, it becomes the plane PM_+. If it descends sidewise towards the axis t^a, it becomes the plane PT_+. If it descends cornerwise, it falls upon the two axes m^a and t^a, and becomes the plane PM_+T_+. And it *cannot* pass out of the horizontal position *without falling* either upon the axis m^a or the axis t^a, or upon m^a and t^a jointly. The position of the plane P is therefore unalterable. Its characters are distinct and cannot be confounded with those of any other plane; and the lengthening or shortening of any of the three axes *of the crystal* has no effect upon the essential characters of the plane P.

217. *Of the Plane* M.—Its essential characters are—to be *vertical;* to be parallel to the axes p^a and t^a; and to cut the axis m^a. It cannot lose one of these characters without ceasing to be the plane M. If it incline ever so little towards the axis p^a, it becomes the plane P_+M. If it incline towards the axis t^a, it becomes the plane MT_+. If it incline cornerwise, it becomes P_+MT_+.

218. *Of the Plane* T.—Its essential characters are—to be *vertical;* to be parallel to the axes p^a and m^a; and to cut the axis t^a. It cannot pass out of its strictly vertical and parallel position, without ceasing to be the plane T, and becoming M_+T, P_+T, or P_+M_+T.

219. *Of the Plane* MT.—Its essential characters are—to be *vertical;* to cut the axes m^a and t^a; and to be parallel to the axis p^a. It may cut the axis m^a and t^a either at *equal* distances from the centre of the crystal, or at *unequal distances,* and may consequently produce many varieties of the plane, as MT, MT_+, and M_+T. But it must never cease to be vertical, to cut the two horizontal axes, and to be parallel to the axis p^a.

220. The foregoing are all the planes that occur upon prisms, and their characters are perfectly distinct from those of the planes that are found upon pyramids. The planes P are distinguished by their horizontal position, and the planes M,T, MT by their vertical position.

221. *Of the Planes* PM.—Their essential characters are—to be *inclined* from the middle of the top and bottom towards the middle of the front and back of the crystal, so as to cut both the axis p^a and the axis m^a, and to be parallel to the axis t^a. No other planes than PM can have these properties, and no variety of the planes PM, produced by extension of the axes p^a or m^a, can cease to possess these essential characters. All the varieties of PM_+ and P_+M, like PM, cut the axes p^a and m^a, and are parallel to the axis t^a.

222. *Of the Planes* PT.—Their essential characters are—to be *inclined* from the middle of the top and bottom towards the middle of the left and right sides of the crystal, so as to cut the axes p^a and t^a, and to be parallel to the axis m^a. No alteration in the relative lengths of the three axes p^a m^a t^a can have any effect upon these characters, and no planes which are without these characters can be denominated PT.

223. *Of the Planes* PMT.—Finally, the planes PMT are *inclined;* they cut all the three axes, and they are parallel to none. They may cut all the axes equally or all unequally, or two of them equally and one otherwise. But *they must cut all the three axes and be parallel to none.* These characters are perfectly distinctive, since none of the other sets of planes cut above *two* axes, and each of them is *parallel* to at least one axis. *The thirteen varieties of* the complement PMT contained in the

eighth column of the Synopsis of Planes, § 200, all possess these essential characters of PMT, and only differ among themselves in respect to the relative distances from the centre of the crystal at which the edges of their planes cut the three axes.

224. These are all the planes that occur upon pyramids. They are distinguished from the planes of prisms by being inclined to the equator, whereas the prismatic planes are either perpendicular to, or parallel with, the equator.

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225. Being now fully acquainted with the positions and mutual relations of the planes contained in the Synopsis, so as not to be liable to mistake one for another, or to confuse them with any thing different, let us turn to the Diagram contained in paragraph 20, and endeavour to trace upon it *a plane* DIFFERENT from those whose properties we have examined. I have depicted in this Diagram the *system of three axes crossing one another at right angles in the centre*, and round about it, what we will at present assume to be an amorphous mass of crystallizable matter. This mass is bounded by the lines numbered 1 2 3 4 5 6 7 8 9 10 11 12.

1st, If we propose to trace a plane parallel to any one of the surfaces of this mass, that is to say, a plane that shall cut any one of the three axes, and be parallel to the other two, we merely repeat the planes P, M, or T.

2dly, If we propose to trace a vertical plane through the points 1 5 10 8, or through the points 2 4 11 7, or through any part of the solid so as to be parallel to the vertical axis p^a, whatever the possible direction of the plane may be as respects the two axes m^a and t^a, we do in every such attempt produce but different varieties of the planes MT, MT_+, and M_+T.

3dly, If we propose to trace a plane that shall cut off the edge 1 M 2 at any possible degree of inclination towards the axes p^a or m^a, or if we attempt to cut through the mass in any direction from the side 2 4 10 8 to the side 1 5 11 7, retaining a parallelism to the axis t^a, we shall, in every case, produce planes that are nothing else than varieties of PM, PM_+, or P_+M.

4thly, If we propose to trace a plane through the points 2 5 11 8 or through the points 1 4 10 7, or through any other part of the mass so as to bisect at any angle the axes p^a and t^a and to be parallel to the axis m^a, we produce in every attempt nothing but varieties of PT, PT_+, and P_+T.

5thly, If, abandoning our attempts to produce new planes by cutting off any *face*, or any *edge*, of the mass 1 to 12, we try what can be done by operating upon its *corners*, we shall find our exertions to be equally fruitless,—

For, if we cut off a portion which leaves a triangular surface of three *equal sides, we simply* produce the plane PMT.

If the surface produced by the section has two sides short and one long, it depends only upon the *direction* of the section, whether we produce the plane P_-MT, PM_-T, or PMT_-, but one of these it *must* be.

If the section has two sides long and one short, it again depends only upon the direction of the section whether we produce the plane P_+MT, PM_+T, or PMT_+, but it must be one of these planes.

If the surface produced by the section has three unequal sides, then, exactly according to the direction of the section, we have the plane P_-MT_+, P_+M_-T, PM_+T_-, P_-M_+T, PM_-T_+, or P_+MT_-, and we cannot have anything else.

Repulsed at all these points, we shall find it to be impossible to attack the solid in any new direction. We cannot imagine a section that shall act otherwise than upon a face, or an edge, or a corner, of the mass figured in the diagram. We have therefore exhausted all the possible cleavages, and are warranted in coming to the conclusion that—

226. *There cannot occur upon any crystal a plane different from those denoted by the symbols* P,M,T, MT. PM, PT, PMT, either written alone, or subscribed by numbers, or by the signs + and –, as represented in the Synopsis, § 200. This is the limit to the variety of planes that can possibly occur upon crystals.

Section VI.—OF CRYSTALLOGRAPHIC NOTATION.

227. Admitting the correctness of the proposition contained in § 226, that " there cannot occur upon any crystal a plane different from those indicated by the symbols P,M,T, MT_x. PM_x, PT_x, $P_xM_xT_x$," then Crystallography—*the Art of Describing Crystals*—consists simply in enumerating the symbols which designate the planes that we observe upon the crystals that we wish to describe. Thus—

A crystal upon which we find the planes P,M,T, is described by the repetition of the symbols P,M,T.

A crystal which has the planes MT.PM, PT, is described by the repitition of the symbols MT.PM,PT.

A crystal which has the planes PMT is described by the repetition of the symbol PMT.

A crystal which has the planes P,M,T,MT.PM,PT,PMT, is described by the repetition of the symbols P,M,T, MT. PM, PT, PMT.

And when the crystals are not equiaxed, we have only to add the signs + or – or $_x$ or a number, as the case may demand, to the letters which refer to the axes whose peculiarities we desire to describe.

228. The symbols are arranged in the order in which they appear in the Synopsis, § 200. Those which denote prismatic planes take precedence of those which denote pyramidal planes, and those in the left hand columns take precedence of those in the right hand columns; so as to produce the following series of symbols :

$P, P_-, P_+, M, M_-, M_+, T, T_-, T_+, MT, MT_+, M_+T. PM, PM_+, P_+M, PT, PT_+, P_+T, PMT, P_-MT, PM_-T, PMT_-, P_+MT, PM_+T, PMT_+, P_-MT_+, P_+M_-T, PM_+T_-, P_-M_+T, PM_-T_+, P_+MT_-.$

This is a universal formula, which comprehends every possible variety of planes, and of which every other symbol must be an abridgement. In making these other symbols by abridgement, the rule to follow is, to *omit the signs of all absent planes, and write the signs of the planes that are present, in the order in which they stand in this universal formula.* Every symbol from P to M_+T, and from PM to P_+MT_- inclusive, is separated from every other by a comma (,)—but every series of prismatic planes is separated from every series of pyramidal planes by a period (.).

The method of denoting the *halves* and *fourths* of sets of planes, has been explained in §§ 19, 22, 23, 24, 26, 88, 101, 125.

The method of denoting the *comparative sizes* of the planes of different sets which occur together on complex combinations has been described in §§ 67 to 69.

The method of describing the lengths of the axes of crystals, as distinguished from the axes of their complements of planes, has been explained in §§ 1 to 14, and § 76.

229. Twin Crystals.—There is a variety of crystallized form, commonly called a *twin crystal*, or *double crystal*, or *macle*, which consists of two crystals piercing one another, or of two halves of a crystal joined together in a position more or less inverted. This variety can be denoted by adding the sign × 2 to the symbol of the complete single crystal, or by enclosing the latter symbol when complex within parentheses and then adding the sign × 2.—*Examples:* PMT × 2, Model 16.—(P_x, T_+, MT_+) × 2. Model 9.—There are many different kinds of these mixed crystals, which could perhaps all be denoted by as many different symbols ; but I question whether it is worth while to burthen our books and memories with different symbols for such a purpose.

230. The following catalogue of the observed forms of the crystals of Fluorspar will serve to illustrate the use of this notation, while it also shows the application of the principles of classification explained in § 213.

Class I. *Complete Prisms.* Order 1. *Square Equator.*

P,M,T. The Cube, Model 1.

Class II. *Complete Pyramids.* Order 1. *Square Equator.*

PMT. The Regular Octahedron. Model 15.

3 P_3MT. The Triakisoctahedron. A form of the same kind but not having the same angles as Model 17. See § 160.

The symbol $3P_3MT$ is equal to P_3MT, PM_3T, PMT_3. See § 200.

PMT, $3p_3mt$. The Octahedron, PMT, combined with the Triakisoctahedron, $3p_3mt$.

The symbols and words that are written in capital letters distinguish the forms that predominate.

CLASS III. *Complete Prisms combined with Incomplete Pyramids.*

ORDER 1. *Square Equator.*

a) The Cube predominant.

P,M,T,mt.pm,pt. The CUBE, P,M,T, combined with the rhombic dodecahedron, mt.pm,pt. Model 27.

P,M,T,mt.pm,pt,pmt. The CUBE, P,M,T, combined with the rhombic dodecahedron, mt.pm,pt, and the octahedron, pmt. Model 32.

Every plane on this model is marked with the symbols of its different forms.

P,M,T,$3p\frac{1}{2}mt$. The CUBE, P,M,T, combined with the icositessarahedron, $p\frac{1}{2}mt,pm\frac{1}{2}t,pmt\frac{1}{2}$. Model 40.

P,M,T,mt.pm,pt,$3p\frac{1}{3}mt$. The CUBE, P,M,T, combined with the rhombic dodecahedron, mt.pm,pt, and the icositessarahedron, $p\frac{1}{3}mt,pm\frac{1}{3}t$, $pmt\frac{1}{3}$.

P,M,T. $6p\frac{1}{4}m\frac{1}{2}t$. The CUBE, P,M,T, combined with the Hexakisoctahedron, $p\frac{1}{4}m\frac{1}{2}t,pm\frac{1}{4}t\frac{1}{2},p\frac{1}{2}mt\frac{1}{4},p\frac{1}{4}mt\frac{1}{2},p\frac{1}{2}m\frac{1}{4}t,pm\frac{1}{2}t\frac{1}{4}$. Model 41.

P,M,T,MT.PM,PT,$3p\frac{1}{3}mt,6p\frac{1}{4}m\frac{1}{2}t,6p\frac{1}{11}m\frac{1}{5}t\frac{1}{3}$. The CUBE, P,M,T, combined with the *Rhombic Dodecahedron*, MT,PM,PT; the icositessarahedron, $3p\frac{1}{3}mt$; the hexakisoctahedron, $6p\frac{1}{4}m\frac{1}{2}t$; and the hexakisoctahedron, $6p\frac{1}{11}m\frac{1}{5}t\frac{1}{3}$.

P,M,T. PMT. The middle crystal betwixt the cube and the octahedron, in which neither form predominates. Model 29.

b) The Octahedron predominant.

p,m,t. PMT. The cube p,m,t, combined with the OCTAHEDRON, PMT. Model 30.

P,M,T,mt.pm,pt,PMT. The *cube*, p,m,t, combined with the rhombic dodecahedron, mt.pm,pt, and the OCTAHEDRON, PMT. Model 33.

p,m,t,MT.PM,PT,PMT,$3p\frac{1}{3}mt$. The cube, p,m,t, combined with the rhombic dodecahedron, mt.pm,pt; the OCTAHEDRON, PMT; and the icositessarahedron, $3p\frac{1}{3}mt$.

ORDER 4. *Rhombo-Rectangular Equator.*

p,m,t,$MT_3,M_3T.PM_8,P_3M,PT_3,P_3T$. The cube, p,m,t, combined with the TETRAKISHEXAHEDRON, $MT_3,M_8T.PM_3,P_3M,PT_3,P_3T$.

P,M,T,$mt_3,m_3t.pm_3,p_3m$,pt ,p_3t. The CUBE P,M,T, combined with the Tetrakishexahedron, $mt_3,m_3t.pm_3,p_3m,pt_3,p_3t$. Model 39.

P,M,T,mt,mt_3,m_3t.pm,pm_3,p_3m,pt,pt_3,p_3t. The CUBE, P,M,T; combined with the rhombic dodecahedron, mt.pm,pt; and the tetrakishexahedron,$mt_3,m_3t.pm_3,p_3m,pt_3,p_3t$.

P,M,T,mt,mt_3,m_3t.pm,pm_3,p_3m,pt,$pt_3,p_3t,3p\frac{1}{2}mt$. The foregoing combination with the addition of the icositessarahedron, $p\frac{1}{2}mt,pm\frac{1}{2}t,pmt\frac{1}{2}$.

p,m,t.$6P\frac{1}{4}M\frac{1}{2}T$. The Cube, P,M,T, combined with the HEXAKISOCTAHEDRON, $6P\frac{1}{4}M\frac{1}{2}T$.

P,M,T,$mt_3,m_3t.pm_3,p_3m,pt_3,p_3t,6p\frac{1}{4}m\frac{1}{2}t$. The CUBE, P,M,T, combined with the tetrakishexahedron, $mt_3,m_3t.pm_3,p_3m,pt_3,p_3t$, and the hex*akisoctahedron*, $6p\frac{1}{4}m\frac{1}{2}t$.

P,M,T,MT,$mt\frac{5}{2}$,$m\frac{5}{2}t$.PM,$pm\frac{5}{2}$,$p\frac{5}{2}m$,PT,$pt\frac{5}{2}$,$p\frac{5}{2}t$,PMT. The CUBE, P,M,T, combined with the *rhombic dodecahedron*, MT.PM,PT; the tetrakishexahedron, $mt\frac{5}{2}$,$m\frac{5}{2}t$.$pm\frac{5}{2}$,$p\frac{5}{2}m$,$pt\frac{5}{2}$,$p\frac{5}{2}t$; and the *octahedron*, pmt.

P,M,T,mt,$mt\frac{3}{2}$,mt_{10},$m\frac{3}{2}t$,$m_{10}t$.pm,$pm\frac{3}{2}$,pm_{10},$p\frac{3}{2}m$,$p_{10}m$,pt,$pt\frac{3}{2}$,pt_{10},$p\frac{3}{2}t$,$p_{10}t$,pmt,$3p\frac{1}{3}mt$,$5(6p_xm_yt_z)$. The CUBE, P,M,T, combined with the rhombic dodecahedron, mt.pm,pt; the tetrakishexahedron, mt_3,m_3t,pm_3,p_3m,pt_3,p_3t; the tetrakishexahedron, mt_{10},$m_{10}t$.pm_{10},$p_{10}m$,pt_{10},$p_{10}t$; the octahedron, pmt; the icositessarahedron, $p\frac{1}{3}mt$,$pm\frac{1}{3}t$,$pmt\frac{1}{3}$; and five different varieties of the hexakisoctahedron, $6p_xm_yt_z$.—This crystal possesses 338 planes.

CLASS IV. *Incomplete Prisms combined with Complete Pyramids.*

ORDER 1. *Square Equator.*

MT.PM,PT. The RHOMBIC DODECAHEDRON. Model 63.

mt.pm,pt,PMT. The rhombic dodecahedron, mt.pm,pt, combined with the OCTAHEDRON, PMT. Model 64.

ORDER 3. *Rhombic Equator.*

MT_3,M_3T.PM_3,P_3M,PT_3,P_3T. The Tetrakishexahedron. General form of Model 68, but with different angles.

ORDER 4. *Rhombo-Quadratic Equator.*

mt.pm,pt,PMT,$3p\frac{1}{3}mt$. The rhombic dodecahedron, mt.pm,pt, combined with the OCTAHEDRON, PMT; and the icositessarahedron, $3p\frac{1}{3}mt$.

mt,mt_3,m_3t.pm,pm_3,p_3m,pt,pt_3,p_3t,PMT. The rhombic dodecahedron, mt.pm,pt, combined with the tetrakishexahedron, mt_3,m_3t.pm_3,p_3m,pt_3,p_3t; and the OCTAHEDRON, PMT.

SECTION VII.—OF CLEAVAGE AND PRIMITIVE FORMS.

231. Certain minerals can be *cleaved* or *mechanically divided* in particular directions, which vary with different substances. The cleavages, or planes produced by this mechanical division of crystals, have properties similar to those of their superficial planes. They are horizontal, vertical, or inclined. They cut one, two, or three of the axes of the crystals; they have definite polaric positions; and they can consequently be denoted by the same symbols which serve to denote the external planes.

232. The cleavages of crystals are sometimes parallel to their external planes and sometimes not so. Thus, when the external planes of a crystal are P,M,T, the cleavages may be P,M,T; MT.PM,PT; or PMT. There is no known connection between external form and internal *cleavage, so that it* is necessary to discover the cleavage or cleavages

of every particular mineral and of every different form by mechanical division. Some minerals afford no cleavage; others afford two or three different kinds of cleavage, in respect of quality or perfection; such as—1, very distinct; 2, less distinct; and 3, indistinct. I propose to denote these varieties by altering the size of the letters that indicate the cleavage; as—

P,M,T	a very distinct cleavage.
MT.PM,PT	a less distinct cleavage.
pmt	an indistinct cleavage.

Numerous examples of the application of these symbols are given in the second part of this work, between pages 16 and 94, where the relations of the cleavage planes to the external planes of crystallised minerals may be seen at a glance.

233. When the cleavages of a mineral are so numerous and so arranged as to constitute two or more prisms of four sides which cross or cut one another, they *cleave* or *cut out* from the crystal, particular geometrical solids, as cubes, octahedrons, rhombohedrons, and so forth. These productions of cleavage have been called *primitive forms*, and it has been assumed by some mineralogists, that they represent the forms of the ultimate molecules of which the whole mass of the crystal from which they are cleaved is composed. But this is an assumption which is entirely incapable of proof, and which is of no use except in so far as it served to render intelligible the systems of crystallography which first brought the term into use. In reality, the *primitive form* of a mineral shows the direction and the number of its cleavages, and nothing more. The use which Romé de L'Isle and Haüy made of the term was, first to give *names* to a variety of "primitive forms," and then to name all other forms according to certain degrees of resemblance which they bore to the forms which had been assumed to be *primitive* forms. The doctrine of primitive forms, therefore, can only be considered as an ingenious artifice of Romé de L'Isle and Haüy, adopted for the purpose of generalising their views of the relation borne by the secondary forms of crystals to one another, which, indeed, is what Haüy expressly admits: "The primitive form (noyau) of a crystal," he says, "is merely a theoretical datum, taken to facilitate the determination of the different crystalline forms belonging to the same substance." *Traité de Cristallographie*, t. i. p. 65. The doctrine of primitive forms was a *use* to which these crystallographers turned their knowledge of the facts derived from the observation of cleavage. They built their theories upon these facts; but although the facts remain true, it does not follow that the theories are true also; and although the doctrine of primitive forms is the part of Haüy's System of Crystallography which has received the most general approbation of mineralogists, I am inclined to believe that the assumption is more injurious than useful, and accordingly I have recommended a system of notation which entirely dispenses with it. The *very common use of the term primitive form*, may, however, induce

some to demand a proof of its alleged injurious tendency: to which I reply by pointing out a few of its practical results, which I think are sufficient to condemn it. One of these results is, that it leads mineralogists to describe forms of minerals which do not exist, namely, the ideal primitives, and to omit to describe the forms which do exist, namely, the forms of actual occurrence, which, being nick-named *secondary*, are treated as if they were of *secondary* or of *no* importance; hence, when a student of mineralogy begins to compare crystallized minerals with the descriptions in his books, he finds the two not to agree, most of the commonly occurring crystals of minerals being summarily dismissed as "secondary forms derived from such or such a primitive form." The student then frequently abandons the study of crystallography in despair. I think it better to describe, as the crystals of each mineral, the forms that *really occur in nature*, and not the forms which, to serve the purposes of a scientific hypothesis, are *assumed to be contained* in the natural forms. In justice to Haüy, it is right to add, that he described in his work on Mineralogy the "secondary" or *real*, as well as the "primitive" or *assumed* forms of minerals; but many of his disciples content themselves with describing the primitive or non-existing forms alone, by which they save trouble to themselves but not to their readers.

Another result is, the difference of opinion produced among mineralogists, and the controversies which result, as to the true primitive form of particular minerals. Thus, the primitive form of iron pyrites is, according to Haüy, the cube, according to Leonhard the pentagonal dodecahedron, and according to Phillips the regular octahedron. This difference of opinion respecting the primitive form of a mineral so well known as iron pyrites, shows how little the character is to be depended upon; while it is fatal to any attempt to introduce a systematic nomenclature, or systematic symbols, to represent complex crystals.

Even the mineralogists who employ the term "primitive form," do so with a constant reliance upon its uncertainty. Thus, Phillips says, "*If* a mineral can be mechanically divided or cleaved in directions which produce only one particular form, that form is denominated its primary or primitive crystal. But some minerals are not so circumstanced...Fluorspar cleaves in four directions, and affords three different forms, a regular octahedron, a regular tetrahedron, and an acute rhomboid; of these, the first has *arbitrarily been selected* as the primary crystal, and *convenience* may be assigned as the reason for the preference...Other substances are cleavable in a still greater number of directions; for instance, blende, from which may be extracted a rhombic dodecahedron, and from this an obtuse rhomboid, an octahedron, an acute rhomboid, and an irregular tetrahedron; in this mineral also, the *choice* of a primary crystal has been *arbitrary*, the rhombic dodecahedron having been *selected*...The arbitrary selections just noticed will suffice to induce the suspicion, that in this department mineralogy has not yet attained perfection; and also to lead the pupil to investigate as he advances in the *science, rather* than to take for granted what is asserted without proving

facts....Other circumstances also exist, sufficient to make us ex- nely cautious on this point. Some minerals to which primary forms e *been assigned*, do not yield, or have not yet been found to yield, regular cleavage in more than one direction, or even not in any ction. In these determinations, one of two modes has been resorted —In the first, thin fragments of the substance have been held up veen the eye and the light; and by this means the extraordinary acity of the Abbé Haüy has enabled him, in several instances, to lare the *probable form* of the primary, from the directions of the rices, or appearances of natural joints which may be observed in the ment; and, in many, these have afterwards proved to be correct. the other mode, the primary form is determined by *analogy*, that is, a *comparison of the forms of the crystals of a mineral with those of r known substances;* but this may in some cases prove a source of r."—R. PHILLIPS, *Introduction to Mineralogy*, article Structure.

he inferences which may be fairly drawn from the above statements these: 1.) The term primitive form sometimes indicates the direction number of the cleavages of a mineral; 2) sometimes the direction number of only a *portion* of the cleavages; and 3) sometimes merely s the crystallographer's opinion of what the cleavage form of a eral would be if the cleavages were more numerous and more distinct nature has happened to make them. It appears from this, that the rine of primitive forms has nothing to recommend it, beyond its umed convenience as an indicator of cleavage, and that when it ends to indicate any thing beyond cleavage, it is perpetually liable to us astray. But the planes produced by cleavage having properties lar to those of the external planes of crystals, and being capable of ation by methods similar to those employed to represent the external es, it follows that we do wrong to adopt a doctrine so uncertain and so fe as this, merely to gain an end which can be better gained without For which reason I have dispensed altogether with the use of the primitive form, and should have dispensed even with the mention , but that I consider it due to those who may wish to know "how I nguish the primitive from the secondary forms," which question has ral times been put to me, to give my reasons for making no such nction. By dispensing with the doctrine in question, we are left at ty to describe every crystal presented to us by nature, according to al aspect, and are not constrained to trim our descriptions to make suit the limited range of a preconceived hypothesis.

14. HAUY'S PRIMITIVE FORMS are as follow:

The Cube.	Model	1.	P,M,T.
Octahedron.		15.	PMT.
Rhombic Dodecahedron.		63.	MT.PM,PT.
Tetrahedron.		117.	$\frac{1}{2}$PMT.
Pentagonal Dodecahedron.		91.	M$\frac{1}{2}$T.P$\frac{1}{2}$M,P$\frac{2}{1}$T.

Haüy's Primitive Forms, Continued:

	Model	
6. Quadratic Prism.	2.	$P\frac{1}{3}$,M,T.
	3.	$P\frac{5}{4}$,M,T.
7. Quadratic Octahedron.	12.	$P\frac{2}{3}$MT.
	13.	$P\frac{5}{2}M,P\frac{5}{2}T$.
8. Right Rectangular Prism.	5.	P_x,M_,T.
9. Rhombic Octahedron.	21.	$P\frac{1\,0}{1\,0}M_1\frac{8}{0}T$.
10. Rectangular Octahedron.	82[b].	$M\frac{4}{3}T.P\frac{5}{4}T$.
11. Rectangular Ditetrahedron.	82[a].	$M_1\frac{6}{0}T.P\frac{7}{1\,0}T$.
12. Right Rhombic Prism.	6.	$P_x,M\frac{4}{3}T$.
13. Oblique Rectangular Prism.	79.	M_,T. $\frac{1}{3}P\frac{3}{7}M$ Zn Ns.
14. Right Rhomboidal Prism.	79[b].	M_,T. $\frac{1}{3}P\frac{1}{3}M$ Zn Ns.
15. Oblique Rhombic Prism.	84.	$M\frac{1\,0}{1\,9}T.\frac{1}{3}P\frac{4}{1\,3}M$ Zn Ns.
	87.	$M\frac{2\,0}{2\,1}T.\frac{1}{3}P\frac{6}{2\,1}T$ Zw Ne.
16. Oblique Rhomboidal Prism.	105.	T, $\frac{1}{3}M\frac{1\,5}{2\,6}$T ne sw. $\frac{1}{3}P\frac{1}{3}M$ Zn Ns.
17. Rhombohedron.	26[a].	$\frac{1}{2}PT$ Zw, $\frac{1}{2}PM\frac{1\,5}{1\,3}T_2$: or R_1.
	26[d].	$\frac{1}{2}P_2T$ Zw, $\frac{1}{3}P_2M\frac{1\,5}{1\,3}T_2$: or R_2.
18. Regular Six-sided Prism.	7.	P_x,T, $M\frac{1\,5}{1\,3}T_2$: or P_x,V.
19. Bipyramidal Dodecahedron.	26.	$P\frac{1\,3}{1\,3}T$, $P\frac{1\,3}{1\,3}M\frac{1\,5}{1\,3}T_2$: or $2R\frac{1\,3}{1\,3}$.

The combination represented by Model 32, which exhibits the Cube, with its edges and solid angles replaced, contains all the planes of *three* of the above-mentioned *primitive forms*, namely, the cube, the rhombic dodecahedron, and the octahedron. This is also the case with Models 33 and 34. In one of these combinations the cube predominates, in another the dodecahedron, in another the octahedron. Which of these three is the *primitive form* of the minerals that the combinations represent? Is it that which predominates? If so, some minerals must have several primitive forms, since it is common to find many combinations of one mineral in which different forms predominate. Is the primitive form to be determined by cleavage? If so, does not this admit, that the term "primitive form" expresses merely the direction and number of the cleavages of minerals?

All the combinations represented by these three models occur among the separate crystals of arsenical grey copper, the cleavage form of which is the dodecahedron, which combination also occurs among its separate crystals. But the dodecahedron also occurs among the separate crystals of antimonial grey copper, and among other crystals of the same variety we find the tetrahedron predominant, but never the cube nor the octahedron. What is the cleavage of antimonial grey copper? There is none! We find a third variety of grey copper, the arsenical-antimonial grey copper, a chemical compound or mixture of the other two kinds. The forms predominant among the crystals of this third variety are the dodecahedron, the tetrahedron, and the hemiicositessarahedron, but never the cube nor the octahedron. What is the cleavage of this variety? No man could foretell it: it is imperfect *octahedral!* But neither *predominant form* nor cleavage has served to guide mineralogists in assign-

ing primitive forms to these minerals, for although we find a tetrahedron assigned to that kind whose cleavage is imperfect octahedral, yet we are told that the arsenical variety with a perfect dodecahedral cleavage, has for primitive form an octahedron. *See* PHILLIPS.

235. MR. BROOKE'S "primary forms" consist of the same variety as Haüy's "primitives," with the exception of Nos. 5, 11, 13, and 19. The Models comprehend examples of all these primitive forms, and I have added the symbols by which they are designated in the present system. In the works of Haüy and Brooke, the planes of *secondary forms* are considered to indicate decrements, or portions cut off from the edges or angles of the primitive forms, and they are indicated by formulæ which show the comparative length of the portions of different edges supposed to be removed by each secondary plane, so that the symbols represent, not a form existing upon the resulting crystal, but one that is supposed to have been cut off and removed.

236. According to the present system of crystallography, every set of possible secondary planes that can occur by replacing any set of similar angles or similar edges on either of the above 19 combinations, must belong to one of the seven forms which are represented by the following symbols:

$$P, M, T, M_x T. P_x M, P_x T, P_x M_y T_z,$$

or to one of those hemihedral or tetartohedral modifications of these forms, which are described in SECTION X.

SECTION VIII.—OF FORMS AND COMBINATIONS.

237. OF FORMS.—The word "Form," separated from its mischievous qualifier "primitive," may be employed very usefully in crystallography. In all the preceding sections, when I have had to speak of the planes indicated by the symbols $P, M, T, M_x T, P_x M, P_x T$, or $P_x M_y T_z$, I have used one of the expressions, *set of planes*, or *complement of planes*. Now, instead of either of these expressions, I propose to employ the word "form," to denote all the planes that a given symbol can indicate. In that case, the form P, for example, will signify two planes; the form MT, four planes; the form PMT, eight planes; the form $\frac{1}{2}$MT, two planes; and the form $\frac{1}{2}$PMT, four planes. Every "set of planes" exhibited in the synopsis of planes, § 200, page 68, is therefore a "Form"; and, let it be particularly remarked, a "form" is *only one set* of planes. With this precise and limited signification, I shall employ this term in the following sections.

238. The definition of the term "form" given in Professor MILLER'S lately published *Treatise on Crystallography*, is as follows: "A form is the figure bounded by a given face and the faces which, by the laws of symmetry of the system of crystallisation, are required to co-exist with it. A form will be *denoted by the* symbol of any one of its faces inclosed

in braces. Thus, the symbol $\{hkl\}$ will be used to express the form bounded by the face (hkl) and its co-existent faces."

In this case, a person can only know the number of co-existent faces belonging to any given "form," when he knows also three other things: 1.) What system of crystallisation it is to which the "form" belongs. 2.) What are the laws of symmetry of that system of crystallisation. 3.) How those laws affect the "form" in question. When he knows all this, he can tell the number of planes belonging to a given "form"; that is to say, the number of times that (hkl) is contained in $\{hkl\}$. But this comprehensiveness of meaning appears to me to make the term difficult of use to students; and for this reason I have set narrower limits to its application, and, I think, rendered its meaning less liable to misconception.

The symbols given in the present work always denote all the planes belonging to a form, except when they have the vulgar fraction $\frac{1}{2}$ or $\frac{1}{4}$ prefixed, or when, as occurs in a few rare cases, the polaric position indicative of a single plane is added to the symbol of its form, as PZ, which means the zenith plane of the form P without the nadir plane.

239. OF COMBINATIONS.—Professor MILLER's definition of the term *combination* is this: "The figure bounded by the faces of any number of forms, is called a combination of those forms." In this definition I concur, for although it was meant to refer to the term "form," as defined by the Professor himself, it serves equally well to denote combinations of the "forms" that are represented by the symbols of the seven sets of planes P,M,T,M_xT.P_xM,P_xT,$P_x M_y T_z$. Therefore,

A "Combination" is a crystal whose faces consist of two or more "Forms."

It is impossible to fix, theoretically, a limit to the number of forms that may occur upon a combination. It is the business of the mineralogist and the chemist, in applying crystallography to their respective sciences, to point out the combinations that really occur, either in the mineral world or among the products of the laboratory.

240. The following is a free translation, adapted to the illustrations which accompany the present work, of GUSTAV ROSE's account of the terms "form" and "combination":—

"The forms of crystals differ essentially in this, that their faces are either all alike, or (leaving parallelism out of the question) partly or entirely unlike. The first are called *simple forms*, the last *compound forms*. The octahedron, Model 15, or the figure on page 41, which is bounded by eight equilateral triangles; the cube, Model 1, or the figure on page 10, which is bounded by six squares; or the six-sided pyramid, Model 26, which is bounded by twelve isosceles triangles, are consequently simple forms; while the ordinary crystal of Galena, Model 29, which is bounded by eight equilateral triangles and six squares; and the quartz crystal, Model 73, which is bounded by twelve isosceles triangles *and six rectangles*, are both compound forms.

"The simple forms differ from one another in the number, the figure, and the relative inclinations of their faces, and they are consequently very different in aspect. Yet the position of their faces, considered in relation to the middle point of the crystal, is always regulated by a determinate law of symmetry. All faces, edges, and corners, with very few exceptions, have faces, edges, and corners parallel to them; and in most cases, one end of a crystal possesses exactly the same faces, edges, and corners that appear on its other end; so that the crystallographer has commonly occasion to study only one end of the crystal. But it does not always follow, that when a simple form has similar faces, it also has similar edges or corners. The contrary has indeed been shown in the examples already adduced, since the octahedron and cube have similar edges and corners, whereas the six-sided pyramid has edges and corners of two different kinds. Hence the term 'simple form' has not the same meaning in crystallography as in geometry. Many simple forms have dissimilar corners and similar edges, as the rhombic dodecahedron, Model 63, or the upper figure on page 13; others have both edges and corners dissimilar, as is the case with the six-sided pyramid; still, in the latter case, the corners are generally symmetrical.

"The simple forms are named after the number and figure of their faces, or after other characteristic peculiarities. The faces which bound the forms are denoted by the same names as the forms themselves; that is to say, the faces of the octahedron are called octahedron faces, the faces of the dodecahedron, dodecahedron faces, &c. In notation, the faces are distinguished by letters or figures; the faces of one and the same simple form receive the same letter or figure, the faces of different forms different letters or figures.

"If we take a compound form and imagine any one set of its similar faces to be enlarged till they alone bound the inclosed space, and obliterate all the dissimilar faces, we then perceive a simple form. If, for example, we take the Galena crystal, Model 29, and in imagination thus enlarge the three-sided faces till they meet one another on all sides, we thereby produce the octahedron, Model 15. If, on the contrary, we enlarge the four-sided faces, we produce the cube, Model 1. Hence, we perceive that a compound form consists of two or more simple forms, or, generally speaking, of as many simple forms as it possesses dissimilar faces. None of these simple forms can naturally appear perfect on the compound, but only portions of each can be visible, and these separated from the other similar portions by intervening portions of dissimilar forms. The faces of each particular form bear a certain relation in respect of magnitude to the faces of the other forms that are present; in some cases being greater or *predominant*, in other cases being smaller or *subordinate*.

241. "As the compound form is a combination of simple forms, it is for that reason generally called a '*combination*.' A given combination is denoted by the names of the simple forms which it comprises, in the writing *of which we begin with* the name of the simple form which is

predominant, and place the subordinate forms afterwards; and when it is particularly necessary, the difference of magnitude is explicitly explained. Thus (P,M,T. pmt*), Model 29, and Model 30, are three different *combinations of the cube*, Model 1, *with the octahedron*, Model 15; and among these, Model 29 is the combination in which the two forms are equipoised; Model (P,M,T. pmt) is that in which the cube predominates, and Model 30 that in which the octahedron predominates."—*Elemente der Krystallographie*, page 2.

242. It will be immediately perceived, that, in consequence of the restricted meaning that I have given to the word "form," many crystals commonly called "forms," or *simple forms*, must on the proposed system be termed *combinations*; as examples of which I may notice the cube, which contains the three forms P,M,T, and the rhombic dodecahedron, which contains the three forms MT, PM, PT, both of which combinations are among the simple forms of Rose and Miller. It is generally of no consequence whether crystals such as these are called forms or combinations; but on the other hand, it is of infinite consequence that our technical terms should have meanings which can be easily found, easily understood, and easily remembered; and such I hope are the meanings given in this section to the terms "form" and "combination."

Methods of Indicating the General Aspect of Combinations.

243. The method of distinguishing the relative sizes of the planes of different forms, which Rose has described, § 241, appears to me to be far less effective than the method which I have described in § 69.

Models.	Rose's Symbols.	Proposed Symbols.
31.*	$(a : \infty a : \infty a) + (a : a : a)$.	P,M,T. pmt.
29.	$(a : \infty a : \infty a) + (a : a : a)$.	P,M,T. PMT.
30.	$(a : a : a) + (a : \infty a : \infty a)$.	p,m,t. PMT.

244. But, indeed, the symbolic description of the relative magnitudes of the planes of different "forms" upon a given "combination," has never been attempted by any crystallographer, as I shall show, by quoting a few specimens of notation from different works.

Hauy.

$P\overset{1}{A}\overset{1}{B}$ provided the "primitive form" is the cube.

$P\underset{1}{B}\overset{1}{B}\underset{1}{A}{}^{1}A^{1}$ provided the "primitive form" is the octahedron.

${}^{1}\underset{1}{A}{}^{1}\overset{3}{e}P^{1}E^{1}$ provided the "primitive form" is the dodecahedron.

* A cube, with its corners very slightly replaced by the planes of the octahedron. Equivalent to Model 31, without the twelve rectangular planes, or Model 38, with four additional triangular planes.

According to Haüy's method of notation, any one of the above three symbols can be used to indicate any one of the following three combinations:

Model 32. P,M,T, mt. pm, pt, PMT
— 33. P,M,T, mt. pm, pt, PMT
— 34. p,m,t, MT. PM, PT, pmt
} All containing the Cube, the Rhombic Dodecahedron, and the Octahedron.

Haüy's symbols, take which of the three you will, do not indicate the general appearance, *the aspect,* of the crystal. They only tell the ***dimensions of certain solids presumed to be cut off, by the planes of the combinations, from the corners or edges of the*** IDEAL SOLIDS ***which Haüy supposed to be contained in the combinations, and which he denominated their " primitive forms."***

MOHS.—The symbol employed by this crystallographer to indicate any one of the three combinations in question, is

H.D.O.

in which H. signifies the hexahedron or cube, D. the dodecahedron, and O. the octahedron. His symbols cannot discriminate one of the combinations from either of the two others, nor does he give any method of doing so except in words at length.

Another Example of comparative Notation:

Model 36. P,M,T, mt. pm, pt, ½pmt
— 34. p,m,t, MT. PM, PT. ½pmt
— 37. p,m,t, mt. pm, pt, ½PMT
{ In this case, half the planes of pmt shown upon model 34 are supposed to be suppressed, viz., those of the Zne Zsw Nnw Nse octants.

Here are three combinations which differ essentially in aspect, although they comprise the same suite of forms. The difference is owing to the predominance in one combination of the Cube, in another of the Dodecahedron, and in the third of the Tetrahedron, as is very well shown by the models and distinctly indicated in the above symbols. Let us see how these three combinations are described by other methods of notation.

WHEWELL.

$2\,(3)\,(1, 0, 0) + 2\,(6)\,(\pm\, 1, 1, 0) + (4)\,(\pm\, 1, 1, 1)$

which symbol answers for every one of the three combinations.

MILLER.

$\{100\}, \{110\}, \kappa\,\{111\}.$

which symbol answers for every one of the three combinations.

G. ROSE.

Model 36. $(a : \infty a : \infty a) + (a : a : \infty a) + \frac{1}{2} r\,(a : a : a)$.
— 34. $(a : a : \infty a) + (a : \infty a : \infty a) + \frac{1}{2} r\,(a : a : a)$.
— 37. $\frac{1}{2} r\,(a : a : a) + (a : a : \infty a) + (a : \infty a : \infty a)$.

The difference consists only in the priority of position which is given to the symbol that indicates the predominant form.

MOHS.

$H.\frac{O}{2}.D.$ *which symbol answers for* every one of the three combinations.

HAÜY.

1 11.011.0
PBA a e E provided the "primitive form" is the cube.

$P\overset{1}{B}B\underset{1}{A^2}\underset{2}{A^2}$ provided the "primitive form" is the tetrahedron.

${}^1\underset{1}{A}{}^1\,{}^1E^1P$ provided the "primitive form" is the dodecahedron.

Any one of these three symbols may be used to indicate any one of the three dissimilar combinations just as the primitive form permits.

MR. BROOKE's symbols for combinations are of the same character as Haüy's, and, like Haüy's, are based upon the suppositious primitive forms of minerals; so that a symbol for any combination of two or three forms, is not founded on crystallographic relations, but depends entirely upon the figure of the ideal primitive form of the mineral.

245. The quotations from Haüy appear to me to show very significantly the mischief that proceeds from making such a doctrine as that of "primitive forms" the basis of a system of notation. Here we have six examples of crystals, all easy of accurate description, for every one of which Haüy has three different and very complicated symbols, none of which give more than a vague idea of the aspect of the given crystals. Indeed the symbols seem to be contrived mainly for the purpose of showing the author's opinion respecting the ideal "primitive forms" of the minerals. This is a complete sacrifice of crystallographic precision upon the altar of theoretical mineralogy.

246. With respect to the subject we have immediately under discussion, namely, the power of denoting the relative magnitudes of different forms existing on the same combination, it may be questioned by some, especially when they see that the most eminent crystallographers have paid little or no attention to it, whether it is a subject that *deserves* attention? The answer to which question is, that any previous want of attention to accuracy in observing facts, or want of power to record them, forms no excuse for continued want of attention, when the value of the facts is made known and the power of recording them provided; and I believe that I may safely add, that no one will glance over the tables in the Second Part of this work without becoming assured that differences in the relative magnitude of forms is a crystallographic character of considerable value, and one which can be indicated with such extreme facility that it would be absurd to neglect it. The silence of former crystallographers respecting the use of symbols capable of indicating differences in the aspect of such combinations as contain forms of similar quality but of unlike magnitude, may be accounted for by supposing them to have been without the symbols, and not without the will to use them; particularly as we find crystallographers who make no distinctions in their symbols, giving special instructions for distinguishing the different combinations by words at length.

SECTION IX.—ZONES.

247. The planes of crystals are ranged upon them in such a manner as to form circular bands or *Zones*, of which there are *five varieties* particularly worthy of attention.

a.) *The Prismatic or Equatorial Zone.*—The planes belonging to this zone are those of the forms M,M_xT,T. Their position is vertical; they are parallel to the axis p^a; and they surround the equator. They are crossed by a brown line on the models of crystals. Their polaric positions are described in §§ 70 to 74.

b.) *The North Zone.*—The planes belonging to this zone are those of the forms P,P_xM,M. They are all parallel to the axis t^a, and therefore pass from left to right parallel to the observer. They surround the north meridian, and they are crossed by a blue line on the models of crystals. Their polaric positions are described in § 122.

c.) *The East Zone.*—The planes belonging to this zone are those of the forms P,P_xT,T. They are all parallel to the axis m^a; and they surround the east meridian. They are crossed by a purple line on the models of crystals. Their polaric positions are described in § 122.

d. e.) *The North-east Zone* and *the North-west Zone.*—The planes belonging to these zones are those of the forms P,M_xT,PMT, P_xMT, $P_x M_y T_z$. Of the forms P and M_xT, I have already spoken. The other planes are parallel to no axis; one half of every given form surrounds the north-east meridian, and occupies the ne and sw quadrants of a crystal; the other half surrounds the north-west meridian, and occupies the nw and se quadrants. The polaric positions of planes belonging to these zones are described in § 124.

Besides these five zones, many others are commonly described, but it does not appear to me that they are of sufficient importance to merit any particular explanation.

248. *Common Properties of the Planes of Zones.*—1.) Every zone is a many-sided endless prism, that cannot produce a crystal, or closed form, until cut or crossed by a form belonging to some other zone. 2.) All the planes of a zone are connected by edges that are perfectly parallel to one another, and to the axis of that zone. 3.) In three of these zones there are two sets of four planes, which all meet one another at an angle of 90°, so as to have a square cross-section. These planes are as follow:—

In the Equatorial Zone : M,T and MT.
Where the cross section is the equator, and the axis is p^a.

In the North Zone : P,M and PM.
Where the cross section is the north meridian, and the axis is t^a.

In the East Zone : P,T and PT.
Where the cross section is the east meridian, and the axis is m^a.

4.) The North-east Zone and North-west Zone also contain two sets of four planes of *remarkable properties*, namely, the combination P,MT, the

cross section of which is a square, and the form PMT, the cross section of which is a rhombus, having angles of 109° 28′ at the opposite sides of the equator, and of 70° 32′ at the Z and N poles. The cross sections of these zones are the north-east and north-west meridians.

249. The forms and combinations just cited are of great importance, since they serve to guide us in finding the positions of all other forms which can occur with them upon complex combinations. Model 32 represents the whole of them, and its symbol is P,M,T, mt. pm, pt, PMT. If we examine the vertical or prismatic zone, the planes of which are indicated on the model by a brown line, we find the form M,T, which make up a square prism, and the form mt, which is a second square prism, both parallel to the axis p^n. On the north zone, which is indicated by a blue line, we find the forms P,M, which make up a square prism, parallel to the axis t^n, and pm, which is a second square prism, parallel to the same axis. On the east zone, indicated by a purple line, we find P,T, producing a square prism, parallel to the axis m^n, and pt, a second square prism, parallel to the same axis. Finally, in each of the octahedral zones, that is to say, in the north-east zone and north-west zone, we find the square prism made up of P,mt, and the rhombic prism of 109° 28′ and 70° 32′, which characterises the form pmt.

250. The interfacial angles of these planes are as follow: The planes of one square prism incline upon those of the other square prism in the same zone at an angle of 135°, while the angle of the inclination of planes of the square prisms upon planes of the rhombic form, pmt, are respectively equal (see §§ 59—61) to the half of 70° 32′ plus 90° (= 125° 16′), and the half of 109° 28′ plus 90° (= 144° 44′). Therefore, the angle of P upon M is 90°; of P upon PM, 135°; P upon T, 90°; P upon PT, 135°; M upon T is 90°; M upon MT, 135°; P upon PMT is 125° 16′; P upon MT, 90°; MT upon PMT is 144° 44′; M upon PMT is 125° 16′; T upon PMT is 125° 16′; which several measurements may be proved by means of the goniometer.

251. All the planes which can be contained upon a combination either belong to one of these five zones, or are symmetrically disposed in the spaces situated betwixt the four meridians. The planes which belong to the equatorial, the north and the east zones, are those of forms which cut either one or two axes, and therefore take the symbols P,M,T,MT,PM, PT. The planes of the two octahedral zones (with the exception of P and M_xT) are those of forms which cut three axes. The forms which have the symbol P_xMT always fall in the direct line of the Znw and Zne zones. The planes of very complex combinations, which do not lie in any of these zones, are the planes of octahedrons that answer to the symbols PM_xT, PMT_x, $P_xM_yT_z$.

252. The object of attending to these zones, is, as I have already said, to facilitate the orderly examination and description of crystals. Instead of beginning the description of a combination with an account of a "primitive form," or a "fundamental form," or with any other *hypothetical matter,* depending upon an accidental property of a mineral, it is

better to proceed on grounds strictly *crystallographical*. It was therefore prescribed in SECTION VI., and with reference to the present nomenclature and distribution of zones, that the symbols employed to denote the forms present upon any combination, should be *ranged in a certain order*. This, then, is a law of the present system, established to promote uniformity in nomenclature. You begin the description of a crystal with the horizontal form P. You proceed to the vertical equatorial zone and you mark down, M,T,MT, M_T, M+T, or any of these forms that happen to be present; and this completes the description of the prismatic portion of the crystal. You next take the forms of the north zone, PM, P_xM, then those of the east zone, PT, P_xT, and finally the forms of the octahedral zones and of the open spaces betwixt the zones, arranging the latter forms in the order of the synopsis § 200, as PMT, P_MT, P+MT, $P_xM_yT_z$. You need only open the second part of this work at random, to find numerous examples of the method that is to be followed.

SECTION X.—THE LAW OF SYMMETRY.

253. In order to be enabled to explain the term "Law of Symmetry" in a satisfactory manner, I shall treat of it in reference to a classification of the forms of crystals under three heads, namely:

1.) HOMOHEDRAL FORMS, or *whole forms*.
2.) HEMIHEDRAL FORMS, or *half forms*.
3.) TETARTOHEDRAL FORMS, or *quarter forms*.

254. The following is GUSTAV ROSE's account of these different forms:—"Most of the simple forms occasionally suffer a peculiar alteration, which is, that the half of their planes, or more rarely the fourth part of them, become so large that they obliterate all the rest. This enlargement and obliteration takes place according to determinate laws, which can be best explained in treating of the forms separately. The result is the production of forms which have only the half or the fourth of the number of planes belonging to the original forms, whence, in contradistinction to the latter, they are called Hemihedral or Half Forms, and Tetartohedral or Quarter Forms; while the original forms are called Homohedral Forms."—*Elemente der Krystallographie, p.* 5.

255. Professor MILLER's account of these forms is this:—"The 'Holohedral Forms' of any system are those which possess the highest degree of symmetry of which the system admits. 'Hemihedral Forms' are those which may be derived from a Holohedral Form, by supposing half of the faces of the latter omitted according to a certain law."—*Treatise on Crystallography, page* 21.

256. Both of these sets of definitions leave the matter entirely open for subsequent explanation, which these authors give in treating of the particular *forms comprehended in* each of their systems of crystallisation.

In the present case, however, I propose to treat this subject with some detail under the general head, and I begin with the following explanations of the three chief terms:—

a.) The *Homohedral Forms* are such as contain whole forms or complete sets of planes, and are designated by the symbols contained in the synopsis, § 200. Examples: P,M,T, MT.PM, PT, PMT.

b.) The *Hemihedral Forms* are such as contain half sets of planes or half forms, and are designated by the symbols contained in the synopsis, with the prefix $\frac{1}{2}$ put to each. Examples: $\frac{1}{2}$P, $\frac{1}{2}$M, $\frac{1}{2}$T, $\frac{1}{2}$MT.$\frac{1}{2}$PM,$\frac{1}{2}$PT, $\frac{1}{2}$PMT.

c.) The *Tetartohedral Forms* are such as contain quarter sets of planes or quarter forms, and are designated by symbols which have the prefix $\frac{1}{4}$. Examples: $\frac{1}{4}$MT.$\frac{1}{4}$PM, $\frac{1}{4}$PT, $\frac{1}{4}$PMT.

Hemihedral and Tetartohedral Forms of different kinds are distinguished by adding to their respective symbols the polaric positions of the planes belonging to each of them, as $\frac{1}{2}$PMT Znw Zse Nne Nsw. This symbol shows that the deficient planes are those situated on the homohedral form at the poles Zne Zsw Nnw Nse.

257. One important point which I beg of the reader to keep in view, in examining the following arguments, is, that many crystals which Rose, Miller, and other crystallographers would call "forms" or "simple forms," are called by me "combinations." Thus, the icositessarahedron, Model 22, is with them a "form" of the octahedral system of crystallisation; but with me it is a "combination" of the three "forms," P$_-$MT, PM$_-$T, PMT$_-$. The ultimate difference is, that they would call such a crystal a "Homohedral Form," while I would call it a "Combination of Homohedral Forms." This explanation will, I hope, prevent ambiguities from obscuring the meaning I wish to convey. *A "Homohedral Form," therefore, is a "Form," according to the definition of that term given in* § 237.

1.) Homohedral Forms.

258. There is a peculiarity common to the planes of all homohedral crystals, which demands our special observation. It is, that every plane of such a crystal is a portion of a four-sided endless prism, the axis of which either coincides with the axis p', or m', or t', or else touches the central point of the crystal where these three axes cross, and cuts them there at an angle which is peculiar to, and characteristic of, every different crystallised substance.

259. Thus, the planes M and T form together a square prism, the axis of which coincides with the vertical axis p'.—MT is another square prism; M$_-$,T and M$_+$,T are rectangular prisms; and M$_-$T and M$_+$T are rhombic prisms; all having axes that coincide with the same vertical axis p', and all being prisms of four sides.

In the same manner, P,M and PM form square prisms; P$_-$,M and *P$_+$,M, rectangular* prisms; and P$_-$M and P$_+$M, rhombic prisms; all

four-sided, and the axes of all of which are coincident with the transverse axis t^a.

Again, the symbols P,T and PT; P_-,T and P_+,T; P_-T and P_+T, denote square, rectangular, and rhombic prisms, all four-sided, and the axes of all of which agree in position with the minor axis m^a.

And finally, all the varieties of the form PMT, consist of two such prisms, square or rhombic, which have axes that are *not* coincident with either p^a, m^a, or t^a, but which cut those axes in the centre of the form at various angles, according to the particular nature of the crystallised substance, by which particular nature, as influencing the power of crystallisation, the dimensions, the angles, and the direction of the cutting prisms appear to be controlled.

260. No single four-sided prism can produce a closed crystal, but *any two such prisms which cross one another at any angle, produce at once a closed form or crystal;* and all homohedral crystals are produced in this manner. I have given an example of the result of this crossing of prisms in the formation of the series of octahedrons, described in §§ 162—199; where I have shown that the same two *cutting prisms*, if the expression is allowable, acting at the same level, and with the same centre, upon the same set of axes, produce either isosceles or scalene octahedrons, and each of these of several different kinds, according as the line of action of the cutting prisms is more or less removed from parallelism with the axis p^a or m^a or t^a.

It is because no single four-sided prism can produce a closed crystal, that none of the symbols of forms, except PMT, ever indicate a complete crystal, § 124. But there are innumerable complete crystals which are entirely free from the form PMT. These crystals, therefore, consist of combinations of other forms, and are so many various examples of the closed crystals produced by the crossing of two or more four-sided prisms.

261. The most complicated crystals of minerals exhibit the same traces of these four-sided cutting prisms, as do the simplest geometrical solids, and we are led by this observation to adopt the following crystallographic hypothesis:—

The planes of homohedral crystals have been formed by *cutting prisms of four sides*—square, rectangular, or rhombic,—which have sometimes acted in a line parallel to one of the three rectangular axes of the crystal, and sometimes across them, but always with their centres upon the central point of the crystal, where the three rectangular axes cross one another.

262. This property of the planes of homohedral forms, of being referable to cutting prisms which *produce planes in sets of four fixed equally around the centre of the crystal*, has given rise to what crystallographers have termed the LAW OF SYMMETRY. They mean to intimate by this term that whenever you find upon a crystal one plane of a form that usually occurs in a set of 4 or 8 planes, you will probably find upon the same crystal *all the other planes* of that form. In virtue of this law it is

held that the top of a crystal is similar to the bottom, the front to the back, the left side to the right side, &c. And it is upon this principle that I have divided the planes into sets of 8, 4, and 2, constituting the series of Forms contained in the Synopsis, § 200.

In order to place the matter clearly before you, I shall re-arrange these Forms below.

Table of all the possible kinds of Homohedral Forms.

Uniaxial Forms:		Biaxial Forms:		Triaxial Forms.	
1.	P.	10.	MT.	19.	PMT.
2.	P_{-}.	11.	$M_{-}T$.		
3.	P_{+}.	12.	$M_{+}T$.	20.	$P_{-}MT$.
				21.	$PM_{-}T$.
4.	M.	13.	PM.	22.	PMT_{-}.
5.	M_{-}.	14.	$P_{-}M$.		
6.	M_{+}.	15.	$P_{+}M$.	23.	$P_{+}MT$.
				24.	$PM_{+}T$.
7.	T.	16.	PT.	25.	PMT_{+}.
8.	T_{-}.	17.	$P_{-}T$.		
9.	T_{+}.	18.	$P_{+}T$.	26.	$P_{-}MT_{+}$.
				27.	$P_{+}M_{-}T$.
				28.	$PM_{+}T_{-}$.
				29.	$P_{-}M_{+}T$.
				30.	$PM_{-}T_{+}$.
				31.	$P_{+}MT_{-}$.

I mean by *Uniaxial* Forms, such as cut one axis and have two planes to the set; by *Biaxial*, such as cut two axes and have four planes to the set; and by *Triaxial*, such as cut three axes and have eight planes to the set.

As the polaric position of every plane of all the above thirty-one Forms has been distinctly described in the foregoing Sections, it is easy to determine, by a slight examination, whether any of the planes of a given Form are absent from a combination or not.

2.) Hemihedral Forms.

263. The Law of Symmetry is a rule which has many exceptions. The planes which commonly occur in sets of four, sometimes occur in sets of two only; and those which should be found in sets of eight, are frequently reduced to four or two. When this is the case, the forms belong to those that are denominated Hemihedral or Tetartohedral,—to the former when they are half forms, and to the latter when they are quarter forms. I propose, in the first place, to take the Hemihedral forms into consideration.

264. The general law which establishes the coincidence of the central *points of the* axes of four-sided cutting prisms, when they cross one

ıer to produce an octahedron, is subject to a very remarkable ano-
. The two rhombic cutting prisms, whose *intersection at the level* (considered irrespectively of the *angle* at which they intersect ınother) produces an octahedron, *sometimes cross one another at dif- t levels,* and produce a form which represents only the half of an ıedron; that is to say, a form which possesses only four planes ad of eight.

5. The *Tetrahedron*, or *Hemioctahedron*, ½PMT, Model 117, is an ıple which illustrates this peculiarity. When this model is placed pright position, its solid angles are at the poles Zne Zsw Nnw its edges are at the poles Z N n e s w, and its planes are at Znw Zse Nne Nsw. The angle formed by the plane Znw upon ›lane Zse, over the pole Z, is 70° 32′. The equator, the north lian, and the east meridian, are all squares, which have their s at the same poles as are occupied by the angles of the same ons of the regular octahedron. There are no planes belonging ıe prismatic zone, the north zone, nor the east zone. There are planes on the north-west zone, and two planes on the north-east ; being together equal to ½ PMT. These relations are seen at nce, by placing the model in position, and examining it in relation ıe coloured lines marked upon it. The north-west and north- meridians of the model are both triangles, having precisely the angles that the north-west and north-east rhombic meridians of the lar octahedron would have, if they were each divided into two gles by the shorter diagonal; that is to say, every triangle has one of 70° 32′, and two equal angles of 54° 44′. There being four s upon the tetrahedron, and its properties being as described, it is ent that this form is the half of the regular octahedron, and that it equently is properly denoted by the symbol ½PMT.

6. I proceed to notice the mode of derivation of this form, to which oregoing description of it is but preliminary.

assume, in the first place, that the two Zenith planes of Model 117 produced by a rhombic cutting prism of 70° 32′ and 109° 28′, ıe shorter diagonal was equal to the Nadir edge of the model, and ıe longer diagonal was twice the length of the axis p[s]. Secondly, ume that its two Nadir planes were produced by a cutting prism of isely the same dimensions as the prism that produced the Zenith s. Thirdly, I assume that these prisms crossed one another *at the angle* as they must have done to produce a regular octahedron; but when they crossed, instead of having both their axes at the same l, they were at such different levels that *the axis or centre of one was at the level of the outer edge of the other prism.* Conse- ıtly, the two prisms, instead of cutting through each other from edge lge, only cut the half of each other, or from one edge to the centre; so but two sides instead of four from each prism were disposed upon the hedral, or rather the hemioctahedral, combination. Hence the result- crystal is the *same that would* be formed by the intersection at a

right angle of two triangular prisms, one of them having a side, and the other an edge, uppermost. The former would represent the *lower* half of the *upper* rhombic cutting prism; the latter the *upper* half of the *lower* cutting prism.

267. OF HEMIHEDRAL FORMS WITH INCLINED PLANES.—One of the most striking characters of the form produced in the manner which I have described, is, that it has *no parallel planes*. Now, there are several combinations of hemihedral forms which have the same property. They are commonly termed *Hemihedral forms with inclined planes*, and, without any exception, their formation may be explained by the hypothesis just applied to the tetrahedron, namely, by the supposition that their individual forms are generated by the intersection of two rhombic prisms, which cut one another at a difference of level equal to half the length of one of the diagonals of the cross-sections of the cutting prisms, which diagonal is sometimes the longer, sometimes the shorter, according to the nature of each form.

268. The hemihedral forms with inclined faces, or rather, the "combinations" produced by such forms, are as follow:—

The Tetrahedron	$\frac{1}{2}$PMT.	Model 117.
The Hemiicositessarahedron	$\frac{1}{2}(3P_{-}MT)$.	Model 119.
The Hemitriakisoctahedron	$\frac{1}{2}(3P_{+}MT)$.	Model 18.
The Hemihexakisoctahedron	$\frac{1}{2}(6P_{-}MT_{+})$.	Model 24.

We see from this list that every variety of octahedral form, which is particularised in the Synopsis of Forms, § 200, is subject to this curious anomaly, and liable to produce combinations of hemihedral forms with inclined planes.

Each of the above-cited hemihedral combinations contains half of the eight planes of all the simple octahedrons that combine to produce the corresponding homohedral combination; and what makes the anomaly still more remarkable, is, that the several octahedrons do not produce single hemihedral forms similar to the tetrahedron, such as $\frac{1}{2}P_{-}MT$, or $\frac{1}{2}P_{+}MT$, or $\frac{1}{2}P_xM_yT_z$, but only combinations of three or six such hemihedral forms; and that we find every hemihedral combination to contain invariably four entire octants out of the eight that complete the homohedral combination, and that these four selected octants are always the same. Thus, on comparing

Model 117,	$\frac{1}{2}$PMT,	with Model 15, PMT,
Model 119,	$\frac{1}{2}(3P_{-}MT)$,	with Model 22, $3P_{-}MT$,
Model 18,	$\frac{1}{2}(3P_{+}MT)$,	with Model 17, $3P_{+}MT$,
Model 24,	$\frac{1}{2}(6P_{-}MT_{+})$,	with Model 23, $6P_{-}MT_{+}$,

it will be found, that every particular hemihedral form (with the exceptions to be stated in § 269) contains all the planes of the corresponding homohedral form that belong to the following octants: Znw Zse Nne Nsw, *and none* of the planes that belong to the octants: Zne Zsw Nnw Nse.

Now, as every simple octahedral form, equiaxed or unequiaxed, contains one plane in every octant, it follows that the hemihedral combinations can contain only half the planes of each of the forms that belong to the different homohedral combinations. There is, consequently, a loss of one half of the planes of each complete set or form ; and this, as above described, I explain by supposing that the rhombic prisms, by whose intersection the forms are produced, cut one another at different levels, the difference being in all cases exactly equal to one half the diameter of each of the two cutting prisms. A result which this hypothesis would indicate, and which a practical examination of crystals shows to be true, is, that all these hemihedral combinations bear a general resemblance to the hemioctahedron or tetrahedron. Thus, Model 119 is a tetrahedron, with a low three-sided pyramid upon each face; Model 18 is also a tetrahedron, with a low three-sided pyramid upon each face; and Model 24 is a tetrahedron, with a low six-sided pyramid upon each face. The general resemblance to the tetrahedron which runs through all these combinations is a parallel to the general resemblance of the respective homohedral combinations to the regular octahedron; and it greatly simplifies our calculations of angles, and references to polaric positions, and other matters, to bear these resemblances in mind.

269. *Right and Left, or Direct und Inverse, Hemihedral Forms.*— It sometimes happens that two hemihedral forms, or combinations, of the same kind, occur upon one crystal. In such a case, one of them contains the planes of the octants Znw Zse Nne Nsw; and the other, the planes of the octants Zne Zsw Nnw Nse. The planes of the first series of octants are *large*, and those of the last series *small;* and this cannot be otherwise; for if the planes of both sets of octants were *equal* as well as *similar*, they would constitute a homohedral form, and not two hemihedral forms. Thus, Model 118 contains two Hemioctahedrons, which, however, do not make up an Octahedron, like Model 15, although the angles of the incidence of all the planes are the same. These two sets of half-forms are called the Right (or Direct) and Left (or Inverse) Hemihedral forms. They can be discriminated in symbols, by writing the Right form in capital letters, and the Left form in small letters, as $\frac{1}{2}$PMT, $\frac{1}{2}$pmt, and by adding Znw to the symbol of the Right form, and Zne to the symbol of the Left form, as $\frac{1}{2}$PMT Znw, $\frac{1}{2}$pmt Zne.

In all cases where only one hemihedral form with inclined faces occurs upon a combination, it is assumed to be the right or direct form, and has assigned to it the positions Znw Zse Nne Nsw, which are indicated in the symbol by the single sign Znw. When both the direct and inverse forms occur upon the same combination, the positions of the first are indicated by Znw, and of the last by Zne.

270. There is no *single hemioctahedron with inclined faces* besides the regular tetrahedron. All the other hemioctahedrons are triple or six-fold, being rather hemihedral *combinations* than hemihedral *forms*. It is sufficient to excite our surprise, to see the action of the cutting prisms in *forming the series of complex* octahedrons which I have described in

§§ 124—199, a series of combinations so extensive, so diversified, and yet so regular. It is matter of still greater astonishment, to find these cutting prisms intersecting one another after a different law, but with precisely the same degree of regularity, to form another series of combinations, the hemioctahedrons. And finally, it is no less wonderful to perceive, as we do upon a close examination of the facts, that all those combinations which appear to contain a right and left hemioctahedron of the same kind, are produced by the identical cutting prisms which form the homohedral and the single hemihedral forms; acting still at the *same angle*, but not at the *level* which produces the homohedral form, nor at the *level* which produces the hemihedral form, but at *a level* INTERMEDIATE *between these two levels!* Hence, two cutting prisms of 109° 28′ and 70° 32′ which intersect one another at a right angle, may, although precisely of the same magnitude, produce three very different forms *according to the level at which they act.* 1.) If they act at the same level, they produce the regular octahedron, Model 15, PMT.—2.) If they act at a difference of level equal to one half the length of the longer diagonal of their cross section, they produce the regular tetrahedron, Model 117, ½PMT.—3.) If they act at any level intermediate between these two levels, they produce the combination of two tetrahedrons, Model 118, ½PMT, ½pmt.

271. OF HEMIHEDRAL FORMS WITH PARALLEL FACES.—The hemihedral forms with parallel faces are of quite a different character from the hemihedral forms with inclined faces. The latter, generally speaking, belong to equiaxed, and the former to unequiaxed, combinations. I believe the only exceptions are a few unimportant unequiaxed tetrahedrons. They do not, therefore, occur together on the same crystals, nor are they ever characteristic of the same mineral or chemical substance. All the hemihedral forms with inclined planes belong to the octahedral zones, with the exception of the remarkable three-sided or hemi-rhombic prisms peculiar to the mineral called tourmaline; whereas, the hemihedral forms with parallel planes occur not only in the octahedral zones, but in the north, the east, and the prismatic zones; as I shall show by examples drawn from each of the zones mentioned.

272. *Hemioctahedrons of the Oblique Prisms.*—The hemioctahedrons of the oblique prisms are formed when a single rhombic cutting prism has acted on a combination so as to produce four planes of a scalene octahedron, but has not been cut by a second prism so as to supply the other four planes requisite to complete the octahedral form. Hemioctahedrons of this kind are extremely numerous, but all the varieties are capable of reduction to four classes:

a.) In this class, the axis of the rhombic cutting prism has passed in the direction of the plane of the north meridian, and from the Nn pole towards the Zs pole. The planes formed on the combination occupy the *positions* Znw Zne Nsw Nse. See Model 115, which is terminated by

the hemioctahedron $\frac{1}{2}P_xM_yT_z$ Znw Zne Nsw Nse; see also Model 103, which contains two dissimilar hemioctahedrons, one of them formed by a cutting prism, whose line of action was nearly parallel to p^a, the other by a cutting prism, whose line of action was nearly parallel to m^a; the planes of the latter being in consequence thrown towards the east zone, and of the former towards the prismatic zone.

b.) The axis of the rhombic prism has again passed in the direction of the plane of the north meridian, but from the Zn pole towards the Ns pole; hence, the planes left upon the combination occupy the positions Zse Zsw Nne Nnw, or precisely the positions left unoccupied by the planes of the forms belonging to class *a.* See Models 57, 71, and 112, all of which exhibit the hemioctahedron $\frac{1}{2}P_xM_yT_z$ Zse Zsw Nne Nnw.

The crystal represented by Model 75 contains the hemioctahedrons of both class *a.* and class *b.*; but they are so nearly of a size and agreement in angles as to appear like a homohedral octahedron. This is an error of the model, because the two hemihedral forms are often very distinct on the mineral which it is intended to represent.

c.) The axis of the rhombic cutting prism has passed in the direction of the plane of the east meridian, and from the Nw pole towards the Ze pole; so that the four planes occupy the positions Znw Zsw Nne Nse. See Model 98, which is terminated by the hemioctahedron $\frac{1}{2}P_xM_yT_z$ Znw Zsw Nne Nse.

d.) The axis of the rhombic cutting prism has again passed in the direction of the east meridian, but from the Zw pole towards the Ne pole, and consequently has formed four planes, which occupy the positions Zne Zse Nnw Nsw, or the inverse positions of the planes of class *c.* See Models 26^a, 26^b, 26^c, 26^d, 72, 114, and 114^a, all of which have the forms $\frac{1}{2}P_xM_yT_z$ Zne Zse Nnw Nsw.

e.) The hemioctahedrons of class *a* occur in combination with those of class *b.*; and those of class *c.* with those of class *d.*; but the hemioctahedrons of the first two classes are never found combined with those of either of the last two classes.

273. *Scalene Hemioctahedrons which have all their four Planes on one Meridian.*—This class of hemihedral forms is not so abundant nor so important as the others. The examples of it are generally considered to be irregular or misshapen crystals, rather than as belonging to a particular class of combinations. As such I may mention the octahedron $\frac{1}{2}$PMT Znw Zse, $\frac{1}{2}$pmt Zne Zsw, having four large and four small planes on different zones, and which is produced by two cutting prisms of similar angles, acting at the same level, and crossing at a right angle; but being, at the time of crossing, of very different relative magnitudes. Also many varieties of Topas, which contain such combinations as P, M_{-}, T, $m_x t$. $\frac{1}{2}P_xM_yT_z$ Znw Zse, $\frac{1}{2}p_xm_yt_z$ Zn^{2}e Zs^{2}w, $p_xm_yt_z$ Zne^2 Zsw^2; and many other octahedrons belonging to the prismatic class of crystals, which are liable to present two hemioctahedrons of different magnitude, but having the *same relations in respect* to their axes. These variations

depend either upon the absence of one of the two cutting prisms necessary to produce the homohedral form, or upon the difference in magnitude of the two cutting prisms, by the intersection of which the form was produced. A different example of hemioctahedrons having all their four planes upon one meridian is afforded by the Scalenohedron.

274. Model 26^f, the *Scalenohedron*, is a combination of three hemioctahedrons, and represents the planes of three similar and equal cutting prisms. The axis of one of these prisms passes from Zw^2 to Ne^2, nearly parallel to t^a, and throws four planes into the positions Z^2n^2e Z^2s^2e N^2n^2w Z^2s^2w, so that they are intersected by the north meridian. The axis of a second prism passes from Zn^2e^2 to Nn^2w^2, and throws four planes into the positions Z^2nw^2 Z^2se^2 N^2nw^2 N^2se^2. The axis of the third prism passes from Zs^2e^2 to Nn^2w^2, and throws four planes into the positions Z^2ne^2 Z^2sw^2 N^2ne^2 N^2sw^2. The three cutting prisms, therefore, produce the hemihedral forms of three very dissimilar scalene octahedrons.

275. The indication of the polaric positions of the different kinds of hemihedral forms described above, is effected by naming the positions of the two zenith planes of each form. Thus: Zne Znw, Znw Zsw, Znw Zse. The planes that are not indicated are parallel to those that are indicated.

276. *Hemihedral Forms of the North Zone.*—The forms P and M become hemihedral, in so far as they frequently appear on a combination differing in size, and therefore in distance from the centre of the crystal, so as, for accurate description, to require the notation:—$P_{-}Z$ and $p_{+}N$; $M_{-}n$ and $m_{+}s$; and sometimes one of the two planes is absent altogether. Hemihedral forms of this kind may be seen in abundant variety on the crystals of commercial alum.

Models 57, 71, 79^b, 79, 84, 103, 105, 109, 112, all represent combinations which include the hemihedral form $\frac{1}{2}P_xM$, and which occupies in all of them the positions Zn Ns.

277. Models 101, 101^a, and 106, represent combinations which contain the hemihedral form $\frac{1}{2}P_xM$ Zn Ns, and in addition another hemihedral form occupying the corresponding inverse positions, as $\frac{1}{2}P_xM$ Zs Nn. The direct and inverse forms in this, as in every case, must be of different magnitudes, else the two hemihedral forms would constitute a homohedral form.

278. *Hemihedral Forms of the East Zone.*—Leaving out of question the forms P already spoken of in § 276, and the form T, which sometimes occurs as Tw alone, or as $T_{-}w$, $t_{+}e$, the only other forms belonging to the east zone are the varieties of P_xT, with the hemihedral forms of which we have now to deal. Models 26^a, 26^b, 26^c, 26^d, 114, 114^a, 72, and 87, all represent combinations which contain the form $\frac{1}{2}P_xT$, and in all of which it occupies the positions Zw Ne.

Model 26^e represents a combination which contains the forms $\frac{1}{2}P_xT$

Zw Ne, and also two varieties of the form $\frac{1}{2}P_xT$ Ze Nw, in addition to three hemioctahedral forms.

279. *Hemihedral Forms of the Equatorial Zone.*—The hemihedral forms $\frac{1}{2}M$ and $\frac{1}{2}T$ being already explained, I have only to speak of the hemihedral form $\frac{1}{2}M_xT$.

Model 105 exhibits the form $\frac{1}{2}M_xT$ holding the positions ne sw.

Model 107 exhibits three varieties of the hemihedral form $\frac{1}{2}M_xT$, which occupy the positions n²w s²e, nw² se², ne sw. The mineral named Topas frequently exhibits $\frac{1}{2}m_{-}t$, $\frac{1}{2}m_{+}t$, and similar forms occur on many prismatic minerals. But the hemihedral forms of this zone are, generally speaking, much less numerous, and much inferior in importance to the hemihedral forms of the four other zones.

280. *Polaric Positions of Hemihedral Biaxial Forms.*—The two planes of each hemihedral form of the three last mentioned zones, namely, $\frac{1}{2}P_xM$, $\frac{1}{2}P_xT$, and $\frac{1}{2}M_xT$, are always parallel to one another, and therefore are on opposite sides of a crystal. They are never in contact, nor inclined to one another.

It is sufficient, in writing the symbols of these forms, to indicate the position of only one of the planes, since the other plane is always parallel to it. Thus:

$\frac{1}{2}M_xT$ nw. $\frac{1}{2}P_xM$ Zn, $\frac{1}{2}P_xT$ Zw.

When there are two dissimilar hemihedral forms in the same zone, the one that has the largest planes has assigned to it the positions just recited; and the one that has the smallest planes, is placed in the opposite quadrants. Thus:

$\frac{1}{2}M_xT$ nw, $\frac{1}{2}m_xt$ ne.
$\frac{1}{2}P_xM$ Zn, $\frac{1}{2}p_xm$ Zs.
$\frac{1}{2}P_xT$ Zw, $\frac{1}{2}p_xt$ Ze.

281. Of Certain Forms that are commonly, but erroneously, called Hemihedral.—Several of the forms which I have described in the foregoing paragraphs, §§ 271—280, are not in general considered to be examples of hemihedral forms; while, on the other hand, there are two octahedral combinations which crystallographers commonly denominate hemihedral forms, but, as it appears to me, without good reason.

The Pentagonal Dodecahedron, Model 91. This combination contains the three forms, $M\frac{1}{2}T$. $P\frac{1}{2}M$, $P\frac{2}{1}T$, presenting four planes on the prismatic zone, four on the north zone, and four on the east zone, all of which planes are perfectly symmetrical. The combination, therefore, contains none but homohedral forms. It is, nevertheless, said to be the hemihedral form of the tetrakishexahedron, Model 68, which combination contains the six homohedral forms $M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$. If the pentagonal *dodecahedron* consisted of $\frac{1}{2}$ ($M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$,

P½T, P$\frac{2}{1}$T), instead of M½T. P½M, P$\frac{2}{1}$T, it would be perfectly correct to call it the half of the tetrakishexahedron; but there is so great a difference between *half the number of the forms*, and *half the planes of all the forms* of a combination, that I think the application of the term hemihedral is, in this instance, quite erroneous. The combination M½T. P½M, P$\frac{2}{1}$T, is a complete crystal, a combination of homohedral forms, and is by no means dependent for its characters upon the other forms which occur upon the tetrakishexahedron. For example, M½T is not the half of M½T, M$\frac{2}{1}$T, but one of two different forms, either of which may, and frequently does, occur in combination without the other, and without being held to be hemihedral. In like manner, we may say that P½M is not the half of P½M, P$\frac{2}{1}$M, nor P$\frac{2}{1}$T the half of P½T, P$\frac{2}{1}$T; but that each is a homohedral form in its own right, and must be so considered, whether it occurs in combination with P$\frac{2}{1}$M and P½T, or without them. And if it be admitted that the forms M½T, P½M, and P$\frac{2}{1}$T are individually homohedral, then it must also be admitted, that the pentagonal dodecahedron is homohedral, inasmuch as a hemihedral combination cannot be produced by homohedral forms.

282. *The Right Hemihexakisoctahedron with parallel faces*, Model 25, which contains the forms P⅓M½T, PM⅓T½, P½MT⅓, (or 3P⅓M½T). This combination is said to be the half of the hexakisoctahedron, Model 23, which contains the forms P⅓M½T,PM⅓T½,P½MT⅓,P⅓MT½,P½M⅓T, PM½T⅓, (or 6P⅓M½T), a different hemihedral form of which has been already explained, namely, the *hemihexakisoctahedron with inclined faces*, Model 24, which contains the forms ½P⅓M½T, ½PM⅓T½, ½P½MT⅓, ½P⅓MT½, ½P½M⅓T, ½PM½T⅓, or in the abridged symbol, ½(6P⅓M½T). This also appears to me to be a piece of needless theoretical complexity. Model 24 represents the true and the only hemihedral form of Model 23, namely, it exhibits a combination of *half the planes of all the forms* that belong to the homohedral combination; whereas Model 25 exhibits a combination of *half the number of homohedral forms* that belong to the homohedral combination. Four of the octants of the hemihedral form with inclined faces, are exactly similar to four of the octants of the homohedral form, and other four are entirely wanting, whereas *all the octants* of what is called the " the hemihedral form with parallel faces," are *totally different* from the octants of the homohedral form. The planes of Model 25, if separately enlarged, would make three complete octahedral forms, or three complete scalene octahedrons, namely, the forms P⅓M½T, PM⅓T½, and P½MT⅓, while the separation and enlargement of the planes of Model 24, would produce six scalene tetrahedrons, namely, ½P⅓M½T, ½PM⅓T½, ½P½MT⅓, and ½P⅓MT½, ½P½M⅓T, ½P⅓MT⅓. This last consideration places in the most striking point of view, the difference in the characters of the two combinations, one of them being decidedly homohedral, and the other decidedly hemihedral.

283. It is singular enough to observe, that, although the pentagonal dodecahedron and the hemihexakisoctahedron with parallel faces, are so little entitled to the character of hemihedral forms, yet crystallographers have made of these two combinations a distinct class of crystals, under the denomination of " Hemihedral Forms with parallel faces." They probably did not know what else to do with them; and, certainly, so long as their *hemihedral* nature is insisted on, they will continue to be difficult of disposal. On the other hand, it is only necessary to admit that " combinations of homohedral forms" are " homohedral combinations," to get immediately quit of the difficulty. But this would render it necessary to cease to apply the term " form" to any solid of more than eight sides, and to substitute for it, in the descriptions of all the equiaxed crystals except PMT, the term " combination."

It has often been mentioned as an extraordinary circumstance, that minerals which possess the form of the pentagonal dodecahedron, such as Iron Pyrites, should not, *like other hemihedral forms*, be pyro-electric. Perhaps the wonder will not appear so great, when it is understood that this form is not essentially, but only theoretically, hemihedral.

3.) Tetartohedral Forms.

284. The forms P,M,T, which consist of only two planes each, cannot occur in tetartohedral forms. But these are, I believe, the only forms exempt from this irregularity. I have seen a crystal of Wernerite, which required the symbol P_{+},M,T, $\frac{1}{4}$MT, and examples of the forms $\frac{1}{4}$PM, $\frac{1}{4}$PT, though still uncommon, can no doubt be produced, as in Alum and Tourmaline; while the tetartohedral varieties of the form $P_x M_y T_z$ are an essential part of the combinations that are called doubly oblique prisms.

The forms $\frac{1}{4}$MT, $\frac{1}{4}$PM, $\frac{1}{4}$PT, consisting only of a single plane, are necessarily unsymmetrical and out of all rule. The form $\frac{1}{4}P_x M_y T_z$ is, however, to a certain extent symmetrical. In the greater number of cases, it forms *a pair of parallel planes*, unlike any other planes on the same combination, and always appearing on crystals which have no right angles. See the Minerals which compose the class of doubly oblique prismatic combinations, Part II. page 91.

The regular octahedron sometimes occurs with one pair of parallel faces very large, and three other pair very small = $\frac{1}{4}$PMT, $\frac{3}{4}$pmt.

The only example that I recollect of tetarto-octahedrons that have the two planes on the same end of the combination, occurs in the case of the crystals of Tourmaline, which are frequently terminated by rhombohedrons that differ at each end, and which consequently present examples of $\frac{1}{4}P_x M$, $\frac{1}{4}P_x T$ and $\frac{1}{4}P_x M_y T_z$. See Part II. pages 59, 60.

None of these cases are of sufficient importance to merit a special investigation.

Discrimination of Homohedral, Hemihedral, and Tetartohedral Forms.

285. PROBLEM: It is demanded, whether the forms on a given combination are homohedral, hemihedral, or tetartohedral.

Place the crystal in position, and imagine it to be divided into octants by the equator, the north meridian, and the east meridian. If the octants are all alike, none but homohedral forms are present. If there are two kinds of octants, a hemihedral form is present. If there are four kinds of octants, a tetartohedral form is present.

286. It only remains to be added, that the planes of all hemihedral and tetartohedral forms incline upon one another, and upon the planes of any homohedral forms with which they occur in combination, at the same angles as do the planes of the corresponding homohedral forms; so that the calculations which serve for homohedral forms, serve equally well for all the fractional forms.

SECTION XI.—A THEORY OF CRYSTALLISATION.

287. The use which I have made of the term *cutting prism* in explaining the Law of Symmetry, will probably meet with objections. It may be represented as absurd to suppose that nature employs a sort of *working tool* in the formation of crystals, or it may be demanded, what evidence I find of the separate existence of these "cutting prisms," other than the appearance of the crystals that they are said to produce?

It was not my intention to enter, in this work, into any discussion relative to the theory of crystallisation; and it was to prevent any expectation that I should discuss that subject, that, at the beginning of the work, I defined crystallography to be merely "the art of describing crystals." I am still of opinion, that a method of crystallography and a theory of crystallisation are things so very different, that they need not be treated of together, and that I might be permitted to employ, *arbitrarily*, the term "cutting prism," as a convenient method of explaining certain facts relating to the symmetry of crystals—of showing the regularity which prevails in the grouping of their planes,—without being constrained to plunge into the details of a theory of crystallisation, of being forced to try to explain, not the derivation of some of the *planes* of crystals, but the mode of the production of the *crystals* themselves. Nevertheless, as I have broken ground by giving a hypothetical explanation of the derivation of certain forms, and as many persons may expect, in a work on crystallography, something more than a mere technical description of the figures of crystals, it will not be going much farther *out of the way, if I* explain just so fully what I mean by *cutting prisms*,

as will render the preceding section intelligible. Whatever comparisons and assumptions I make, having merely this end in view, are only therefore to be considered as *a figurative method of describing crystals*. To pretend to give a true theory of crystallisation, would be greatly to overleap the present bounds of physical science.

288. The planes of crystals which we may conceive to be produced by the action of *four-sided prisms*, crossing or cutting one another at different angles and in different directions, are enumerated in § 259 as those which constitute the forms P,M,T, M_xT, P_xM, P_xT, and $P_xM_yT_z$. Now these forms are represented in § 200, as all the forms that are known in crystallography; and, in SECTION V., I have shown that they represent *all the forms that can possibly occur upon crystals*. It follows thence, that these FOUR-SIDED CUTTING PRISMS are the *generators of all the forms that can appear upon crystallised combinations*—in other words, that they are the "primitive" or "original forms" whence the planes of all crystals are derived. For this reason, and in order to have a convenient term for common use, I venture to propose for these generating prisms the name of EIDOGENS, from the words, *eidos*, "a form," and *genomai*, "to generate." I should be better pleased with the term PRIMITIVE FORM, but unfortunately that term has acquired a meaning which unfits it for my present purpose. By EIDOGEN, then, is meant a four-sided prism of indefinite length.

289. Let us proceed to investigate the origin of the "eidogen" and the manner in which it works.

290. I assume that the particles of all crystals, originally, and at the period of the production of the crystals, were in a state of mobility; being dissolved in a liquid, fused by heat, or suspended in some kind of gas; that the movements of the particles depended on the directing power of electricity; and that each particle or molecule, whether chemically simple or compound in its nature, whether a single physical atom or a group of atoms, was endowed with polarity, having what we may conveniently denote, in the terms proposed by Mr. Graham, a CHLOROUS *pole* and a ZINCOUS *pole*, which poles had severally the power of inducing polarity in adjacent mobile particles, and of attracting dissimilar poles and repelling similar poles. I proceed to investigate the phenomena of crystallisation in reference to the case of a saline solution.

291. A saline solution, at a certain state of concentration, begins to yield crystals. Before the crystals appear in the solid state, it is probable that they are completely formed in the liquid state; for the smallest visible crystal is as perfect in its forms as is the largest crystal that ever existed. At different temperatures, any given salt may combine with different quantities of water, or be subject to other changes in composition, and produce different crystallisable compounds. The formation of each of these compounds may give rise to the production of a certain amount of electrical power, which is now known to be attendant both on the exercise of the power of chemical combination and on the act of crystallisation. The crystallisable particles set in motion by

electricity, arrange themselves in the order prescribed by their polarity, so that a single row of particles presents the following arrangement:

1.) CZCZCZCZCZCZCZCZCZ

But longitudinal arrangement is not the only one of which particles thus acted upon by electricity are capable. The attractive and repulsive forces must act laterally as well as longitudinally; and may, therefore, produce a lateral arrangement of particles somewhat in this order:

```
2.)      C
        CZC
         C
```

where the central Z represents the zincous pole of one of the electrified particles of the longitudinal series.

But the induced polarity would not be at an end, when the grouping of particles had proceeded thus far. The central zincous pole would now have a chlorous pole on each side of it in the longitudinal series, and four chlorous poles about it in the lateral series. The strength of this attraction would immediately bring the zincous poles of a number of other particles into the combination, and produce a series which may be conceived to increase by accession of particles as follows:

```
3.)           4.)     Z             5.)     C
                     ZCZ                   CZC
    Z               ZCZCZ                 CZCZC
   ZCZ             ZCZCZCZ               CZCZCZC
  ZCZCZ           ZCZCZCZCZ             CZCZCZCZC
   ZCZ             ZCZCZCZ               CZCZCZC
    Z               ZCZCZ                 CZCZC
                     ZCZ                   CZC
                      Z                     C
```

in which 3.) represents the next stage to 2.), while 4.) and 5.) represent different views of a subsequent stage—where 5.) is the next plane of particles to 4.) in a longitudinal series of similar lateral extent.

In this manner a single row of electrified particles may be supposed to become connected with a multitude of other particles; every particle in the longitudinal series inducing polarity in all the mobile particles around it, and becoming the centre of a lateral plane of particles.

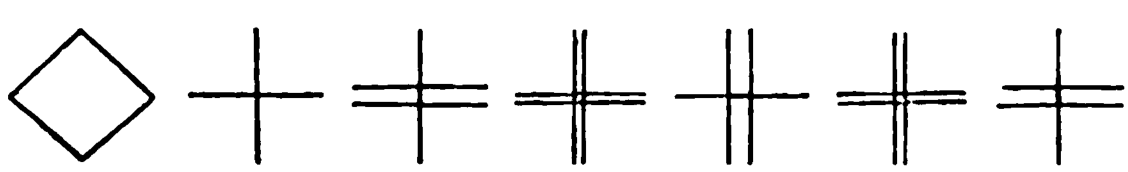
292. But there may also be a power which regulates the extent to which the polarity induced by any given particle of a crystallising substance may proceed laterally, and this power I take to be magnetism. The same power which changes the poles of a magnetic needle when an electrical current is passed across it, may rationally be supposed to regulate the disposition of the particles of a crystallising salt which have been set into motion by an electrical current; and there are many reasons to believe that this is the case. By way of giving an explicit view of this matter, I shall assume, that when an electrical current has disposed a series of crystallising particles in the order shown by the figures 1.) to 5.), that there is produced a series of lateral magnetic *currents, often* but not always at right angles to the electrical cur-

rent, and tending *to regulate the limits within which the power of crystallisation shall operate.* Thus I suppose that across each longitudinal electrical series, two magnetic axes are formed; the one extending from e to w, the other from n to s in figure 6.), and that

```
                 s
                 C
                 Z
                 C
6.)   e ZCZCZCZCZCZCZCZCZ w
                 C
                 Z
                 C
                 n
```

the magnetic currents pass from Z in the centre, which is a pole of the longitudinal series, towards the poles n and s, the extremities of one axis, and spread thence towards e and w, the extremities of the inverse axis, in order to complete the circle and return to Z. Will it be said that this is an extravagant assumption? Is it a whit more extravagant than the assumption so commonly agreed to, that the magnetic effluvia passes from the two ends of a bar magnet to re-enter the magnet by other poles situated in intermediate parts of it? But the truth of the latter assumption, it will be urged, can be proved by experiment. When iron filings are agitated on a sheet of paper laid over a bar magnet, they arrange themselves in the order of the magnetic currents. Good, I reply; and when a slice of a transparent crystallised substance, cut properly from an eidogen, is examined by a polariscope, the particles of the mineral are seen to be arranged in an order that wonderfully corresponds with the figure of the magnetic curves, and which renders visible, in the most surprising and most beautiful manner, the two magnetic axes for whose existence I contend. No one who attentively considers the black crosses and curves exhibited by minerals when examined in polarised light, can, I think, hesitate to ascribe them to the effects produced by the arrangement of the particles of the crystallised mineral, if not in the order of the magnetic curve, at least in an order which bears a great resemblance to it.

But here is an experiment which appears to throw light on this subject:

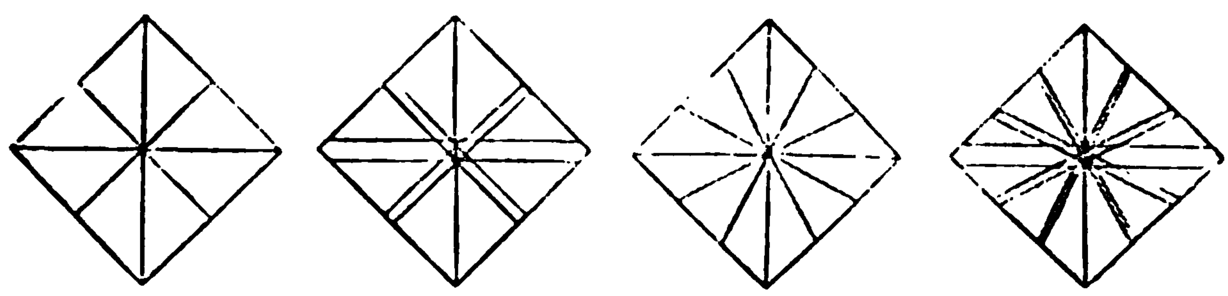


R x 0° 45° 90° 135° 180°

Take a piece of transparent calcareous spar of the size of figure R, and place it over a black cross made with thin lines and of the form of figure x. The crystal may either be placed with a plane flat on the paper, or held with four sides in a vertical position, so that when you look down upon the paper through the upper plane, the point of sight shall be in a *line with the axis of* the prism. Place the longer diagonal

of the terminal plane of the crystal upon the horizontal line of the cross, figure x. Then look through the crystal at the cross: it will appear like fig. 0°. Turn the crystal horizontally 45°: the cross will then appear like fig. 45°. Turn the crystal other 45°: the cross will appear like fig. 90°. Turn the crystal other 45°: the cross will appear like fig. 135°. Turn the crystal other 45°: the cross will appear like fig. 180°. All these changes take place in turning the crystal through half a circle. If you turn it through the other half circle, a similar set of changes take place, and at 360°, the final view of the cross is exactly like the first, fig. 0°.

The following experiment is equally curious:



A B C D

If you hold the rhombic prism of calcareous spar over a double cross making angles of 45° at the centre, as shown by figure A, keeping as before the longer diagonal of the terminal plane over the horizontal line of the figure, the cross appears as in figure B. If you turn the crystal round horizontally, changes take place in the figure of the cross similar to those described above, and which are very decided at every 45°, until at 90° the figure is exactly the reverse of what it is at 0°, the broad double band being then vertical.

If you hold the crystal over a figure containing lines which make angles of 30° round the centre, as shown by figure C, the appearance produced at 0°, that is, when the longer diagonal of the terminal plane of the crystal is placed over the horizontal line of the figure, is exactly like figure D.

The phenomenon exhibited in these experiments is commonly known under the appellation of the double refraction of light. But the particular point to which I wish to draw attention is, the proof afforded by these experiments, that the particles of the crystal which are situated in the direction of the *longer diagonal* of the terminal plane, have different properties from the particles which are situated in the direction of the *shorter diagonal*, and that the particles which are situated at intermediate points have properties in agreement with the ratio of their proximity to one or other of these diagonals; in other words, that the properties of the particles of the crystal depend directly upon their magnetic relations, and that double refraction is one of these properties, and crystalline form another.

The fact that electricity and magnetism are directly concerned in the operation of crystallisation, is satisfactorily made out by numerous experiments, such as the production of perfectly regular crystals of metallic

copper, metallic silver, red oxide of copper, and other insoluble substances by means of simple voltaic circles of very low power.

293. I conceive, then, that an EIDOGEN is formed thus, and has the properties here recited:

1.) There is an arrangement of particles in longitudinal order, of greater or less extent, according to the mass of matter present, and to the degree of electrical excitement produced by the temperature, the circumstances attendant on the act of crystallisation, and the particular properties of the matter under operation.

2.) That there is a contemporaneous exertion of magnetic power, which regulates the arrangement of the particles laterally, and restricts the indefinite combination that would result from the continuous and unchecked propagation of electric polarity by induction among an infinite number of crystallisable particles.

3.) That the two magnetic axes, ns and ew, fig. 6), are of variable length relative to one another, and are individually regulated by the special nature of the crystallising substance, and by the accidents which modify each act of crystallisation.

4.) That the passage of the magnetic current from pole to pole situated at the ends, and at proportional distances throughout the length, of the magnetic axes, regulates the form of the eidogen.

5.) That innumerable eidogens may exist in a given liquid, without the appearance of a single solid crystal.

6.) That when an eidogen is completely formed, its electricity becomes latent.

7.) That the eidogens may be pierced by one another, and crossed in all directions, without being necessarily destroyed.

8.) But that the formation of some one eidogen, produced by the action of a very powerful electrical force, may take place at the expense of several other eidogens.

9.) That crystals are formed by the crossing of different eidogens, which thus cut out closed forms or crystals, bounded by planes of a determinate figure, which crystals may still remain liquid, and be subject to farther truncation or intersection by other eidogens, previous to their solidification.

10.) That crystals become solid in virtue of the exercise of cohesive attraction, which, on the separation of the solvent occasioned by any sufficient physical cause, gives coherence to the solid particles, and terminates for a period the electric action and the mobility which results from it.

11.) That different chemical substances, or possibly different physical groups of particles, regulate, not only the relative dimensions of the magnetic axes, but also the direction, intensity, and variations of the electrical current; so that from a given centre there may proceed numerous eidogens, of determinate dimensions, in determinate number, and proceeding in *determinate directions, varying* with the physical or chemical properties *of a given substance.*

12.) That the plane in which the magnetic axes lye, may sometimes be inclined, and not at right angles, to the electrical current.

13.) That a solid crystal, apparently consisting of one eidogen, is, in fact, a congeries of eidogens, which can often be mechanically separated into numerous smaller masses of eidogens, by planes passing parallel to some of the faces of the eidogens, or to the magnetic plane, or to one of the magnetic axes; which planes of separation are commonly called "planes of cleavage."

14.) That although the solidification of a crystal suspends, it does not annihilate, the electric properties of its eidogens, which can be recalled into action by change of temperature and other physical forces.

15.) That the alterations which take place in the size or figure of a crystal, after its solidification, depend upon the joint action of the chemical and mechanical forces to which the crystal is subject.

a.) *Chemical Action.*—If the crystal is placed in a solution which, owing to continuous evaporation, reduction of temperature, or other cause, is in a crystallisable state, new eidogens will form around the crystal, corresponding in figure, number, and direction to the nature of the substance, the figure of the solid crystal, and the intensity of the electrical excitement. The origin of these new eidogens is the chemical action which takes place between the particles that constitute the crystal and those that exist in the solution, which several particles I conceive to have the same disposition to combine with the liquid of the solution that two separate quantities of any base have to combine with a qantity of acid incompetent to saturate the two quantities of base. The commencement of this chemical action on the liquid by the outer particles of the crystal, induces polarity in the surrounding mobile particles, and originates the new eidogens. Every plane of the solid crystal being part of an eidogen, is immediately coated by an addition to that eidogen, and this coating is followed by a second or third according to the continuance of the action. Thus, if figure *a* represents the form of a cross-section of the solid crystal supposed to be submitted to action, figure *b* may represent it with one cleavage plane added, and figure *c* with two such planes.

a.)
Z
ZCZ
Z

b.)
C
CZC
CZCZC
CZC
C

c.)
Z
ZCZ
ZCZCZ
ZCZCZCZ
ZCZCZ
ZCZ
Z

The solidification of these coatings, or additional layers of particles, is produced by the same force that effects the solidification of the original crystal itself.

If, on the contrary, the solid crystal is placed in a solution which is not saturated with the same substance, or which is exposed to increase of temperature, then no increase in the size of the crystal is produced, for the chemical action which takes place between the crystal and the *solution*, overcomes the cohesive attraction which binds the particles of

the crystal together. The crystal dissolves in the liquid, and the crystallisation is detroyed.

b.) The *mechanical forces* act differently. When a crystal is freely suspended in a crystallising solution, the eidogens generated by the resulting chemical action take those positions around the crystal which the electric and magnetic currents direct. But where any mechanical obstruction comes in the way, the progress of crystallisation ends; for, as the liquid eidogens have no power to overcome mechanical obstructions, they proceed no farther in the direction of such obstructions. Hence, the face upon which a crystal lies in a pan, or by which it is affixed to an insoluble substance, can receive no addition. Hence also the diversity of incomplete and irregular crystals produced by the crystallisation both of salts and minerals between masses of inert solid matter, or in holes and confined situations, where the liquid eidogens had no opportunity to extend and arrange themselves freely and symmetrically.

Mr. Spencer's very curious experiment of producing, by voltaic agency, veins of metallic copper amidst a porous mass of stucco, shows, in my opinion, very decidedly, the influence of mere mechanically obstructions in retarding the arrangement of eidogens, and so preventing the formation of regular crystals.

294. Let us now examine a few cases in crystallography with reference to the foregoing hypothesis.

a.) Ehrenberg, in examining crystallising solutions under a powerful microscope, saw no commotion in the liquid, and no appearance of any arrangement of particles, preceding the actual formation of the solid crystal (Pogg. *Ann.* Bd. 35); but I do not conceive this to militate against the hypothesis of the previous formation of liquid eidogens, because in examining a transparent solution with common light, it was *à priori* unlikely that he should see the solid particles in movement. It is probable that a similar examination of a crystallising solution with polarised light, would show the existence of the eidogens.

b.) Ehrenberg, in observing the crystallisation of common salt, first saw flat six-sided tables, which, on the subsequent appearance of cubical crystals, melted away. This is a phenomenon something like the suppositious destruction of one eidogen by another, noticed in § 293, No. (8).

c.) This hypothesis readily accounts for the formation of prisms of all dimensions, their lateral dimensions or angles being determined (1)—(4) by the electric or magnetic properties of each substance; the word substance meaning either a chemical substance, or a group of physical atoms; it being impossible to determine whether the phenomenon is chemical or physical.

d.) The formation of prisms with oblique terminations is accounted for by (12), where the plane of the magnetic axes is supposed to be inclined to the electric current. It is, of course, alike rational to suppose the inclination of this plane to be in the direction of its shorter as of its

longer diagonal; so that this hypothesis explains equally well the production of the two different kinds of oblique prisms.

e.) The hypothesis also readily explains the production of pyramids or octahedrons formed by the intersection of two, three, six, twelve, or more eidogens; forming pyramids of eight, twelve, twenty-four, forty-eight, or any greater number of planes; four planes being the quantity produced by every eidogen that acts upon the same centre. See No. (11). § 293.

Am I asked for evidence in support of the hypothesis laid down in (11), namely, that different substances have an inherent power to regulate the number, direction, and intensity of the electric and magnetic currents? The evidence is, that exactly such combinations of eidogens exist, as would be produced if the hypothesis were true. You read and interpret a cypher, and you naturally infer that you have found the key to it. Besides, what extravagance is there in the supposition that, at a given centre, where an electric current is admitted to exist, such a system of vibrations may come into play as suffices to turn the electric current several times at right angles, or at equal angles, and simultaneously to change the magnetic poles? Suppose the phenomenon to become visible to the eye, would it appear more astonishing than is the symmetrical arrangement of iron filings by magnetic power, or the symmetrical arrangement of common dust upon a plate of glass, which takes place when you apply a fiddle-bow to the edge of the glass?

f.) The production of hemihedral forms with inclined faces, such as the tetrahedron, may be explained by the intersection of two eidogens having similar proportions and magnitudes, but situated at different levels; the amount of the difference of level being always equal to one half of the width of the intersecting eidogens. A hemihedral form with inclined faces contains, therefore, planes derived from the half of two eidogens. As it is reasonable to imagine that there is a systematic circulation of the electric and magnetic currents in every complete eidogen, one might suppose that a form containing incomplete eidogens would have peculiar electrical relations; and this supposition is proved by experiment to be well grounded, for very nearly every crystal that contains hemihedral forms with inclined faces, possesses the singular property of *pyro-electricity;* that is to say, it becomes *electric with polarity upon every change of temperature,* as witness tourmaline, boracite, and the like.

The phenomena described in § 270, the singular relations which hold between the octahedron, the tetrahedron, and the combination of two tetrahedrons, are easily explained according to this hypothesis. If the reader will merely read § 270, and substitute throughout the word *eidogen* for the term "cutting prism," the theory of the formation of the crystals in question will be instantly comprehended.

g.) The production of hemitrope crystals is explained by supposing a liquid eidogen, or a liquid crystal (9), to be affected by a play of attractions in such a manner that one half of it, namely, the portions marked *sew, or sen, separates* from the other half, and after turning round 180°, is

forced, by a contrary attraction, to rejoin the other half in an inverted position. I do not mean to convey the idea that the half of an eidogen,

```
       s                 s                  s
       CZ                CZ                C  Z
       ZC                ZC                Z  C
     CZCZCZ            CZCZCZ            CZC  ZCZ
e    ZCZCZC  w     e             w    e  ZCZ  CZC  w
       CZ              ZCZCZC              C  Z
       ZC                CZ                Z  C
       n                 ZC                  n
                         n
```

or the half of a crystal, is turned round in a mass; but rather, that all the poles on one side of an eidogen or a crystal becoming changed, as they possibly could be by induced electricity from an accidental source, every particle of the compound necessarily changes its position; and when every particle is inverted, of course the whole is inverted.

h.) The occurrence of crystals of fluor spar, gypsum, calcareous spar, and many other minerals, intersecting one another in all directions, is to be explained by (7—9), that the eidogens, while liquid, can readily intersect one another, or even intersect crystals already formed, but which remain in a state of liquidity.

i.) "It is a fact worthy of attention, that the amphiboles which contain alumina, or those of which the composition is most complicated, are almost always found crystallized with secondary facets; while the grammatites, of more simple constitution, present only primitive facets."—Von Bonsdorff, *Annals Phil.* Oct. 1822. If different groups of physical or chemical atoms produce, as stated in (11), different eidogens, it is only what we might expect, that compounds which contain the greatest variety of different groups should present facets (planes) derived from the greatest variety of eidogens.

j.) "If we take a mass of alum of sufficient size, all traces of whose exterior crystalline form have been removed by cutting and grinding, and expose it to the solvent power of water, the fluid will act upon it at first in all directions alike; but as the water approaches its point of saturation, the force of heterogeneous adhesion diminishes, and is nearly balanced by the force of homogeneous cohesion, which latter only yields ultimately in those directions of least resistance, which are determined by the regular structure of crystalline arrangement. Under these circumstances its surface will become embossed with the forms of octahedrons and sections of octahedrons, and an immense variety of geometrical figures stamped, as it were, or carved upon its substance."—Daniel, *Chemical Philosophy*, page 76. The engravings which accompany the above paragraph show, that a mass of alum thus operated upon, consisted of a congeries of rhombic eidogens. This, with other examples quoted by Professor Daniel, tend to justify the statement made in § 293, No. (13).

k.) Many unequiaxed crystals, when exposed to a moderate heat, change their figure slightly; the explanation of which is, that at different temperatures, the same substance develops a different quantity of electromotive power, and produces different eidogens. When, therefore, a

crystal is heated, the magnetic poles are changed; one of the magnetic axes lengthens and the other shortens, and as the particles move in a corresponding ratio, there is necessarily a change effected in the form of the eidogens, and therefore in the inclination of the planes of the crystal. Equiaxed crystals, produced by the action of several similar and equal eidogens, expand in all directions equally when heated, the alteration which takes place in one eidogen being compensated by that which takes place equally in all that are present.

l.) Most crystals when strongly heated, fly into pieces, and several kinds may thus be separated into distinct crystals entirely different from the crystals from which they are produced. The explanation of this phenomenon is the same as the foregoing: a change of temperature alters the electrical properties of the substance, and changes its magnetic axes. New eidogens are formed, and the old ones are rent to pieces, that the particles may move into their new positions. Or supposing the heated crystal to consist of a congeries of eidogens, these may be supposed to have the property of combining at different temperatures into differently grouped combinations.

m.) The regular octahedron is frequently found having four planes in the direct octahedral zone much larger than the four complementary planes in the inverse zone (= ½PMT Znw Zse, ½pmt Zne Zsw), in consequence of which there are edges instead of solid angles at the Z and N poles, and the resulting form has the appearance of the combination $M\frac{7}{10}T \cdot P\frac{7}{10}T$. This variety is common in the mineral called magnetic iron ore. The scalene octahedron peculiar to sulphur, $P\dagger\frac{9}{10}M\frac{8}{10}T$, Model 21, is liable to the same irregularity, and consequently produces the combination ½ $P\dagger\frac{9}{10}M\frac{8}{10}T$ Znw Zse, ½ $p\dagger\frac{9}{10}m\frac{8}{10}t$ Zne Zsw, in which also there are edges instead of solid angles at the poles Z and N.

The explanation of this irregularity is extremely easy; we need only suppose that the two eidogens whose intersection produced each homohedral octahedron, though of the usual dimensions and situated at the usual levels, happened at the moment of action to be of different magnitudes. We then perceive at a glance how the different faces came to be of different sizes; the large planes being necessarily produced by the small eidogens, and the small planes by the large eidogens.

n.) It is equally easy to explain the difference which is so frequently observed in the size of the planes of the cube, for example, when replacing the solid angles of the octahedron, which planes should be p,m,t. PMT, but which are sometimes P Z, p N, M n, M s, T e, T w. PMT, or exhibit other irregularities equally remarkable.

The explanation of this irregularity is, that the square eidogens which form the different planes of the cube, have not their axes exactly upon the same centre as the axes of the eidogens which produce the octahedron. The consequence is, that the corners of the octahedron are cut off unequally, according to the distance of the extremity of each point from the centre of the square eidogens.

o.) *The same* explanations, or one or other of them, apply to nearly all

the cases of forms and combinations which exhibit unequal extension of their planes; as, for example, the case where the pyramidal dodecahedron, Model 26, exhibits four planes in one zone much larger, or much smaller, than its other eight planes, and the case where the prismatic zone, T,M_T, exhibits the unsymmetrical planes T_w, $t_{+}e$, M_T. In these, and a multitude of similar examples, which will immediately suggest themselves to a person accustomed to look over crystallised minerals, we can readily explain the phenomenon by one of the foregoing suppositions, namely, that the eidogen which produced the unequal planes was of different magnitude from the corresponding eidogens in other zones, or that it had its axis on a different centre.

p.) The eidogens which produce the combinations of the oblique prismatic system have the property of moving all in one plane; that is to say, their axes are stellate. I have shown in PART II., page 78, that crystals of this class may be divided into two kinds, namely:

a.) Those which have M_T. $\frac{1}{2}P_xM$ Zn, $\frac{1}{2}P_xM_yT_z$ Zne Znw.
b.) Those which have M_T. $\frac{1}{2}P_xT$ Zw, $\frac{1}{2}P_xM_yT_z$ Znw Zsw.

I explain the forms of class *a*), by supposing the axes of all the eidogens to lye in the plane of the north meridian, and those of class *b*) to lye in the plane of the east meridian. There is, I think, no example of a hemihedral combination belonging to the oblique prisms which has the axes of part of its eidogens in one of these meridians and part in the other.

q.) The combinations of the prismatic system, which has only homohedral forms, that is to say, only forms which exhibit all the four planes of every rhombic eidogen, and all the eight planes of every octahedron, have eidogens with axes in several planes. Thus, Model 110, containing the forms M,M_xT, P_xM, P_xT, has three rhombic eidogens, whose axes are situated at right angles to one another, and lie in the planes of the north meridian, the east meridian, and the equator.

r.) When the axes of several eidogens which occur upon one combination, are *in one plane*, the ratio of the two magnetic axes can only be expressed by a high fraction; whereas, when the axes of the eidogens of a combination occur *in many planes*, the ratio of the magnetic axes can be expressed by a very simple fraction. The same remark holds good respecting the relation between the magnetic axes and the longitudinal electric axis of an eidogen. Thus, it will be found, that the relations of the axes p^2 m^2 t^2 of all the forms of the octahedral system, are expressed in very simple numbers: see Part II., page 15; while the relations of the axes of the forms belonging to the oblique prismatic system, can only be expressed by very complex fractions: see Part II., pages 77—90. And for the same reason, the characteristics of the forms of the pyramidal system, Part II., page 32, are simpler than those of the forms of the prismatic system, Part II., page 61.

It may be inferred from this, that substances belonging to different classes of crystallisation possess an inherent power to regulate the in-

tensity of the electric and magnetic forces, each according to its class, and that the number of vibrations which the electric and magnetic currents may make on a given centre, as well as the figure and dimensions of the resulting eidogens, depend upon the intensity of the forces engendered by particular substances, according to the exact quantum of which there may be greater or less length in the electric axis, more or less inequality in the magnetic axes, and more or less variety in the number of oscillations.

But apart from all explanation, the fact is not a little extraordinary, that the complete, the symmetrical, the complex combinations of the octahedral system of crystallisation, should all have forms whose axes can be named with very simple numbers, while, in proportion as the forms upon the several other classes of combinations become less complete and less symmetrical, and the combinations less complex, the axes of the forms require to be represented by fractions that are more and more complex and irregular.

This brief explanation of a Theory of Crystallisation, being merely intended to generalise the views taken in the preceding three sections, of the causes of the symmetry of forms and combinations, it must not be expected to present a full and exact account of all the phenomena of crystallisation. I have, in the present essay, merely noticed a few of the most striking facts respecting crystals and crystallisation, and given a running commentary thereon, in allusion to the above hypothesis. No great harm can arise if my theoretical views are altogether erroneous, because the system of crystallography which I have proposed, is in no respect dependent upon this particular theory, nor upon any theory relating to the philosophy of crystallisation; but is based entirely upon the geometrical relations of crystalline forms. But, I will add, that as this theory has arisen in my mind during the investigation of the geometrical relations of the forms of crystals, so I think it adapted to give very important general views of the nature and structure of crystals, and of the geometrical relations of their forms; and I am satisfied, that, even if the theory is wrong, it is adapted, as a mere collection of arbitrary data, greatly to facilitate the study of crystallography.

SECTION XII.—THE USE OF SPHERICAL TRIGONOMETRY IN CRYSTALLOGRAPHY.

295. The trigonometrical calculations made in the foregoing SECTIONS, have been after formulæ derived from *Plane* Trigonometry and without the aid of logarithms. I explained in §§ 37—62, as much of plane trigonometry as was necessary for the purpose then in view, and I have throughout adhered to the methods there explained. We have, however, other calculations to make now, for which plane trigonometry is insufficient, and the making of which without the aid to be derived from logarithms, would be extremely wearisome. I propose, therefore, to call in the assistance of spherical trigonometry and of logarithms, to afford the additional mathematical help which the Crystallographer requires. It is not my intention to explain at length the doctrines of trigonometry. I shall only take such information from that science as answers my present purpose, reduce the extracts to a series of rules or formulæ, and show how those rules are to be worked with the help of logarithmic tables.

296. I think it right, however, to warn the reader, that I find it to be a difficult task to explain briefly, and yet in a popular manner, the application of trigonometry to crystallography; and although I shall endeavour to make the explanation intelligible, it is likely I shall not succeed completely. In that case, it will remain for the reader to seek mathematical information from persons who teach it professionally; and I can assure every one who is not acquainted with geometry, algebra, and trigonometry, that a few weeks spent in learning the elements of these three branches of science, will greatly facilitate his subsequent study of crystallography. To direct the course of those who may be inclined to act upon this advice, I shall give, in this section, a table of all the formulæ employed in the calculation of spherical triangles, and which are applicable to crystallography; the methods of working which formulæ ought to be thoroughly understood. The necessary mathematical knowledge can be readily acquired, not only from a crystallographer who understands mathematics, but from *a mathematician who is not conversant with crystallography*, which is a consideration of much importance; for it is far easier to procure personal instruction in mathematics than in crystallography. It will, therefore, clear away much doubt and difficulty, if I show the student what he ought to learn, and the mathematician what he has to teach, in reference to this subject. The crystallographer who knows how to work the formulæ that are given in this section, and those which depend upon the principles explained in § 16 and §§ 37—62, has sufficient mathematical knowledge to enable him to proceed with the study of his main subject; but he will be infinitely better prepared for his task, *if he can find the leisure* or the opportunity to make himself

master of the theory of trigonometry, by investigating the derivation of these formulæ, and the demonstration of their correctness. Such points as these lye altogether out of my path, and will be passed without farther notice.

297. The formulæ given below include many that are not used in the present work, but which will often be found useful by the student who investigates the systems of other crystallographers. The table of square roots, and others, given in this Section or at the end of the volume, are appended for the same reason. I found these tables useful to me while I was engaged in studying and compiling the Second Part of this work, and I have therefore printed them as likely to be useful to my readers when they consult the authorities that I have so frequently quoted.

298. TABLE OF TRIGONOMETRICAL FORMULÆ

FOR CALCULATING THE RELATIONS BETWEEN THE SIDES AND ANGLES OF TRIANGLES.

A.) RIGHT-ANGLED SOLID TRIANGLES,

Where angle C = 90°.

No.	Given.	Sought.	Equations.	Logarithmic Equations.
1.	A,a	b	$\sin b = \frac{\tan a}{\tan A}$	$\log \sin b = \log \tan a + 10 - \log \tan A.$
2.	A,a	B	$\sin B = \frac{\cos A}{\cos a}$	$\log \sin B = \log \cos A + 10 - \log \cos a.$
3.	A,a	c	$\sin c = \frac{\sin a}{\sin A}$	$\log \sin c = \log \sin a + 10 - \log \sin A.$
4.	A,B	a	$\cos a = \frac{\cos A}{\sin B}$	$\log \cos a = \log \cos A + 10 - \log \sin B.$
5.	A,B	b	$\cos b = \frac{\cos B}{\sin A}$	$\log \cos b = \log \cos B + 10 - \log \sin A.$
6.	A,B	c	$\cos c = \cot A \cot B$	$\log \cos c = \log \cot A + \log \cot B - 10.$
7.	A,b	a	$\tan a = \tan A \sin b$	$\log \tan a = \log \tan A + \log \sin b - 10.$
8.	A,b	B	$\cos B = \cos b \sin A$	$\log \cos B = \log \cos b + \log \sin A - 10.$
9.	A,b	c	$\tan c = \frac{\tan b}{\cos A}$	$\log \tan c = \log \tan b + 10 - \log \cos A.$
10.	a,B	A	$\cos A = \cos a \sin B$	$\log \cos A = \log \cos a + \log \sin B - 10.$
11.	a,B	b	$\tan b = \tan B \sin a$	$\log \tan b = \log \tan B + \log \sin a - 10.$
12.	a,B	c	$\tan c = \frac{\tan a}{\cos B}$	$\log \tan c = \log \tan a + 10 - \log \cos B.$
13.	a,b	A	$\tan A = \frac{\tan a}{\sin b}$	$\log \tan A = \log \tan a + 10 - \log \sin b.$
14.	a,b	B	$\tan B = \frac{\tan b}{\sin a}$	$\log \tan B = \log \tan b + 10 - \log \sin a.$
15.	a,b	c	$\cos c = \cos a \cos b$	$\log \cos c = \log \cos a + \log \cos b - 10.$
16.	A,c	a	$\sin a = \sin c \sin A$	$\log \sin a = \log \sin c + \log \sin A - 10.$
17.	A,c	B	$\cot B = \frac{\cos c}{\cot A}$	$\log \cot B = \log \cos c + 10 - \log \cot A.$
18.	A,c	b	$\tan b = \tan c \cos A$	$\log \tan b = \log \tan c + \log \cos A - 10.$

No.	Given.	Sought.	Equations.	Logarithmic Equations.
19.	a,c	A	$\sin A = \dfrac{\sin a}{\sin c}$	$\log \sin A = \log \sin a + 10 - \log \sin c.$
20.	a,c	B	$\cos B = \dfrac{\tan a}{\tan c}$	$\log \cos B = \log \tan a + 10 - \log \tan c.$
21.	a,c	b	$\cos b = \dfrac{\cos c}{\cos a}$	$\log \cos b = \log \cos c + 10 - \log \cos a.$
22.	B,b	A	$\sin A = \dfrac{\cos B}{\cos b}$	$\log \sin A = \log \cos B + 10 - \log \cos b.$
23.	B,b	a	$\sin a = \dfrac{\tan b}{\tan B}$	$\log \sin a = \log \tan b + 10 - \log \tan B.$
24.	B,b	c	$\sin c = \dfrac{\sin b}{\sin B}$	$\log \sin c = \log \sin b + 10 - \log \sin B.$
25.	B,c	A	$\cot A = \dfrac{\cos c}{\cot B}$	$\log \cot A = \log \cos c + 10 - \log \cot B.$
26.	B,c	a	$\tan a = \tan c \cos B$	$\log \tan a = \log \tan c + \log \cos B - 10.$
27.	B,c	b	$\sin b = \sin c \sin B$	$\log \sin b = \log \sin c + \log \sin B - 10.$
28.	b,c	A	$\cos A = \dfrac{\tan b}{\tan c}$	$\log \cos A = \log \tan b + 10 - \log \tan c.$
29.	b,c	a	$\cos a = \dfrac{\cos c}{\cos b}$	$\log \cos a = \log \cos c + 10 - \log \cos b.$
30.	b,c	B	$\sin B = \dfrac{\sin b}{\sin c}$	$\log \sin B = \log \sin b + 10 - \log \sin c.$

B.) OBLIQUE-ANGLED SOLID TRIANGLES.

No.	Given.	Sought.	Equations.	Logarithmic Equations.
31.	A,a,B	b	$\sin b = \dfrac{\sin B \sin a}{\sin A}$	$\text{Log} \sin b = \log \sin B + \log \sin a - \log \sin A$
32.	A,a,B	C	$\sin (C - x) = \dfrac{\cos A \sin x}{\cos B}$ where $\tan x = \dfrac{\cot B}{\cos a}$ Then, $C = C - x + x$	$\text{Log} \tan x = \log \cot B - \log \cos a + 10$ $\text{Log} \sin (C - x) = \log \cos A + \log \sin x - \log \cos B.$
33.	A,a,B	c	$\sin (c - x) = \dfrac{\tan B \sin x}{\tan A}$ where $\tan x = \tan a \cos B$ Then, $c = c - x + x.$	$\text{Log} \tan x = \log \tan a + \log \cos B - 10.$ $\text{Log} \sin (c - x) = \log \tan B + \log \sin x - \log \tan A$
34.	A,a,b	B	$\sin B = \dfrac{\sin b \sin A}{\sin a}$	$\text{Log} \sin B = \log \sin b + \log \sin A - \log \sin a$
35.	A,a,b	C	$\sin (C + x) = \dfrac{\tan b \sin x}{\tan a}$ where $\tan x = \tan A \cos b$ Then, $C = C + x - x.$	$\text{Log} \tan x = \log \tan A + \log \cos b - 10$ $\text{Log} \sin (C + x) = \log \tan b + \log \sin x - \log \tan a$
36.	A,a,b	c	$\cos (c - x) = \dfrac{\cos a \cos x}{\cos b}$ where $\tan x = \tan b \cos A$ Then, $c = c - x + x.$	$\text{Log} \tan x = \log \tan b + \log \cos A - 10$ $\text{Log} \cos (c - x) = \log \cos a + \log \cos x - \log \cos b$
37.	A,B,C	a	$\sin \tfrac{1}{2} a = \sqrt{\dfrac{-\cos S \cos(S-A)}{\sin B \sin C}}$ where $S = \tfrac{1}{2} (A + B + C)$	$\text{Log} \sin \tfrac{1}{2} a = \tfrac{1}{2}[\log \cos S + \log \cos (S - A) - (\log \sin B + \log \sin C) + 20]$

No.	Given.	Sought.	Equations.	Logarithmic Equations.
38.	A,B,C	a	$\cos a=\frac{2\cos\frac{1}{2}(A+x)\cos\frac{1}{2}(A-x)}{\sin B\sin C}$ where $\cos x = \cos B\cos C$	Log $\cos x = \log\cos B + \log\cos C - 10$. Log $\cos a = \log 2 + \log\cos\frac{1}{2}(A+x) + \log\cos\frac{1}{2}(A-x) - (\log\sin B + \log\sin C) + 10$
39.	A,B,c	a	$\tan\frac{1}{2}(a+b)=\tan\frac{1}{2}c\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}$ $\tan\frac{1}{2}(a-b)=\tan\frac{1}{2}c\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}$ Then, $a=\frac{1}{2}(a+b)+\frac{1}{2}(a-b)$	Log $\tan\frac{1}{2}(a+b) = \log\tan\frac{1}{2}c + \log\cos\frac{1}{2}(A-B) - \log\cos\frac{1}{2}(A+B)$ Log $\tan\frac{1}{2}(a-b) = \log\tan\frac{1}{2}c + \log\sin\frac{1}{2}(A-B) - \log\sin\frac{1}{2}(A+B)$
40.	A,B,c	b	Same as No. 39 Then, $b=\frac{1}{2}(a+b)-\frac{1}{2}(a-b)$	
41.	A,B,c	C	$\sin C=\frac{\sin c\sin A}{\sin a}$ a is first found by No. 39.	Log $\sin C = \log\sin c + \log\sin A - \log\sin a$
42.	A,B,c	C	$\cos C=\frac{\cos A\sin(B-x)}{\sin x}$ where $\cot x = \tan A\cos c$	Log $\cot x = \log\tan A + \log\cos c - 10$ Log $\cos C = \log\cos A + \log\sin(B-x) - \log\sin x$
43.	A,B,c	a	$\cot a=\frac{\cot c\cos(B-x)}{\cos x}$ where $\cot x = \tan A\cos c$	Log $\cot x = \log\tan A + \log\cos c - 10$ Log $\cot a = \log\cot c + \log\cos(B-x) - \log\cos x$
44.	A,B,c	b	Similar to No. 43, B and A changing places.	
45.	a,b,C	A	$\tan\frac{1}{2}(A+B)=\cot\frac{1}{2}C\frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}$ $\tan\frac{1}{2}(A-B)=\cot\frac{1}{2}C\frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}$ Then, $A=\frac{1}{2}(A+B)+\frac{1}{2}(A-B)$	Log $\tan\frac{1}{2}(A+B) = \log\cot\frac{1}{2}C + \log\cos\frac{1}{2}(a-b) - \log\cos\frac{1}{2}(a+b)$ Log $\tan\frac{1}{2}(A-B) = \log\cot\frac{1}{2}C + \log\sin\frac{1}{2}(a-b) - \log\sin\frac{1}{2}(a+b)$
46.	a,b,C	B	Same as No. 45. Then, $B=\frac{1}{2}(A+B)-\frac{1}{2}(A-B)$	
47.	a,b,C	c	$\sin c=\frac{\sin C\sin a}{\sin A}$ A is first found by No. 45.	Log $\sin c = \log\sin C + \log\sin a - \log\sin A$
48.	a,b,C	A	$\tan A=\frac{\tan C\sin x}{\sin(b-x)}$ where $\tan x = \tan a\cos C$	Log $\tan x = \log\tan a + \log\cos C - 10$ Log $\tan A = \log\tan C + \log\sin x - \log\sin(b-x)$.
49.	a,b,C	B	Similar to No. 48, a and b changing places.	
50.	a,b,C	c	$\cos c=\frac{\cos b\cos(a-x)}{\cos x}$ where $\tan x = \tan b\cos C$	Log $\tan x = \log\tan b + \log\cos C - 10$. Log $\cos c = \log\cos b + \log\cos(a-x) - \log\cos x$
51.	a,b,c	A	$\cos A=\frac{2\sin\frac{1}{2}(x+a)\sin\frac{1}{2}(x-a)}{\sin b\sin c}$ where $\cos x = \cos b\cos c$	Log $\cos x = \log\cos b + \log\cos c - 10$ Log $\cos A = \log 2 + \log\sin\frac{1}{2}(x+a) + \log\sin\frac{1}{2}(x-a) - (\log\sin b + \log\sin c) + 10$
52.	a,b,c	A	$\sin\frac{1}{2}A=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b\sin c}}$ where $s=\frac{1}{2}(a+b+c)$	Log $\sin\frac{1}{2}A = \frac{1}{2}[\log\sin(s-b) + \log\sin(s-c) - (\log\sin b + \log\sin c) + 20]$

C.) QUADRANTAL SOLID TRIANGLES.

Where side $c = 90°$.

A solid triangle which has one of its *sides* a quadrant, is called a quadrantal solid triangle.

For the solution of the several cases of quadrantal solid triangles, only two quantities are required to be given, besides the side of 90°.

The equations which contain the sign – may give either the desired angle or its supplement. See §§ 330, 331.

No.	Given.	Sought.	Equations.	Logarithmic Equations.
53.	A,a	B	$\sin B = \frac{\tan A}{\tan a}$	$\log \sin B = \log \tan A + 10 - \log \tan a$.
54.	A,a	b	$\sin b = \frac{\cos a}{\cos A}$	$\log \sin b = \log \cos a + 10 - \log \cos A$.
55.	A,a	C	$\sin C = \frac{\sin A}{\sin a}$	$\log \sin C = \log \sin A + 10 - \log \sin a$.
56.	A,B	a	$\tan a = \frac{\tan A}{\sin B}$	$\log \tan a = \log \tan A + 10 - \log \sin B$.
57.	A,B	b	$\tan b = \frac{\tan B}{\sin A}$	$\log \tan b = \log \tan B + 10 - \log \sin A$.
58.	A,B	C	$\cos C = -\cos A \cos B$	$\log \cos C = \log \cos A + \log \cos B - 10$.
59.	A,b	a	$\cos a = \cos A \sin b$	$\log \cos a = \log \cos A + \log \sin b - 10$.
60.	A,b	B	$\tan B = \tan b \sin A$	$\log \tan B = \log \tan b + \log \sin A - 10$.
61.	A,b	C	$\tan C = -\frac{\tan A}{\cos b}$	$\log \tan C = \log \tan A + 10 - \log \cos b$.
62.	a,B	A	$\tan A = \tan a \sin B$	$\log \tan A = \log \tan a + \log \sin B - 10$.
63.	a,B	b	$\cos b = \cos B \sin a$	$\log \cos b = \log \cos B + \log \sin a - 10$.
64.	a,B	C	$\tan C = -\frac{\tan B}{\cos a}$	$\log \tan C = \log \tan B + 10 - \log \cos a$.
65.	a,b	A	$\cos A = \frac{\cos a}{\sin b}$	$\log \cos C = \log \cos a + 10 - \log \sin b$.
66.	a,b	B	$\cos B = \frac{\cos b}{\sin a}$	$\log \cos B = \log \cos b + 10 - \log \sin a$.
67.	a,b	C	$\cos C = -\cot a \cot b$	$\log \cos C = \log \cot a + \log \cot b - 10$.
68.	A,C	a	$\sin a = \frac{\sin A}{\sin C}$	$\log \sin a = \log \sin A + 10 - \log \sin C$.
69.	A,C	B	$\cos B = -\frac{\cos C}{\cos A}$	$\log \cos B = \log \cos C + 10 - \log \cos A$.
70.	A,C	b	$\cos b = -\frac{\tan A}{\tan C}$	$\log \cos b = \log \tan A + 10 - \log \tan C$.
71.	a,C	A	$\sin A = \sin C \sin a$	$\log \sin A = \log \sin C + \log \sin a - 10$.
72.	a,C	B	$\tan B = -\tan C \cos a$	$\log \tan B = \log \tan C + \log \cos a - 10$.
73.	a,C	b	$\cot b = -\frac{\cos C}{\cot a}$	$\log \cot b = \log \cos C + 10 - \log \cot a$.
74.	B,b	A	$\sin A = \frac{\tan B}{\tan b}$	$\log \sin A = \log \tan B + 10 - \log \tan b$.
75.	B,b	a	$\sin a = \frac{\cos b}{\cos B}$	$\log \sin a = \log \cos b + 10 - \log \cos B$.
76.	B,b	C	$\sin C = \frac{\sin B}{\sin b}$	$\log \sin C = \log \sin B + 10 - \log \sin b$.

77.	B,C	A	$\cos A = -\frac{\cos C}{\cos B}$	$\log\cos A = \log\cos C + 10 - \log\cos B.$
78.	B,C	a	$\cos a = -\frac{\tan B}{\tan C}$	$\log\cos a = \log\tan B + 10 - \log\tan C.$
79.	B,C	b	$\sin b = \frac{\sin B}{\sin C}$	$\log\sin b = \log\sin B + 10 - \log\sin C.$
80.	b,C	A	$\tan A = -\tan C \cos b$	$\log\tan A = \log\tan C + \log\cos b - 10.$
81.	b,C	a	$\cot a = -\frac{\cos C}{\cot b}$	$\log\cot a = \log\cos C + 10 - \log\cot b.$
82.	b,C	B	$\sin B = \sin C \sin b$	$\log\sin B = \log\sin C + \log\sin b - 10.$

D.) RIGHT-ANGLED PLANE TRIANGLES.

These are printed with the other formulæ chiefly for the sake of convenience in reference. The subjoined figure represents *lineally* several of those important functions of an angle, which the Table of Sines, &c. gives *numerically*. They have been already explained in §§ 37—62, but may still be advantageously recited here.

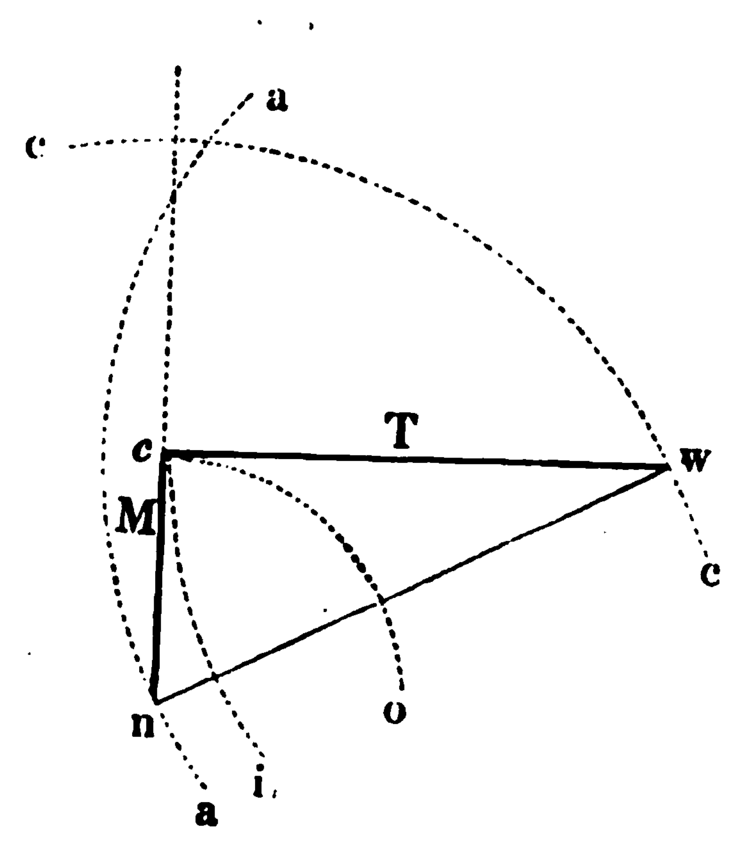
Let the arc a a be part of a circle which has w for its centre, and the angle *c*wn for its limits. Then *c*n is its sine and *c*w its cosine.

Let *c* i be an arc parallel to the arc a a, and having the same centre, w, and the same limits. Then *c* n is its tangent, *c* w its cotangent, and n w its secant.

Let c c be an arc with n for its centre, and the angle *c*nw for its limits. Then *c* w is its sine and *c* n its cosine.

Let *c* o be an arc parallel to the arc c c, and having the same centre, n, and the same limits. Then *c* w is its tangent, *c* n its cotangent, and n w its secant.

The sines of the angles of a triangle are proportionate to the opposite sides.



The figure n*c*w represents a right-angled plane triangle; it also represents the quadrant of an equator. *c* or n*c*w is the right angle and the centre of the equator. M and T are axes perpendicular to one another. $M = m^2$ $T = t^2$.

The angles n, or *c*nw, and w, or *c*wn, are of variable magnitude; but one of them is always the complement of the other, so that if, in any right-angled triangle, either n or w be given, then n, *c* and w are all given; because the sum of the three angles of a triangle is equal to two right angles. Hence:

$$c = 90°;\ n = 90° - w;\ \text{and } w = 90° - n.$$

No.	Given.	Sought.	Equations.	Logarithmic Equations.
83.	c,M,T	n	$\tan n = \frac{T}{M}$	Log tan n = log T + 10 - log M
84.	c,M,T	n	$\cot n = \frac{M}{T}$	Log cot n = log M + 10 - log T,
85.	c,M,T	w	$\tan w = \frac{M}{T}$	Log tan w = log M + 10 - log T
86.	c,M,T	w	$\cot w = \frac{T}{M}$	Log cot w = log T + 10 - log M

[Put T or $t^a = 1$ (unity), then M or $m^a = \begin{cases} \text{tan angle w, or} \\ \text{cot angle n} \end{cases}$]

No.	Given.	Sought.	Equations.	
87.	n	m^a	$m^a = \cot n$	Axis t^a being 1.0.
88.	w	m^a	$m^a = \tan w$	
89.	m^a	n	$n = \cot m^a$	
90.	m^a	w	$w = \tan m^a$	

E.) OBLIQUE-ANGLED PLANE TRIANGLES.

Let the three angles be called A, B, C, and the three sides, situated respectively opposite to the three angles, a, b, c.

There must always be three quantities given, and one of these a side.

If two of the angles A and B be given, the third C is found thus: $C = 180° - (A + B)$.

No.	Given.	Sought.	Equations.	Logarithmic Equations.
91.	A,B,	C	$C = 180° - (A + B)$	
92.	A,a,b	B	$\sin B = \frac{b \sin A}{a}$ When A is less than 90° and a less than b, the angle B may be either that given in the Table of Sines, or 180° - B.	$\log \sin B = \log b + \log \sin A - \log a$.
93.	A,a,b	C	Find B by 92; then C by 91.	
94.	A,a,b	c	Find C by 93; then c by 95.	
95.	A,a,C	c	$c = \frac{a \sin C}{\sin A}$	$\log c = \log a + \log \sin C - \log \sin A$
96.	A,B,b	a	$a = \frac{b \sin A}{\sin B}$	$\log a = \log b + \log \sin A - \log \sin B$
97.	A,B,b	c	$c = \frac{b \sin C}{\sin B}$	$\log c = \log b + \log \sin C - \log \sin B$
98.	A,b,c	a	$\sin x = \frac{2\sqrt{bc.} \cos \frac{1}{2} A}{b + c}$ $a = (b + c) \cos x$	$\log \sin x = \log 2 + \frac{1}{2} \log b + \frac{1}{2} \log c + \log \cos \frac{1}{2} A - \log (b + c)$, $\log a = \log (b + c) + \log \cos x - 10$
99.	A,b,c	B	$\tan \frac{1}{2}(B - C) = \frac{b - c}{b + c} \cot \frac{1}{2} A$	$\log \tan \frac{1}{2}(B - C) = \log (b - c) + \log \cot \frac{1}{2} A - \log (b + c)$

Now, $\frac{1}{2}(B + C) = 90° - \frac{1}{2}A$;
because $A + B + C = 180°$
and $\frac{1}{2}(A + B + C) = 90°$.
Then,
$B = \frac{1}{2}(B+C) + \frac{1}{2}(B - C)$

100.	A, b, c	C	Same as No. 99. Then, $C = \frac{1}{2}(B + C) - \frac{1}{2}(B - C)$	
101.	a, b, c	A	$\sin \frac{1}{2} A = \sqrt{\left\{\frac{(s-b)(s-c)}{b c}\right\}}$ where $s = (a + b + c)$	$\log \sin \frac{1}{2} A = \frac{1}{2} [\log (s - b) + \log (s - c) - \log b - \log c] + 10$
			Another method,	
102.	a, b, c	A	$\cos \frac{1}{2} A = \sqrt{\left\{\frac{s.(s-a)}{b c}\right\}}$ where $s = \frac{1}{2}(a + b + c)$	$\log \cos \frac{1}{2} A = \frac{1}{2} [\log s + \log (s - a) - \log b - \log c] + 10$

F.) MISCELLANEOUS EQUATIONS.

These equations are sometimes employed to shorten and simplify the Formulæ of the preceding divisions, as will be shown hereafter by examples. See § 335.

No.	Equations.	Logarithmic Equations.
103.	$\tan A = \frac{\sin A}{\cos A}$	$\log \tan A = \log \sin A + 10 - \log \cos A.$
104.	$\cot A = \frac{\cos A}{\sin A}$	$\log \cot A = \log \cos A + 10 - \log \sin A.$
105.	$\cos A = \cot A \sin A$	$\log \cos A = \log \cot A + \log \sin A - 10.$
106.	$\sin A = \tan A \cos A$	$\log \sin A = \log \tan A + \log \cos A - 10.$
107.	$\tan A \cot A = 1$	$\log \tan A + \log \cot A = 20.0000.$
108.	$\tan A = \frac{1}{\cot A}$	$\log \tan A = 20.0000 - \log \cot A.$
109.	$\cot A = \frac{1}{\tan A}$	$\log \cot A = 20.0000 - \log \tan A.$
110.	$\sec A = \frac{1}{\cos A}$	$\log \sec A = 20.0000 - \log \cos A.$
111.	$\operatorname{cosec} A = \frac{1}{\sin A}$	$\log \operatorname{cosec} A = 20.0000 - \log \sin A.$

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299. *Explanation of such Algebraic Terms and Characters as are employed in this work.*

a.) An *Equation* is an algebraic expression of the equality existing between two given quantities. Thus, $\sin b = \frac{\tan a}{\tan A}$ (Formula 1) is an *Equation*, in which it is intimated that the quantity represented by the term $\sin b$ is equal to the quantity represented by the term $\frac{\tan a}{\tan A}$. In this and in all equations, the two equivalent quantities are separated by the sign $=$, which is thence called *equal to*, or the sign of equality.

b.) $+$ *Plus*, the sign of addition. Two quantities between which it is placed are to be added together.

c.) — *Minus*, the sign of subtraction. The latter of two quantities between which it is placed, is to be subtracted from the former.

Thus, $\log \sin b = \log \tan a + 10 - \log \tan A$ (Formula 1) means, *that the quantity* represented by $\log \sin b$ is equal to the quantity which

is produced by adding 10 to log tan a, and deducting log tan A from the joint sum.

If a quantity appear without a sign, $+$ is understood.

d.) $\times$ The sign of ***Multiplication.*** The quantities between which it is placed are to be multiplied together. Multiplication is also frequently denoted by a point (.) placed between two quantities, and still more frequently by mere juxtaposition, without any intervening sign. Thus, $\cos c = \cot A \cot B$ (Formula 6) signifies, that the quantity represented by $\cos c$ is equal to the quantity represented by cot A multiplied by the quantity represented by cot B. If the formula were $\cos c = \cot A \times \cot B$, or $\cos c = \cot A \,.\, \cot B$, it would have the same meaning.

e.) $\div$ The sign of ***Division.*** The former of two quantities between which it is placed, is to be divided by the latter. But the division of one quantity by another is more frequently represented by placing the dividend over the divisor with a line between them, in which case the expression is called a fraction. Thus, in $\sin b = \frac{\tan a}{\tan A}$ (Formula 1) the fraction $\frac{\tan a}{\tan A}$ denotes the product of the division of the quantity represented by tan a by the quantity represented by tan A.

f.) $\surd$ The ***radical sign*** signifies that the quantity before which it is placed, is to have the square root extracted. See § 333.

g.) A vinculum $\overline{\quad\quad}$, the parenthesis ( ), or the brace { }, is used to collect together quantities which are to be spoken of or employed as a single quantity. Thus, $(C-x)$ in Formula 32, means the single quantity produced by deducting the quantity x from the quantity C. Sometimes a single quantity contains several other single quantities, each composed of several quantities; in which case, both parentheses and braces are employed for the sake of distinctness. See the logarithmic equation, No. 37, where the single quantity enclosed in [ ], in order to be divided by the term $\frac{1}{2}$, is composed of several quantities, each of which require to be enclosed by ( ). The extent of the action of the radical sign is sometimes denoted by the vinculum, which is a horizontal line placed over the sign affected, and joined to the upper point of the radical sign, as shown in Formula 37. This, however, is not essential, and in modern works it is generally omitted, as shown in Formula 52.

h.) ∞ The sign of ***Infinity.*** The quantity to which it is attached is of an unlimited value.

i.) ***Positive*** quantities are such as have the sign $+$ before them; ***negative*** quantities, such as have the sign — before them. When no sign is quoted, the quantities are positive.

k.) ***Abridged Terms indicating the Functions of Angles.***

nat sin = natural sine.	log sin = logarithmic sine.
nat cos = natural cosine.	log cos = logarithmic cosine.
nat tan = natural tangent.	log tan = logarithmic tangent.
nat cot = *natural cotangent.*	log cot = logarithmic cotangent.

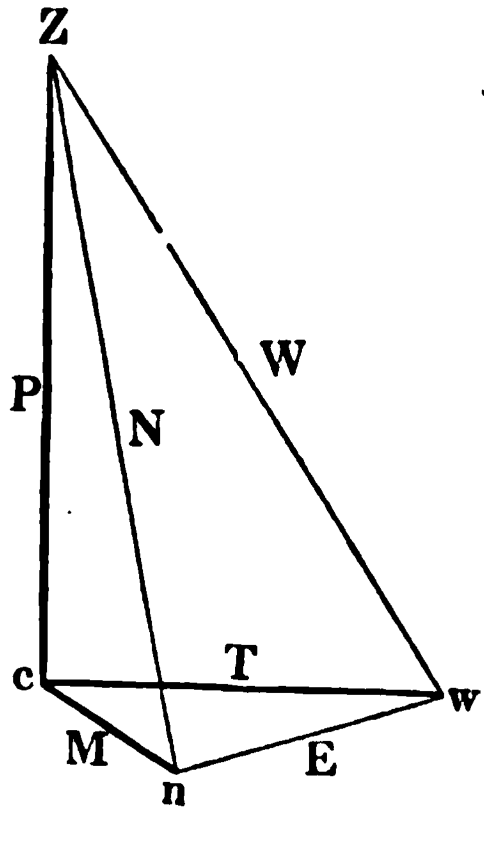
Whenever a function is named without the prefix *log*, it is understood to indicate a *natural* number, whether it is prefixed by *nat* or not.

OF SOLID TRIANGLES, *Spherical Triangles*, or *Trisolid Angles*.

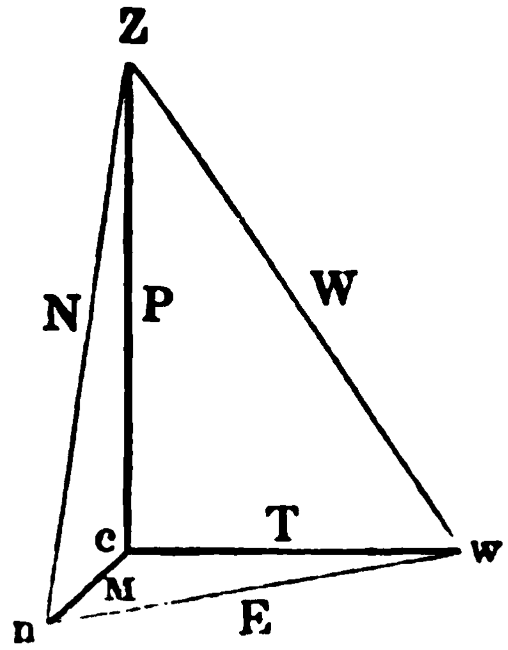
300. The chief use of the Formulæ contained in the Table, § 298, is in calculating the relations existing between the external angles and the axes $p^a\ m^a\ t^a$ of the different varieties of pyramids, in order that, if a symbol be given, such as PMT, $P\frac{1}{3}MT$, or $P\frac{2}{3}MT$, the inclination of the planes and edges may be deduced from the characteristic of the symbol; or if the number of faces of a pyramid and their inclinations be given, a characteristic and symbol may be found to represent the length of the axes.

301. If an octahedron, such as is represented by Model 21, $P\frac{10}{10}M\frac{8}{10}T$, be supposed to be divided into eight parts or octants, by sections parallel to the equator and to the north and east meridians, (see § 187), every one of these octants will be a tetrahedron or solid of four faces, similar to the subjoined figures.

1.



2.

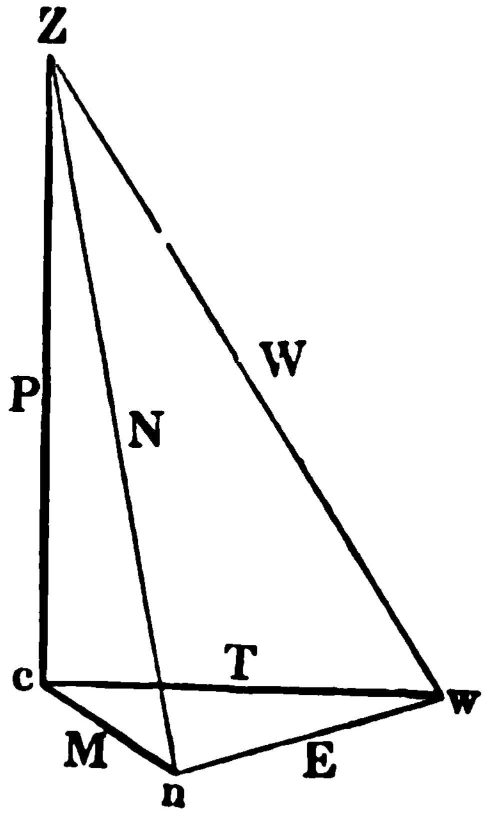


302. The octant chosen for representation, by way of example, is the Znw octant, or that whose external plane touches the Zenith, north and west poles. In figure 1, it is represented as if seen from Zne; in figure 2 as if seen from Znw; in other respects the octants differ only in the apparent length of their various edges. The line P in both represents the Zenith portion of the axis p^a, M the north portion of m^a, and T the west portion of t^a. The line N is the Zn edge of the north meridian, W the Zw edge of the east meridian, E the nw edge of the equator. The point c is the centre of the crystal, Z the Zenith pole, n the north pole, w the west pole. The planes Zcn, Zcw, ncw, are right-angled triangles, and are always right-angled at c, whatever may be the comparative lengths of the three lines P,M,T. The shape of the plane Znw, and the value of its three plane angles, vary with every different "Form." *When the axes* are all equal (PMT), the plane Znw is an equilateral

le; when there is one short and two long, or one long and two axes, (P_MT or P+MT), the plane is an isosceles triangle; and the axes are all unequal ($P_xM_yT_z$), the plane is a scalene triangle. relations may be observed on Models 15, 12, 13, and 21. The acial angle, or inclination of the three planes Zcn, Zcw, ncw, upon nother, across the edges P,M,T, is always 90°. The inclination of one of the three right-angled planes upon the oblique-angled plane ferent, and *the difference of the inclination of each right-angled to the oblique-angled plane bears a constant relation to the lengths three edges* P,M,T. This is the principle upon which the calcu- of spherical trigonometry depend.

L A *spherical*, or a SOLID TRIANGLE, which term I shall employ ference, consists of *six essential parts*, any *three* of which being 1, the others may be found. These *six parts* are the three edges les and the three planes or *sides*, for angles and sides are the terms use of by mathematicians. The three angles are commonly ed by the three capital letters, A, B, C, and the three sides by the letters *a*, *b*, *c*. When one of the angles (edges) of a solid triangle ght angle, it is always denoted by the letter C, and A and B then the two other angles. The sides *a*, *b*, *c* are named after the to which they are respectively opposed. Thus, side *c* is the which is opposite to the angle C, side *a* is the plane opposite to A, and side *b* the plane opposite to angle B. Hence, every angle side of a different name, and every side an angle of a different situated on each hand of it. The point where the three sides and angles all meet is called the centre or vertex of the solid triangle.

:. Every octant produced by the division " form" $P_xM_yT_z$, presents *four* solid les, as shown in the annexed figure.



If *c* is the centre, we have the angles T, and the sides *p*, *m*, *t*, respectively ite to each angle.—2.) If Z is the cen- e have the angles P, N, W, and the p, *n*, *w*.—3.) If *n* is the centre, we have ngles N, M, E, and the sides *n*, *m*, *e*.— nd if *w* is the centre, we have the W, T, E, and the sides *w*, *t*, *e*.

. Now, on the principle stated above, three quantities out of these six which ndispensable to a solid triangle, are the other three can be found. This case even when the solid triangle right angle; and when it *has* a right angle, we require to know wo other quantities, to be put in a position to calculate all the rest. as I have said, is the principle upon which trigonometry depends, e Formulæ which I have given in the Table are simply methods of *this principle into practice*.

306. It will be seen on examining the Formulæ, that the calculations are performed by means of what are called the FUNCTIONS *of an angle*, namely, the terms tangent, sine, &c., which I have already explained in §§ 37—62. It is only necessary to add here a few words respecting the Logarithmic Equations by which the Formulæ are to be worked. The advantages presented by logarithmic over natural or ordinary numbers, in so far as regards the calculations we have now in view, are chiefly these two: *multiplication* is performed by merely adding the multiplicand to the multiplier, and *division* by merely subtracting the divisor from the dividend; by which easy processes we arrive at the same results as by performing a very tedious multiplication or division with ordinary numbers. The Equations which are to be worked by Logarithms are marked so in the Table, where they are always accompanied by equations adapted to be worked by natural numbers.

307. In the Tables of Logarithms referred to in page 16, and in the short Table of Sines and Tangents appended to this work, there are given the numbers which represent as well the logarithmic as the natural tangents, and other functions, which puts it in the student's power to work the equations by either of these methods at his pleasure.

It may not be out of place to add here the following brief account of the *properties of Spherical or Solid Triangles*.

1. A spherical triangle which has one of its sides a quadrant, is called a quadrantal spherical triangle.

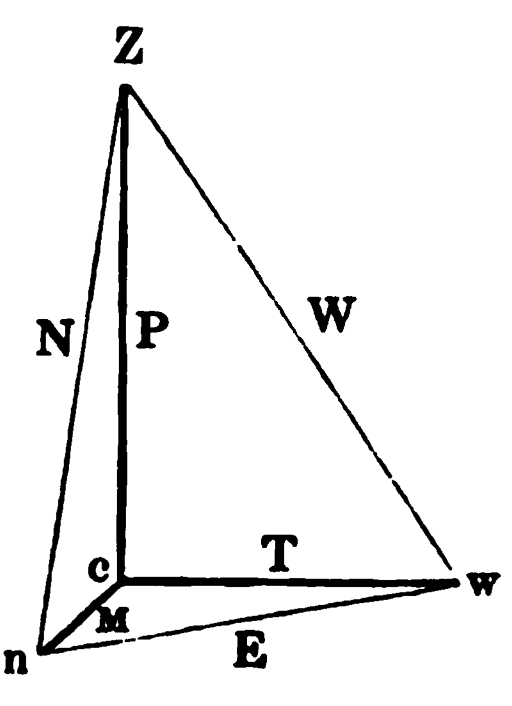
2. If two sides are quadrants, the opposite angles are right angles, and if the three sides are quadrants, the three angles are right angles.

3. If two angles of a spherical triangle are equal, the opposite sides are equal, and conversely, if two sides are equal the opposite angles are equal.

4. The greatest side of a spherical triangle has the greatest angle opposite to it.

5. The sum of the three sides of a spherical triangle is always less than 360°.

6. The sum of the three angles of a spherical triangle is always less than six right angles, and always greater than two right angles.



308. *Description of the Formulæ.*—The Formulæ are so arranged, that any one of them may be readily found, which is adapted to resolve a given question. This point will perhaps be best illustrated by an example:—Suppose you were to examine Model 15, and find the inclination of the faces across the terminal or oblique edges to be in all cases 109° 28′. This, in reference to the octant in the figure, gives you a solid triangle which has Z for its vertex, and in which angle P = 90° (which is always the angle

across every *axis* of an octant), angle N $= \frac{109^\circ 28'}{2} = 54^\circ\ 44'$, and angle W also $= \frac{109^\circ 28'}{2} = 54^\circ\ 44'$. Now, as the right angle, § 303, is always called C, and as the two other angles are called A and B, the quantities given in this case are the three angles A, B, C; and, of course, the quantities which we can find are the three sides *a, b, c*. Let us find each of these quantities. It is evidently of no importance whether the three angles are called N, W, P, or A, B, C, or whether the three sides are called *n, w, p*, or *a, b, c*. The chief point to be remembered is that C is always to be the angle of 90°, that A and B are the angles on each side of it, and that the sides *c, a, b*, are respectively opposite to angles of the same name. Hence, if angle C is identified with P, M, or T, namely, with one of the three axes of the octahedron, then side *c* is the Znw plane of the form, sides *a* and *b* are identical either with quadrants of the north and east meridians, or of one of these meridians and the equator; while angles A and B are the angles of incidence of the plane Znw on the north and east meridian, or on one of these meridians and the equator.

We, therefore, in this case take angle C to be in the place of P, A in the place of N, and B in the place of W. Angle C is given $= 90^\circ$, A $= 54^\circ\ 44'$, and B $= 54^\circ\ 44'$; and we have, with these data, to find the value of the three sides or plane angles, PZW $= a$, PZN $= b$, and NZW $= c$.

309. We begin the investigation of this problem by referring to the TABLE OF FORMULÆ, § 298, Class A.) RIGHT-ANGLED TRIANGLES, and we look down the second column, under the head of *Given* (Quantities), till we come to A, B, which suits the present case. As the given quantities are all arranged alphabetically, it is easy to find what we want. When we come to A, B, we perceive that Formulæ 4, 5, 6, give us the methods of finding what is sought for, namely, the value of the three sides *a, b, c*, by three separate calculations.

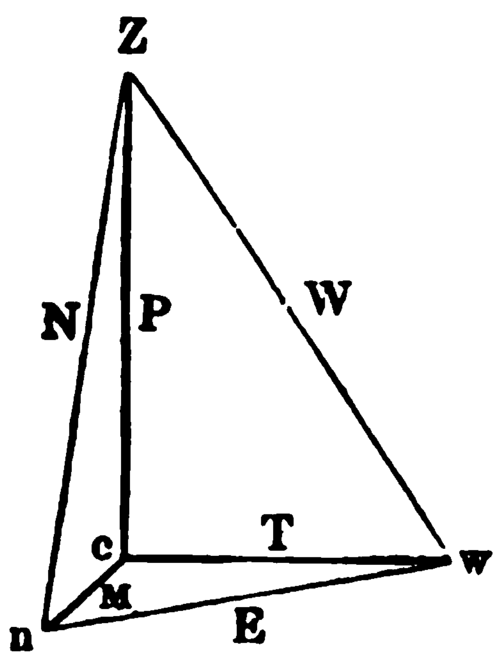
310. *Formulæ* 4. Given, angles A, B; sought, side *a*.

Equation: $\cos a = \frac{\cos A}{\sin B}$. This means that the natural cosine of side *a* is equal to the natural cosine of angle A divided by the natural sine of angle B. Now, according to the Table,* the natural cosine of 54° 44′ is .5774, and the natural sine is .8165. Then

$$\begin{array}{r} .8165\ )\ .5774\ (.7072 \\ \underline{57155} \\ 58500 \\ \underline{57155} \\ 13450 \\ 16330 \end{array}$$

* This refers to HUTTON's Large Table of Sines; because the Table appended to this work does not contain the *natural*, but only the *logarithmic* sines and cosines, which, with the natural and logarithmic tangents and cotangents, are, with a few trifling exceptions, all the functions of an angle that are necessary for working the trigonometrical Formulæ which are *employed in the present work.*

The product is .7072, which is the natural cosine of the angle answering to side *a*. Having found this, we look in the Table of Natural Cosines for the number .7072, which is easily found, because the numbers are in regular order, and we find it to indicate an angle of 45°. This, then, is the angle of the side PZN or *b*, or the side PZW or *a*, in the annexed figure of the octant, for in this case both side *a* and side *b* are alike, because angle A and angle B are alike. If now we want to know the relation of axis p^a either to axis m^a or to axis t^a, we have only to look at the tangent or cotangent of 45°, which we perceive, to be unity, or the same as radius, whence it follows that the axes p^a m^a t^a of the Form PMT represented by Model 15, and whose interfacial angles are found, by measurement, to be 109° 28′,—are all of like length.



311. It is now necessary to examine the Logarithmic Equation of Formula 4, which is:

$$\log \cos a = \log \cos A + 10 - \log \sin B.$$

This means, that the logarithmic cosine of side *a* is equal to the logarithmic cosine of angle A first added to 10, and then having the logarithmic sine of angle B subtracted from it. Angles A and B are each 54° 44′, which angle we look for in the Table, and take its logarithmic cosine and sine as follows:

$$\begin{array}{rr}
\text{Log cos A} = 54^\circ\, 44' = & 9.7615 \\
\text{Add} & 10. \\
\hline
& 19.7615 \\
-\text{Log sin B} = 54^\circ\, 44' = & 9.9119 \\
\hline
\text{Product} = \log \cos a = & 9.8496
\end{array}$$

We next look in the Table of Logarithmic Cosines for the number 9.8496, which is found to be the cosine of 45°, as was determined by the natural numbers in § 310.

312. In practice, the *adding of* 10 in the above formal manner is dispensed with, so that the operation is performed as follows:

$$\begin{array}{rr}
10 + \text{Log cos A} = 54^\circ\, 44' = & 19.7615 \\
-\text{Log sin B} = 54^\circ\, 44' = & 9.9119 \\
\hline
\log \cos a = 45^\circ = & 9.8496
\end{array}$$

313. There does not appear in this example to be much gained by the employment of logarithmic instead of ordinary numbers; but it is very different with cases in which there are a greater number of quantities to deal with, as in the working of the equations relative to oblique-

angled triangles. The examples of the latter, which will be given in the next SECTION, will render very evident the advantages afforded by the method of working by logarithms.

314. *Formula* 5. Given, angles A, B; sought, side *b*. This equation need not be worked, because the case is precisely similar to the preceding, and the value of side *b* is already obtained.

315. *Formula* 6. Given, angles A, B; sought, side *c*.

Equation: $\cos c = \cot A \cot B$.

This means, cosine *c* is equal to cotangent A multiplied by cotangent B.

$$\begin{array}{lr} \text{Natural cotangent A} = 54^\circ\ 44' = & .7072 \\ \text{Natural cotangent B} = 54^\circ\ 44' = & .7072 \\ \hline & 14144 \\ & 49504 \\ & 495040 \\ \hline \text{Product} = \text{nat} \cos c = & .50013184 \end{array}$$

We look for this product in the Table of Natural Cosines, and find .5000 against 60°, which is the nearest sum to it, so that side *c*, or the plane angle n Z w in the figure of the octant, is 60°.

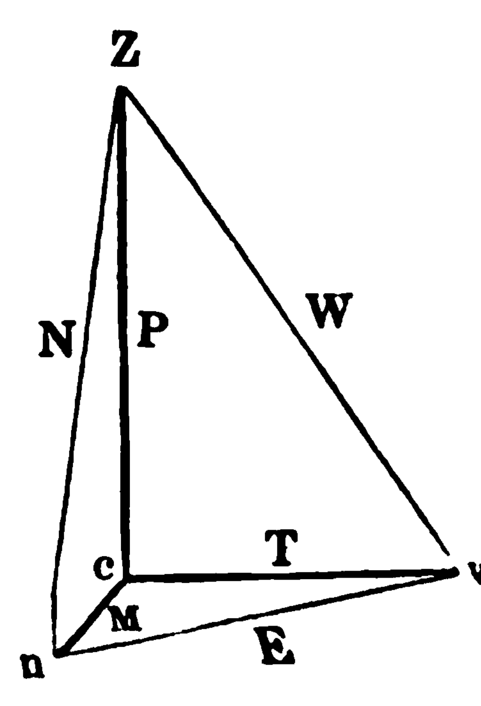
316. *The same by logarithms:*

$$\text{Log} \cos c = \log \cot A + \log \cot B - 10.$$

$$\begin{array}{lr} \text{Log cot A} = 54^\circ\ 44' = & 9.8495 \\ \text{Log cot B} = 54^\circ\ 44' = & 9.8495 \\ \hline \text{Product} = \text{Log} \cos c = & 9.6990 \end{array}$$

We deduct 10 by merely scoring the pen through the first figure of the product, or better by omitting 10 in the adding together of the two sums, and then we look for the result, 9.6990, in the Table of Logarithmic Cosines, where we find it against 60°, which is therefore the value of side *c*.

317. In § 300, I mentioned a problem which is the reverse of those we have just examined. The problem is this: Given a symbol, such as P½MT, to find the interfacial angles of the form which it denotes.



a.) To find the angle across a terminal edge. —Let the figure represent a solid triangle with Z as its vertex. P½MT intimates that the axes are $p_1^a\ m_2^a\ t_2^a$. This gives for the octant the value of 1 for P, 2 for M, 2 for T. Now as we have the value of P and M, we have the value of the angle NZP, = side *a*; and as we have the value of P and T, we have the value of the angle WZP = side *b*. As m^a and t^a are alike, so also are side *a* and side *b*, and the value of these sides in degrees is obtained by looking in the Table

of Natural Tangents for the degree which answers to tangent 2.0 or to cotangent 0.5, either of which quantities suit the case in question, where P is twice the length of either M or T. We thus find the value of side *a* and side *b* in degrees, to be each 63° 26′. Therefore, the problem is as follows:

Given, sides a, b, (C, the right angle being understood); *sought, angle* A, which is the angle across N or W, or the half of the angle across one of the terminal edges. We look for *a, b* among the given quantities in Table A.) § 298, and find at No. 13 the Formula required, which is:

$$\text{Log tan A} = \text{log tan } a + 10 - \text{log sin } b.$$
$$10 + \text{Log tan } a = 63^\circ\, 26' = 20.3010$$
$$- \text{Log sin } b = 63^\circ\, 26' = 9.9515$$
$$\text{Product} = \text{log tan A} = 10.3495$$

This product is the log tan of 65° 54½′; the double of which, or 131° 49′, is the value of the angle across a terminal edge of the form P½MT.

b.) To find the angle across the equator.

In this case, we take the point n for the vertex of a triangle. The quantities given are then, the angle N n M = side *a*, the value of which is the degree that answers to tangent 0.5 or cotangent 2.0, namely, 26° 34′, and the angle M n E = side *b*, which, as M c T is a right angle and M and T are of equal length, is an angle of 45°. The point now to be determined, is the angle across the edge E = angle A. The problem is therefore: *Given, sides a, b; sought, angle* A, which is the same as problem *a.*), *Formula* 13, namely,

$$\text{Log tan A} = \text{log tan } a + 10 - \text{log sin } b.$$
$$10 + \text{Log tan } a = 26^\circ\, 34' = 19.6990$$
$$- \text{Log sin } b = 45^\circ \quad = 9.8495$$
$$\text{Product} = \text{log tan A} = 9.8495$$

This product is the log tan of 35° 16′; the double of which, or 70° 32′, is the value of the angle across the equator of P½MT.

c.) To find the angle across a terminal edge and across the equator, both from one solid triangle.

For this we employ the same quantities as in problem *b.*), and we calculate the value of angle B.

Formula 14, *Given, sides a, b; sought, angle* B.

$$\text{Log tan B} = \text{log tan } b + 10 - \text{log sin } a.$$
$$10 + \text{Log tan } b = 45^\circ\ \ 0' = 20.0000$$
$$- \text{Log sin } a = 26^\circ\, 34' = 9.6505$$
$$\text{Product} = \text{Log tan B} = 10.3495$$

This product, as shown in problem *a.*), is the log tan of 65° 54½′, which *proves the angle* over a terminal edge of the form P½MT to be 131° 49′.

Thus, by problems *b.*) and *c.*) together, we find both the required interfacial angles from one solid triangle.

d.) *Another way to find the interfacial angles across the equator and across a terminal edge of the form* P$\frac{1}{2}$MT, *from one solid triangle.*

We take the point Z of the octant for the vertex of the triangle, and we suppose the octant divided into two equal and similar portions by a plane equal to the north-west meridian, and which passes through the points ZcE, dividing the edge or angle P exactly into two equal edges or angles. As the angle across the edge P is 90°, so the angle of the new edge produced by dividing P into two equal edges, must be 45°. The right angle in this new solid triangle, is across the edge produced by the division of the plane Znw, that is to say, across an edge that runs from Z to E. In other words, angle C is the angle of incidence of the plane Znw upon the north-west meridian, or new plane produced.

The data we now have to help our calculations are these:—The right angle C. The plane NZP, equal, as found in problem *a.*), to 63° 26′, which plane being, not as before *adjacent* to, but now *opposite* to, the right angle C, must be called side *c*. And thirdly, the new angle produced by the division of the edge P, which angle = 45°, may be called angle A.

From these data we have to determine the value of angle B, which is equal to N or to half the terminal edge of P$\frac{1}{2}$MT, and the value of side *b*, which is the new plane produced, and which gives the inclination of the plane nZw to the axis p^s, and, therefore, the complement of its inclination to the equator.

These two problems are as follow:

1. *Given, angle* A *and side c; sought, angle* B. *Formula* 17.
2. *Given, angle* A *and side c; sought, side b. Formula* 18.

Formula 17. Log cot B = log cos *c* + 10 — log cot A.

$$\begin{aligned} 10 + \text{Log cos } c &= 63^\circ\ 26' = 19.6505 \\ -\ \text{Log cot A} &= 45^\circ\ \ 0' = 10.0000 \\ \hline \text{Product} = \text{log cot B} &= 9.6505 \end{aligned}$$

This product is the log cot of 65° 54½′, which shows the angle across the terminal edge to be 131° 49′, as previously found by problem *a.*).

Formula 18. Log tan *b* = log tan *c* + log cos A — 10.

$$\begin{aligned} \text{Log tan } c &= 63^\circ\ 26' = 10.3010 \\ +\ \text{Log cos A} &= 45^\circ\ \ 0' = 9.8495 \\ \hline \text{Product} - 10 = \text{Log tan } b &= 10.1505 \end{aligned}$$

This product is the log tan of 54° 44′, which is the inclination of the external plane of the form P$\frac{1}{2}$MT to the axis p^s. Of course, the complement of this angle, namely, 35° 16′, is the inclination of the same plane to the equator; and double the complement, or 70° 32′, is the angle across the equator, measured from a plane of the upper pyramid upon a plane of the lower pyramid; which determination agrees with that *derived from problem b.*).

318. I shall now bring forward two or three tables which contain information that will be required in subsequent investigations.

TABLE OF LOGARITHMS OF NUMBERS,

FROM 1 to 100, WITH INDICES.

1	0.0000	21	1.3222	41	1.6128	61	1.7853	81	1.9085
2	0.3010	22	1.3424	42	1.6233	62	1.7924	82	1.9138
3	0.4771	23	1.3617	43	1.6335	63	1.7993	83	1.9191
4	0.6021	24	1.3802	44	1.6435	64	1.8062	84	1.9243
5	0.6990	25	1.3979	45	1.6532	65	1.8129	85	1.9294
6	0.7782	26	1.4150	46	1.6628	66	1.8195	86	1.9345
7	0.8451	27	1.4314	47	1.6721	67	1.8261	87	1.9395
8	0.9031	28	1.4472	48	1.6812	68	1.8325	88	1.9445
9	0.9542	29	1.4624	49	1.6902	69	1.8388	89	1.9494
10	1.0000	30	1.4771	50	1.6990	70	1.8451	90	1.9542
11	1.0414	31	1.4914	51	1.7076	71	1.8513	91	1.9590
12	1.0792	32	1.5052	52	1.7160	72	1.8573	92	1.9638
13	1.1139	33	1.5185	53	1.7243	73	1.8633	93	1.9685
14	1.1461	34	1.5315	54	1.7324	74	1.8692	94	1.9731
15	1.1761	35	1.5441	55	1.7404	75	1.8751	95	1.9777
16	1.2041	36	1.5563	56	1.7482	76	1.8808	96	1.9823
17	1.2305	37	1.5682	57	1.7559	77	1.8865	97	1.9868
18	1.2553	38	1.5798	58	1.7634	78	1.8921	98	1.9912
19	1.2788	39	1.5911	59	1.7709	79	1.8976	99	1.9956
20	1.3010	40	1.6021	60	1.7782	80	1.9031	100	2.0000

319. EXPLANATION:—Several of the Formulæ contained in § 298, require the use of what are called Logarithms of Numbers, which I have therefore printed in the above table for the convenience of reference. The four Formulæ, Nos. 83 to 86, are inserted in the Table, rather with a view to illustrate the nature of Right-Angled Plane Triangles, than as Formulæ for common use. But it will perhaps be proper to explain in what manner they are to be used when needful, and how their employment is frequently rendered unnecessary.

320. The cross section of any one of the forms $M_xT.P_xM, P_xT$, is a rhombus, one-fourth of which is a right-angled plane triangle, similar to the figure in Table D.) page 124. The hypothenuse, or longest side, of of this triangle is always a diagonal of a plane of M_xT. P_xM, or P_xT, while the two sides of the triangle are either p^a and m^a, p^a and t^a, or m^a and t^a. And the index $_x$ of the symbols M_xT. P_xM, P_xT, is simply the exponent of the comparative lengths of the two axes of each form, or of the two legs of the plane triangle.

321. Now, in the Tables of Crystallised Minerals contained in Part II., the algebraic index $_x$ is replaced, as often as information afforded the means, by an arithmetical index, which shows the *precise comparative length of every pair of axes.* Thus, at page 66, we find one of the

crystals of Carbonate of Lead described as $M\frac{6}{10}T$. $P\frac{7}{10}T$, Model 82^a. This means, that the equator or cross section of $M\frac{6}{10}T$ is a rhombus, one-fourth of which is a right-angled plane triangle whose leg m^a is $= 6$, and leg $t^a = 10$, and that the east meridian or cross section of $P\frac{7}{10}T$ is a rhombus, one-fourth of which is a right-angled plane triangle whose leg p^a is $= 7$, and leg $t^a = 10$. This relation is also stated in the affix to the symbol, by the term—" Axes: $p^a_7 m^a_6 t^a_{10}$." With this information before us, we can calculate the following six particulars relating to the combination shown by Model 82^a: 1.) the angle across the pole n, and 2.) the angle across the pole w, both on the edge of the equator; 3.) the angle across the pole Z, and 4.) the angle across the pole w, both on the edge of the east meridian; 5.) the angle across the edge Znw, from a plane of $M\frac{6}{10}T$ upon a plane of $P\frac{7}{10}T$; and finally, 6.) all the plane angles of $M\frac{6}{10}T$. $P\frac{7}{10}T$, as shown upon Model 82^a. The 5th and 6th of these calculations are made by means of an *oblique-angled solid triangle*, on which account I shall work the operations in full, because all the examples of solid triangles hitherto worked were of right-angled solid triangles, and there are certain peculiarities relating to oblique-angled solid triangles which have still to be explained.

In describing Model 82^a, I have therefore to explain, first, the calculation of right-angled plane triangles, and secondly, the calculation of oblique-angled solid triangles.

322. Calculation of Right-Angled Plane Triangles.

1.) *To find the obtuse angle of* $M\frac{6}{10}T$ *at the pole* n. Let *c* be the centre of the crystal. Then, *Formula* 83; *given*, *c*,M,T; *sought*, *angle* n. M is given above $= 6$, and $T = 10$. Therefore:

$$\text{Log tan n} = \log T + 10 - \log M$$
$$= \log 10 + 10 - \log 6.$$

Here we find the use of the Logarithms of Numbers contained in the above table.

$$\begin{aligned} 10 + \log T &= 10 = 11.0000 \\ - \log M &= \ \ 6 = \ \ \underline{0.7782} \\ \text{Log tan n} &= 10.2218 = 59^\circ\ 2'. \end{aligned}$$

The product, 10.2218, is the log tan of 59° 2′, the double of which, = 118° 4′, is the angle across the pole n measured on the edge of the equator.

2.) *To find the acute angle of* $M\frac{6}{10}T$ *at the pole* w. The angle across the pole w on the edge of the equator is the supplement of the angle across the pole n, or 180° — 118° 4′ = 61° 56′.

3. *To find the obtuse angle of* $P\frac{7}{10}T$ *at the pole* Z. As P is given at 7 and T at 10, *we make the following* calculation:

Formula 84. *Given*, c, P, T; *sought, angle* n.

$$\log \cot n = \log P + 10 - \log T.$$
$$= \log 7 + 10 - \log 10.$$
$$10 + \log P = 7 = 10.8451$$
$$- \log T = 10 = 1.0000$$
$$\log \cot n = 9.8451 = 55°.$$

The product, 9.8451, is the log cot of 55°, the double of which, = 110°, is the angle over Z on the edge of the east meridian.

4.) *To find the acute angle of* $P\frac{7}{10}T$ *at the pole* w. The angle across the pole w from Zw to Nw is therefore 180° — 110° = 70°.

The above results will be found to agree very closely with measurements taken by means of the goniometer, round the equator and the east meridian of Model 82[a], in the direction of the brown and purple lines marked upon the model.

TABLE OF INDICES.

323. *To find the external angles of the Biaxial Forms* $M_xT.P_xM,P_xT$, *when the value of the index* $_x$ *is known, as in* $M\frac{6}{10}T$. $P\frac{7}{10}T$, *without performing the trigonometrical calculations.*

This is done by means of the *Table of Indices or Characteristics*, printed in the opposite page.

324. EXPLANATION:—This table is adapted for the instant conversion of indices expressed in vulgar fractions, as $\frac{6}{10}$ and $\frac{7}{10}$, into decimal fractions, as .6000 and .7000, which fractions are the natural tangents or natural cotangents of the external angles of the Biaxial Forms that correspond to the given axial relations denoted by the symbols. This will be best explained by an example:

1.) *To find the external angles of* $M\frac{6}{10}T$.—Look for the *numerator* 6 among the large figures in the upper horizontal line of the table. Pass your eye down the column below figure 6, till you come to the figure which is on the level of the *denominator* 10, contained among the large figures in the outer vertical column of the table. The decimal fraction found at the point of intersection of the two lines of figures, is that which expresses *six tenths of unity*, namely, .6000. This decimal fraction is the value of axis m^a when axis t^a is 1.0000. Therefore, by *Formulæ* 89 and 90, the angle of which .6000 is the cotangent, namely, 59° 2′, is the angle at the pole n, and the angle of which .6000 is the tangent, namely, 30° 58′ (See the Table of Tangents) is the angle at the pole w of a triangle, or quadrant of the equator, of the form $M\frac{6}{10}T$; and twice these angles, or 59° 2′ × 2= 118° 4′, and 30° 58° × 2 = 61° 56′, are the external angles of $M\frac{6}{10}T$ required by the problem.

325. This example shows that the Table of Indices is adapted to obviate a great deal of calculation, by exhibiting the direct relation of a vulgar fraction constituting the index of a symbol to the tangent or cotangent which shows the external angles of the Form to which it belongs. All the indices of the symbols of Biaxial Forms can be treated *in the same* manner as those of the Form $M\frac{6}{10}T$ above examined. Thus,

TABLE OF INDICES, OR CHARACTERISTICS.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.00	11.00	12.00	13.00	14.00	15.00	16.0	17.0	18.0	19.0	20.0	1
2	.5000	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	5.500	6.000	6.500	7.000	7.500	8.00	8.50	9.00	9.50	10.0	2
3	.3333	.6667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	3.667	4.000	4.333	4.667	5.000	5.33	5.67	6.00	6.33	6.67	3
4	.2500	.5000	.7500	1.000	1.250	1.500	1.750	2.000	2.250	2.500	2.750	3.000	3.250	3.500	4.750	4.00	4.25	4.50	4.75	5.00	4
5	.2000	.4000	.6000	.8000	1.000	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800	3.000	3.20	3.40	3.60	3.80	4.00	5
6	.1667	.3333	.5000	.6667	.8333	1.000	1.167	1.333	1.500	1.667	1.833	2.000	2.167	2.333	2.500	2.67	2.83	3.00	3.17	3.33	6
7	.1429	.2857	.4286	.5714	.7143	.8571	1.000	1.143	1.286	1.429	1.571	1.714	1.857	2.000	2.143	2.29	2.43	2.57	2.71	2.86	7
8	.1250	.2500	.3750	.5000	.6250	.7500	.8750	1.000	1.125	1.250	1.375	1.500	1.625	1.750	1.875	2.00	2.13	2.25	2.37	2.50	8
9	.1111	.2222	.3333	.4444	.5555	.6667	.7778	.8889	1.000	1.111	1.222	1.333	1.444	1.555	1.667	1.78	1.89	2.00	2.11	2.22	9
10	.1000	.2000	.3000	.4000	.5000	.6000	.7000	.8000	.9000	1.000	1.100	1.200	1.300	1.400	1.500	1.60	1.70	1.80	1.90	2.00	10
11	.0909	.1818	.2727	.3636	.4545	.5455	.6364	.7273	.8182	.9091	1.000	1.091	1.182	1.273	1.364	1.45	1.55	1.64	1.73	1.82	11
12	.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167	1.000	1.083	1.167	1.250	1.33	1.42	1.50	1.58	1.67	12
13	.0769	.1538	2308	.3077	.3846	.4615	.5385	.6154	.6923	.7692	.8462	.9231	1.000	1.077	1.154	1.23	1.31	1.38	1.46	1.54	13
14	.0714	.1429	.2143	.2857	.3571	.4286	.5000	.5714	.6429	.7143	.7857	.8571	.9286	1.000	1.071	1.14	1.21	1.29	1.36	1.43	14
15	.0667	.1333	.2000	.2667	.3333	.4000	.4667	.5334	.6000	.6667	.7333	.8000	.8667	.9333	1.000	1.07	1.13	1.20	1.27	1.33	15
16	.0625	.1250	.1875	.2500	.3125	.3750	.4375	.5000	.5625	.6250	.6875	.7500	.8125	.8750	.9375	1.00	1.06	1.12	1.19	1.25	16
17	.0588	.1176	.1765	.2353	.2941	.3529	.4118	.4706	.5294	.5882	.6471	.7059	.7647	.8235	.8824	.941	1.00	1.06	1.12	1.18	17
18	.0556	.1111	.1667	.2222	.2778	.3333	.3889	.4444	.5000	.5556	.6111	.6667	.7222	.7778	.8334	.889	.944	1.00	1.05	1.11	18
19	.0526	.1053	.1579	.2105	.2632	.3158	.3684	.4210	.4737	.5263	.5790	.6316	.6842	.7368	.7896	.842	.895	.947	1.00	1.05	19
20	.0500	.1000	.1500	.2000	.2500	.3000	.3500	.4000	.4500	.5000	.5500	.6000	.6500	.7000	.7500	.800	.850	.900	.950	1.00	20
21	.0476	.0952	.1429	.1905	.2381	.2857	.3333	.3810	.4286	.4762	.5238	.5714	.6190	.6667	.7144	.762	.809	.857	.905	.952	21
22	.0455	.0909	.1364	.1818	.2273	.2727	.3182	.3636	.4091	.4545	.5000	.5455	.5909	.6364	.6818	.727	.773	.818	.864	.909	22
23	.0435	.0870	.1304	.1739	.2174	.2609	.3043	.3478	.3913	.4348	.4783	.5217	.5652	.6087	.6521	.696	.739	.783	.826	.870	23
24	.0417	.0833	.1250	.1667	.2083	.2500	.2917	.3333	.3750	.4167	.4583	.5000	.5417	.5833	.6250	.667	.708	.750	.792	.833	24
25	.0400	.0800	.1200	.1600	.2000	.2400	.2800	.3200	.3600	.4000	.4400	.4800	.5200	.5600	.6000	.640	.680	.720	.760	.800	25

given, Sulphate of Barytes, Part II., page 68, combination P, M$\frac{4}{5}$T, Model 6; required, the equatorial angle at the pole n. Referring to the Table of Indices, and in the horizontal line of 5ths, and below the numerator 4, we find .8000. This is the cotangent of 51° 20′, or half the angle of incidence of the plane ne upon the plane nw of the Form M$\frac{4}{5}$T, which angle is 51° 20′ × 2 = 102° 40′.

The Table of Indices presents no great advantages in the cases of such simple vulgar fractions as those which I have quoted, but it will be found to be very convenient when the fractions are such as $\frac{16}{17}$, $\frac{11}{18}$, $\frac{5}{11}$, or other odd numbers which cannot be readily converted into decimal fractions by mental calculation without the help of the pen.

326. *The axes of Forms belonging to a given zone are multiples of one another, for the same Mineral.*—The Table of Indices presents another peculiarity, which it is useful to remember in examining complex crystals of minerals. Whenever a combination is found, which contains several rhombic forms in one zone, as, for example, M$\frac{2}{3}$T, m$\frac{4}{3}$t, m$\frac{6}{3}$t, Muriate of Copper, then a single horizontal line of the table contains the length of the variable axis of all the forms of the given zone peculiar to that mineral. Thus, if we take the mineral just quoted, and refer to the line of *thirds* among the *denominators*, then under *numerator* 2, we find the cotangent which shows the angle of a triangle of M$\frac{2}{3}$T; under *numerator* 4, the cotangent which shows the angle of a triangle of m$\frac{4}{3}$t; under *numerator* 6, the cotangent which shows the angle of a triangle of m$\frac{6}{3}$t. These cotangents are .6667, 1.333, and 2.000, and the corresponding angles are 56° 19′, 36° 52′, and 26° 34′; so that the interfacial angles of the plane ne upon nw of these respective vertical forms, peculiar to Muriate of Copper, are 112° 38′, 73° 44′, and 53° 8′. This view of the mutual relations of the axes of all the forms belonging to any single zone of a particular mineral, is frequently of important service in correcting imperfect measurements, and supplying data when defective.

327. *Construction of Symbols for Biaxial and Triaxial Forms.*—It will be seen, that besides giving a Table of Indices, I have added two columns of Indices to the Table of Sines and Tangents. These are intended to be used in the *construction of symbols*, as I shall show by an example.—Given, model 82ª, with the symbol M_xT. P_xT, and the measurements ne on nw = 118° 4′, Ze on Zw = 110°. Required, the value of $_x$ and $_x$ expressed in vulgar fractions.

a.) Divide 118° 4′ by 2, and find the cotangent of the result: $\frac{118° 4'}{2}$ = 59° 2′, cot. .6000. This is the value of m^s, when t^s is 1.0; and against this angle you find, in the outer column of the Table of Sines, the vulgar fraction $\frac{3}{5}$, which is the value of $_x$ in M_xT.

b.) Divide 110° by 2, and find the cotangent of the result: $\frac{110°}{2}$ = 55°, cot .7002. This is the value of p^s, when t^s is 1.0, and against this angle in the Table of Sines you find the vulgar fraction $\frac{7}{10}$, which is the value of $_x$ in P_xT.

The symbol M_xT. P_xT is, therefore, equal to M$\frac{3}{5}$T. P$\frac{7}{10}$T. But as

T is of the same length in M_xT and P_xT, it is better to change $\frac{3}{5}$ for its synonyme $\frac{6}{10}$, and write $M\frac{6}{10}T$. $P\frac{7}{10}T$.

c.) In forming symbols for Triaxial Forms, it is necessary to bear in mind that there are three relations to be shown, namely, the length of p^a, m^a, and t^a, or the distance of the poles Z,n,w, from the centre of the triaxial form. Let the general symbol be $P_xM_yT_z$, and the value of x, y, z be 1, 2, 3. Then we have

$$P : M :: 1 : 2$$
$$P : T :: 1 : 3$$
$$M : T :: 2 : 3$$

This is equal to $p_1^a m_2^a t_3^a$ or $P_1M_2T_3$; but as it is desirable that t^a should, as frequently as possible, be made unity, it is better to write $P\frac{1}{3}M\frac{2}{3}T$, which indicates the same relations.

328. Calculation of Oblique-angled Solid Triangles.

5.) *To find the angle across the edge* Znw *of* Model 82^a, *or the inclination of a plane of* $M\frac{6}{10}T$ *to a plane of* $P\frac{7}{10}T$.

Take the Zn pole of the combination as the vertex of an oblique-angled solid triangle. This pole is at the point where the two upper planes of P_xT meet the two front planes of M_xT. The solid triangle which I shall take consists of the following parts: the north meridian, which is a rectangle, with an angle of 90° at the pole Zn. Call this side *c* of the solid triangle. Then side *a* will be the plane P_xT Zw, and side *b* will be the plane M_xT nw. Agreeably to this arrangement, angle A will be the inclination of M_xT nw on the north meridian; angle B will be the inclination of the plane P_xT Zw on the north meridian; and angle C will be the inclination of M_xT nw upon P_xT Zw across the edge Znw, which is the angle required in the problem.—Of the parts here named, we know the value of $c = 90°$, of $A = 59° 2'$, of $B = 55°$, the two latter being found by problems 1) and 3), § 322. The present problem is, consequently,—*given, side c and angles A, B; required, angle C*, and this can be solved either by *Formula* 41, or *Formula* 42. We have therefore a choice between two methods of calculation, and we naturally ask, what is to guide us in choosing betwixt them? If we compare formulæ 41 and 42 with one another, we perceive that the former is considerably the simpler of the two; but that it requires a given quantity, which is to be found by a preliminary calculation. This preliminary calculation is, as directed in the Formula, to be effected by means of Formula 39. Upon referring to this Formula, we find that the preliminary calculation is much longer than either 41 or 42; so that Formula 41 becomes altogether considerably longer than Formula 42; and for this reason—the results by each being the same—we should prefer to work our calculation by Formula 42. But there is yet another thing to be taken into consideration, in judging of the comparative merits of these two Formulæ. Although by problem 5.) we seek only to find angle *C of the oblique-angled* solid triangle that we have described,

there is afterwards another calculation to make, problem 6.), in order to learn what are the plane angles of the external planes of Model 82ª. Now, it appears from a comparative examination of the model and the solid triangle, that two of the plane angles of the model are equivalent to side a and side b, of the same solid triangle whose angle C is equivalent to the angle required in problem 5.) It appears, also, that these two sides are given by the solution of the equation contained in Formula 39, with its subordinate equation, Formula 40; consequently, if we resolve the equations contained in Formulæ 39, 40, and 41, we solve problems 5.) and 6.) together; whereas, if we begin by solving problem 5.) with the short Formula No. 42, we shall still have to resolve the equations in Formulæ 39 and 40, or in the two additional Formulæ, Nos. 43 and 44, in order to be able to solve problem 6.) It is therefore advisable to solve problem 5.) by means of Formulæ 39 and 41, instead of employing the short Formula No. 42.

329. *Formula* 39. *Given*, A, B, c; *Sought*, a.

First Equation:

$$\text{Log}\tan\tfrac{1}{2}(a+b) = \log\tan\tfrac{1}{2}c + \log\cos\tfrac{1}{2}(\text{A}-\text{B}) - \log\cos\tfrac{1}{2}(\text{A}+\text{B}).$$

Second Equation:

$$\text{Log}\tan\tfrac{1}{2}(a-b) = \log\tan\tfrac{1}{2}c + \log\sin\tfrac{1}{2}(\text{A}-\text{B}) - \log\sin\tfrac{1}{2}(\text{A}+\text{B}).$$

Then, $a = \tfrac{1}{2}(a+b) + \tfrac{1}{2}(a-b)$

A is given $= 59^\circ\ 2'$; B $= 55^\circ$; $c = 90^\circ$.

It is advisable to commence a calculation of this nature by drawing out a full plan of it in accordance with the Formula, and in the manner represented in the first of the two opposite columns. It is *always possible to do this*, because every step of the calculation is set forth in the Formula, and it is proper to do it with the greatest care, since this is the most important part of the process. When the plan is ready, you resort to the Table of Sines and Tangents, and extract the functions, as shown by the second of the two columns opposite. In the present example, you take first the log tan of angle 45°, and write it both in the first and second equation. Then you look for the log cos of angle 2° 1′, and write it in the first equation, and while the Table of Sines is still open at the place of 2° 1′, you extract its log sin, and write it in the second equation. Finally, you turn to angle 57° 1′ in the Table of Sines, and extract its log cos for the first equation, and its log sin for the second equation. You already perceive one of the uses to be derived from beginning a calculation of this kind by drawing out a full plan of it, which use is, to be enabled to avoid the trouble of turning up repeatedly the same angle in thé Table of Sines, by extracting at once all the functions of the same angle which may be required at different stages of the calculation. Another use of drawing out a plan of the calculation is, that it promotes an orderly method of working, and prevents the accidental substitution of multiplication for division, or other confusion of the quantities, which is apt to occur when this precaution is neglected.

The two equations being prepared in the manner above described, the two additions and two subtractions are readily made in agreement with the Formula. You have then only to look in the Table of log tangents for the angles answering to 10.2638 and 8.6227, which you find to be 61° 25′ and 2° 24′, and to add these together, as prescribed in the final equation, to find the result of the equation, and the desired quantity of the problem, which is, $a = 63° 49'$. Hence, the calculation is the simplest thing imaginable; the entire difficulty of proceeding consisting, in fact, in properly choosing the parts of the solid triangle, and carefully drawing up the plan of the operation.

Details of the First Equation:

$$
\begin{array}{rll}
\log\tan\frac{1}{2}c = & 45° & = \\
A = & 59° 2' & \\
B = & 55° & \\
\hline
(A-B) = & 4° 2' & \\
+\log\cos\frac{1}{2}(A-B) = & 2° 1' & = \\
\hline
A = & 59° 2' & \\
B = & 55° & \\
\hline
(A+B) = & 114° 2' & \\
-\log\cos\frac{1}{2}(A+B) = & 57° 1' & = \\
\hline
\log\tan\frac{1}{2}(a+b) = & & =
\end{array}
$$

Details of the Second Equation:

$$
\begin{array}{rll}
\log\tan\frac{1}{2}c = & 45° & = \\
+\log\sin\frac{1}{2}(A-B) = & 2° 1' & = \\
\hline
-\log\sin\frac{1}{2}(A+B) = & 57° 1' & = \\
\hline
\log\tan\frac{1}{2}(a-b) = & & =
\end{array}
$$

Final Equation:

$$
\begin{array}{rl}
\frac{1}{2}(a+b) = & \\
+\frac{1}{2}(a-b) = & \\
\hline
a = &
\end{array}
$$

Details of the First Equation:

$$
\begin{array}{rlr}
\log\tan\frac{1}{2}c = & 45° 0' = & 10.0000 \\
A = & 59° 2' & \\
B = & 55° & \\
\hline
(A-B) = & 4° 2' & \\
+\log\cos\frac{1}{2}(A-B) = & 2° 1' = & 9.9997 \\
\hline
 & & 19.9997 \\
A = & 59° 2' & \\
B = & 55° & \\
\hline
(A+B) = & 114° 2' & \\
-\log\cos\frac{1}{2}(A+B) = & 57° 1' = & 9.7359 \\
\hline
\log\tan\frac{1}{2}(a+b) = & 61° 25' = & 10.2638
\end{array}
$$

Details of the Second Equation:

$$
\begin{array}{rlr}
\log\tan\frac{1}{2}c = & 45° 0' = & 10.0000 \\
+\log\sin\frac{1}{2}(A-B) = & 2° 1' = & 8.5464 \\
\hline
 & & 18.5464 \\
-\log\sin\frac{1}{2}(A+B) = & 57° 1' = & 9.9237 \\
\hline
\log\tan\frac{1}{2}(a-b) = & 2° 24' = & 8.6227
\end{array}
$$

Final Equation:

$$
\begin{array}{rl}
\frac{1}{2}(a+b) = & 61° 25' \\
+\frac{1}{2}(a-b) = & 2° 24' \\
\hline
a = & 63° 49'
\end{array}
$$

Upon applying the goniometer on Model 82[a] to one of the plane angles of the faces of $P_x T$ at the point where they touch the north meridian, you find the above result of 63° 49′ to agree precisely with that afforded by mechanical measurement.

This preliminary calculation being effected, we proceed now to find angle C of the solid triangle, as required by problem 5.), page 141.

330. *Formula* 41. $\log\sin C = \log\sin c + \log\sin A - \log\sin a$.

A is given $= 59° 2'$; c is given $= 90°$; a is found by Formula 39 $= 63° 49'$.

$$
\begin{array}{rlr}
\log\sin c = & 90° = & 10.0000 \\
+\log\sin A = & 59° 2' = & 9.9332 \\
\hline
 & & 19.9332 \\
-\log\sin a = & 63° 49' = & 9.9530 \\
\hline
\log\sin C = & 72° 50' = & 9.9802
\end{array}
$$

The product is 72° 50′, which, according to the Formula, should be the angle required by problem 5.), across the edge Znw of Model 82ᵃ, from a plane of M$\frac{6}{10}$T upon a plane of P$\frac{7}{10}$T. But if you apply the goniometer to Model 82ᵃ, in the given direction, and measure the angle mechanically, you will find the angle to be, not 72° 50′, but 107° 10′. Here, then, is a disagreement which shows something to be wrong. Is it the Formula, the calculation, or the mechanical measurement? You repeat the mechanical measurement and find it correct; you examine the calculation and find it just. You conclude, then, that the error lies in the Formula, and your conclusion is right. Here is the difficulty: *the sine of an angle and that of its supplement are the same.* 107° 10′ is the supplement of 72° 50′, and 9.9802 is the log sin both of 72° 50′ and 107° 10′. This is called in trigonometry the *ambiguous case*, and it occurs whenever the given quantities of a solid triangle are *two sides with an angle opposite to one of them*, as *a*,*b* with A; or, as in the present case, *a*,*c* with A;—or when the given quantities are *two angles with a side opposite to one of them*, as A,B with *a*; and it may be suspected to occur whenever the product of an equation is a *sine* or a *cosine*. There is no possibility of altering or improving the Formulæ in such a manner as to get rid of this difficulty, so that the student must remember it to be a difficulty of possible occurrence, and accordingly be prepared to judge from associated circumstances, or to determine by mechanical measurement, which of the two angles given by such a calculation, is the one which ought to be adopted as the true angle.

Among the equations belonging to quadrantal solid triangles, Table C.), § 298, there are twelve which contain the negative sign (—), and which, consequently, give ambiguous products. It is impossible to lay down brief and yet trustworthy rules to show in what cases the product of these equations is the *true angle*, and in what it is the *supplement* of the true angle. I believe, therefore, that the checking of the calculation by the mechanical measurement of a crystal, or the observation of the general rules given in § 307, will be better worth the reader's attention than any special rules on the subject. The following observations may, however, guide him in particular cases:—

1.) The cosine, tangent, and cotangent of an angle greater than 90° and less than 180°, is *negative*, or has the sign —. 2.) The same functions of an angle less than 90° are *positive*. 3.) The cosine of an arc and its supplement are equal with different signs. 4.) When positive signs are multiplied together, the product is positive, but when a positive sign is multiplied by a negative sign, the product is negative. 5.) When — is written before an expression in a Formula, the sign of the expression is to be changed. Thus, for instance, in

Formula 80. $\tan A = -\tan C \cos b$.

Suppose $C = 80°$ and $b = 105°$. Then, tan C is *positive*, because 80° is less than 90°, but cos *b* is *negative*, because 105° is greater than 90°. These two quantities, one being positive and one negative, multi-

plied into each other, produce a negative result, but then, as — is written before this result in the Formula, the sign is to be changed into +. Therefore, tan A being positive, it must be less than 90°. If tan A had come out negative, we must have taken the supplement of the angle found in the tables.

331. *Solution of the foregoing problem by means of a Quadrantal Solid Triangle.*

To find the angle across the edge Znw of Model 82ᵃ, or the inclination of a plane of $M\frac{6}{10}T$ upon a plane of $P\frac{7}{10}T$. That is to say, see page 141,—*Given, side c and angles* A, B; *required, angle* C.

Side c = 90°; angle A = 59° 2′; angle B = 55°. In virtue of the existence of a *side* of 90° on this solid triangle, it may be referred to the class of quadrantal solid triangles, contained in Table C, § 298, where the Formula which contains the given conditions is

$$\text{No. 58: } \cos C = -\cos A \cos B.$$

$$\log \cos C = \cos A + \cos B - 10.$$

$$\begin{array}{rl} \cos A = 59^\circ\ 2' = & 9.7114 \\ + \cos B = 55^\circ \quad = & 9.7586 \\ \hline \log \cos C = 72^\circ\ 50' = & 9.4700 \end{array}$$

The product is 72° 50′; but, for the reason stated in the note to Table C, § 298, we may either take this angle or its supplement, = 180° — 72° 50′ = 107° 10′. We find, by the mechanical measurement of the model, that it is the larger angle that must be taken, and the observations contained in § 330 lead us to the same conclusion; for cos A = 59° 2′ and cos B = 55° are both positive, because the angles are both less than 90° (obs. 2.) Their product is positive, for the reason given in obs. 4). But the existence of the negative sign in the Formula changes the product from positive to negative, according to obs. 5.) Hence, cos C = 9.4700 is negative, and we take as the true angle, not the one given against cos 9.4700 in the table of cosines, but its supplement.

It appears from this last investigation, that when an oblique-angled solid triangle has a side = 90°, *the calculation of the triangle is as easy as if it were a right-angled solid triangle.*

332. Problem 6.) We have still to investigate *the plane angles of the faces of Model* 82ᵃ.

a.) Of these, we have already determined one of the two similar plane angles on the upper part of the face of $P\frac{7}{10}T$ Zw. This, by Formula 39, was found to be 63° 49′. The two similar plane angles of this face are, therefore, together equal to 127° 38′: consequently, the acute plane angle at the pole w is 180° — 127° 38′ = 52° 22′, which agrees with approximate measurement by the goniometer.

b.) To find one of the plane angles of the face $M\frac{6}{10}T$ nw, we employ, for the reason stated in § 328, *Formula* 40: $b = \frac{1}{2}(a + b) - \frac{1}{2}(a - b)$.

By Formula 39, page 143, we have $\frac{1}{2}(a+b) = 61° 25'$
and $\frac{1}{2}(a-b) = 2° 24'$

Therefore, $b = 59° 1'$

This, as proved by the goniometer, is the plane angle of $M\frac{6}{10}T$ nw at the pole Zn. There is an equal plane angle at the pole Nn. These two angles are together equal to 118° 2′. The plane angle at the pole w is = 180° — 118° 2′ = 61° 58′, which agrees with approximate measurement by the goniometer.

The plane angles of the faces of Model 82[a] are thus all found.

c.) But it is unnecessary to resort to Formula 39, even to determine the plane angles of Model 82[a], because, in consequence of the presence of *a side of* 90° on the solid triangle chosen for examination, the sides *a* and *b* of that solid triangle can, like the angle C, be found by the easy method of solving *quadrantal* instead of *oblique-angled* solid triangles, by which means a good deal of trouble is avoided.

Given, A = 59° 2′; B = 55°; *c* = 90°; to find *a* and *b*.

Formula 56. Log tan *a* = log tan A + 10 — log sin B.

$$\begin{aligned} 10 + \log\tan A &= 59°\ 2' = 20.2218 \\ -\log\sin B &= 55° \quad = 9.9134 \\ \hline \log\tan a &= 63°\ 49' = 10.3084 \end{aligned}$$

Formula 57. Log tan *b* = log tan B + 10 — log sin A.

$$\begin{aligned} 10 + \log\tan B &= 55° \quad = 20.1548 \\ -\log\sin A &= 59°\ 2' = 9.9332 \\ \hline \log\tan b &= 59°\ 1' = 10.2216 \end{aligned}$$

TABLE OF SQUARE ROOTS.

2	1.4142	12	3.4641	22	4.6904	32	5.6569	42	6.4807
3	1.7321	13	3.6056	23	4.7958	33	5.7446	43	6.5574
4	2.0000	14	3.7417	24	4.8990	34	5.8310	44	6.6332
5	2.2361	15	3.8730	25	5.0000	35	5.9161	45	6.7082
6	2.4495	16	4.0000	26	5.0990	36	6.0000	46	6.7823
7	2.6458	17	4.1231	27	5.1962	37	6.0828	47	6.8557
8	2.8284	18	4.2426	28	5.2915	38	6.1644	48	6.9282
9	3.0000	19	4.3589	29	5.3852	39	6.2450	49	7.0000
10	3.1623	20	4.4721	30	5.4772	40	6.3246	50	7.0711
11	3.3166	21	4.5826	31	5.5678	41	6.4031	60	7.7460

333. Explanation. Some crystallographers are in the practice of stating certain relations of lines to one another, by means of square roots of numbers. I think it an inconvenient practice, and do not follow it; but since the practice is in use, it requires explanation. Haüy, for ex-

ample, in speaking of the crystal of Carbonate of Lead, which is represented by Model 82^a, says, that normals to the poles Z, n, and w, are related as the numbers $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{8}$. To find the meaning of this statement, we take the square roots of these numbers from the above table as follows:—

$$\begin{aligned} \text{Normal to Z} &= \tfrac{1}{2} \text{ Axis p}^a = \sqrt{2} = 1.4142 \\ \text{Normal to n} &= \tfrac{1}{2} \text{ Axis m}^a = \sqrt{3} = 1.7320 \\ \text{Normal to w} &= \tfrac{1}{2} \text{ Axis t}^a = \sqrt{8} = 2.8284 \end{aligned}$$

According to these measurements, the forms on Model 82^a are $M\frac{1}{2}\frac{.7320}{.8284}T$. $P\frac{1}{2}\frac{.4142}{.8284}T$. The indices of these symbols do not quite agree with the indices of $M\frac{6}{10}T$. $P\frac{7}{10}T$; but that is nothing, for neither do the given square roots agree exactly with Haüy's own measurements of the external angles of the crystal. The point which I am explaining is simply the method of converting the square roots of numbers, wherever they occur, into decimal fractions, by means of the above Table.

334. *Precautions to be taken by the Crystallographer in the Analysis of Crystallographic Combinations.*

The examples quoted in the course of this section show that there are often many ways of finding a desired result. *The judgment of the crystallographer must in all cases be exercised, to choose from among numerous trigonometrical methods, the one which is best adapted to his purpose.* When a problem is proposed, he must in every case first consider its conditions, and the connection between the given parts and those which he is called upon to determine. In dividing a crystal or model into solid triangles, for the purpose of calculation, he should take care to choose such solid triangles as are easiest of calculation, and such as are not liable to give ambiguous results. This being attended to, and the problem fixed upon, the known quantities should be denoted by the algebraic symbols A, B, C, a, b, c, or such of them as are necessary; and as many independent equations be then prepared as there are unknown quantities to be found. The resolution of these equations gives the solution of the problem. It is impossible to give general rules for selecting those quantities which will afford the easiest and most concise methods of solving any problem; but so far as this is practicable, it will be done, as respects Minerals, in the next SECTION, where a multiplicity of examples will be brought forward to illustrate the various methods of analyzing the crystals of every particular class. Yet after all, a good deal must be left to the skill and attention of the student, and to the exercise of that experience which he will gradually acquire by examining and imitating the explanatory examples.

All the 30 Formulæ relating to right-angled triangles, and all the 30 which relate to quadrantal triangles, are worked either on the model of No. 1 or No. 6, and with the same facility as the recited examples. This is not the case, however, with the Formulæ relating to oblique-angled triangles, which are more complex and more diversified, although the

arithmetical operations are essentially the same. The logarithmic equations alone are worked, because they are much easier than the others. The quantities are either added or subtracted, according as they are prefixed by $+$ or $-$. There is no other more difficult arithmetical process to go through than that, and the most troublesome thing which it is necessary to attend to, is the right grouping of different quantities into what I may call the compound quantities that are placed within parenthesis ( ) or braces { }, or are affected by the radical sign $\surd$, or by the fraction $\frac{1}{2}$. All these particulars, however, are very distinctly marked in the different Formulæ, and will be illustrated by examples in the next section.

335. *Abridgement of Formulæ.*—It is sometimes possible, as I shall have occasion to show in the next section, to abridge the labour attending these calculations, by certain modifications of some of the foregoing formulæ. This happens in the case of one of the examples which I have taken to illustrate the nature of the trigonometrical calculations. It also happens in all cases where the horizontal axes of a pyramid (m^{2} t^{2}) are equal to one another, whether they agree in length with p^{2} or not. The case alluded to is where the angles A and B of a right-angled solid triangle are both of the same value, and enter into the equation No. 4, namely,

$$\cos a = \frac{\cos A}{\sin B}.$$

Now, as A and B are alike, this formula is equivalent to the following:

$$\cos a = \frac{\cos A}{\sin A}.$$

But by Formula 104, we find that $\frac{\cos A}{\sin A} = \cot A$, so that Formula 4 can be abridged to $\cos a = \cot A$. We look, therefore, in the Table of Natural Cotangents for 54° 44′, the value of A, and we find its cotangent to be .7072, which is precisely the product of $\frac{\cos A}{\sin B}$ when worked at length, as in § 310. The abridged Formula thus obtained, is one that is very extensively used in crystallography, as will appear in the sequel.

336. The manner in which I have brought this subject under the reader's attention, will, I hope, not only enable him to make considerable use of the Formulæ empirically, but put it in his power to consult a mathematician with advantage, because the latter can adjust his instructions to the object which this statement places in view.

SECTION XIII.—AN INQUIRY INTO THE VARIETY OF FORMS AND COMBINATIONS WHICH OCCUR UPON THE CRYSTALS OF MINERALS.

337. I take as the basis of this inquiry, a work entitled, "*Elemente der Krystallographie, nebst einer tabellarischen Uebersicht der Mineralien nach den Krystallformen, von* GUSTAV ROSE. *Zweite Auflage, Berlin,* 1838." The reasons which induce me to make choice of this work are these :—It contains a full account of a system of crystallography, which, in contradistinction to Haüy's system, I may call the *German system of crystallography*. This system was invented by WEISS of Berlin, and has been adopted by NAUMANN, MOHS, and ROSE in Germany, by MILLER in England, and by other distinguished mineralogists. It is the system of crystallography, which, slightly modified by different writers, is now in most general use throughout Europe. ROSE's work is, moreover, the latest published continental treatise on this science, and it is the production of one of the most eminent of living mineralogists. It may therefore be held to contain a fair representation of the crystallography of the present age.

338. ROSE's, or rather WEISS's, SYSTEM, is founded on the assumption of what are termed, "*Six Systems of Axes of Crystallisation,*" which systems bear the following titles:

G. ROSE:	*Literal translation:*
1.) Das reguläre System.	1.) The regular system.
2.) Das zwei- und einaxiage.	2.) Two-and-one-axed.
3.) Das drei- und einaxige.	3.) Three-and-one-axed.
4.) Das ein- und einaxige.	4.) One-and-one-axed.
5.) Das zwei- und eingliedrige.	5.) Two-and-one-membered.
6.) Das ein- und eingliedrige.	6.) One-and-one-membered.

Professor MILLER's names for these six systems are as follow:

1.) The Octahedral System of Crystallisation.
2.) The Pyramidal System.
3.) The Rhombohedral System.
4.) The Prismatic System.
5.) The Oblique Prismatic System.
6.) The Doubly Oblique Prismatic System.

In describing these six systems, I shall employ Professor MILLER's titles in preference to those afforded by the literal translation of ROSE's.

339. There are three topics to handle in the following inquiry:

First, I have to show, after ROSE, what forms and combinations constitute the crystals of the mineral world, and how they are classified, both according to his method and to mine.

Secondly, I have to give the mathematical proofs of the separate identity of the several forms, and to show how the different combinations are to be mathematically analysed.

Thirdly, I have to prove that the System of Crystallography which is recommended in the present work, is suitable for the exact and convenient description of every combination and form thus analysed and identified, and, therefore, suitable for all the purposes of the mineralogist.

Let it not be supposed, however, that I am about to give a full account of Rose's book: that is not my object. What I purpose to do is, to recite the forms and combinations which he adduces as constituting each system of crystallisation, and then to explain and illustrate these forms and combinations according to the methods developed in the foregoing sections of this treatise. I shall describe the same objects that Rose describes, but in entirely different terms.

340. The principles upon which the classification of crystals into the above-named six systems is effected, are as follow:

All natural crystals may be divided into two classes—the EQUIAXED and the UNEQUIAXED crystals. The equiaxed are those whose axes are all alike, as $p^a\, m^a\, t^a$; the unequiaxed, those whose axes are dissimilar, as $p^a_x\, m^a\, t^a$, or $p^a_x\, m^a_y\, t^a_z$. This distinction refers to *Combinations*, § 239, not to *Forms*, § 237.

1.) Now, it is an *ultimate fact* in mineralogy, that when a mineral produces an equiaxed combination, it produces no combinations that are unequiaxed. This may be considered a universal rule, subject to a few trifling exceptions. The equiaxed minerals, or the crystals whose axes are $p^a\, m^a\, t^a$, are therefore made to constitute the regular system, or *Octahedral System of Crystallisation*, which is called *octahedral*, because the regular octahedron is one of its most important crystals. This is the first of the six systems of crystallisation.

2.) When the unequiaxed crystals are compared with one another, we find that we can select a class whose axes are $p^a_x\, m^a\, t^a$; that is to say, a class of combinations whose equatorial axes m^a and t^a are both alike, but different from the vertical axis p^a. It is again an *ultimate fact* in mineralogy, that a mineral which produces a single combination of this two-and-one-axed kind, never produces a combination whose axes have any other relation than this. The two-and-one-axed class of crystals is therefore a second system of crystallisation, and as the crystals commonly called Square-based Pyramids, Models 12 and 13, are very important forms of this system, it has thence been termed the *Pyramidal System of Crystallisation.*

3.) All the other unequiaxed crystals have the relation of $p^a_x\, m^a_y\, t^a_z$; that is to say, their three rectangular axes are all different from one another. In this peculiarity, the crystals of the remaining four systems agree; but, at the same time, they disagree in several other very important particulars.

For example, we can select from them a class of crystals, whose axes *have always the* relation of $p^a_x\, m^a_{15}\, t^a_{13}$ or $p_x\, m^a_{14}\, t^a_{13}$, and which, if they are

prisms, have 6, 12, or 24 vertical planes, or if they are pyramids, have 3, 6, or 3^x planes meeting at the poles Z and N. The rhombohedron, Model 26*; the hexagonal prism, Model 7; the twelve-sided prism, Model 10; and the six-sided pyramid, Model 26, present examples of this class. We are again guided by the observance of an *ultimate fact* in mineralogy, which is, that a mineral which exhibits any one of the forms here named, may exhibit any of the others, but never can produce a combination whose axes are different from $p^a_x m^a_{15} t^a_{13}$ or $p^a_x m^a_{14} t^a_{13}$. This is the third system of crystallisation, and as the rhombohedron is a very important crystal of this system, it has thence been termed the *Rhombohedral System of Crystallisation.* Its other designation of the three-and-one-axed system, is due to the circumstance that WEISS and his followers describe the forms of this system in reference to a system of *three* similar equatorial axes which cross in the centre of the equator at an angle of 60°, instead of a system of two equatorial axes which cross at an angle of 90°. In this particular, the present system of crystallography differs from theirs essentially.

4.) Pursuing the examination of the unequiaxed crystals, we find it impossible to classify them any farther by reference to their axes, which are always $p^a_x m^a_y t^a_z$, but not $p^a_x m^a_{15} t^a_{13}$ nor $p^a_x m^a_{14} t^a_{13}$. We therefore shift our ground, and classify them upon a different principle.

All the rest of the unequiaxed crystals whose north zone, east zone, north-east zone, and north-west zone, present only *Homohedral Forms,* §§ 257—262; constitute the fourth system of crystallisation, which, because it contains the minerals that produce the extensive family of rhombic and rectangular prisms, is called the *Prismatic System of Crystallisation.*

5.) The unequiaxed crystals whose north zone, east zone, north-east zone, and north-west zone, present *Hemihedral Forms,* §§ 272—280, constitute the fifth system of crystallisation, which is called the *Oblique Prismatic System of Crystallisation,* because it contains the minerals that produce prisms with single terminal planes of the east zone or north zone, set obliquely on the vertical prisms.

6.) The unequiaxed crystals whose north-east zone and north-west zone present *Tetartohedral Forms,* § 284, constitute the sixth system of crystallisation, which is called the *Doubly Oblique Prismatic System of Crystallisation,* because all the planes, both prismatic and pyramidal, appear as if they were parallel to three axes that cross each other obliquely in every direction.

The classification of the minerals of the last three systems depends upon *ultimate facts* in mineralogy similar to those which lead to the classification of the minerals of the first three systems. A mineral which presents a combination belonging to the 4th, 5th, or 6th system, never presents a combination belonging to any other system. This is a broad statement, liable, like all general rules, to particular exceptions, yet not to such exceptions as render the general statement untrustworthy.

Such are *the "six systems of axes of crystallisation."* They all

rest upon what I have called an *ultimate fact*, that a mineral which produces a combination belonging to one of the six systems, never produces a combination belonging to any of the other systems. This curious phenomenon cannot be accounted for: the fact is as inexplicable as extraordinary; but it affords a very excellent basis for the classification of crystallised minerals.

Each of these six systems contains a certain number of characteristic forms and combinations, which I shall now proceed to examine in detail, taking the name of each system of crystallisation as the title of a separate chapter.

I. THE OCTAHEDRAL SYSTEM OF CRYSTALLISATION.

341. The character of the Forms belonging to this system, as given by Rose, is this,—They have three Axes, which are all equal, and placed at right angles to one another.

Rose's enumeration of the Forms belonging to this system of crystallisation, is as follows:—

A. Homohedral Forms:

1. The Octahedron, ...	Model 15.	PMT.
2. The Cube, ...	— 1.	P,M,T.
3. The Rhombic Dodecahedron, ...	— 63.	MT.PM, PT.
4. The Icositessarahedron, ...	— 22.	$3P_-MT$.
5. The Triakisoctahedron, ...	— 17.	$3P_+MT$.
6. The Tetrakishexahedron, ...	— 68.	$\begin{cases} M_-T, M_+T. P_-M, \\ P_+M, P_-T, P_+T, \end{cases}$
7. The Hexakisoctahedron, ...	— 23.	$6P_-MT_+$.

B. Hemihedral Forms:

1. The Tetrahedron, ...	Model 117.	$\frac{1}{2}$ PMT.
2. The Hemiicositessarahedron, ...	— 119.	$\frac{1}{2}(3P_-MT)$.
3. The Hemitriakisoctahedron, ...	— 18.	$\frac{1}{2}(3P_+MT)$.
4. The Hemihexakisoctahedron with inclined faces, ...	— 24.	$\frac{1}{2}(6P_-MT_+)$.
5. The Pentagonal Dodecahedron, ...	— 91.	$M_-T. P_-M, P_+T$.
6. The Hemihexakisoctahedron with parallel faces, ...	— 25.	$3P_-MT_+$.

I have added to Rose's name of each crystal the number of the Model, and the new symbol which is intended to represent it. These crystals are "Forms," according to Rose's explanation of that term, inasmuch as each of them contains none but similar and equal planes; but, with the exception of Models 15 and 117, they are all "Combinations," according to the new definition of that term given in § 239, since, with the two exceptions named, every one of them contains several of the "Forms" *that are enumerated* in § 200. See Section VIII.

NORMALS.

342. Besides the three rectangular axes, which I call $p^s m^s t^s$, but which ROSE calls *a*, *a*, *a*, there are other four lines, or axes, of great importance in the consideration of the combinations that belong to this system. These are the lines that pass through the centre of the crystal, and connect the eight corners of the cube, and which are perpendicular to the eight faces of the octahedron. These are sometimes called *Hexahedron axes*, sometimes *Normals to the Octahedron faces*, and sometimes *Trigonal axes*. The POLES in which these lines terminate are described in § 21, pages 6, 7, as Znw Zne Zse Zsw Nnw Nne Nsw Nse. The line which passes from any one of these poles to the centre of the crystal will, throughout this section, be called the NORMAL of that pole; hence, a line betwixt c and 2, figure in page 6, will be the Znw *normal*.

Less important than the four lines which connect the corners of the cube, but still of considerable utility, are the six lines which connect the centres of the edges of the cube, and which, therefore, terminate at the poles nw ne se sw Zn Zs Ns Nn Zw Ze Ne Nw. See § 21. These lines will also, with a view to avoid the use of the word axis, be termed *normals*, and the length of each normal will be a line from the pole at the surface of the crystal, to c, its centre. Thus, a line betwixt c and M, figure in page 6, will be the Zn normal.

It will probably be occasionally convenient to consider all the 26 poles that are marked on the figure in page 6, and enumerated in § 21, to be the terminations of normals, and, in order that they may be referred to when necessary, either individually or in groups, I shall give them the following names:—

Unipolar Normals.—The normals that touch the poles Z N n e w s. They meet the centres of the planes of the cube, the corners of the octahedron, and the four-faced angles of the rhombic dodecahedron.

Bipolar Normals.—The normals that touch the poles nw ne se sw Zn Zs Nn Ns Ze Zw Ne Nw. They meet the centres of the edges of the cube, the centres of the edges of the octahedron, and the centres of the planes of the rhombic dodecahedron.

Tripolar Normals.—The normals that touch the poles Znw Zne Zse Zsw Nnw Nne Nse Nsw. They meet the corners of the cube, the centres of the planes of the octahedron, and the three-faced angles of the rhombic dodecahedron.

Relation of these Normals to the Angles of the Homohedral Forms of the Octahedral System of Crystallisation.

In Models 22 and 23, all the 26 normals terminate in solid angles. In Model 17 and 68, only the unipolar and tripolar normals terminate in solid angles, while the bipolar normals terminate in edges. In Model 63, the unipolar and tripolar normals terminate in solid angles, and the bipolar normals in planes. In Model 15, the unipolar normals terminate in solid angles, the bipolar normals in edges, and the tripolar normals in planes. And in Model 1, the unipolar normals terminate in planes, the bipolar normals in edges, and the tripolar normals in solid angles.

ROSE's Catalogue of the Minerals that belong to the octahedral system, is given in Part II., pages 3—12.

A symbolic catalogue of the Forms and Combinations which are presented by the crystals of each of these minerals, is given in Part II., pages 15—32. The Table in Part II., page 15, is a synopsis of the Forms and Combinations which belong to the system.

1. THE OCTAHEDRON. Model 15. PMT.

344. This form is figured and fully described at page 41. ROSE's symbol for it is $(a : a : a)$, which intimates that each face cuts the three rectangular axes in a similar manner.

It is indispensable to an octahedron, that its equator, north meridian, and east meridian, shall be squares, and that the angles of these sections shall be at the poles Z N n e s w. It follows that the angle of inclination of any edge to either of the axes $p^2\, m^2\, t^2$ is $\frac{90^\circ}{2} = 45^\circ$. The north-east and north-west meridians are rhombuses of 109° 28′. Therefore, the angle over any edge is 109° 28′, and the angle of incidence of two planes over a solid angle is 70° 32′. The plane angles of the faces are all 60°.

According to the principles of classification explained in SECTION IV. the octahedron is a complete pyramid with a square equator. The minerals which occur in this form are quoted at page 100, Part II., being in number no fewer than thirty-six.

ROSE gives the following method of indicating single planes of the octahedron:—“ It is useful in many cases to denote each of the eight faces of the octahedron by a particular mark. With this intent, we indicate the front half of the horizontal axis, or the part turned towards the observer, by $a_{,}$, the back part by $a'_{,}$, the right half of the horizontal axis which is parallel to the observer by $a_{,,}$, the left half by $a'_{,,}$, the upper half of the vertical axis by $a_{,,,}$, the lower half of it by $a'_{,,,}$; the signs for the eight faces of the octahedron are then as follow,” (column 1):—

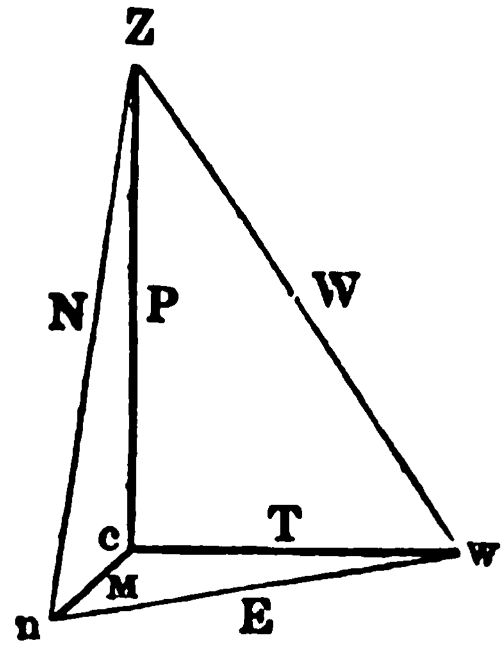
I have added, in a separate column, the signs by which I propose to effect the same end.

	ROSE'S SIGNS.	NEW SIGNS.
1.)	$(a_{,} : a_{,,} : a_{,,,})$	PMT Znw.
2.)	$(a'_{,} : a_{,,} : a_{,,,})$	PMT Zsw.
3.)	$(a'_{,} : a'_{,,} : a_{,,,})$	PMT Zse.
4.)	$(a_{,} : a'_{,,} : a_{,,,})$	PMT Zne.
5.)	$(a_{,} : a_{,,} : a'_{,,,})$	PMT Nnw.
6.)	$(a'_{,} : a_{,,} : a'_{,,,})$	PMT Nsw.
7.)	$(a'_{,} : a'_{,,} : a'_{,,,})$	PMT Nse.
8.)	$(a'_{,} : a'_{,,} : a'_{,,,})$	PMT Nne.

There is an error in Rose's sign, No. 8.), where $a'_{,}$ should be $a_{,}$. I copy it as it stands in the original, because it shows the difficulty of getting such marks correctly printed. It is still more difficult for any body to *remember* them.

It was the sight of this Table that induced me to introduce into crystallography the use of the astronomical terms Zenith, Nadir, north, east, south, west, equator and meridian, which I have found to aid the memory so much in remembering polaric positions, and so *greatly to facilitate* the description of particular sections and zones, that I should be very *unwilling to give* up the use of them.

345. PROBLEM. *Given, the symbol* PMT; *sought, the interfacial angle of the plane* Znw *upon the plane* Nnw.



Divide the form PMT into octants, § 301, take the Znw octant, and find, by the following methods, the inclination of the plane Znw to the equator, which is half the required angle of Znw or Nnw.

The axes of PMT are $p^a\, m^a\, t^a$, or $P = 1$, $M = 1$, $T = 1$. The given octant is, therefore, the simplest form of a right-angled solid triangle. Take pole n for its vertex. Then you have given, the following three quantities: M = the right angle = C; the plane c n w = 45° = side a; and the plane c n Z = 45° = side b. The angle required is that across the edge E, which, being opposite to the given part called side b, is angle B. Hence, this problem is the same as—*Given, sides* a, b; *sought, angle* B:

Formula 14. log tan B = log tan b + 10 — log sin a.

$$\begin{aligned} 10 + \log\tan b &= 45^\circ &&= 20.0000 \\ -\log\sin a &= 45^\circ &&= 9.8495 \\ \hline \log\tan B &= 54^\circ\, 44' &&= 10.1505 \end{aligned}$$

Therefore, 54° 44′ is the angle across the edge marked E in the figure of the octant, or the inclination of the plane Znw to the equator. Then, 54° 44′ × 2 = 109° 28′, is the interfacial angle of Znw on Nnw.

346. PROBLEM. *Given, the symbol* PMT; *sought, the interfacial angle of the plane* Znw *upon the plane* Zne.

This problem is the same as the foregoing, except that the required angle is that formed by two planes meeting at the north meridian instead of the equator.

a.) Proceed as before, but instead of Formula 14, take Formula 13; for the problem now is, *Given, sides* a, b; *sought, angle* A.

Formula 13. log tan A = log tan a + 10 — log sin b.

$$\begin{aligned} 10 + \log\tan a &= 45^\circ &&= 20.0000 \\ -\log\sin b &= 45^\circ &&= 9.8495 \\ \hline \log\tan A &= 54^\circ\, 44' &&= 10.1505 \end{aligned}$$

Therefore, 54° 44′ is the inclination of the plane Znw to the north meridian; and twice that angle, = 54° 44′ × 2 = 109° 28′, is the interfacial angle of Znw on Zne.

b.) *Another method of working this problem*, is by means of the quadrantal solid triangle, No. 58. Put $c = 90°$ = angle of the equator at the north pole, *measured from* the edge ne upon the edge nw. Then,

A = 54° 44′ and B = 54° 44′, will be half the inclination of the plane Zne on Nne and of Znw on Nnw; and C will be the inclination of Zne on Znw demanded in the problem.

Now, according to the principles explained in § 331, angles A and B, being both under 90°, are both positive, and the sum produced by their multiplication together is also positive, but since this product is prefixed by the sign — in the Formula, it must be changed from positive to negative. Hence, the angle found by resolving this equation will not be the angle contained in the table, but its supplement. These considerations remove the ambiguity which rests upon Formula 58.

Formula 58. log cos C = log cos A + log cos B — 10.

$$\begin{aligned} \log\cos A &= 54°\,44' = 9.7615 \\ +\log\cos B &= 54°\,44' = \underline{9.7615} \\ \log\cos \text{supplement of } C &= 70°\,32' = 9.5230 \\ \text{Therefore, } C &= 109°\,28'; \end{aligned}$$

because 180° — 70° 32′ = 109° 28. See §§ 330, 331.

347. PROBLEM. *Given the symbol* PMT, *sought the value of each of the three plane angles of the face* Znw.

a.) Take the same solid triangle as in problem § 345, and observe that the part now required is the *side opposite to the right angle*, that is to say, *side c.* Hence the problem is: *Given, sides a, b; sought, side c,* namely:

Formula 15. log cos *c* = log cos *a* + log cos *b* — 10.

$$\begin{aligned} \log\cos a &= 45° = 9.8495 \\ +\log\cos b &= 45° = \underline{9.8495} \\ \log\cos c &= 60° = 9.6990 \end{aligned}$$

Therefore, each of the plane angles required is 60°.

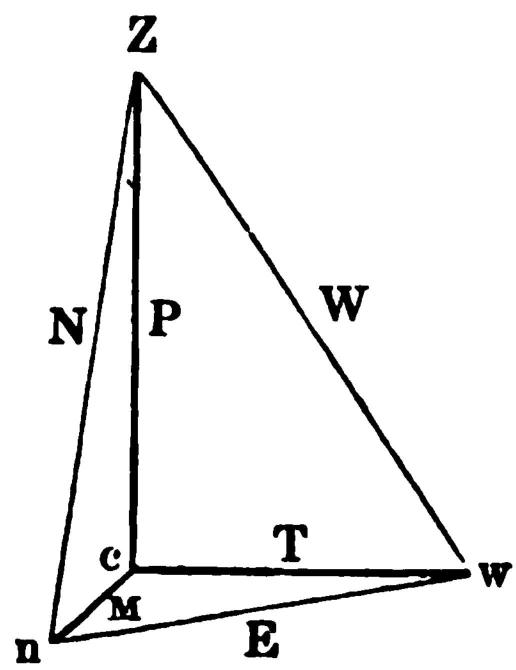
b.) Take the quadrantal solid triangle described in § 346, *b.*), and employ Formula 56, log tan *a* = log tan A + 10 — log sin B.

$$\begin{aligned} 10 + \log\tan A &= 54°\,44' = 20.1505 \\ -\log\sin B &= 54°\,44' = \underline{9.9119} \\ \log\tan a &= 60° \quad = 10.2386 \end{aligned}$$

c.) Here it may be noticed, that when the three axes, $p^a m^a t^a$, of the form PMT are all alike, the three plane angles of an external face of the form are also all alike; when the axes are $p^a_x m^a t^a$, or $p^a m^a_x t^a$, or $p^a m^a t^a_x$, that is to say, two of them alike but different from the third, then the plane angles are also two alike but different from the third; and when the three axes are $p^a_x m^a_y t^a_z$, or all unlike, then the three plane angles are also all dissimilar. Hence the distinction of ***regular***, ***isosceles***, and ***scalene octahedrons.***

348. PROBLEM. *Given*, Model 15 *with the symbol* P_xMT; *required, the value of the characteristic* $_x$.

Take, with the goniometer, the inclination of the plane Znw upon the planes Nnw and Zne. Both are 109° 28′. Then take the Znw octant of the form as a solid triangle with pole n for its vertex. The parts measured of the solid triangle are the edge M or right angle = C; the edge N $= \frac{109° 28'}{2}$ = 54° 44′ = angle A; and the edge E = $\frac{109° 28'}{2}$ = 54° 44′ = angle B. The part required is the plane angle Znc, which, being a side of the solid triangle, and opposite to angle B, is side *b*. We require the plane angle Znc, because the natural tangent of that angle is the value of $_x$ in the problem. Hence the problem to be worked is this: *Given, angles* A,B; *sought, side b: Formula* 5; namely, $\cos b = \frac{\cos B}{\sin A}$, which Formula can be simplified by Formula 104, (see § 335), into $\cos b = \cot A$. As we have here only to compare two quantities with one another, we do not need the aid of logarithms, but content ourselves with referring to the table of natural numbers, where we find:



$$\text{nat cot A} = 54° 44' = .7072$$
$$\text{nat cos } b = 45° 00' = .7072$$

That is to say, we first look in the column of natural cotangents for the angle 54° 44′ and find its natural cotangent to be .7072. Then we look in the column of natural cosines for the number .7072, and find its angle to be 45°. This, then, is the value of angle Znc, and the natural tangent of this angle is 1.0 or unity; so that P_xMT means PMT.

You will observe, that you can, if you please, use the logarithmic functions, but it is without gaining any advantage. Thus:

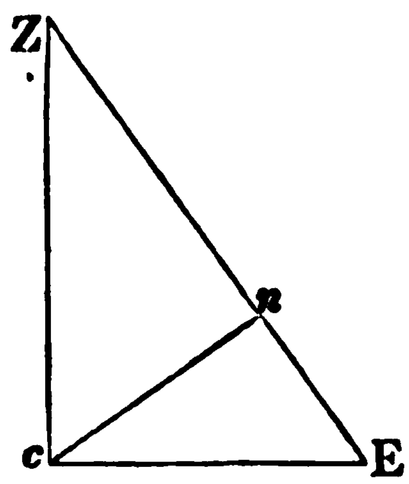
$$\text{log cot A} = 54° 44' = 9.8495$$
$$\text{log cos } b = 45° 00' = 9.8495$$

349. PROBLEM. *Given*, Model 15, *with the symbol* PMT. *Required, the inclination of the* Znw normal, *to the axis* p^a, *to the equator, to the* Znw *plane, and to the other* normals.

The Znw quadrant of the north-west meridian of the form PMT is a right-angled triangle, having an angle of 54° 44′ where the plane Znw meets the equator, and an angle of 35° 16′ where it meets the pole Z. A perpendicular dropped from the hypothenuse or longest side of this triangle, upon the right angle at the centre of the crystal, divides the triangle into two triangles having angles similar to those of the undivided triangle.

Put *Zc* = axis p^a; *Ec* = diagonal of the equator; ZE = Znw quadrant of the *north-west meridian*. Then Z*c*E = 90°; cZE = 35° 16′; ZEc

= 54° 44′. Let *nc* be the perpendicular dropped from the side ZE. Then the triangles *Zcn* and *cEn* will have similar angles, namely, 54° 44′, the angle at *n* being always 90°. But the line *nc* will have the same relation to m^a and t^a as to p^a, assuming the line *Zc* to represent either p^a, m^a, or t^a. Therefore, *n* is the centre of the Znw plane of the form PMT. Therefore, it is the Znw pole, and the line *nc* is the Znw normal. Consequently, the plane Znw is perpendicular to the Znw normal, and the inclination of this normal to axis p^a is the angle *Zcn* = 54° 44′ or the complement of 35° 16′; the inclination of it to the equator is the angle *Ecn* = 35° 16′ or the complement of 54° 44′; and its inclination to the Nnw normal (or to the normal of any adjoining plane of PMT) is twice the angle *nc*E, or 35° 16′ × 2 = 70° 32′.



It follows from these results, that the inclination of every one of the eight tripolar normals is:

To an adjacent UNIPOLAR *normal* = 54° 44′,
To an adjacent BIPOLAR *normal* = 35° 16′,
To an adjacent TRIPOLAR *normal* = 70° 32′.

2. The Cube. Model 1. P,M,T.

350. This combination is described in §§ 21—25. Rose's symbol for it is $(a : \infty a : \infty a)$.—According to the principles of classification explained in Section IV., the cube is a complete prism with a square equator. The minerals which occur in this shape are quoted at page 97, Part II., being thirty-five in number.

The method of proving trigonometrically the value of the plane angles of Model 1, and of finding the inclination of its planes and edges to the Znw normal, and the inclination of that normal to the axis p^a, is given in § 363.

351. Combinations of the Cube, P,M,T, with the Octahedron, PMT.

a.) Model 29. P,M,T. PMT. Rose's symbol for which is $(a : \infty a : \infty a) + (a : a : a)$.—The middle crystal between the cube and the octahedron. A complete prism combined with an incomplete pyramid. The minerals which occur in this shape are quoted at page 105, Part II.

b.) Model 31.* (See note page 88). P,M,T, pmt. Rose's symbol is $(a : \infty a : \infty a) + (a : a : a)$. The cube with its corners slightly truncated by the planes of the octahedron. A complete prism with an incomplete pyramid. The minerals which occur in this form are quoted at page 104, Part II.

c.) Model 30. p,m,t.PMT. Rose's symbol is $(a : a : a) + (a : \infty a : \infty a)$. The octahedron with its angles replaced by the planes of the cube. A

complete prism with an incomplete pyramid. The minerals which occur in this shape are quoted at page 106, Part II.

352. PROBLEM. *Given,* Model 29, *with the symbol* P,M,T, P_xMT; *required, the value of the index* $_x$.

When the apex of an octahedron is truncated by the horizontal plane PZ, the inclination of every terminal edge and terminal plane upon the horizontal plane is equal to 90°, added to the inclination of the terminal edge or terminal plane to the axis p^s. The inclination of a terminal edge or terminal plane to the axis p^s, is half the inclination of two opposite terminal edges or terminal planes measured over the pole Z.

The inclination of a plane of the form PMT to the axis p^s is 35° 16′ (= 70° 32′ ÷ 2). And 35° 16′ + 90° = 125° 16′. Take now, with the goniometer, the inclination of one of the triangular planes of Model 29 upon each of the square planes which surround it. The measurement is in every case 125° 16′. Therefore, P,M,T, P_xMT, contains the planes of the cube and the regular octahedron, and the value of $_x$ is unity.

A difference of magnitude in the planes P,M,T, and PMT, makes no difference in the angle at which they incline upon one another. Thus, the angles of P,M,T upon pmt, are the same as those of p,m,t upon PMT, as will be found on measuring Models 31 and 30.

353. *Another method of showing the interfacial angles of the planes of* P,M,T *upon those of* PMT.—It will be proved in § 363, *b.*), that the planes of PZ, Mn, Tw, incline upon the Znw normal at an angle of 35° 16′. But the plane PMT Znw is perpendicular to the Znw normal, § 349. Therefore, § 61, the inclination of the planes PZ, Mn, Tw, to the planes PMT Znw which they surround, must be 90° + 35° 16′ = 125° 16′.

3. THE RHOMBIC DODECAHEDRON. Model 63. MT. PM, PT.

354. This combination is described in §§ 21 B, 31, 89, and 103. Rose's symbol for it is: $(a : a : \infty a)$. The rhombic dodecahedron is an incomplete prism with a complete pyramid. The minerals which occur in this shape, 28 in number, are quoted at page 110, Part II.

The measurements over the poles Z, N, n, e, w, s, are as follow:—The angle formed by two opposite planes = 90°, by two opposite edges = 109° 28′; as may be proved by the goniometer. We may consider the rhombic dodecahedron to be a cube, having a four-faced pyramid upon each of its planes, the shorter diagonals of the planes of the dodecahedron showing the form of the base of the pyramid; or we may consider it to be an octahedron with a three-faced pyramid upon each of its planes, the longer diagonals of the planes of the dodecahedron showing the form of the base of the pyramid. The six apices of the four-faced pyramids are united by p^s m^s t^s: the eight apices of the three-faced pyramids, by the tripolar normals, Znw, &c.

355. PROBLEM. *Given, the combination* MT. PM, PT. Model 63; *required, the angle of incidence of the plane* PM Zn *upon the plane* PT Zw; *that is to say, the angle across an edge of the combination.*

Suppose the Znw octant of Model 63 to be divided vertically into two equal portions by the north-west meridian, and one of these portions, namely, that which contains part of the plane PM Zn, to be taken as a solid triangle with pole Z for its vertex. A *side* of this triangle will be the Zn quadrant of the north meridian, which, as the meridian is a square, gives 45° for the value of this side. Call it *side a.* As the north meridian is at right angles to the form PM, that edge or angle of the solid triangle produced between plane PM Zn and *side a*, will be angle C = 90°. Then angle B will be the edge or angle at axis p^a where the north meridian and the north-west meridian intersect one another at an angle of 45°; and angle A (opposite side a) will be half the inclination of the plane PM Zn upon the plane PT Zw, which is the unknown quantity sought. The quantities given are, therefore, side $a = 45°$, and angle $B = 45°$; and we have to find angle A. This case is answered by Formula 10; namely,

$$\cos A = \cos a \sin B\text{: or } \log \cos A = \log \cos a + \log \sin B - 10.$$

$$\begin{aligned} \log \cos a = 45° &= 9.8495 \\ + \log \sin B = 45° &= 9.8495 \\ \hline \log \cos A = 60° &= 9.6990 \end{aligned}$$

Angle A being half the inclination of plane PM Zn upon plane PT Zw, it follows that the angle demanded is twice 60° = 120°.

356. PROBLEM. *Given, the combination,* MT. PM, PT, Model 63; *required, the inclination of the edges between* PM *and* PT, *that is to say, the oblique edges of the four-faced pyramid, to the axis* p^a.

Take the solid triangle made use of in the last problem, and with the given data, find side c (opposite angle C), which is the Znw quadrant of the north-west meridian. The problem is: *Given, side a and angle* B; *sought, side c; Formula* 12: $\tan c = \frac{\tan a}{\cos B}$, or

$$\log \tan c = \log \tan a + 10 - \log \cos B.$$

$$\begin{aligned} 10 + \log \tan a = 45° &= 20.0000 \\ - \log \cos B = 45° &= 9.8495 \\ \hline \log \tan c = 54° \, 44' &= 10.1505 \end{aligned}$$

This product, 54° 44′, is the desired angle, showing the inclination of the terminal edges to axis p^a.

It is proved in the same manner, that the edges between MT and PM incline upon the axis m^a at an angle of 54° 44′; and that the edges between MT and PT incline upon the axis t^a at the same angle. Of course, the inclination of any two of these edges to one another over the pole Z, n, or w, is 54° 44′ × 2 = 109° 28′, as was assumed in § 354.

357. PROBLEM. *Given, the combination* MT.PM, PT, Model 63; *required, the plane angles of its external faces.*

Take the solid triangle made use of in the foregoing two problems, and with the given data, find side b (opposite angle B); which angle is the half of the acute plane angle of a face at the pole Z.

The problem therefore is: *Given, side a and angle* B; *sought, side b.*

$$\text{Formula 11. } \log \tan b = \log \tan B + \log \sin a - 10.$$

$$\begin{array}{rll} \log \tan B = 45^\circ & = & 10.0000 \\ + \log \sin a = 45^\circ & = & 9.8495 \\ \hline \log \tan b = 35^\circ\, 16' & = & 9.8495 \end{array}$$

Twice this product, or $35^\circ\, 16' \times 2 = 70^\circ\, 32'$, is the value of the acute plane angle at the pole Z, and the supplement of $70^\circ\, 32'$ ($= 180^\circ - 70^\circ\, 32'$) $= 109^\circ\, 28'$, is the value of the obtuse plane angle at the pole Znw.

358. *Another method of solving the foregoing problem.*

Take the same solid triangle as before, with the same vertex, pole Z, but employ angle B $= 45^\circ$, and angle A, found by the problem in § 355, or by measurement of the model, to be $= 60^\circ$. Then seek side b, using Formula 5:

$$\log \cos b = \log \cos B + 10 - \log \sin A.$$

$$\begin{array}{rll} 10 + \log \cos B = 45^\circ & = & 19.8495 \\ - \log \sin A = 60^\circ & = & 9.9375 \\ \hline \log \cos b = 35^\circ\, 16' & = & 9.9120 \end{array}$$

The product is necessarily the same as that of the last problem.

359. PROBLEM. *Given, the combination* MT. PM, PT, Model 63; *required, the plane angles of the faces of the combination, and the inclination of the edges and planes to the tripolar normals, or specially, the inclination of the three edges and of the three planes contained in the* Znw octant, *to the* Znw normal.

CALCULATION OF THREE-FACED PYRAMIDS.

Hold Model 63 in such a position that the pole Znw is placed in the position of pole Z, and then look from Zenith down upon the model. You will perceive a six-sided prism terminated by a three-faced pyramid, the Znw normal being in the position of the axis p^a. Now, suppose the model to be divided into three sections, by vertical planes passing through the three terminal edges, and meeting at the Znw normal, and suppose also that each of these sections is divided into two smaller sections, by other vertical planes passing in the direction of the shorter diagonals of the three rhombic terminal planes, and also meeting at the Znw normal. Take one of these *sixths of the crystal* as a solid triangle, having the pole Znw for its vertex, and observe what parts of the triangle *are known, and what* are unknown. In the first place, the bisec-

tion of the rhombic terminal planes, by a plane perpendicular to them, produces an edge of 90°, which we will call angle C; secondly, the division of the crystal into six equal portions, by planes, which all meet at a vertical axis under the pole Z, namely, the Znw normal, produces a vertical edge of 60°, as part of each section of the crystal, which edge we will call angle A; and, thirdly, as the angle across each terminal edge of the three-faced pyramid is known to be 120°, § 355, the half of it = 60°, is the value of the third edge of the triangle, which may be called angle B. We know, therefore, the three *angles* of the solid triangle: A and B being each = 60°, and C = 90°; and with these data we can find the three sides of the solid triangle, which are respectively opposite to the three angles.—Now, side *a* of this solid triangle is half the obtuse plane angle of an external plane of Model 63; side *b* is an inner vertical plane, which shows the inclination of an external plane to the Znw normal; and side *c* is another inner verticle plane, which shows the inclination of a terminal edge to the Znw normal. We have therefore three equations to resolve:

a.) *Given, angles* A, B; *sought, side a. Formula* 4. $\cos a = \frac{\cos A}{\sin B}$, which, (as A and B are both = 60°), *Formula* 104 transforms to

$$\cos a = \cot A.$$

$$\begin{aligned} \text{nat cot A} &= 60^\circ &&= .5774 \\ \text{nat cos } a &= 54^\circ\, 44' &&= .5774 \end{aligned}$$

Hence the obtuse plane angles of the faces of Model 63 are 109° 28′, and, consequently, its acute plane angles are 70° 32′, (= 180′ — 109° 28′) as was found by the Problem in § 357.

b.) *Given, angles* A, B; *sought, side b, Formula* 5. This Formula admits the same abbreviation as the last, and gives the same result, namely, 54° 44′, which is the inclination of a plane of Model 63 upon the Znw normal.

c.) *Given, angles* A, B; *sought, side c.*

Formula 6. $\log \cos c = \log \cot A + \log \cot B - 10.$

$$\begin{aligned} \log \cot A &= 60^\circ &&= 9.7614 \\ + \log \cot B &= 60^\circ &&= 9.7614 \\ \hline \log \cos c &= 70^\circ\, 32' &&= 9.5228 \end{aligned}$$

This product, 70° 32′, is the inclination of an edge of Model 63, to the Znw normal. The following is a check on the correctness of this calculation:—The inclination of a plane to the vertical axis, added to the inclination of an edge to the vertical axis, must be equal to the inclination of a plane to an edge measured over that vertical axis. Now, 54° 44′ + 70° 32′ = 125° 16′, which, on applying the goniometer to Model 63, will be found to be the exact inclination of a plane to an edge measured over the trisolid angle at Znw.

360. There is a peculiarity in the results afforded by Equations *b.*) *and c.*), which is extremely important in respect to certain calcula-

tions, which will have to be considered in another Section, regarding the crystals that are called rhombohedrons. When Model 63 is held in the position described in § 359, with the pole Znw in the position of the pole Z, the Model has the same planes, and occupying the same polaric position as the planes belonging to Model 71, which is a combination of the unequiaxed rhombohedral class. But the peculiarity to which I wish to direct your attention is *the relation which holds between the inclination of* A PLANE *and of* AN EDGE *of a three-faced pyramid to the* AXIS *which is perpendicular to the trisolid angle where the three planes and three edges meet.* In the case of Model 71, and of all the rhombohedrons, this perpendicular axis is p^a; but in the case of Model 63, this axis is the Znw normal. This difference, however, does not affect the relations which the edges and planes in question bear to the line perpendicular to the trisolid angle, where the edges and planes all meet.

361. Suppose now that the *plane of the pyramid,* which we have found to incline on the vertical axis, (Equation *b,*) at an angle of 54° 44′, inclines also on the equator of the combination, it must do so at an angle of 35° 16′. Suppose, also, that the *edge of the pyramid* which (Equation *c,*) inclines upon the vertical axis at an angle of 70° 32′, inclines at the other end upon the equator, the angle formed there must be 19° 28′. If we call the vertical axis p^a, and the line along the equator, which is touched by the edge and plane in question, t^a, and if we put $p^a = 1$, then the distance from the centre of t^a to the point where it touches the inclined plane will be tan 54° 44′ = 1.4141, and the distance to the point where it touches the inclined edge will be tan 70° 32′ = 2.8291. These relations may be expressed by the symbols $P\frac{10000}{14141}T$, and $P\frac{10000}{28291}T$. From this observation we draw the following conclusion:—

362. *When a pyramid consists of three equal and similar planes, separated by three equal and similar edges, the point where one of the inclined edges touches the equator is* TWICE AS FAR FROM THE FOOT OF THE VERTICAL AXIS *as is the point where the equator is touched by the diagonal of one of the inclined planes.* In other words, *the cotangent of the inclination of a plane of a three-faced pyramid to the vertical axis, is twice the cotangent of the inclination of an edge to the same axis.* Thus:

Inclination of plane, 54° 44′, cot .7072
Inclination of edge, 70° 32′, cot .3535

363. PROBLEM. *To find the inclination of the planes* PZ, Mn, Tw, *of the* Cube, Model 1, *and of the three edges which separate them, to the* Znw normal.

The principle laid down in § 362 will be found to apply even to the cube, if it is placed in such a position as to resemble a rhombohedron.

Hold Model 1, with the trisolid angle Znw, in the place of the pole Z, and then suppose the model to be divided into six similar vertical portions, by six *vertical sections,* proceeding from the central axis out-

wards. See § 359. Take one of the resulting *sixths* as a solid triangle, having the pole Znw for its vertex. You will then have, *as given parts*, angle C $=90°$ = inclination of a terminal plane to a section perpendicular thereto; angle A $=60°$ = inclination of two adjacent vertical sections at the vertical axis; and angle B $=45°$ = half the inclination between two external planes, across an external edge. With these data you can find side a = half the plane angle at Znw of an external face; side b = inclination of an external plane to the Znw normal; and side c = inclination of an external edge to the Znw normal.

a.) Given, angles A, B; *sought, side a.*

Formula 4. $\log \cos a = \log \cos A + 10 - \log \sin B.$

$$\begin{array}{rcl} 10 + \log \cos A = 60° & = & 19.6990 \\ - \log \sin B = 45° & = & 9.8495 \\ \hline \log \cos a = 45° & = & 9.8495 \end{array}$$

Hence the plane angles of Model 1 are each twice 45°, or 90°.

b.) Given, angles A, B; *sought, side b.*

Formula 5. $\log \cos b = \log \cos B + 10 - \log \sin A.$

$$\begin{array}{rcl} 10 + \log \cos B = 45° & = & 19.8495 \\ - \log \sin A = 60° & = & 9.9375 \\ \hline \log \cos b = 35°\ 16' & = & 9.9120 \end{array}$$

This is the inclination of the external planes to the Znw normal. Then, as PZ inclines to this normal at an angle of 35° 16′, the normal itself must incline to the nw vertical edge of the model, and also to the axis p^a, at an angle equal to 90° — 35° 16′, or 54° 44′, because PZ is at right angles to p^a, and also to the vertical edge nw. These relations are corroborated by the following equation:—

c.) Given, angles A, B; *sought side c.*

Formula 6. $\log \cos c = \log \cot A + \log \cot B - 10.$

$$\begin{array}{rcl} \log \cot A = 60° & = & 9.7614 \\ + \log \cot B = 45° & = & 10.0000 \\ \hline \log \cos c = 54°\ 44' & = & 9.7614 \end{array}$$

This is the inclination of any edge of the form P,M,T, to the Znw normal.

The product of equation *b.*) = 35° 16′, and that of equation *c.*) = 54° 44′, are together equal to 90°, which the goniometer will show to be the angle of inclination of any plane of Model 1, upon any edge, measured over a solid angle.

As the tangent of 35° 16′ is 0.7072, and the tangent of 54° 44′ is 1.4141, we may express the products of equations *b.*) and *c.*) in symbols as follows:—$P\frac{10000}{07072}T$ and $P\frac{10000}{14141}T$, in which expression the reader will again observe the curious relations pointed out in §§ 361, 362; since *07072 is the half* of 14141, as 14141 is the half of 28291.

364. COMBINATIONS OF THE RHOMBIC DODECAHEDRON, MT.PM,PT, WITH THE OCTAHEDRON, PMT.

Model 64. mt. pm, pt, PMT.

Model 65. MT. PM, PT, pmt.

Both of these combinations are incomplete prisms combined with complete pyramids. The minerals which occur in these forms are contained in Class 4, Order 1, Genus 1, in Part II., page 110.]

365. PROBLEM. *Given, the combination* MT.PM,PT,PMT; *required, the inclination of a plane of* PMT *upon a plane of* MT, PM, *or* PT.

The *edges* of PMT have the same polaric positions, and the same relations to p^a m^a t^a, as have the *planes* of MT. PM, PT; consequently, the planes of MT. PM, PT, replace the edges of PMT evenly; and, § 61, the incidence of a plane of PMT upon a plane of MT, PM, or PT, is the half of 109° 28′ + 90° = 144° 44′, as will be found on applying the goniometer to Models 64 and 65.

366. *Another method of solving this problem.*—The plane PMT Znw, which replaces the trisolid angle at the Znw pole of MT. PM, PT, is exactly perpendicular to the Znw normal, § 349. Now the planes of MT. PM, PT. incline upon this normal at an angle of 54° 44′, § 359, *equation* 2.) Hence they must incline upon PMT at an angle of 54° 44′ + 90° = 144° 44′.

367. COMBINATIONS OF THE CUBE P,M,T, WITH THE RHOMBIC DODECAHEDRON, MT. PM, PT.

Model 27. P,M,T, MT. PM, PT.

Model 28. p,m,t, MT. PM, PT.

Model 36.* P,M,T, mt. pm, pt. (*This supposes the hexagonal planes of ½pmt shown on four of the corners of the cube to be away.)

These combinations are complete prisms with incomplete pyramids. The minerals which occur in these shapes are placed in Class 3, Order 1, Genus 1, Groups *a* and *b*, Part II., pages 104, 105.

Analysis of these Combinations:

As any two opposite planes of MT. PM, PT meet at the poles Z, N, n, e, s, or w, at an angle of 90°, and as the planes of P,M,T are perpendicular to the axes which connect these poles, it follows, that the planes of P,M,T cut those of MT. PM, PT at an angle of $\frac{90°}{2}$ + 90° = 135°, which measurement with the goniometer shows to be correct.

368. COMBINATIONS CONTAINING P,M,T, WITH MT. PM, PT, AND PMT.

Model 31. P,M,T, mt. pm, pt, PMT.

— 32. *The same with the Forms marked.*

— 33. P,M,T, mt. pm, pt, PMT.

— 34. p,m,t, MT. PM, PT, pmt.

These combinations are complete prisms with incomplete pyramids.

See the minerals described in Part II., pages 105, 106. Class 3, Order 1, Genus 1, Groups *a*, *b*, *c*.

The planes of these combinations show the inclinations of the planes of every one of the three combinations upon the planes of both the others.

The reader may take instrumentally the angles round the equator and the four meridians of each of these three models, and see that they agree with the angles of the different forms, and also that their aggregate value is in every case such as it should be, according to the principle explained in §§ 79—85.

4. THE ICOSITESSARAHEDRON, P_MT, PM_T, PMT_: or 3P_MT.

Varieties of this combination:

P½MT, PM½T, PMT½: or 3P½MT.

P⅓MT, PM⅓T, PMT⅓: or 3P⅓MT.

Model 22 is 3P½MT.

369. This combination is described in §§ 128—146. ROSE's symbol for it is: $(a : a : \text{m}\, a)$ or for the variety exhibited by Model 22, and described as 3P½MT, his symbol is: $(a : a : \frac{1}{2}a)$. The icositessarahedron is a complete pyramid with a rhombic equator, and falls into Class 2, Order 3, Genus 1. The minerals which occur in this shape are quoted in Part II., page 102.

The icositessarahedron may be considered as a rhombic dodecahedron, having a flat scalene pyramid with a rhombic base, resting upon each plane.

The edges on Model 22 are of two kinds as respects their length. Those which meet at the tripolar normals are *short*, and will be called *s* in the following problems. Those which meet at the unipolar normals are *long*, and will be called *l*.

370. PROBLEM. *Given*, Model 22, *with the angle across a long edge, l. Required, the inclination of the edge l to the axis* p^a, *and the value of the index* _ *in the symbol* P_MT.

a.) Let the angle across the edge $l = 131° \; 49'$.

Take the Znw octant of Model 22 as a solid triangle with pole Z for its vertex. Then the vertical edge at the junction of the north and east meridians will be angle C $= 90°$; angle A will be $\frac{1}{2}\, l = \frac{131° \; 49'}{2} = 65° \; 54\frac{1}{2}'$; angle B will be the same; and the part required is side *a*, which is the Zn or Zw portion of one of the meridians, and the inclination of a long edge to the axis p^a. As angle A and angle B are similar quantities, the *Formula* to be used is No. 4 modified by No. 104, or

$$\cos a = \cot \text{A}.$$

$$\text{nat cot A} = 65° \; 54\tfrac{1}{2}' = .4471$$

$$\text{nat cos } a = 63° \; 26' \;\; = .4472$$

Therefore, the inclination of the edge *l* to p^a is 63° 26′.

This is the measurement which proves that the four upper and four *lower planes* of Model 22 require the symbol P½MT; for the cotangent

of 63° 26′, the inclination of an edge to the axis p^a, is .5000. It is proved in the same manner, that the eight planes at the poles n, s, require the symbol PM½T, and that the eight planes at the poles e, w, require the symbol PMT½ or P⅔M⅔T. The accuracy of these calculations is checked by the direct application of the goniometer to any two edges of the model which meet at any one of the six poles Z, N, n, e, w, s, where the angle must be 63° 26′ × 2 = 126° 52′.

b.) Let the angle across the edge *l* = 144° 54′. The solution of this problem gives 71° 34′ for the required angle, and ⅓ for the value of the index _ in 3P_MT. See § 145.

I leave the working of this problem to the reader.

371. PROBLEM. *Given, the symbol* P½MT, PM½T, PMT½; *required, the angle across a long edge.*

This problem is the reverse of the preceding problem. Take the Znw octant of the combination as a solid triangle, with the pole Z for its vertex. The symbol P½MT shows the axes to be $p^a_5 m^a_{10} t^a_{10}$. Look, therefore, for the angle of which $\frac{5}{10}$ or .5 is the cotangent. This angle is 63° 26′, and it represents the inclination of the north and east meridians, that is to say, of two different terminal edges of the pyramid, to the axis p^a. These parts are sides *a* and *b* of the solid triangle, which have angle C = 90° between them. The part of the triangle to be found is, therefore, angle A or B, one of which represents the inclination of a plane to the north meridian, and the other, the inclination of a plane to the east meridian. Hence, the problem is as follows: *Given, sides a, b; sought, angle* A.

Formula 13. log tan A = log tan *a* + 10 — log sin *b*.

$$\begin{aligned} 10 + \log\tan a &= 63°\ 26' = 20.3010 \\ -\log\sin b &= 63°\ 26' = 9.9515 \\ \hline \log\tan A &= 65°\ 54\tfrac{1}{2}' = 10.3495 \end{aligned}$$

Twice this product, or 65° 54½′ × 2 = 131° 49′, is the required angle across a long edge.

372. PROBLEM. *Given, the symbol* P½MT, PM½T, PMT½; *required, the angle across an edge l.* Answer, 144° 54′. This problem is left for the reader to work.

373. PROBLEM. *Given,* Model 22, *with the angle across a short edge, s; required, the inclination of the three short edges, and of the three planes between them, to the* Znw *normal; also, the plane angle of the faces at the pole* Znw.

The principle upon which the solution of this problem depends, is explained in §§ 359—363, which sections relate to the analysis of three-faced *pyramids.*

Let the angle across the edge $s = 146° 27'$.

Form a solid triangle, with the pole Znw for its vertex, and having an angle $C = 90°$, an angle $A = 60°$, and an angle $B = \frac{146° 27'}{2} = 73° 13\frac{1}{2}'$.

With these data, find the following three parts of the solid triangle:

side a = half the plane angle at Znw of one of the external faces of the crystal.

side b = inclination of the external planes of the Model to the Znw normal.

side c = inclination of the external edges of the Model to the Znw normal.

a.) Given, angles A, B; *sought, side a.*

Formula 4. $\log \cos a = \log \cos A + 10 - \log \sin B.$

$$\begin{array}{rll} 10 + \log\cos A = 60° & = & 19.6990 \\ - \log \sin B = 73° 13\frac{1}{2}' & = & 9.9811 \\ \hline \log\cos a = 58° 31' & = & 9.7179 \end{array}$$

Twice this product, or $58° 31' \times 2 = 117° 2'$, is the plane angle of each face of Model 22 at the pole Znw.

b.) Given, angles A, B; *sought, side b.*

Formula 5. $\log \cos b = \log \cos B + 10 - \log \sin A.$

$$\begin{array}{rll} 10 + \log\cos B = 73° 13\frac{1}{2}' & = & 19.4603 \\ - \log \sin A = 60° & = & 9.9375 \\ \hline \log\cos b = 70° 32' & = & 9.5228 \end{array}$$

This product, 70° 32′, is the inclination of the external planes to the Znw normal.

c.) Given, angles A, B; *sought, side c.*

Formula 6. $\log \cos c = \log \cot A + \log \cot B - 10.$

$$\begin{array}{rll} \log\cot A = 60° & = & 9.7614 \\ + \log \cot B = 73° 13\frac{1}{2}' & = & 9.4792 \\ \hline \log\cos c = 79° 58\frac{1}{2}' & = & 9.2406 \end{array}$$

This product, $79° 58\frac{1}{2}'$, is the inclination of the external edges to the Znw normal.

There is a direct and easy check over the accuracy of these calculations. According to the principle stated in § 362, the tangent of the product of equation *c.*) should be a line twice the length of the tangent of the product of equation *b.*) Now the tangent of 70° 32′ is 2.8291, and the tangent of $79° 58\frac{1}{2}'$ is 5.6569, which is its double within a fraction. If the calculations were made with tables reckoning *seconds*, the products would be exact, instead of merely approximate, as in this and many other examples of brief calculation in this work.

374. **Problem.** The same as the preceding, but with the angle across the short edge $s = 129° 31'$. See § 146. I leave the solution of this problem as an exercise for the reader.

375. **Problem.** *With the information contained in* §§ 369—373, *to find all the external plane angles of* Model 22, P½MT, PM½T, PMT½.

a.) The plane angle at the pole Znw is found, by § 373 *a.*), to be 117° 2′. Call this *a*.

b.) To find the plane angle at the pole Z, take the solid triangle described in § 370 *a.*), in which are given, $C = 90°$; $A = 65° 54\frac{1}{2}'$; and $B = 65° 54\frac{1}{2}'$. The part required is side *c*. Hence:

Formula 6. $\log \cos c = \log \cot A + \log \cot B - 10.$

$$
\begin{array}{rl}
\log \cot A = 65° 54\frac{1}{2}' = & 9.6505 \\
+ \log \cot B = 65° 54\frac{1}{2}' = & \underline{9.6505} \\
\log \cos c = 78° 28' = & 9.3010
\end{array}
$$

This product, 78° 28′, is the plane angle at the pole Z. Call this *b*.

c.) The plane angles at the poles Zn and Zw are both alike. Call each of them *c*.

I have shown in § 82, that the four angles of a plane of four sides, which description applies to the faces of Model 22, are together equal to 360°. But we have found one angle, *a*, to be 117° 2′, and another angle, *b*, to be 78° 28′. Therefore, each of the remaining angles, *c*, must be $\frac{1}{2}[360° - (117° 2' + 78° 28') = 164° 30'] = 82° 15'$. Thus:

$$
\begin{array}{lcl}
\text{Angle at pole Znw} & = & 117° \ \ 2' \\
\quad — \qquad \text{Z} & = & 78° 28' \\
\quad — \qquad \text{Zn} & = & 82° 15' \\
\quad — \qquad \text{Zw} & = & \underline{82° 15'} \\
\text{All the four angles} & = & 360° 00'
\end{array}
$$

d.) ***Problem.*** The plane angle *c* can also be found by a direct operation, when you know the angle across both the external edges *s* and *l*. Example: The angle across *s* is 146° 27′. Call the half of it $B = 73° 13\frac{1}{2}'$. The angle across *l* is 131° 49′. Call the half of it $A = 65° 54\frac{1}{2}'$. Take a solid triangle with the pole Zn for its vertex, and let planes pass through the edges *s* and *l*, and intersect each other at the Zn normal. Then C will be the angle produced by the junction of these sections at the Zn normal; A will be half the Z^2n edge *l*; B half the Z^2n^2w edge *s*; and *c*, the plane angle required. This problem can be solved by

Formula 6. $\log \cos c = \log \cot A + \log \cot B - 10.$

$$
\begin{array}{rl}
\log \cot A = 65° 54\frac{1}{2}' = & 9.6505 \\
+ \log \cot B = 73° 13\frac{1}{2}' = & \underline{9.4792} \\
\log \cos c = 82° 15' = & 9.1297
\end{array}
$$

This *product is confirmatory of* the accuracy of the calculation given in c.)

376. PROBLEM. *Given, the symbol* 3P½MT; *required, the inclination of the planes to the axis* p^2.

a.) First find half the angle across an edge *l*, by means of problem § 371. Call this angle B = 65° 54½′.

b.) Take the Znw octant of Model 22, and divide it into two vertical portions by the north-west meridian. Choose the portion adjoining the north meridian as a solid triangle, with the pole Z for its vertex. Then B is the inclination of the external plane to the north meridian = 65° 54½′; C is the inclination of the Z^2nw plane of the model upon the north-west meridian; *c* is the Zn quadrant of the north meridian = 63° 26′, see § 371; and *b* is the Znw quadrant of the north-west meridian, or the inclination of the plane Z^2nw to the axis p^2, which is the quantity demanded in the problem. Therefore, we have *given*, B, *c*; *to find*, *b*.

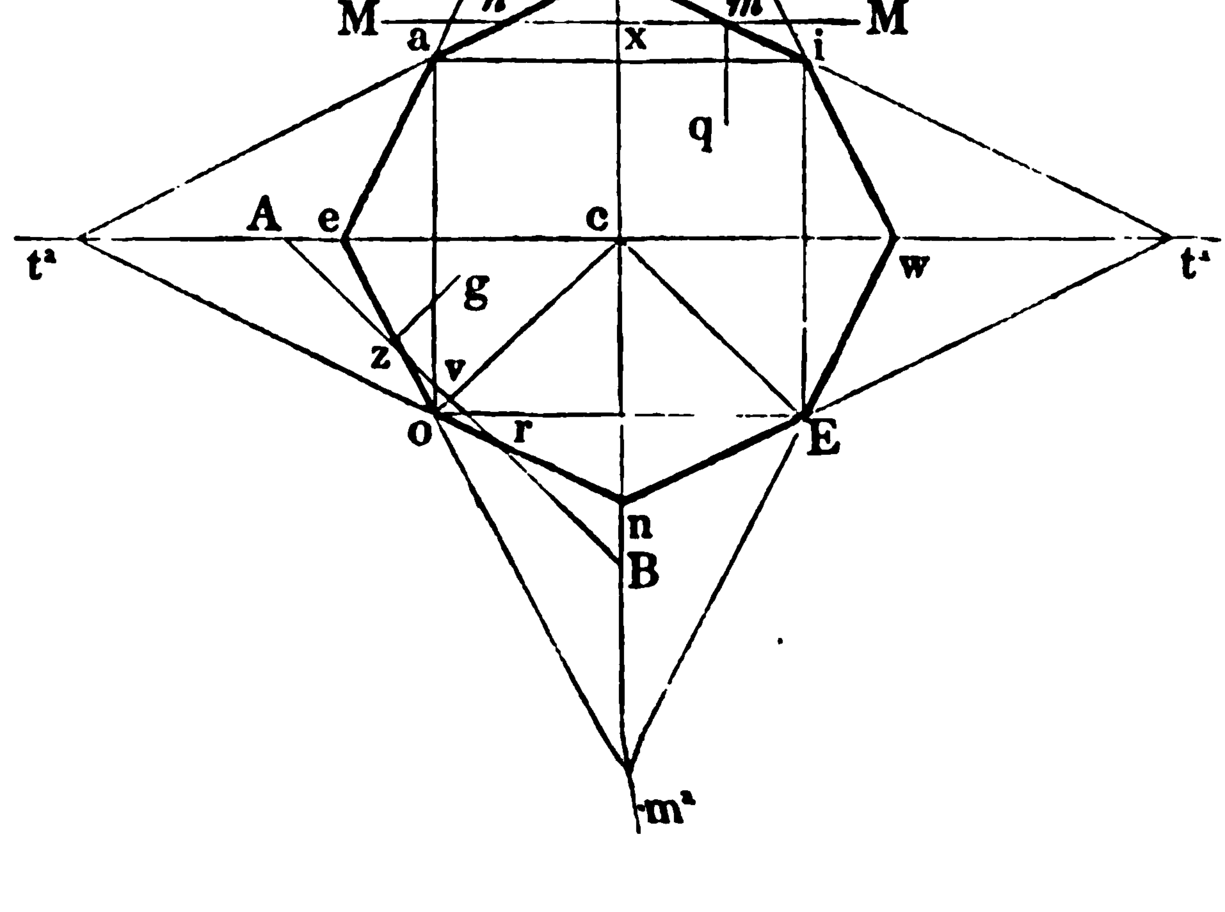
Formula 27. log sin b = log sin c + log sin B — 10.

$$\begin{aligned} \log \sin c &= 63^\circ\ 26' &= 9.9515 \\ + \log \sin B &= 65^\circ\ 54\tfrac{1}{2}' &= 9.9604 \\ \hline \log \sin b &= 54^\circ\ 44' &= 9.9119 \end{aligned}$$

Therefore, 54° 44′ is the inclination of a plane of P½MT to p^2, and 54° 44′ × 2 = 109° 28′ is the inclination of the plane Z^2nw to Z^2se over the pole Z, as given in § 146.

377. PROBLEM. *Given, the symbol* 3P½MT; *required, the angle across a short edge, s.*

a.) The equator of this combination is eight-sided, or is shaped like n e s w in the annexed figure, where the three angles of the triangle ncE



are together equal to 180°; namely, angle ncE = 45°, or half the right angle ncw; angle cnE, found by problem § 370 = 63° 26′; and angle cEn = 180° — (45° + 63° 26′) = 71° 34′.

The north meridian and east meridian of the combination 3P½MT, are exactly like the equator. Compare the octagon in this figure with the three principal sections of Model 22.

Find, by problem § 371, the inclination of an external plane upon the plane figured nEc, which is the same angle as the inclination of an external plane to the north meridian = 65° 54½′.

Then take a solid triangle, having the pole nw, marked E in the figure, for its vertex; the plane angle nEc = 71° 34′ for side *a*; the inclination of the north-west meridian to the plane nEc = 90° for angle C; and the inclination of the external plane PM½T Zn²w to the plane nEc = 65° 54½′. for angle B. With these data we can find angle A, which is the inclination of the plane PM½T Zn²w to the north-west meridian, or half the angle across a short edge, the quantity demanded in the problem. Hence the problem for solution is: *Given, a*,B; *to find* A.

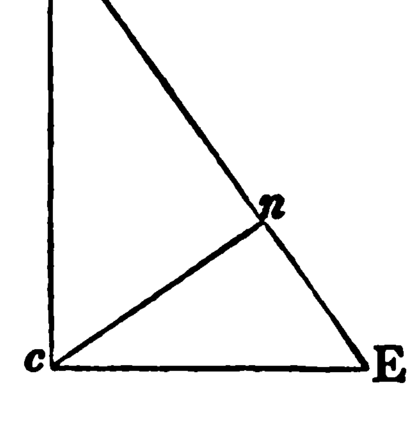
Formula 10. log cos A = log cos *a* + log sin B — 10.

$$\begin{aligned} \log\cos a &= 71°\ 34' &&= 9.5000 \\ +\ \log\sin B &= 65°\ 54\tfrac{1}{2}' &&= 9.9604 \\ \hline \log\cos A &= 73°\ 13\tfrac{1}{2}' &&= 9.4604 \end{aligned}$$

Twice this product = 73° 13½′ × 2 = 146° 27′ is the angle across a short edge of Model 22, 3P½MT.

b.) *Another method of solving this problem.*

The inclination of the Znw normal to the axis p², § 349, is 54° 44′. The inclination of the plane P½MT Z²nw to the axis p², § 376, is also 54° 44′. Therefore, the inclination of the plane P½MT Z²nw to the normal Znw, or the angle Znc in the annexed figure, is 180° — (54° 44′ + 54° 44′) = 70° 32′.



Take a solid triangle such as is described in § 373, with the pole Znw for its vertex; the angle 70° 32′, or the inclination of a plane to Znw, for side *b*; an interior vertical edge = 60°, for angle A; the inclination of an external plane to a section through its shorter diagonal = 90°, for angle C; and with these data, find angle B, which is half the angle across a short edge proceeding from the pole Znw, and half the quantity demanded in the problem. You have, therefore,

Given, A,*b*; *to find* B.

Formula 8. log cos B = log cos *b* + log sin A — 10.

$$\begin{aligned} \log\cos b &= 70°\ 32' &&= 9.5228 \\ +\ \log\sin A &= 60° &&= 9.9375 \\ \hline \log\cos B &= 73°\ 13\tfrac{1}{2}' &&= 9.4603 \end{aligned}$$

Twice this product, or $73° 13\frac{1}{2}' \times 2 = 146° 27'$, is the required angle across a short edge of 3P½MT, Model 22.

378. **Problem.** *Given, the symbol* 3P⅓MT; *required, the angle across a short edge, s.* The answer is given in § 146. The reader may work the problem according to either of the methods described in § 377.

379. **Problem.** *To find the angle across a long edge of* Model 22, 3P½MT, *when the angle across a short edge is given* $= 73° 13\frac{1}{2}'$.

a.) *First Method.*—Find by problem § 373, equation *b.*), the inclination of the plane Z³nw to the Znw normal. Let this be 70° 32′. The inclination of the same plane to the axis p^{s} is $180° - (70° 32' + 54° 44') = 54° 44'$. See § 377, *b.*)

Take a solid triangle consisting of half the Znw octant of Model 22, with the pole Z for its vertex, and the edge where the plane Z³nw meets the north-west meridian for its right angle = C. Then, angle A is the vertical edge where the north meridian cuts the north-west meridian, which angle is therefore 45°; side *b* is the inclination of the plane Z³nw to the axis p^{s} = 54° 44′; and angle B is the inclination of the plane Z³nw to the north meridian, or half the desired angle across a long edge of Model 22. You have, therefore, this problem to solve:

Given, A, *b*; *to find* B.

Formula 8. $\log \cos B = \log \cos b + \log \sin A - 10.$

$$\begin{aligned} \log \cos b &= 54° 44' = 9.7615 \\ + \log \sin A &= 45° \quad\;\; = 9.8495 \\ \hline \log \cos B &= 65° 54' = 9.6110 \end{aligned}$$

Twice this product, or $65° 54' \times 2 = 131° 48'$, is the angle across a long edge of the model. This product is, however, ½′ too little, as the true doubled angle is 131° 49′. This error is occasioned by the brevity of the logarithmic numbers employed in these calculations, and by want of attention to divisions smaller than minutes.

b.) *Second Method.*—Find by problem § 373, equation *c.*), the inclination of a short edge of Model 22 to the Znw normal. Let this be 79° 58½′. The inclination of the Znw normal to the equator is 35° 16′, § 349. Therefore, the inclination of the equator to the short edge of the model which connects the poles Znw and nw is $180° - (79° 58\frac{1}{2}' + 35° 16') = 64° 45\frac{1}{2}'$.

Take now the solid triangle described in equation *a.*), § 377, with the pole nw for its vertex, and angle C = 90° as there described; side *a* found as above = 64° 45½′; and angle B, given in the problem (½ angle across *s*) = 73° 13½′. With these data, find angle A, which is half the angle across a long edge of the model.

Formula 10. log cos A = log cos a + log sin B — 10.

$$\begin{aligned} \log\cos a &= 64^\circ\ 45\tfrac{1}{2}' = 9.6299 \\ +\log\sin B &= 73^\circ\ 13\tfrac{1}{2}' = 9.9811 \\ \hline \log\cos A &= 65^\circ\ 54' \quad = 9.6110 \end{aligned}$$

This product is the same as that given by the first method of calculation.

c.) *Third Method.*—The same as *b*, and with the same solid triangle, but with a change in the quantities. Take now B = 73° 13½′ as given in the problem, and b = 71° 34′, as determined in § 377, equation *a.*) With these data, find A by

Formula 22. log sin A = log cos B + 10 — log cos b.

$$\begin{aligned} 10 + \log\cos B &= 73^\circ\ 13\tfrac{1}{2}' = 19.4603 \\ -\log\cos b &= 71^\circ\ 34' \quad = \ \ 9.5000 \\ \hline \log\sin A &= 65^\circ\ 54' \quad = \ \ 9.9603 \end{aligned}$$

380. PROBLEM. *With the information contained in* §§ 369—379, *to determine all the angles of the equator and of the four meridians of* Model 22.

a.) The equator, the east meridian, and the north meridian are all alike, so that it will only be necessary to examine the equator. Turn to the figure in page 170. The equator is represented by the thick lines in that figure. The four angles marked n e s w are all alike, and the four angles marked a i E o are all alike. The whole angles are together equal to 1080°, see § 82. It is proved in § 370, that half the angle marked n in the figure is 63° 26′. Therefore, n is 126° 52′, and the four angles marked n e s w are together equal to 126° 52′ × 4 = 507° 28′. Deducting this sum from 1080°, we have 572° 32′ for the value of the four angles marked a i E o. Dividing 572° 32′ by 4, we have 143° 8′ for the value of each of the angles last mentioned. See § 79. In problem § 377, equation *a.*), we found the value of half the angle marked E in the figure to be 71° 34′, and since 71° 34′ × 2 = 143° 8′, we have in that determination a proof of the correctness of the present calculation.

b.) The north-east and north-west meridians are alike, so that we need only calculate the angles of one of them, namely, the north-west. We have here eight angles in all, but there are angles of three different kinds. Examine the Model. At poles Z and N, the angle is twice the inclination of a plane to axis p^4. Therefore, § 376, 109° 28′. At poles nw and se, the angle is twice the inclination of a short edge of the model to the equator. Therefore, § 379, *b.*), 129° 31′. At poles Znw, Zse, Nnw, and Nse, the angle is the inclination of a plane to a short edge over the tripolar normal. Therefore, § 373, *b.*) and *c.*), = 70° 32′ + 79° 58½′ = 150° 30½′. If these angles are correct, their aggregate sum must be 1080°.

Proof.

$$\begin{array}{lcl} 109^\circ\ 28' \times 2 &=& 218^\circ\ 56' \\ 129^\circ\ 31' \times 2 &=& 259^\circ\ \ 2' \\ 150^\circ\ 30\tfrac{1}{2}' \times 4 &=& 602^\circ\ \ 2' \\ \hline & & 1080^\circ\ 00' \end{array}$$

381. COMBINATIONS OF THE ICOSITESSARAHEDRON WITH THE RHOMBIC DODECAHEDRON.

MT. PM, PT, 3p½mt.

mt. pm, pt, 3P½MT. Model 69.

These combinations are incomplete prisms with complete pyramids. The minerals that are found in these forms are quoted at page 112, Part II. in Class 4, Order 4, Genus 1.

Analysis.

The diagonals of the faces of 3P½MT, which connect the two dissimilar angles, have the same positions as the edges of the rhombic dodecahedron, MT. PM, PT. Therefore, when the two combinations occur together, the edges of the latter combination are replaced by the planes of the former, as shown on Model 69.

As the angle across an edge of MT. PM, PT is 120°, § 355, the inclination of a plane of 3P½MT upon any plane of MT. PM, PT, must be $= \frac{120^\circ}{2} + 90^\circ = 150^\circ$.

382. COMBINATIONS OF THE ICOSITESSARAHEDRON WITH THE CUBE.

Model 39. P,M,T. 3p½mt. This combination is a complete prism with an incomplete pyramid. Minerals, Part II., page 105.

p,m,t. 3P½MT. Similar to Model 22, with the solid angles at Z N n e s w truncated by p,m,t. Minerals, Part II., page 108.

Analysis.

The inclination of a plane of P,M,T to an adjoining plane of 3P½MT is

$$54^\circ\ 44' + 90^\circ = 144^\circ\ 44',$$

in which 54° 44′ is the inclination of a plane of P½MT to p[a], as found by problem § 376.

383. COMBINATION OF 3P½MT WITH PMT AND P,M,T.

P,M,T. PMT, 3p½mt. Minerals, Part II., page 108.

Analysis.

The diagonals of the planes of Model 22, which connect the poles Zn Zw and nw, have the same position as the edges of the combination P,M,T, PMT, Model 29. Therefore, the planes of 3P½MT truncate the edges of that combination, when all the three combinations occur upon the same solid.

The inclination of a plane of PMT to an adjoining plane of 3P½MT is

$$70^\circ\ 32' + 90^\circ = 160^\circ\ 32',$$

in which 70° 32′ is the inclination of a plane of 3P½MT to the Znw *normal, § 373, b.)*

384. COMBINATIONS OF $3P\frac{1}{3}MT$ WITH OTHER FORMS.

The combination $3P\frac{1}{3}MT$ occurs very frequently subordinate, rarely predominant, and scarcely ever in an isolated condition. ROSE quotes the following as its most characteristic combinations:

P,M,T. $3p\frac{1}{3}mt$.,......................	Minerals,	Part II.,	page	105.
MT. PM, PT, $3p\frac{1}{3}mt$	—	—	—	111.
PMT, $3p\frac{1}{3}mt$	—	—	—	102.
pmt, $3P\frac{1}{3}MT$	—	—	—	102.
MT. PM, PT, pmt, $3p\frac{1}{3}mt$	—	—	—	112.
MT. PM, PT, PMT, $3p\frac{1}{3}mt$...........	—	—	—	111.
P,M,T, MT. PM, PT, $3p\frac{1}{3}mt$	—	—	—	105.

These combinations may be all investigated according to the methods described in §§ 369—383. It is therefore unnecessary to give the details of their analysis.

5. THE TRIAKISOCTAHEDRON, P_+MT, PM_+T, PMT_+: or $3P_+MT$.

Varieties of this combination:

$P\frac{3}{2}MT$, $PM\frac{3}{2}T$, $PMT\frac{3}{2}$: or $3P\frac{3}{2}MT$.
P_2MT, PM_2T, PMT_2: or $3P_2MT$.
P_3MT, PM_3T, PMT_3: or $3P_3MT$.
Model 17 is $3P_2MT$.

385. This combination is described in §§ 147—160. ROSE's symbol for it is $(a : a : 2a)$. The triakisoctahedron is a complete pyramid with a square equator. The Minerals which occur in this shape are described at page 101, Part II., in Class 2, Order 1, Genus 1. There are three known varieties of the combination, namely, $3P\frac{3}{2}MT$, $3P_2MT$, and $3P_3MT$, most of which commonly occur subordinately, and rarely either predominant or isolated. This combination is that which, in the language of the older crystallographers, is said to *bevel the edges* of the octahedron.

There are two kinds of edges on this combination, namely, *long edges* which connect the poles of p^2 m^2 t^2, and *short edges*, which meet, three at each tripolar normal, and four at each unipolar normal.

The equator, the north meridian, and the east meridian, of this combination, are all squares. Therefore, the long edges incline to p^2, m^2, and t^2, at an angle of 45°.

386. PROBLEM. *Given,* Model 17, $3P_2MT$, *with an angle across a long edge* = 141° 3′. *Required, the angle across a short edge.*

a.) Suppose Model 17 to be divided into eight portions by the four meridians, or by vertical sections passing through the eight edges that meet at the pole Z. Take the octant which contains the plane PMT_+ Z^2n^2w for a solid triangle, with the pole Z for its vertex. This plane is stamped with P and M on the model. You have, here, an oblique-angled solid triangle, where C is half the angle demanded across a short edge; B is *an angle of 45°* formed by the meeting of the north meridian

with the north-west meridian at the axis p^a; A is angle of 70° 31½′, being half the angle across a long edge; and c is a side of 45°, being the Zn quadrant of the north meridian, or the inclination of the Zn long edge of the model to p^a. The problem is, therefore, *Given*, A = 70° 31½′, B = 45°, c = 45°; *required*, C, and the Formula which answers to it is No. 41 with No. 39, as explained in § 328; but the problem can also be solved by Formula 42, which I shall employ in preference, for the sake of varying the examples:

Formula 42. $\begin{cases} \log \cot x = \log \tan A + \log \cos c - 10. \\ \log \cos C = \log \cos A + \log \sin (B - x) - \log \sin x. \end{cases}$

First Equation:

$$\begin{array}{rcl} \log \tan A = 70^\circ\, 31\tfrac{1}{2}' & = & 10.4515 \\ + \log \cos c = 45^\circ & = & 9.8495 \\ \hline \log \cot x = 26^\circ\, 34' & = & 10.3010 \end{array}$$

Second Equation:

$$\begin{array}{rcl} \log \cos A = 70^\circ\, 31\tfrac{1}{2}' & = & 9.5230 \\ B = 45^\circ & & \\ x = 26^\circ\, 34' & & \\ \hline + \log \sin (B - x) = 18^\circ\, 26' & = & 9.5000 \\ \hline & & 19.0230 \\ - \log \sin x = 26^\circ\, 34' & = & 9.6505 \\ \hline \log \cos C = 76^\circ\, 22' & = & 9.3725 \end{array}$$

Twice this product, or 76° 22′ × 2 = 152° 44′, is the angle across a short edge of Model 17. See § 160.

b.) *Another Method.*—Half the difference between the angle across a long edge and 109° 28′, is the complement of the inclination of a plane to the Znw normal of 3 $P_{+}MT$. Find this first, and then calculate the angle across a short edge, by the method described in § 359. That is to say, from the angle across a long edge, take the octahedral angle 109° 28′. Divide the residue by 2. The complement of this last product is the inclination of a plane of $3P_{+}MT$ to the Znw normal.

Illustration.—Model 17 resembles a regular octahedron with a flat three-faced pyramid fixed upon each plane. Consequently, the angle across a long edge of this model, includes the angle across the edge of the regular octahedron, together with the angles at which two of these flat pyramids incline upon two different faces of the included octahedron. Therefore, the angle across a long edge of the model, minus 109° 28′, or the octahedron edge, and also minus half the residue, is equal to the inclination of one of the flat pyramid faces upon one of the faces of the included octahedron. Then, again, the Znw normal is perpendicular to a plane of the regular octahedron, so that the complement of the inclina-

tion of the pyramidal plane upon the face of the octahedron, is the inclination of the same plane upon the Znw normal.

Put the angle across a long edge = 141° 3′. Then 141° 3′ — 109° 28′ = 31° 35′. And $\frac{31° 35'}{2}$ = 15° 47½′. Complement of 15° 47½′ = 74° 12½′. This is the inclination of a plane of Model 17 upon the Znw normal.

Now, form a solid triangle, containing a *sixth* of Model 17, with the pole Znw for its vertex, in the manner described in § 359. The *known* parts of the solid triangle are then, angle C = 90°, formed by the inclination of a plane of the model to a section perpendicular to that plane; angle A = 60°, formed where two sections meet at the Znw normal; and side *b*, the inclination of a plane of the model to the Znw normal, already found to be 74° 12½′. With these data, you have to find angle B, which is half the desired angle across a short edge of the model.

Given, A = 60°; *b* = 74° 12½′; *to find*, B.

Formula 8. log cos B = log cos *b* + log sin A — 10.

$$\begin{array}{rll} \log \cos b &= 74° \ 12\tfrac{1}{2}' &= 9.4348 \\ + \log \sin A &= 60° &= 9.9375 \\ \hline \log \cos B &= 76° \ 22' &= 9.3723 \end{array}$$

Twice this product, or 76° 22′ × 2 = 152° 44′, is the desired angle across a short edge of the model. See § 160.

c.) *Another example of Method b.*—Put the angle across a long edge = 129° 31′, which agrees with the combination 3P$\frac{3}{2}$MT. To find the angle across a short edge.

129° 31′ — 109° 28′ = 20° 3′. The half of it = 10° 1½′. Its complement is 79° 58½′, which is the inclination of a plane of 3P$\frac{3}{2}$MT upon the Znw normal. Now, form a solid triangle as directed in *b.*), and take the same Formula, but change the value of *b*.

Given, A = 60°; *b* = 79° 58½′; *to find*, B.

Formula 8. log cos B = log cos *b* + log sin A — 10.

$$\begin{array}{rll} \log \cos b &= 79° \ 58\tfrac{1}{2}' &= 9.2407 \\ + \log \sin A &= 60° &= 9.9375 \\ \hline \log \cos B &= 81° \ 19\tfrac{3}{4}' &= 9.1782 \end{array}$$

Twice this product, or 81° 19¾′ × 2 = 162° 39½′ is the angle across a short edge of the combination 3P$\frac{3}{2}$MT. See § 160.

d.) The reader may work, according to either of these methods, the following problem: Given, $3P_3MT$, with the angle across a long edge = 153° 28′; required, the angle across a short edge. The answer is given in § 160.

387. PROBLEM. *Given*, Model 17, $3P_3MT$, *with the angle across a short edge; to find the angle across a long edge.*

a.) Form a solid triangle, as described in § 386, *b.*), with the pole Znw for its vertex, and *calculate* the inclination of the planes of Model 17 to

the Znw normal. Then double the complement of the discovered angle, and add to it 109° 28′. The product is the desired angle across a long edge of the model.

b.) *Illustration.*—Put the angle across a short edge = 152° 44′. The known parts of the solid triangle will then be as follows: angle C = 90°; angle A = 60°; angle B = $\frac{152° 44'}{2}$ = 76° 22′. And the part required, namely, the inclination of a plane to the Znw normal, will be side *b*. The problem is, therefore, *Given*, A, B; *to find*, *b*.

Formula 5. log cos *b* = log cos B + 10 — log sin A.

$$\begin{aligned} 10 + \log\cos B &= 76°\ 22' &&= 19.3724 \\ -\log\sin A &= 60° &&= 9.9375 \\ \hline \log\cos b &= 74°\ 12\tfrac{1}{2}' &&= 9.4349 \end{aligned}$$

The complement of 74° 12½′ is 15° 47½′, twice which is 31° 35′, and this added to 109° 28′ produces 141° 3′, which is the angle across a long edge of Model 17. This problem is the reverse of that given in § 386, and the calculation is therefore also reversed.

c.) Put the angle across a short edge = 142° 8′, as it is found to be in the combination, $3P_3MT$. This gives the equation:

log cos *b* = log cos 71° 4′ + 10 — log sin 60°.

$$\begin{aligned} 10 + \log\cos 71°\ 4' &= 19.5112 \\ -\log\sin 60° &= 9.9375 \\ \hline \log\cos b = 68° &= 9.5737 \end{aligned}$$

Complement of 68° = 22°. Twice 22° = 44°. This, added to 109° 28′ is 153° 28′. In ROSE's *Krystallographie*, page 27, this angle is stated 158° 28′, in mistake for 153° 28′.

d.) Put the angle across a short edge = 162° 39½′, which is the angle of the combination $3P\frac{3}{2}MT$. Find the angle across a long edge. I leave this problem for the reader to work: the answer is given in §§ 160 and 386 *c*.)

388. PROBLEM. *Given*, Model 17, $3P_3MT$, *with the angle across a long edge, to find the inclination of a short edge to the* Znw *normal.*

Proceed as in § 386, *b*.) to form a solid triangle with pole Znw for its vertex. Then observe that you have this problem to solve. *Given*, A,B; *to find c*, Formula 6; or else, *Given*, A, *b*; *to find*, *c*, Formula 9; or else, *Given*, B, *b*; *to find*, *c*, Formula 24. Any one of these Formulæ will answer the purpose. I shall take

Formula 6. log cos *c* = log cot A + log cot B — 10.

$$\begin{aligned} \log\cot A &= 60° &&= 9.7614 \\ +\log\cot B &= 76°\ 22' &&= 9.3848 \\ \hline \log\cos c &= 81°\ 57' &&= 9.1462 \end{aligned}$$

Therefore, 81° 57′ is the inclination of a short edge to the Znw normal.

Control over the accuracy of this calculation:—This control is effected on the principle stated in § 362, that the cotangent of the inclination of a plane of a three-faced pyramid to the vertical axis, is twice the cotangent of the inclination of an edge of the pyramid to the same axis. Now, we find by § 386, *b.*), that the inclination of *a plane* to the Znw normal of Model 17, which is the vertical axis in this case, is 74° 12½′. The cot of this angle is .2828. Half this sum is .1414, which is the cot of 81° 57′, proved, by § 388, to be the inclination of *an edge* to the given vertical axis.

389. PROBLEM. *Given, a triakisoctahedron,* Model 17, $3P_{+}MT$; *required, the plane angles of its external faces.*

a.) Let the symbol be $3P_2MT$.

This problem is solved by a solid triangle having the pole Znw for its vertex. There must, therefore, *be given,* the angle across one of the edges, or the inclination of a plane or of an edge to the Znw normal. If nothing is given, the angle across a short edge is taken with the goniometer. Put this angle = 152° 44′, as found by problem § 386, *a* or *b.*)

Form a solid triangle, as described in § 386, *b.*), having the following known parts, namely: angle C = 90°; angle A = 60°; and angle B = 76° 22′. The part of the triangle to be determined is side *a*, which is half the plane angle of one of the faces of Model 17 at the pole Znw. This problem can be solved by

Formula 4. $\log \cos a = \log \cos A + 10 - \log \sin B.$

$$\begin{array}{rll} 10 + \log \cos A = 60^\circ & = & 19.6990 \\ -\log \sin B = 76^\circ\ 22' & = & 9.9876 \\ \hline \log \cos a = 59^\circ\ 2' & = & 9.7114 \end{array}$$

Twice this product, or 59° 2′ × 2 = 118° 4′, is the obtuse plane angle of the faces of Model 17. The two acute angles are 180° — 118° 4′ = 61° 56′, or each separately is 30° 58′.

b.) Let the symbol be $3P_3MT$.

c.) Let the symbol be $3P\frac{3}{2}MT$.

First find the angle across a short edge, as described in preceding paragraphs, and then finish the calculation according to the above model.

390. PROBLEM. *Given, the symbol* $3P_2MT$; *required, a.*) *the angle across a long edge, b.*) *the angle across a short edge, c.*) *the inclination of a plane to the* Znw *normal, d.*) *the inclination of a short edge to the* Znw *normal, e.*) *the obtuse plane angle of a face, and f.*) *the acute plane angle of a face.*

The symbol $3P_2MT$ signifies the three forms P_2MT, PM_2T, PMT_2. It is the combination represented by Model 17. The crystals denoted by the symbols $3P_3MT$ and $3P\frac{3}{2}MT$, have the same number of faces and the same general aspect as this model, but differ among themselves in all their angles.

e and f.) The plane angles of a face are found by the problem in § 389, when the inclination of a short edge to the Znw normal is known.

d.) The inclination of a short edge to the Znw normal is found by the problem in § 388, when you know the inclination of a plane to the Znw normal.

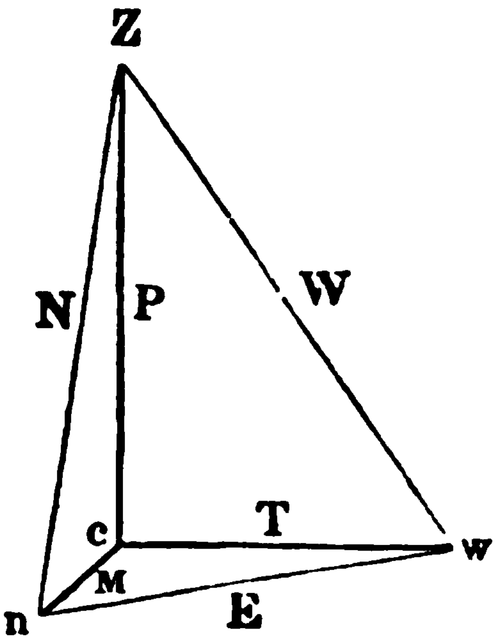
c.) The inclination of a plane to the Znw normal is found by the problem in § 386 *b.*), when you know the angle across a long edge.

b.) The angle across a short edge is found by the problem in § 386, when you know the angle across a long edge.

a.) Therefore, all the required angles can be found, if you first find the angle across a long edge, by the method described in the next paragraph.

391. *To find the angle across a long edge of* $3P_2MT$, *when the symbol alone is given.*

The long edges of the combination are those which bound the equator and the north and east meridians. The planes whose longest edges meet at the equator are those of the form P_2MT. The angle across a long edge may therefore be said to be the inclination of plane Zn^2w^2 to plane Nn^2w^2 of the form P_2MT. Let the annexed figure represent the Znw octant of this form. Then P,M,T, are the axes $p_1^2m_1^1t_1^1$ of P_2MT. The line E is the nw edge of the equator, and the inclination of the plane Zn^2w^2 to the plane ncw across the edge E, is *half the angle across a long edge of the model*, demanded in the problem. If you take this octant as a solid triangle with pole n for its vertex, then you have the angle across the edge M = 90° = angle C; the plane angle cnw = 45° = side *a*; and the plane angle Znc, which answers to tan 2.0000 = 63° 26′ = side *b*. With these data, you have to find angle B, (opposite side *b*), which is the angle across the edge E. Hence the problem to be solved is, *Given, a, b; to find,* B.



Formula 14. log tan B = log tan *b* + 10 — log sin *a*.

$$\begin{aligned} 10 + \log \tan b = 63°\ 26' &= 20.3010 \\ -\log \sin a = 45° &= \underline{\ 9.8495} \\ \log \tan B = 70°\ 31\tfrac{1}{2}' &= 10.4515 \end{aligned}$$

Twice this product, or 70° 31½′ × 2 = 141° 3′, is the angle across a long edge of the combination $3P_2MT$.

The angle across a long edge of the combination $3P\frac{3}{2}MT$, or $3P_3MT$, can be found in the same manner.

392. PROBLEM. *Given,* Model 17, *with the symbol* P_+MT, PM_+T, PMT_+; *required, the value of the index* $_+$.

a.) Let the angle across a long edge be measured and found = 141° 3′. Observe, that the combination consists of three acute square-based octahedrons, one resting upon the equator, one upon the north meridian, and one upon the east meridian. In every case, the long edges divide two planes belonging to the same octahedron.

Begin with the octahedron whose planes are attached to the equator. This is P_+MT. Half the given angle is 70° 31½′. This angle expresses the inclination of a plane to the equator. The equator is square, therefore the inclination of the nw edge of the equator to the axis m^a is 45°.

Form a right-angled solid triangle with pole n for its vertex, and in which angle C = 90°, is the inclination of the equator to the north meridian; angle A = 70° 31½′, the given inclination of the Znw plane to the equator; and side *b* = 45°, the inclination of the equator to the axis m^a. With these data, you can find side *a*, which is the inclination of the Zn edge of the form P_+MT to the axis m^a, no part of which edge is visible upon the model, because it is replaced by the form PMT_+. This problem, *Given*, A, *b*; *to find*, *a*, can be solved as follows:

Formula 7. log tan *a* = log tan A + log sin *b* — 10.

$$\begin{array}{rll} \log\tan A = 70^\circ\,31\tfrac{1}{2}' & = & 10.4515 \\ +\ \log\sin b = 45^\circ & = & \underline{\ 9.8495} \\ \log\tan a = 63^\circ\,26' & = & 10.3010 \end{array}$$

This product, 63° 26′, is the inclination of the Zn edge of P_+MT to m^a at the pole n. The tangent of 63° 26′ is 2.0000, which is the length of the axis p^a when m^a is 1.0. Therefore, the value of the index $_+$ in P_+MT is 2, and the symbol is P_2MT.

In the same manner, it is easy to prove, that the planes which surround the north meridian require the symbol PMT_2, or $P\tfrac{1}{2}M\tfrac{1}{2}T$, and that the planes which surround the east meridian require the symbol PM_2T.

b.) Let the angle across the long edge = 129° 31′.

c.) Let the angle across the long edge = 153° 28′.

These two problems are to be treated in the same manner as problem *a.*) The answers are

$$P\tfrac{3}{2}MT,\ PM\tfrac{3}{2}T,\ PMT\tfrac{3}{2}:\ \text{or}\ 3P\tfrac{3}{2}MT.$$
$$P_3MT,\ PM_3T,\ PMT_3:\ \text{or}\ 3P_3MT.$$

393. COMBINATIONS OF THE TRIAKISOCTAHEDRON WITH OTHER FORMS.

MT. PM, PT, 3P½MT, 3p⅔mt. This combination somewhat resembles Model 69, only the rhombic dodecahedron is predominant, and the icositessarahedron subordinate. The planes of the triakisoctahedron replace the remaining edges of the icositessarahedron which meet at the tripolar normals. Minerals, Part II. page 112.

PMT, $3p_2mt$. The *bevelled* octahedron. Minerals, Part II. page 100.

MT. PM, PT, PMT, $3p_3mt$. The combination represented by Model 65, with the addition of three narrow planes around each face of the regular octahedron. Minerals, Part II. page 110.

P,M,T, PMT, $3p_2mt$, $3p_3mt$. The cube, with each corner replaced by the octahedron, and also by six other planes, inclining two on each edge of the cube. Minerals, Part II. page 105.

394. *Angles across the edges of the above combinations.*

The inclination of $3P_+MT$ upon MT. PM, PT, is half the angle across a long edge of $3P_+MT$ added to 90°.

The inclination of $3P_+MT$ upon PMT, is half the inclination of a plane to a tripolar normal of $3P_+MT$, added to 90°.

6. THE TETRAKISHEXAHEDRON, M_-T, M_+T. P_-M, P_+M, P_-T, P_+T.

Varieties of this Combination:

$M\frac{2}{3}T$, $M\frac{3}{2}T$. $P\frac{2}{3}M$, $P\frac{3}{2}M$, $P\frac{2}{3}T$, $P\frac{3}{2}T$.
$M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$.
$M\frac{2}{5}T$, $M\frac{5}{2}T$. $P\frac{2}{5}M$, $P\frac{5}{2}M$, $P\frac{2}{5}T$, $P\frac{5}{2}T$.
$M\frac{1}{3}T$, $M\frac{3}{1}T$. $P\frac{1}{3}M$, $P\frac{3}{1}M$, $P\frac{1}{3}T$, $P\frac{3}{1}T$.
$M\frac{1}{5}T$, $M\frac{5}{1}T$. $P\frac{1}{5}M$, $P\frac{5}{1}M$, $P\frac{1}{5}T$, $P\frac{5}{1}T$.
$M\frac{1}{10}T$, $M\frac{10}{1}T$. $P\frac{1}{10}M$, $P\frac{10}{1}M$, $P\frac{1}{10}T$, $P\frac{10}{1}T$.

Model 68 is MT_2, M_2T. PM_2, $'P_2M$, PT_2, $''P_2T$:
or $M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$.

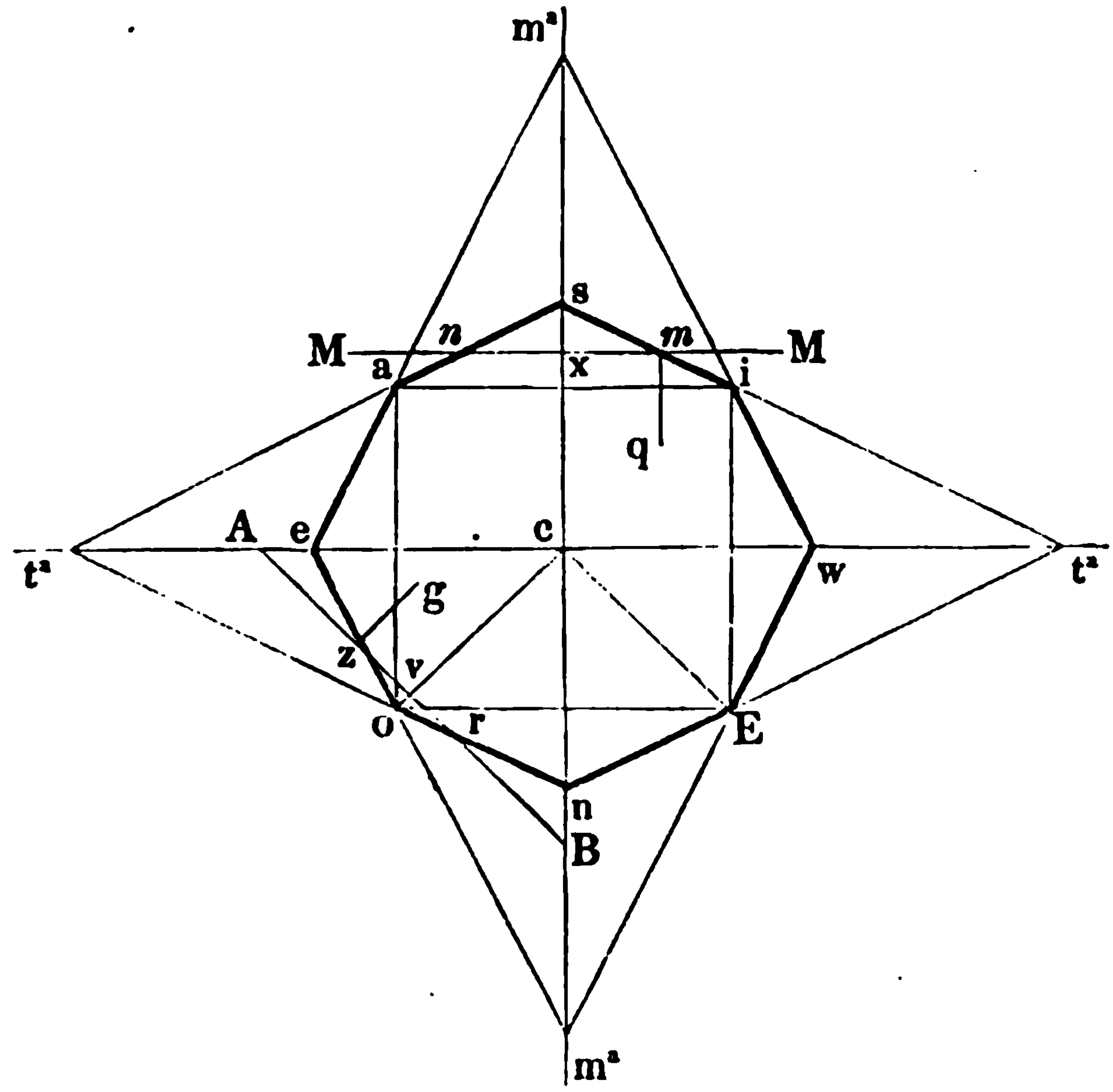
395. This combination is described in §§ 111—113. ROSE's symbol for Model 68 is $(2\,a : a : \infty a)$. The tetrakishexahedron resembles a cube that has a four-sided square-based pyramid upon each of its planes. It has two kinds of edges, namely, 12 long edges which connect the tripolar normals, and pass through the poles of the bipolar normals, and 24 short edges which connect the unipolar with the tripolar normals. It is an incomplete prism with a complete pyramid, and has a rhombic equator. Hence, the minerals which it represents are contained in Class 4, Order 3, Genus 1, Part II., page 112. Of the six varieties enumerated above, those having the indices $\frac{1}{2}$ and $\frac{1}{3}$ occur in an isolated state, but the rest only subordinately.

396. PROBLEM. *Given,* $M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$, Model 68, *with the angle across a long edge; required, the angle across a short edge.*

Let the angle across a long edge be 143° 8′. In this case, the equator of the combination resembles the octagon n o e a s i w E drawn in the following diagram; so that the equator of Model 22 and 68 are similar.

Upon comparing this diagram with Model 68, we perceive that the thin straight lines between a i o and E in the diagram, have the same positions as the long edges on the model, which pass through the terminations of the bipolar normals and connect the tripolar normals. They

are, therefore, the edges of the cube, upon the planes of which the six flat square-based pyramids of the tetrakishexahedron are assumed to be superimposed. Now, the angle across a long edge, say, for example, the angle n E w in the diagram is equal to the right angle o E i, plus



the angle o E n, plus the angle i E w. Therefore, if the angle n E w is 143° 8′, the angle o E n or i E w is $\frac{143^\circ\ 8' - 90^\circ}{2} = 26^\circ\ 34'$. This angle is the complement of the angle c n E in the diagram, which is consequently 63° 26′, and which represents the inclination of the planes of Model 68 to the unipolar normals, or to the axes $p^a\ m^a\ t^a$.

a.) Suppose the uppermost pyramid of Model 68 to be divided by the four meridians into eight sections. Take one of these as a solid triangle, with pole Z for its vertex. Then you have angle C = 90°, the edge formed by the intersection of the north or east meridian with an external plane; side a = 63° 26′, inclination of a plane to the axis p^a; and angle B = 45°, formed by the intersection of the north-west meridian with the east or north meridian. With these data, you have to find angle A, which is half the angle across a short edge of the model, or across an oblique or terminal edge of the flat pyramid.

Formula 10. log cos A = log cos a + log sin B — 10.

$$\begin{aligned} \log\cos a &= 63^\circ\ 26' = 9.6505 \\ +\ \log\sin B &= 45^\circ \quad\ \ = 9.8495 \\ \hline \log\cos A &= 71^\circ\ 34' = 9.5000 \end{aligned}$$

Twice this prodnct, or 71° 34′ × 2 = 143° 8′, is the required angle across a short *edge of Model 68.*

This model is, however, not sufficiently well made to demonstrate this fact instrumentally. I mention this circumstance, lest the reader should be puzzled by the non-agreement of the mechanical measurement with the result of the reckoning.

b.) *Another Method.*—When you know the inclination of a plane to the base of the flat pyramid, found above = 26° 34′, take a solid triangle with the pole Znw for its vertex. Then you have angle C = 90°, the inclination of the base to the north-west meridian; the above described angle of 26° 34′ for angle B; and the plane angle c E o in the diagram = 45°, for side *a*. With these data, you have to find angle A, which is half the angle across a terminal edge of the pyramid.

Formula 10. log cos A = log cos *a* + log sin B — 10.

$$\begin{aligned} \log\cos a &= 45^\circ &&= 9.8495 \\ +\ \log\sin B &= 26^\circ\ 34' &&= 9.6505 \\ \hline \log\cos A &= 71^\circ\ 34' &&= 9.5000 \end{aligned}$$

Twice 71° 34′ = 143° 8′, is the angle demanded.

397. PROBLEM. *Given*, $M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$, Model 68, ***with the angle across a short edge; required, the angle across a long edge.***

The given angle across a short edge is 143° 8′.

The operations described in the foregoing problem are here to be reversed, since we now have given the angle across a terminal edge of a square-based pyramid, and are required to determine thence the angle across the equator.

$\frac{143^\circ\ 8'}{2}$ = 71° 34′ = half the angle across the terminal edge. Call this angle A.

Take the solid triangle described in § 396 *b.*), with pole Znw for its vertex. The given parts are now A = 71° 34′, *a* = 45°, and C = 90°. With these data, you have to find B, which is the inclination of a plane of the pyramid to the base. All these quantities are fully described in the preceding paragraph, so that it would be needless to go into farther details. You have, therefore, *given*, A,*a*; *to find*, B.

Formula 2. log sin B = log cos A + 10 — log cos *a*.

$$\begin{aligned} 10 + \log\cos A &= 71^\circ\ 34' &&= 19.5000 \\ -\ \log\cos a &= 45^\circ &&= 9.8495 \\ \hline \log\sin B &= 26^\circ\ 34' &&= 9.6505 \end{aligned}$$

This product, 26° 34′, is the same as the angle o E n of the diagram in § 396. To find the angle n E w in the same diagram, which is the required angle across a long edge, as already explained in § 396, it is necessary to add to angle o E n the angles o E i and i E w, or, in other words, we must double the product of the equation and add 90°. Thus:

$$26^\circ\ 34' \times 2 = 53^\circ\ 8' + 90^\circ = 143^\circ\ 8'.$$

The angles across the short edges of $M\frac{1}{2}T, M\frac{2}{1}T. P\frac{1}{2}M, P\frac{2}{1}M, P\frac{1}{2}T, P\frac{2}{1}T$, are the same as those across the long edges. This is not the case with any other variety of the tetrakishexahedron, the angles of all of which may, however, be calculated in the same manner as those of the variety which I have chosen as an example.

398. PROBLEM. *Given*, $M\frac{1}{2}T, M\frac{2}{1}T. P\frac{1}{2}M, P\frac{2}{1}M, P\frac{1}{2}T, P\frac{2}{1}T$, Model 68, *with the angle across a short edge; required, the inclination of the planes to the axes* p^2 m^2 and t^2.

The given angle is 143° 8′.

Divide the uppermost flat square-based pyramid into eight sections, by the meridians which pass through the four terminal edges, and across the four terminal planes. Take one of the resulting *eighths* as a solid triangle with pole Z for its vertex. Then you have given, angle C = 90° = inclination of an external plane to the north meridian; angle B = 45° = intersection of the north meridian with the north-west meridian at axis p^2; and angle A = $\frac{143° 8'}{2}$ = 71° 34′ = half the angle across the terminal edge that is divided by the north-west meridian. With these data, you have to find side *a*, which is half the upper portion of the north meridian, or the inclination of a plane to the axis p^2. Therefore, you have *given*, A, B; *to find*, *a*.

Formula 4. log cos *a* = log cos A + 10 — log sin B.

$$\begin{aligned} 10 + \log\cos A &= 71°\ 34' = 19.5000 \\ -\log\sin B &= 45° \quad\ = \ \ 9.8495 \\ \hline \log\cos a &= 63°\ 26' = \ \ 9.6505 \end{aligned}$$

This product, 63° 26′, is the inclination of a plane of Model 68 to any one of the axes p^2 m^2 t^2. See § 396, equation *a*.), which is the counterpart of the present problem, and which shows the method of determining the inclination of a plane to an axis, when the given quantity is *the angle across a long edge.*

399. PROBLEM. *Given*, $M\frac{1}{2}T, M\frac{2}{1}T. P\frac{1}{2}M, P\frac{2}{1}M, P\frac{1}{2}T, P\frac{2}{1}T$, Model 68, *with the angle across a long edge* = 143° 8′*; required, the inclination of the planes to the axes* p^2 m^2 t^2.

a.) One method of solving this problem is contained in § 396, where the required angle is shown to be 63° 26′.

b.) *Another Method.*—As the equator of the model is eight-sided, all its angles must amount to 1080°, and since it has four angles of one kind (across the unipolar normals), and four of another kind (across the bipolar normals), one angle of each kind must together be equal to $\frac{1080°}{4}$ = 270°. Therefore, the angle across a unipolar normal is 270° minus the angle across a bipolar normal. Applying this principle to the present example, where the angle across a bipolar normal is given at 143° 8′, you find the angle across a unipolar normal to be 270° — 143° 8′ = 126° 52′.

The half of this angle $= \frac{126^\circ\ 52'}{2} = 63^\circ\ 26'$, is the required inclination of a plane of the crystal to an axis.

400. PROBLEM. *Given*, $M\frac{1}{1}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$, Model 68, *with the angle across a short edge; required the inclination of the short edges to the axes* p^a m^a and t^a.

The given angle across the short edge is 143° 8′.

Divide the flat square-based pyramid into four sections, by the north-east and north-west meridians, which pass through the terminal edges of the pyramid. Take one of these sections as a solid triangle, with pole Z for its vertex. The parts given are angle $C = 90°$, the intersection of the two meridians at axis p^a; angle $A = 71° 34'$, half the angle across a terminal edge; and angle B, the same. The part required is side a or side b, either of which shows the inclination of an edge to an axis. This problem, *given*, A,B; *to find*, a, can be solved by Formula 4. $\cos a = \frac{\cos A}{\sin B}$. But since A and B are similar quantities, the equation is reduced by *Formula* 104, to $\cos a = \cot A$.

$$\begin{aligned} \text{nat } \cot A &= 71^\circ\ 34' = 9.5228 \\ \text{nat } \cos a &= 70^\circ\ 32' = 9.5228 \end{aligned}$$

This product, 70° 32′, is the inclination of a short edge of Model 68 to any one of the three axes, p^a m^a or t^a. Twice the product, or $70° 32' \times 2 = 141° 4'$, is the inclination of two opposite edges across an axis.

401. PROBLEM. *Given*, $M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$, Model 68, *with the angle across a long edge* = 143° 8′; *required, the inclination of the short edges to the axes* p^a m^a t^a.

When the inclination of the short edges to the axes of this combination are to be found from *the angle across a long edge*, proceed as follows:

Take the solid triangle described in § 396, a), in which are known, $C = 90°$; $a = 63° 26'$; and $B = 45°$; and with these data, find side c, which is the inclination of a terminal edge of the pyramid to axis p^a.

Formula 12. $\log \tan c = \log \tan a + 10 - \log \cos B$

$$\begin{aligned} 10 + \log \tan a &= 63^\circ\ 26' = 20.3010 \\ -\log \cos B &= 45^\circ \quad\ \ = 9.8495 \\ \hline \log \tan c &= 70^\circ\ 32' = 10.4515 \end{aligned}$$

402. PROBLEM. *Given*, $M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$, Model 68, *with the angle across a short edge; required the plane angles of the external faces of the combination.*

The given angle across the short edge is 143° 8′.

a.) To find the obtuse plane angle at pole Z. Take the solid triangle described in § 400, and with the given angles, find side c, which is the obtuse plane angle at pole Z.

Given, $A = 71° 34'$; $B = 71° 34'$; $C = 90°$; required, c.

Formula 6. log cos c = log cot A + log cot B — 10.

$$\begin{array}{rl} \log\cot A = 71^\circ\ 34' = & 9.5228 \\ +\ \log\cot B = 71^\circ\ 34' = & 9.5228 \\ \hline \log\cos c\ = 83^\circ\ 37' = & 9.0456 \end{array}$$

b.) ***Another way to find the obtuse plane angle at pole Z.*** Take the solid triangle described in § 396, *a.*), with pole Z for its vertex, and in which the given parts are C = 90°; A = 71° 34′; and B = 45°; as there described. With these data, you have to find b, which is half the obtuse plane angle at pole Z.

Formula 5. log cos b = log cos B + 10 — log sin A.

$$\begin{array}{rl} 10 + \log\cos B = 45^\circ \quad = & 19.8495 \\ -\ \log\sin A = 71^\circ\ 34' \ = & 9{,}9771 \\ \hline \log\cos b\ = 41^\circ\ 48\tfrac{1}{2}' = & 9.8724 \end{array}$$

Twice this product = 41 48½′ × 2 = 83° 37′, is the obtuse plane angle at pole Z.

c.) *To find the acute plane angles of the faces of* Model 68. Each face is a triangle, of which one angle is found to be 83° 37′, and the other two angles are therefore necessarily together equal to 180° — 83° 37′ = 96° 23′, or each of them is equal to 48° 11½′. These results are corroborated by those of the next problem.

403. PROBLEM. *Given*, M$\tfrac{1}{1}$T, M$\tfrac{2}{1}$T, P$\tfrac{1}{2}$M, P$\tfrac{2}{1}$M, P$\tfrac{1}{2}$T, P$\tfrac{2}{1}$T, Model 68, ***with the angle across a short edge*** = 143° 8′, ***and the angle across a long edge*** = 143° 8′; ***required, the plane angles of the external faces, and the inclination of the short and long edges to the tripolar normals.***

CALCULATION OF SIX-FACED PYRAMIDS.

a.) *To find the acute plane angle of the faces at pole* Znw.

Put pole Znw, Model 68, in the place of pole Z. Suppose the Model to be divided by vertical planes that pass through the six edges which meet at pole Znw. These vertical planes will intersect one another at the Znw normal, and cut out sections of the crystal, having each an interior edge or angle of the value of 60° (= 360° ÷ 6). Take one of these sections as a solid triangle, with pole Znw for its vertex, and call the interior angle of 60° angle C. Then angle A will be half the angle across a long edge = 71° 34′, and angle B will be half the angle across a short edge = 71° 34′. In the present case, these two angles have the same value, but in general they have different values; the calculation, however, is performed in the same way, whether the value of A and B is the same or not. With angles A, B, C, given as above, you have to find side c, which is the acute plane angle of an external face and one of the angles demanded in the problem. The quantities constituting this equation form an oblique-angled solid triangle, on which account the calculation is longer than it would be if the given parts formed only a right-

angled solid triangle. It is quite unnecessary to calculate any of the parts of the tetrakishexahedron by means of oblique-angled solid triangles, because, as I have shown, it is possible to divide the six-faced pyramid out of which the oblique-angled solid triangle is taken, into a series of right-angled solid triangles. But I give the equations contained in the present problem, in order to show the reader what can be done with a six-faced pyramid by means of oblique-angled solid triangles, in cases where reduction of the given pyramid to right-angled solid triangles is impossible.

The Formula to be employed in resolving this equation is No. 37. To make the given parts suit the terms in this equation, where a instead of c is the part to be found, it is necessary to change the designation of the interior angle of 60° to A, when B will represent half the angle across a short edge, and C half the angle across a long edge, each of them being 71° 34′. The equation is then as follows:—

Formula 37. $\text{Sin } \frac{1}{2}a = \sqrt{\frac{-\cos S \cos (S-A)}{\sin B \sin C}}$, where $S = \frac{1}{2}(A+B+C)$.

$\text{Log} \sin \frac{1}{2} a =$

$\frac{1}{2} [\log \cos S + \log \cos (S - A) + 20 - (\log \sin B + \log \sin C)]$

A = 60°	S = 101° 34′	180°
B = 71° 34′	A = 60°	S = 101° 34′
C = 71° 34′	(S — A) = 41° 34′	Suppt. of S = 78° 26′
2)203° 8′		
S = 101° 34′		

log cos S =	78° 26′	=	— 9.3021
+ log cos (S — A) =	41° 34′	=	9.8740
+ 20		=	39.1761
— { log sin B = 71° 34′ = 9.9771; + log sin C = 71° 34′ = 9.9771 }		=	19.9542
			2)19.2219
sin ½ a =	24° 5¾′	=	9.61095

Twice this product, or 24° 5¾′ × 2 = 48° 11½′, is the acute plane angle required, which result agrees with that found by problem § 402, c). This angle being one of two similar acute angles on each face of the model, they are together equal to 96° 23′, and the third or obtuse plane angle of each face is 180° — 96° 23′ = 83° 37′, as found in § 402, a).

As this is one of the cases in which the calculation gives an ambiguous result, it is proper to remind the reader of the cautions that were given on this score in § 330. The first product obtained in the calculation is S = 101° 34′. This angle being greater than 90° is negative. You *take therefore* the cosine of its supplement, and prefix the sign — to it. *The next product* is (S — A) = 41° 34′, which being less than 90° is po-

sitive. The multiplication of the negative and positive quantities together, produces a negative result; but the sign of this result must be changed to positive, because the Formula employed contains the negative sign. Consequently, the product 19.1761 is a positive quantity, and as the quantities afterwards employed in the calculation are all positive, so the final result is positive.

b.) To find the inclination of the short edges to the tripolar normals. Take the same solid triangle as in equation *a*), with the given parts, C = 60°; B = 71° 34′; and A = 71° 34′. Then, if B is taken as half the angle across a short edge, the part to find is *a*. The Formula to be used is No. 37, as before.

$$\begin{aligned} S &= 101^\circ\ 34' \\ A &= \ \ 71^\circ\ 34' \\ \hline (S - A) &= \ \ 30^\circ \end{aligned}$$

$$\begin{array}{rcrcr} \log\cos S = & 78^\circ\ 26' & = & - & 9.3021 \\ + \log\cos(S - A) = & 30^\circ & = & & 9.9375 \\ \hline & + 20 & = & & 39.2396 \\ - \left\{ \begin{array}{l} \log\sin B = 71^\circ\ 34' = 9.9771 \\ + \log\sin C = 60^\circ \quad = 9.9375 \end{array} \right\} & & = & & 19.9146 \\ \hline & & & & 2)19.3250 \\ \hline \sin \tfrac{1}{2}\, a = & 27^\circ\ 22' & = & & 9.6625 \end{array}$$

Twice this product, or 54° 44′, is the inclination of a short edge of the model to the Znw normal.

c.) To find the inclination of the long edges to the tripolar normals. Take the same solid triangle as in equation *b*), but put B equal to half the angle across a long edge, and A equal to half the angle across a short edge. Then *a* will be the inclination of a long edge to the tripolar normal. But since angle A and angle B are both alike, the result of this equation must come out the same as that of equation *b*): in other words, both the short edges and the long edges of Model 68 incline upon the tripolar normal, at an angle of 54° 44′. This coincidence is accidental, and does not occur with the other tetrakishexahedrons.

d.) Proof of the correctness of the calculations contained in equations b) and c). The inclination of the short edge of Model 68 to axis p^s was found by § 400, to be 70° 32′. The complement of this angle, added to the inclination of plane PZ to the Znw normal, is the inclination of the given short edge to that normal. Now, the complement of 70° 32′ is 19° 28′, and the inclination of PZ to the Znw normal was found by § 363 *b*) to be 35° 16′. Then 19° 28′ + 35° 16′ = 54° 44′. Finally, the inclination of a long edge of Model 68 to the tripolar normal, is evidently the same as the inclination of a vertical edge of the cube to that normal; and this angle was found by § 363 *c*) to be 54° 44′. The product of

54° 44′ + 54° 44′, is 109° 28′, which will be found, by approximate measurement with the goniometer, to be the inclination of a short edge to a long edge of Model 68, across the pole Znw.

404. PROBLEM. *Given, the symbol* $M_{-}T$, $M_{+}T$. $P_{-}M$, $P_{+}M$, $P_{-}T$, $P_{+}T$, *with the angle across a long edge* = 126° 52′; *Required, the value of the two characteristics,* — *and* +.

a.) Find by problem § 399, equation *b*), the inclination of the planes of the combination to the axes $p^a m^a t^a$. 270° — 126° 52′ = 143° 8′ and $\frac{143° 8'}{2}$ = 71° 34′. This is the inclination a plane of $M_{-}T$ to m^a, or of $M_{+}T$ to t^a. Therefore, the cotangent of 71° 34′ gives the value of the sign —, and its tangent the value of the sign +. The first is = $\frac{1}{3}$, and the second = $\frac{3}{1}$. Hence the combination is $M\frac{1}{3}T$, $M\frac{3}{1}T$. $P\frac{1}{3}M$, $P\frac{3}{1}M$, $P\frac{1}{3}T$, $P\frac{3}{1}T$.

b.) *Given, the index; required, the corresponding angle.* If, on the contrary, the part given is the symbol, $M\frac{2}{3}T$, $M\frac{3}{2}T$. $P\frac{2}{3}M$, $P\frac{3}{2}M$, $P\frac{2}{3}T$, $P\frac{3}{2}T$, and you are required to tell the angle across a long edge, you first seek in the Table of Indices, page 139, for the value of the indices $\frac{2}{3}$ and $\frac{3}{2}$, which you find to be:

$$\frac{2}{3} = .6667 \qquad\qquad \frac{3}{2} = 1.500$$

The first of which is the cotangent, and the second the tangent of 56° 18½′. Twice this angle = 112° 37′, is the angle across pole n, from plane $M\frac{2}{3}T$ ne to plane $M\frac{3}{2}T$ nw. Hence the angle across a long edge is 270° — 112° 37′ = 157° 23′. See § 113.

405. COMBINATIONS OF THE TETRAKISHEXAHEDRON WITH OTHER FORMS.

MT, m$\frac{2}{3}$t, m$\frac{3}{2}$t. PM, p$\frac{2}{3}$m, p$\frac{3}{2}$m, PT, p$\frac{2}{3}$t, p$\frac{3}{2}$t, 3P$\frac{1}{2}$MT.

A combination somewhat resembling Model 69, but in which the rhombic dodecahedron predominates, and in which the planes of the tetrakishexahedron replace those edges of the icositessarahedron which lye on the equator, and on the north and east meridians, the edges of combination converging towards the axes $p^a m^a t^a$.

Minerals: Garnet from Friedberg.

MT, m$\frac{1}{2}$t, m$\frac{2}{1}$t. PM, p$\frac{1}{2}$m, p$\frac{2}{1}$m, PT, p$\frac{1}{2}$t, p$\frac{2}{1}$t, 3P$\frac{1}{2}$MT.

A combination similar to Model 69, but having the edges of the icositessarahedron that meet at the unipolar normals replaced by very narrow tangent planes, which are the planes of the tetrakishexahedron.

Minerals: Garnet from Dognatzky.

P,M,T, m$\frac{1}{3}$t, m$\frac{3}{1}$t. p$\frac{1}{3}$m, p$\frac{3}{1}$m, p$\frac{1}{3}$t, p$\frac{3}{1}$t. Model 45.

p,m,t, M$\frac{1}{3}$T, M$\frac{3}{1}$T. P$\frac{1}{3}$M, P$\frac{3}{1}$M, P$\frac{1}{3}$T, P$\frac{3}{1}$T.

The bevelled cube: in the first example the cube, and in the second example the tetrakishexahedron predominant. The first occurs most generally. Examples: Fluorspar from Alston Moor. The second is *presented by* Fluorspar from Bohemia.

P,M,T, MT, m$\frac{2}{5}$t, m$\frac{5}{2}$t. PM, p$\frac{2}{5}$m, p$\frac{5}{2}$m. PT, p$\frac{2}{5}$t, p$\frac{5}{2}$t, PMT.
P,M,T, MT, m$\frac{1}{3}$t, m$\frac{3}{1}$t. PM, p$\frac{1}{3}$m, p$\frac{3}{1}$m, PT, p$\frac{1}{3}$t, p$\frac{3}{1}$t, PMT.
P,M,T, MT, m$\frac{1}{5}$t, m$\frac{5}{1}$t. PM, p$\frac{1}{5}$m, p$\frac{5}{1}$m, PT, p$\frac{1}{5}$t, p$\frac{5}{1}$t, PMT.

Three combinations similar to Model 31, but having all the edges betwixt the cube and the rhombic dodecahedron replaced by the planes of the tetrakishexahedron.

Minerals: Green Fluorspar from Cumberland and Native Copper from Siberia in the first form; Fluorspar from Cumberland in the second form; and Red Oxide of Copper from Siberia in the third form.

406. *Analysis of the foregoing Combinations.*—In all these combinations, the tetrakishexahedron occurs in company with the cube or the rhombic docecahedron, which renders the analysis extremely easy. The inclination of a plane of the tetrakishexahedron to a plane of the cube, is the inclination of a plane to an axis added to 90°. The inclination of a plane of the tetrakishexahedron to a plane of the rhombic dodecahedron, is half the angle across a long edge added to 90°.

Examples: The angle of M upon m_t, Model 45, is 161° 34′. Deducting 90°, we have 71° 34′ = inclination of a plane of m_t to m^a. The cotangent of 71° 34′ is $\frac{1}{3}$. Hence, Model 45 is P,M,T, m$\frac{1}{3}$t, m$\frac{3}{1}$t. p$\frac{1}{3}$m, p$\frac{3}{1}$m, p$\frac{1}{3}$t, p$\frac{3}{1}$t.

407. The inclination of a plane of m_t upon a plane of MT, in a combination containing the rhombic dodecahedron and a tetrakishexahedron, is 146° 18½′. Deducting 90° we have 56° 18½′, which is half the angle across a long edge of the tetrakishexahedron. The difference between this product and 135°, say, 135° — 56° 18½′ = 78° 41½′, is the inclination of a plane of m_t of that form to m^a. The cotangent of 78° 41½′ is $\frac{1}{5}$. Therefore, the tetrakishexahedron contained in the given combination is, m$\frac{1}{5}$t, m$\frac{5}{1}$t. p$\frac{1}{5}$m, p$\frac{5}{1}$m, p$\frac{1}{5}$t, p$\frac{5}{1}$t.

7. The Hexakisoctahedron, or *Six-fold Octahedron.*

P_-MT_+, P_+M_-T, PM_+T_-, P_-M_+T, PM_-T_+, P_+MT_-: or $6P_-MT_+$.

Varieties of this Combination:

6P$\frac{1}{3}$M$\frac{1}{2}$T.
6P$\frac{1}{4}$M$\frac{1}{3}$T.
6P$\frac{1}{4}$M$\frac{1}{2}$T.
6P$\frac{3}{11}$M$\frac{3}{5}$T.
6P$\frac{1}{7}$M$\frac{1}{3}$T.

Model 23 is 6P$\frac{1}{7}$M$\frac{1}{3}$T.

408. The hexakisoctahedron is described in §§ 194—197. It is a complete pyramid with a rhombic equator. The only minerals which present the hexakisoctahedron as a separate crystal, are Garnet = 6P$\frac{1}{7}$M$\frac{1}{3}$T, and Diamond = 6P$\frac{1}{3}$M$\frac{1}{2}$T. See Part II. page 102. The other varieties occur only subordinately.

The edges of this combination are of three kinds as regards their length. *The longest edges* connect the unipolar and tripolar normals;

the *middle edges* connect the unipolar and bipolar normals; and the *shortest edges* connect the bipolar and tripolar normals. The angles across these different edges of all the known varieties of the combination, are given in § 197.

From the description of this combination given in § 194, it will be seen, that its unipolar normals have, in reference to the external planes of the solid, the character of the principal axis of a dioctahedron or eight-sided pyramid; its bipolar normals, the character of the principal axis of a rhombic pyramid; and its tripolar normals, the character of the principal axis of a scalenohedron, or scalene six-sided pyramid. Advantage will be taken of these peculiarities in the following calculations.

409. PROBLEM. *Given*, Model 23, $6P\_MT_{+}$, *with the angle across a long edge* = 158° 47′, *and the angle across a middle edge* = 165° 2′; *required, the inclination of the middle edge to the axis* p^z, *and to the* Zn bipolar normal.

a.) Grant the uppermost flat eight-sided pyramid of Model 23, which contains the Zenith forms of the forms P_MT_{+} and $P\_M_{+}T$, to be divided into eight sections by the four meridians which pass through its terminal edges. Take the Z'n'w octant as a solid triangle, with pole Z for its vertex. Name the given parts as follows:—Let the angle formed by the intersection of the north and north-west meridians, be angle C = 45°; let half the long edge = $\frac{158° 47'}{2}$ = 79° 23½′ be angle A; and let half the middle edge = $\frac{165° 2'}{2}$ = 82° 31′, be angle B. The part required, namely, the inclination of the middle edge to p^z, will then be side *a*. These quantities constitute an oblique-angled spherical triangle, the solution of which requires Formula 37.

$$\sin \tfrac{1}{2} a = \sqrt{\frac{-\cos S \cos (S-A)}{\sin B \sin C}}, \text{ where } S = \tfrac{1}{2}(A+B+C).$$

$\log \sin \tfrac{1}{2} a =$

$$\tfrac{1}{2}\{\log \cos S + \log \cos (S - A) + 20 - (\log \sin B + \log \sin C)\}.$$

$$\begin{array}{lll}
A = 79° 23\tfrac{1}{2}' & \qquad & S = 103° 27\tfrac{1}{4}' \\
B = 82° 31' & & A = 79° 23\tfrac{1}{2}' \\
C = 45° & & \overline{} \\
\overline{} & & S - A = 24° \; 3\tfrac{3}{4}' \\
2)206° 54\tfrac{1}{2}' & & \\
\overline{S = 103° 27\tfrac{1}{4}'} & &
\end{array}$$

$$\begin{array}{rr}
\log \cos \text{ Supplement of } S = 76° 32\tfrac{3}{4}' = - & 9.3667 \\
+ \log \cos (S-A) = 24° \; 3\tfrac{3}{4}' = & 9.9605 \\
 & \overline{} \\
+ 20 = & 39.3272 \\
- \left\{ \begin{array}{l} \log \sin B = 82° 31' = 9.9963 \\ + \log \sin C = 45° \quad = 9.8495 \end{array} \right\} = & 19.8458 \\
 & \overline{} \\
 & 2)19.4814
\end{array}$$

Twice this product, or $33° 24' \times 2 = 66° 48'$, is the required inclination of the middle or Z^2n edge to the axis p^2.

b.) When the arithmetical indices are given with the symbol, as $6P\frac{1}{7}M\frac{1}{3}T$, instead of $6P_{-}MT_{+}$, then it is easier to find the inclination of the middle edge to the axis p^2 by the problem contained in § 411, which is worked by a right-angled solid triangle; but when the symbol contains only the signs $_{-}$ and $_{+}$, two equations must be solved, §§ 409, 410, for the purpose of finding the arithmetical equivalent of these two signs.

c.) The inclination of the middle edge to the bipolar normal Zn, is 135° minus its inclination to the unipolar normal, namely, $135° - 66° 48' = 68° 12'$. See §§ 412, 418.

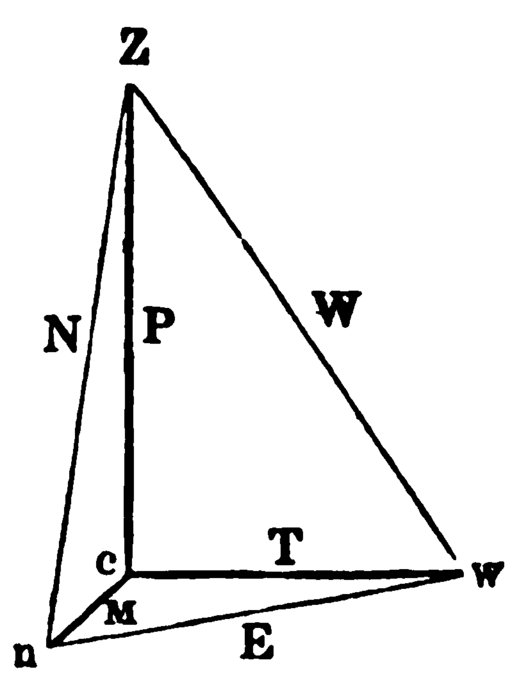
410. PROBLEM. *With the information contained in the preceding problem, to find the value of the indices* $_{-}$ *and* $_{+}$ *in the symbols* $P_{-}MT_{+}$, $P_{-}M_{+}T$, *which characterise the dioctahedron or eight-sided pyramid of* Model 23.

CALCULATION OF EIGHT-SIDED PYRAMIDS.

This problem is of considerable importance, as it is used, not only in the investigation of the pyramids of the hexakisoctahedron, but also of those of the dioctahedrons of the pyramidal system of crystallisation.

a.) We had given, in the last problem, the angle across the middle edge, half of which is $= 82° 31'$, and we obtained as the product of an equation, the inclination of that edge to axis $p^2 = 66° 48'$. Now it will be observed, that this angle is on the north meridian, and that its cotangent will give the relation of the two axes, p^2 and m^2, to the two planes that lie on each side of the middle edge. This cotangent is .4286 or $\frac{3}{7}$, which intimates that the axis p^2 bears to the axis m^2 the relation of 3 to 7.

b.) To complete our knowledge of the axes of the octahedron to which these two planes belong, we have next to find the relation of p^2 and m^2 to t^2. With this in view, we take the Znw octant of an assumed octahedron, and use it as a right-angled solid triangle with pole Z for its vertex. We know the value of the angle across N, because it is given in the problem $= 82° 31'$, and we know the value of the plane angle nZc, because it is the inclination of the edge N to axis p^2, found above $= 66° 48'$. Call the first datum angle A, and the second, side *b*. The edge or angle P, between the planes nZc and wZc, will then be angle $C = 90°$. What we now want to learn is, the value of side *a*, which is the plane angle wZc in the annexed diagram, since this angle will give us the required relation of axis p^2 to axis t^2. It is evident that this angle can be *found by the following* calculation:



Given, $A = 82° 31'$; $b = 66° 48'$; $C = 90°$; *to find*, a.

Formula 7. $\log \tan a = \log \tan A + \log \sin b - 10.$

$$\log \tan A = 82° 31' = 10.8815$$
$$+ \log \sin b = 66° 48' = 9.9634$$
$$\log \tan a = 81° 52' = 10.8449$$

This angle, 81° 52', is the inclination to the axis p^a of an edge which does not appear on Model 23, because it is replaced by an accompanying octahedron; but it is the Zw boundary of the simple scalene octahedron, whose Zn boundary has been found to incline on p^a at an angle of 66° 48'. Hence the cotangent of the product of the last equation gives the demanded relation of axis p^a to axis t^a, and we find in the table that the cotangent of 81° 52' is .1429 or $\frac{1}{7}$.

c.) By *a.*) we found the relation of the axes, p^a to m^a = 3 to 7; and now in *b.*) we find the relation of p^a to t^a = 1 to 7; or putting $p^a = 3$ in order to agree with the former product, and multiplying $\frac{1}{7}$ by 3, we have $\frac{3}{21}$, or p^a to t^a = 3 to 21. This makes the relation of the three axes to one another to be $p^a_3 m^a_7 t^a_{21}$, and gives for the octahedron the symbol $P\frac{3}{21}M\frac{7}{21}T$. But the index $\frac{3}{21}$ is equal to $\frac{1}{7}$, and the index $\frac{7}{21}$ is equal to $\frac{1}{3}$, so that the symbol $P\frac{3}{21}M\frac{7}{21}T$, can be reduced to the simpler synonymous expression of $P\frac{1}{7}M\frac{1}{3}T$. This, therefore, is the translation of P_MT_{+}, while the translation of $P\_M_{+}T$, the symbol of the associated form, is $P\frac{1}{7}MT\frac{1}{3}$.

d.) There are, on Model 23, six pyramids of eight sides, such as that which we have just investigated. All of them converge to a point over a unipolar normal. Their angles are all alike. They can be calculated by the same methods as are employed above, and they give similar results as regards their axial relations. The only difference is, that every pair of pyramids affects the three axes differently from the other two pair. Thus, the pyramids which converge at the poles Z and N, give the symbol $P\frac{1}{7}M\frac{1}{3}T$, $P\frac{1}{7}MT\frac{1}{3}$; those which converge at the poles n and s, give the symbols $PM\frac{1}{7}T\frac{1}{3}$, $P\frac{1}{3}M\frac{1}{7}T$; and those which converge at the poles e and w, give the symbols $P\frac{1}{3}MT\frac{1}{7}$, $PM\frac{1}{3}T\frac{1}{7}$. These differences being attended to, all the varieties of the hexakisoctahedron may have their indices calculated from their angles in the manner here explained.

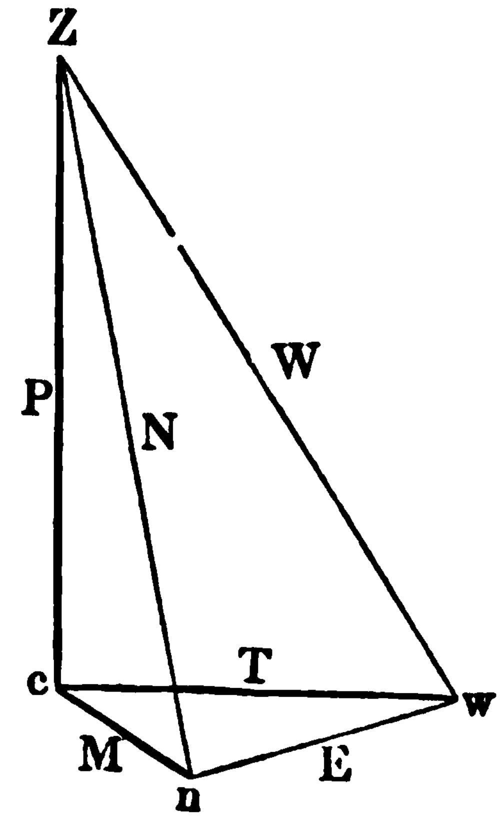
The calculation of the dioctahedron requires two angles to be given. These are assumed above, to be those across the two terminal edges. I shall presently explain what steps are to be taken when one of the given angles is that across a horizontal edge.

411. PROBLEM. *Given, the symbol* $6P\frac{1}{3}M\frac{1}{2}T$; *required, the angle across the* Z^2n *or middle terminal edge of the combination.*

Calculation of the Angles from the Symbol of a Hexakisoctahedron.

The reader of the foregoing problem will perceive that the solution of *this problem* merely requires a reversal of the process followed there.

The present operation is as follows: You take one of the octahedrons of $6P\frac{1}{3}M\frac{1}{2}T$, namely the first, $P\frac{1}{3}M\frac{1}{2}T$. You begin by finding the relations of the axes; next, the plane angles of the north and east meridians of the simple octahedron; and finally, the inclination of the north meridian to the external plane, which inclination is half the required angle across the middle or Z^2n edge.



$P\frac{1}{2}M\frac{1}{3}T$ is equal to $P_2M_3T_6$, the axial relations of which are p^a to $t^a = \frac{1}{3} = .3333$, and p^a to $m^a = \frac{2}{3} = .6667$. Taking these fractions for cotangents, you have .3337 cot 71° 34′, and .6667 cot 56° 18½′. Consider these angles to be two sides of an octant or right-angled solid triangle; call 71° 34′ side *a*, and 56° 18½′ side *b*. Then angle A will be half the required angle across the Z^2n edge of the given hexakisoctahedron.

Formula 13. log tan A = log tan *a* + 10 — log sin *b*.

$$\begin{aligned} 10 + \log\tan a &= 71^\circ\,34' &&= 20.4772 \\ -\log\sin b &= 56^\circ\,18\tfrac{1}{2}' &&= 9.9201 \\ \hline \log\tan A &= 74^\circ\,30' &&= 10.5571 \end{aligned}$$

This product is the inclination of the north meridian to one of the adjacent planes of the uppermost flat eight-sided pyramid in the given hexakisoctahedron. Twice the product, or 74° 30′ × 2 = 149°, is the angle across a middle or Z^2n edge. See § 197.

This calculation is the first that must be made whenever the symbol of a hexakisoctahedron is known, and the external angles are to be calculated from it. The products afforded are the inclination of a middle edge to axis p^a and the angle across the middle edge. With these two data, all the other angles of the hexakisoctahedron can be calculated by means of the various problems given in this section. On the other hand, whenever any two angles of a hexakisoctahedron are given, it is possible to deduce from them those particular angles which are afforded by the present problem, and from which, by the process given in problem § 410, the value of the characteristics of the symbol of the hexakisoctahedron can be readily determined. These two problems are therefore of great importance as regards the hexakisoctahedron, since one of them serves as a guide from the crystal to the symbol, and the other from the symbol to the crystal.

412. PROBLEM. *Given*, Model 23, $6P\frac{1}{7}M\frac{1}{2}T$; *required, the inclination of the middle edge to the unipolar normal* Z *or axis* p^a, *and to the bipolar normal* Zn; *required also, the plane angles of the north meridian, the east meridian, and the equator, of* Model 23.

a.) First find, by the process described in problem § 411, the inclination of the middle edge to axis p^a at the unipolar normal Z. For this purpose, you take the single octahedron $P\frac{1}{7}M\frac{1}{3}T$, two planes of which meet at the middle edge in question, and thus intimate that the inclination of the middle edge to p^a is, in fact, the inclination of the edges of these planes to that axis.

$P\frac{1}{7}M\frac{1}{3}T$ is equal to $P_3M_7T_{21}$, because, $\frac{1}{7} = \frac{3}{21}$, and $\frac{1}{3} = \frac{7}{21}$; so that the relation of p^a to m^a is $= \frac{3}{7}$. On looking at the Table of Indices, page 139, you find the cotangent corresponding to $\frac{3}{7}$ to be .4286, which gives 66° 48′ for the inclination of the middle edge to axis p^a.

b.) The unipolar and bipolar normals cut one another in the centre of the crystal at an angle of 45°. A line which connects the terminations of a unipolar and a bipolar normal, forms therefore a triangle with these two normals; which triangle has one angle of 45°, and two other angles equal together to 180° — 45° = 135°. Now we have found one of these angles, namely, that at pole Z, to be 66° 48′. Hence the other angle is 135° — 66° 48′ = 68° 12′. This is therefore the inclination of a middle edge of Model 23 to the bipolar normal Zn.

c.) *Another method of reckoning the inclination of the middle edge to the bipolar normal.* Twice the inclination of the midddle edge to the unipolar normal Z, is the inclination of two opposite middle edges over the pole Z. Therefore, 66° 48′ × 2 = 133° 36′.

The north meridian of Model 23 is an octagon similar to figure n o e a s i w E in page 183, having four similar angles of one kind at the terminations of the unipolar normals, and four similar angles of another kind at the terminations of the bipolar normals. For the reasons stated in § 82, two of these angles, or one of each kind, are together equal to 270°. Now, in the present case, we know that an angle over a unipolar normal is 133° 36′. Wherefore an angle over a bipolar normal must be 270° — 133° 36′ = 136° 24′, half of which sum = $\frac{136° \, 24'}{2}$ = 68° 12′, must be the inclination of the middle edge to the Zn normal.

d.) *To find the plane angles of the equator, &c.* It follows from the foregoing investigation, that the equator, the north meridian, and the east meridian, have alternate angles of 133° 36′ at the unipolar normals, and 136° 24′ at the bipolar normals, which relations agree with the geometrical characters of the given octagonal sections, since (133° 36′ + 136° 24′) × 4 = 1080°, which is the aggregate value of the angles of an octagon.

413. PROBLEM. *Given,* Model 23, $6P\_MT_{+}$, *with the angle across a long edge* = 158° 47′, *and the angle across a middle edge* = 165° 2′; *required, the inclination of the long edge to axis* p^a.

Form a solid triangle, similar to that employed in the equation § 409, but change the designation of the angles as follows. We have, as before, three angles or edges given, and are required to find a side. In the former example, the required side was part of the north meridian, but in *the present* example, it is part of the north-west meridian. To make the

problem suit the Formula, this side must nevertheless be called side a; in which case, angle A will be half the angle across the middle edge $= 82° 31'$; angle B will be half the angle across the long edge $= 79° 23\frac{1}{2}'$; and angle C will be the interior angle of 45°, described in § 409. Hence the problem is:

Given, $A = 82° 31'$; $B = 79° 23\frac{1}{2}'$; $C = 45°$; *to find*, a.

Formula 37. $\sin \frac{1}{2} a = \sqrt{\frac{-\cos S \cos (S - A)}{\sin B \sin C}}$, where $S = \frac{1}{2}(A + B + C)$.

$\log \sin \frac{1}{2} a =$

$\frac{1}{2} [\log \cos S + \log \cos (S - A) + 20 - (\log \sin B + \log \sin C)]$

$$\begin{array}{rl}
A = & 82° 31' \\
B = & 79° 23\frac{1}{2}' \\
C = & 45° \\
\hline
2\,) & 206° 54\frac{1}{2}' \\
\hline
S = & 103° 27\frac{1}{4}' \\
\text{Supplement of } S = & 76° 32\frac{3}{4}'
\end{array}
\qquad
\begin{array}{rl}
S = & 103° 27\frac{1}{4}' \\
A = & 82° 31' \\
\hline
S - A = & 20° 56\frac{1}{4}'
\end{array}$$

$$\begin{array}{rr}
\log \cos S = 76° 32\frac{3}{4}' = - & 9.3667 \\
+ \log \cos (S - A) = 20° 56\frac{1}{4}' = & 9.9703 \\
\hline
+ 20 \quad = & 39.3370 \\
- \left\{ \begin{array}{l} \log \sin B = 79° 23\frac{1}{2}' = 9.9925 \\ + \log \sin C = 45° \quad = 9.8495 \end{array} \right\} = & 19.8420 \\
\hline
2\,) & 19.4950 \\
\hline
\sin \frac{1}{2} a = 33° 59\frac{3}{4}' = & 9.7475
\end{array}$$

Twice this product, or $33° 59\frac{3}{4}' \times 2 = 67° 59\frac{1}{2}'$, is the required inclination of the long edge of Model 23 to axis p^s.

Twice $67° 59\frac{1}{2}'$, or 135° 59', is the inclination of two opposite long edges to one another, measured over the pole Z.

414. PROBLEM. *Given*, Model 23, $6P\frac{1}{4}M\frac{1}{2}T$; *required, the inclination of the long edge to axis* p^s.

Find by problems § 412 *a.*), and § 411, the inclination of the middle edge to axis p^s, and the inclination of a plane to the north meridian. Call the first $c = 66° 48'$, and the second $A = 82° 31'$. Take the oblique-angled solid triangle described in § 409, but with different angles and different designations to the angles. Put a, the required side, equal to the inclination of the long edge to axis p^s. Then, B will be the interior angle of 45°. The given quantities are now two angles and an intermediate side, and you are required to find a side opposite to angle A. These quantities form the following equation:

Given, $A = 82° 31'$; $B = 45°$; $c = 66° 48'$; *to find*, a.

Formula 43. $\begin{cases} \log \cot x = \log \tan A + \log \cos c - 10. \\ \log \cot a = \log \cot c + \log \cos (B - x) - \log \cos x. \end{cases}$

$$\begin{array}{rlr} \log \tan A = 82° 31' = & 10.8815 & B = 45° \\ + \log \cos c = 66° 48' = & 9.5954 & x = 18° 26' \\ \hline \log \cot x = 18° 26' = & 10.4769 & B - x = 26° 34' \end{array}$$

$$\begin{array}{rr} \log \cot c = 66° 48' = & 9.6321 \\ + \log \cos (B - x) = 26° 34' = & 9.9515 \\ \hline & 19.5836 \\ - \log \cos x = 18° 26' = & 9.9771 \\ \hline \log \cot a = 67° 59\tfrac{1}{2}' = & 9.6065 \end{array}$$

This product, 67° 59½', agrees with the product of problem § 413.

415. PROBLEM. *Given*, Model 23, $6P\tfrac{1}{7}M\tfrac{1}{3}T$, *with the inclination of a long edge to the axis* $p^a = 67° 59\tfrac{1}{2}'$; *required, the inclination of the same edge to the* tripolar normal Znw.

The tripolar normal Znw, the unipolar normal Z, or semi-axis p^a, and the long edge of Model 23, which connects pole Z with pole Znw, form together a triangle. In this, the inclination of the unipolar to the tripolar normal is 54° 44', § 349; the inclination of the unipolar normal to the long edge is given = 67° 59½'; and the inclination of the long edge to the tripolar normal is necessarily equal to 180° — (54° 44' + 67° 59½') = 57° 16½'; because 57° 16½' + 54° 44' + 67° 59½' = 180° = the three angles of a triangle. In problem § 424, the inclination of the long edge to the tripolar normal is found to be 57° 16½' by another process.

416. PROBLEM. *Given*, Model 23, $6P\tfrac{1}{7}M\tfrac{1}{3}T$; *required, the angle across the* Z^anw *or long terminal edge of the combination.*

First use problems § 411, § 412 *a*.), to find the inclination of the middle edge to axis p^a, and half the angle across the middle edge. The latter is given in § 409 = 82° 31'; the former is found by the same problem to be = 66° 48'; of course, the products of calculations by means of the methods given in § 412 *a*.) and § 411, would be the same as these.

Take the oblique-angled solid triangle described in § 409, having pole Z for its vertex, and an interior angle of 45°. Call this A. Then, B will be 82° 31'; the required angle across a long edge will be C; and the above described side of 66° 48' between A and B will be c. Hence we have:

Given, $A = 45°$; $B = 82° 31'$; $c = 66° 48'$; *to find*, C.

This requires *Formula* 42.

$$\log \cot x = \log \tan A + \log \cos c - 10.$$
$$\log \cos C = \log \cos A + \log \sin (B - x) - \log \sin x.$$

$$\begin{array}{lrl} \log \tan A = 45^\circ & = 10.0000 & \quad B = 82^\circ\ 31' \\ + \log \cos c = 66^\circ\ 48' = & 9.5954 & \quad x = 68^\circ\ 30' \\ \hline \log \cot x = 68^\circ\ 30' = & 9.5954 & \quad B - x = 14^\circ\ 1' \end{array}$$

$$\begin{array}{rl} \log \cos A = 45^\circ & = 9.8495 \\ + \log \sin (B - x) = 14^\circ\ 1' & = 9.3842 \\ \hline & 19.2337 \\ - \log \sin x = 68^\circ\ 30' & = 9.9687 \\ \hline \log \cos C = 79^\circ\ 23\tfrac{1}{2}' & = 9.2650 \end{array}$$

Twice this product, or $79^\circ\ 23\frac{1}{2}' \times 2 = 158^\circ\ 47'$, is the required angle across the Z^2nw or long terminal edge of Model 23. § 197.

417. PROBLEM. *Given,* Model 23, $6P\frac{1}{7}M\frac{1}{3}T$; *required, the plane angle at pole* Z *of one of its external faces.*

Call the required plane angle a. Then the solid triangle employed in § 416, with the same given quantities, may be employed to solve the present problem, with the help of Formula 43.

Given, $A = 45^\circ$; $B = 82^\circ\ 31'$; $c = 66^\circ\ 48'$; *to find,* a.

Formula 43. $\log \cot x = \log \tan A + \log \cos c - 10.$

$$\log \cot a = \log \cot c + \log \cos (B - x) - \log \cos x$$

$$x = 68^\circ\ 30'; \ B - x = 14^\circ\ 1'. \text{ See § 416.}$$

$$\begin{array}{rl} \log \cot c = 66^\circ\ 48' & = \ \ 9.6321 \\ + \log \cos (B - x) = 14^\circ\ 1' & = \ \ 9.9869 \\ \hline & 19.6190 \\ - \log \cos x = 68^\circ\ 30' & = \ \ 9.5641 \\ \hline \log \cot a = 41^\circ\ 23\tfrac{1}{2}' & = 10.0549 \end{array}$$

This product, $41^\circ\ 23\frac{1}{2}'$, is the required plane angle of a face of Model 23, at pole Z.

418. PROBLEM. *Given,* Model 23, $6P\_MT_+$, *with the angle across a middle edge* $= 165^\circ\ 2'$, *and the angle across a short edge* $= 136^\circ\ 47'$; *required, the inclination of the middle edge to the* Zn normal.

The Zn normal of Model 23 has the character of the principal axis of a scalene pyramid, as respects the four planes which surround the pole Zn. Divide this pyramid into four sections by planes passing through the terminal edges, and meeting at the Zn normal. Take one of these sections as a right-angled solid triangle, with pole Zn for its vertex. Then you have, angle $C = 90^\circ$ = intersection of two dividing planes at the Zn normal; angle $B = 82^\circ\ 31'$ = half the angle across a middle edge; and *angle* $A = 68^\circ\ 23\frac{1}{2}'$ = half the angle across a short edge.

With these data, you have to find side a, which is the required inclination of the middle edge to the Zn normal. The problem is, therefore:

Given, A $= 68° 23\frac{1}{2}'$; B $= 82° 31'$; C $= 90°$; *to find*, a.

Formula 4. log cos a = log cos A + 10 — log sin B.

$$\begin{aligned} 10 + \log\cos A = 68° 23\tfrac{1}{2}' &= 19.5662 \\ -\log\sin B = 82° 31' &= 9.9963 \\ \hline \log\cos a = 68° 12' &= 9.5699 \end{aligned}$$

This product, 68° 12′, is the required inclination of the middle edge to the Zn normal.

The inclination of the middle edge to axis p² is = 135° — 68° 12′ = 66° 48′. § 412. Hence the data given in this problem are sufficient to lead to a knowledge of the arithmetical value of the signs $_-$ and $_+$ in the symbol P_-MT_+. See § 410.

419. PROBLEM. *Given*, Model 23, $6P\frac{1}{7}M\frac{1}{3}T$, *with the inclination across a middle edge* = 165° 2′, *and the inclination across a short edge* = 136° 47′; *required, the inclination of the short edge to the* Zn *bipolar normal.*

Take the right angled solid triangle employed in the last problem, and with the same given quantities seek for side b, which is the required inclination of a short edge to the Zn normal.

Given, A $= 68° 23\frac{1}{2}'$; B $= 82° 31'$; C $= 90°$; *to find*, b.

Formula 5. log cos b = log cos B + 10 — log sin A.

$$\begin{aligned} 10 + \log\cos B = 82° 31' &= 19.1147 \\ -\log\sin A = 68° 23\tfrac{1}{2}' &= 9.9684 \\ \hline \log\cos b = 81° 57' &= 9.1463 \end{aligned}$$

This product, 81° 57′, is the inclination of the short edge of Model 23 to the Zn bipolar normal.

420. PROBLEM. *Given, the same data as in the last two problems; required, the plane angle at pole* Zn *of an external face of the model.*

Use the same solid triangle as in § 419.

Given, A $= 68° 23\frac{1}{2}'$; B $= 82° 31'$; C $= 90°$; *to find*, c.

Formula 6. log cos c = log cot A + log cot B — 10.

$$\begin{aligned} \log\cot A = 68° 23\tfrac{1}{2}' &= 9.5978 \\ +\log\cot B = 82° 31' &= 9.1185 \\ \hline \log\cos c = 87° 1' &= 8{,}7163 \end{aligned}$$

This product, 87° 1′, is the plane angle of the faces at pole Zn.

421. PROBLEM. *Given*, Model 23, $P\frac{1}{7}M\frac{1}{3}T$, *with the angle across a middle edge* = 165° 2′, *and the angle across a long edge* = 158° 47′; *required, the angle across a short edge.*

Find by the methods given in § 409 or § 412, the inclination of the middle edge to the Zn normal. Put it $= 68° 12'$. Then take the right angled solid triangle described in § 418. Use these quantities: $C = 90°$ = the interior angle; $B = 82° 31'$ = half the angle across the middle edge; $a = 68° 12'$ = inclination of the middle edge to the Zn normal. Then seek A = inclination of an external face to the section passing through the short edges. Twice A will be the required angle across a short edge of the model.

Given, $a = 68° 12'$; $B = 82° 31'$; $C = 90°$; *to find*, A.

Formula 10. $\log \cos A = \log \cos a + \log \sin B - 10$.

$$\begin{array}{rl} \log \cos a = 68° 12' & = 9.5698 \\ + \log \sin B = 82° 31' & = \underline{9.9963} \\ \log \cos A = 68° 23\tfrac{1}{2}' & = 9.5661 \end{array}$$

Twice this product, or $68° 23\tfrac{1}{2}' \times 2 = 136° 47'$, is the required angle across a short edge of the model.

The angle across a short edge can be found, when only the symbol is given. You first find by problem § 411, the angle across a middle edge; and the inclination of that edge to p^s; then by problem § 412, the inclination of the middle edge to the Zn normal; and finally, by problem § 421, the angle across the short edge.

422. Problem. *Given*, Model 23, P_MT_+, *with the angle across a short edge* $= 136° 47'$, *and the angle cross a middle edge* $= 165° 2'$; *required, the angle across a long edge.*

Use the right angled solid triangle described in § 418, to find the inclination of the middle edge to the Zn normal, which is $68° 12'$. The difference between $68° 12'$ and $135° = 66° 48'$, is the inclination of the middle edge to axis p^s. The auxiliary angle of 135°, introduced here, is the supplement of the inclination of the unipolar normal to the bipolar normal. See § 412, *b*).

You now take an oblique angled solid triangle, formed as described in § 416. You have then, A, the interior angle of 45°; $B = 82° 31'$ = half the angle across a middle edge given in this problem; and $c = 66° 48'$, he inclination of the middle edge to axis p^s. With these data, and the formula quoted in § 416, you find $C = 79° 23\tfrac{1}{2}'$. This product is half the required angle across the long edge, which angle is 158° 47'.

423. Problem. *Given*, Model 23, $P\tfrac{1}{7}M\tfrac{1}{3}T$, *with the inclination of a short edge to the* Zn *bipolar normal, as found by problem* § 419, $= 81° 57'$; *required, the inclination of the short edge to the tripolar normal* Znw.

The bipolar normal Zn meets the tripolar normal Znw, in the centre of the crystal at an angle of 35° 16'. These two normals, in conjunction with a short edge of the model, complete a triangle. It follows, that

since the inclination of the short edge to the bipolar normal is 81° 57′, its inclination to the tripolar normal must be = 180° — (35° 16′ + 81° 57′) = 62° 47′; because 35° 16′ + 81° 57′ + 62° 47′ = 180°.

Hence, 180° — 35° 16′ = 144° 44′, is the inclination of a short edge to a unipolar normal added to its inclination to a tripolar normal. If one of these angles is known, the other is found by taking the known angle from 144° 44′. Thus, 144° 44′ — 81° 57′ = 62° 47′, as determined above.

424. PROBLEM. *Given,* Model 23, P_MT_{+}, *with the angle across a long edge* = 158° 47′, *and the angle across a short edge* = 136° 47′; *required, the inclination of a long edge to the* Znw *normal.*

Put the Znw normal in the place of a principal axis, and suppose the model to be divided into six portions by sections passing through the long and short edges, and meeting at the Znw normal. Each of these six portions will have an interior angle of 60° where two sections meet at the Znw normal. Take one of these portions as a solid triangle, with pole Znw for its vertex. Then the given parts of the solid triangle are C = 60° = interior angle; B = $\frac{158° 47'}{2}$ = 79° 23½′ = half the angle across a long edge; and A = $\frac{136° 47'}{2}$ = 68° 23½′ = half the angle across a short edge. With these data, you can find *a*, which is the inclination of a long edge to the Znw normal.

Given, A = 68° 23½′; B = 79° 23½′; C = 60°: *to find, a.*

The precautions necessary to be observed in using the following Formula, have been described in § 403.

Formula 37. $\sin \frac{1}{2} a = \sqrt{\frac{-\cos S \cos (S - A)}{\sin B \sin C}}$, where $S = \frac{1}{2}(A + B + C)$

log sin ½ *a* =

½{log cos S + log cos (S — A) + 20 — (log sin B + log sin C)}

$$\begin{array}{rl} A = & 68° \ 23\frac{1}{2}' \\ B = & 79° \ 23\frac{1}{2}' \\ C = & 60° \\ \hline & 2)207° \ 47' \\ \hline S = & 103° \ 53\frac{1}{2}' \\ \text{Supplement of } S = & 76° \ \ 6\frac{1}{2}' \end{array} \qquad \begin{array}{rl} S = & 103° \ 53\frac{1}{2}' \\ A = & 68° \ 23\frac{1}{2}' \\ \hline S - A = & 35° \ 30' \end{array}$$

$$\begin{array}{rr} \log \cos S = 76° \ 6\frac{1}{2}' = & -\ 9.3804 \\ + \log \cos (S - A) = 35° \ 30' = & 9.9107 \\ \hline + 20 = & 39.2911 \\ -\left\{\begin{array}{l} \log \sin B = 79° \ 23\frac{1}{2}' = 9.9925 \\ + \log \sin C = 60° \qquad = 9.9375 \end{array}\right\} = & 19.9300 \\ \hline & 2)19.3611 \\ \hline \log \sin \frac{1}{2} a = 28° \ 38\frac{1}{2}' = & 9.68055 \end{array}$$

Twice this product, or $28° \, 38\frac{1}{2}' \times 2 = 57° \, 16\frac{1}{2}'$, is the required inclination of a long edge of Model 23 to the tripolar normal Znw. See § 415.

425. *Check on the accuracy of this Calculation.*—The north-west meridian of Model 23 is an octagon, with three kinds of angles. The angle at Z and N is 135° 59′, § 413. The angle at the tripolar normals is equal to the inclination of the long edge to the tripolar normal = 57° 16½′, § 424, added to the inclination of the short edge to the same normal = 62° 47′, § 423; equal together to 120° 3½′. The angle at the bipolar normals is twice the inclination of a short edge to the bipolar normal, or $81° \, 57' \times 2 = 163° \, 54'$, § 419. Consequently, the eight angles are as follows:—

$$\begin{aligned} 135° \, 59' \quad \times 2 &= 271° \, 58' \\ 120° \quad 3\tfrac{1}{2}' \times 4 &= 480° \, 14' \\ 163° \, 54' \quad \times 2 &= 327° \, 48' \\ \hline & \quad 1080° \end{aligned}$$

426. PROBLEM. *Given,* Model 23, 6P½M½T, *with the angle across a long edge* = 158° 47′, *and the angle across a short edge* = 136° 47′; *required, the inclination of a short edge to the* Znw *tripolar normal, and to the* Zn *bipolar normal.*

a.) Take the same oblique-angled solid triangle and the same Formula, as were used in problem § 424, but change the designations of the angles as follows, in order to make angle A fall opposite the required side, which the Formula calls side *a*.

Given, $A = 79° \, 23\frac{1}{2}'$; $B = 68° \, 23\frac{1}{2}'$; $C = 60°$; *to find, a.*

Refer to § 424 for the preamble of the operation.

$$\begin{aligned} S &= 103° \, 53\tfrac{1}{2}', \quad \text{as found in § 424.} \\ A &= 79° \, 23\tfrac{1}{2}' \\ \hline S - A &= 24° \, 30' \end{aligned}$$

$$\begin{aligned} \log \cos S = 76° \; 6\tfrac{1}{2}' &= - \quad 9.3803 \\ + \log \cos (S - A) = 24° \, 30' &= \quad\;\; 9.9590 \\ \hline + 20 &= \quad 29.3393 \\ -\left\{ \begin{array}{l} \log \sin B = 68° \, 23\tfrac{1}{2}' = 9.9684 \\ + \log \sin C = 60° \quad\;\; = 9.9375 \end{array} \right\} &= \quad 19.9059 \\ \hline 2\,) & \;\; 19.4334 \\ \hline \log \sin \tfrac{1}{2} a = 31° \, 23\tfrac{1}{2}' &= \quad\;\; 9.7167 \end{aligned}$$

Twice this product, or $31° \, 23\frac{1}{2}' \times 2 = 62° \, 47'$, is the required inclination of a short edge to the tripolar normal. This agrees with the calculation in § 423.

b.) The inclination of a short edge to the bipolar normal Zn, is 144° 44′ — 62° 47′ = 81° 57′. See § 423.

427. PROBLEM. *Given,* Model 23, $6P\frac{1}{7}M\frac{1}{3}T$, *with the angle across a long edge* = 158° 47′, *and the angle across a short edge* = 136° 47′; *required, the plane angle of the external faces at pole* Znw.

Employ the same oblique-angled solid triangle, and the same Formula, as in §§ 424, 426; but change the designations of the angles as follows:

$$\textit{Given, } A = 60°;\ B = 79°\ 23\tfrac{1}{2}';\ C = 68°\ 23\tfrac{1}{2}';\ \textit{to find, } a.$$

$$\begin{aligned} S &= 103°\ 53\tfrac{1}{2}' \\ A &= \ \ 60° \\ \hline S - A &= \ \ 43°\ 53\tfrac{1}{2}' \end{aligned}$$

$$\begin{aligned} \log\cos S = 76°\ \ 6\tfrac{1}{2}' &= -\ 9.3803 \\ +\ \log\cos (S - A) = 43°\ 53\tfrac{1}{2}' &= \quad 9.8577 \\ \hline +\ 20 &= \quad 39.2380 \\ -\left\{\begin{array}{l} \log\sin B = 79°\ 23\tfrac{1}{2}' = 9.9925 \\ +\ \log\sin C = 68°\ 23\tfrac{1}{2}' = 9.9684 \end{array}\right\} &= \quad 19.9609 \\ \hline &\quad 2)19.2771 \\ \hline \log\sin \tfrac{1}{2}a = 25°\ 47\tfrac{1}{2}' &= \quad 9.63855 \end{aligned}$$

Twice this product, or 25° 47½′ × 2 = 51° 35′, is the required plane angle of the external faces at pole Znw.

428. PROBLEM. *Given,* Model 23, $6P\frac{1}{7}M\frac{1}{3}T$, *with the angle across a long edge* = 158° 47′, *and the angle across a short edge* = 136° 47′; *required, the angle across a middle edge.*

First, find by problem § 426 *b.*), the inclination of a short edge to the bipolar normal Zn. Call this a = 81° 57′.

Then take a right-angled solid triangle, such as is described in § 418, with pole Zn for its vertex, and having for its angles, half the angle across a short edge, half the angle across a middle edge, and the angle formed by the intersection of planes passing through the middle and short edges, and meeting at the Zn normal. The value of this last angle is 90°. Call this angle C. The value of half the angle across a short edge is given in the problem at $\frac{136°\ 47'}{2}$ = 68° 23½′. Call this angle B. Then the inclination of the short edge to the Zn normal will be side a = 81° 57′. With these data, you can find half the angle across a middle edge, since it will be angle A of the same right-angled solid triangle. The problem is, therefore:

$$\textit{Given, } a = 81°\ 57';\ B = 68°\ 23\tfrac{1}{2}';\ C = 90°;\ \textit{to find, } A.$$

Formula 10. log cos A = log cos a + log sin B — 10.

$$\begin{aligned} \log\cos a = 81°\ 57' &= 9.1462 \\ +\ \log\sin B = 68°\ 23\tfrac{1}{2}' &= 9.9684 \\ \hline \log\cos A = 82°\ 31' &= 9.1146 \end{aligned}$$

Twice this product, or $82° \ 31' \times 2 = 165° \ 2'$, is the required angle across a middle edge.

429. PROBLEM. *Given*, Model 23, $6P\frac{1}{7}M\frac{1}{5}T$; *required, the plane angles of its external faces.*

The plane angles of the faces of Model 23 have been found to be as follows:

At pole Z,	§ 417 =	41° 23½′
At pole Zn,	§ 420 =	87° 1′
At pole Znw,	§ 427 =	51° 35′
		179° 59½′

The aggregate sum, 179° 59½′, is ½′ less than it ought to be. This arises from inattention to fractions of angles smaller than half minutes, and from the brevity of the logarithmic numbers employed. If the angles were always reckoned to *seconds*, and the logarithmic numbers carried out to 7 decimal places, these occasional errors would not occur.

430. COMBINATIONS CONTAINING THE HEXAKISOCTAHEDRON.

MT. PM, PT, $3P\frac{1}{2}MT$, $6p\frac{1}{3}m\frac{1}{2}t$.
MT. PM, PT, $3P\frac{1}{2}MT$, $6p\frac{1}{4}m\frac{1}{3}t$.
These combinations are represented by Model 69, with the addition of narrow planes replacing the edges between the planes of the rhombic dodecahedron and those of the icositessarahedron. Minerals: Garnet, &c., Part II., page 112.

P,M,T. $6p\frac{1}{4}m\frac{1}{2}t$. Model 40.
P,M,T. $6p\frac{1}{7}m\frac{1}{5}t$.
P,M,T. $6p\frac{3}{11}m\frac{3}{5}t$.
The cube, with the solid angles replaced each by six scalene triangular planes. Minerals: Fluorspar, Part II., page 105.

P,M,T, MT. PM, PT, $3p\frac{1}{3}mt$, $6p\frac{1}{4}m\frac{1}{2}t$, $6p\frac{3}{11}m\frac{3}{5}t$.—The cube predominant, having the edges replaced by the planes of the rhombic dodecahedron, and the solid angles replaced each by fifteen small planes. Minerals: Fluorspar, Part II., page 105.

431. *Analysis of these Combinations.*—In all these combinations, the planes of the hexakisoctahedron are placed in such a manner that measurements can be taken across two of the edges of the hexakisoctahedron. In the combination which Model 69 partially resembles, these edges will be the middle and short edges, because the long edge is replaced by a plane of P_MT. In the combination which Model 40 resembles, the edges to be measured are the long edge and the short edge, since the middle edge is here replaced by the plane PZ. Measurements across any two edges of the hexakisoctahedron, are sufficient to guide us to a knowledge *of its symbol,* as I have demonstrated in §§ 408—429. When

the known angles are those across a middle edge and a short edge, we use problems §§ 409, 410; when they are those across a long edge and a short edge, we use problems §§ 428, 409, and 410.

I proceed next to the investigation of ROSE's Hemihedral Forms of the Octahedral System of Crystallisation, the general characters of which have been already fully explained in §§ 263—270.

I. THE TETRAHEDRON, or *Hemioctahedron*.

Model 117. ½PMT.

432. This form is described in § 265. ROSE's symbol for it is ½r (*a* : *a* : *a*). According to the principles of classification explained in Section IV., it is an incomplete pyramid, since it has none but inclined planes, and yet is without solid angles at the poles Z and N. It has a square equator. The minerals which occur in this form are quoted at page 122, Part II. in Class 6, Order 1, Genus 1.

433. The unipolar normals terminate in the middle of the edges of this form; the bipolar normals, in the middle of the lines drawn on the faces of the model so as to connect the poles of the unipolar normals. The tripolar normals terminate in the centre of each of the four faces, and in each of the four solid angles of the model. The lines which connect the unipolar normals of this form, and which are drawn on Model 117 in coloured ink, show the position of the edges of the octahedron.

434. PROBLEM. *Given,* Model 117, ½PMT, *with the angle across every edge* $= 70° 32'$; *required, the plane angle of the faces.*

a.) Take one of the solid angles of the form as an oblique angled solid triangle, and seek the value of a plane angle by means of Formula 37. As the faces of Model 117 are equilateral triangles, we have *given,* A, B, C each $= 70° 32'$; *to find, a.* The answer will of course be 60°.

b.) A shorter method of calculation is, to divide one of the solid angles of the form into two equal sections, by a plane assumed to pass through one of the edges and across one of the planes of the model. Taking one of these sections as a solid triangle, the problem is this: *Given,* $A = \frac{70° 32'}{2} = 35° 16'$; $B = 70° 32'$; $C = 90°$; *to find, a.* In this case, *a* is half the required plane angle.

Formula 4. $\log \cos a = \log \cos A + 10 - \log \sin B.$

$$\begin{aligned} 10 + \log \cos A &= 35° 16' = 19.9119 \\ - \log \sin B &= 70° 32' = 9.9744 \\ \hline \log \cos a &= 30° = 9.9375 \end{aligned}$$

Twice this product, or $30° \times 2 = 60°$, is the required plane angle.

435. PROBLEM. *Given,* Model 117, ½PMT, *with the angles across*

dge = 70° 32′; *required, the inclination of the tripolar normals terminate in a plane, to axis* p², *and the inclination of those which ate in a solid angle, to an edge of the model.*

The inclination of the tripolar normals Znw Zse Nne Nsw to axis ound by the process given in § 349. It is 54° 44′. The inclina- the four other tripolar normals to an edge is 35° 16′. This is t, from the consideration, that the north-west meridian of Model ivided into two portions by the vertical axis p², is exactly equal to ıre in § 349.

Another Method.—Take the solid angle of the model as a three- pyramid, and assume it to be divided into six portions by the pro- scribed in § 359. One of these portions, taken as a right angled iangle, will have the following known parts: A = 60°; B = 35° 16′; 0°. With these you can find, a = half a plane angle of one of es; b = inclination of an external plane to the tripolar normal; = inclination of an edge to a tripolar normal.

nula 4. log cos a = log cos A + 10 — log sin B.

$$\begin{aligned} 10 + \log\cos A &= 60^\circ &&= 19.6990 \\ -\log\sin B &= 35^\circ\,16' &&= 9.7615 \\ \hline \log\cos a &= 30^\circ &&= 9.9375 \end{aligned}$$

nula 5. log cos b = log cos B + 10 — log sin A.

$$\begin{aligned} 10 + \log\cos B &= 35^\circ\,16' &&= 19.9119 \\ -\log\sin A &= 60^\circ &&= 9.9375 \\ \hline \log\cos b &= 19^\circ\,28' &&= 9.9744 \end{aligned}$$

nula 6. log cos c = log cot A + log cot B — 10.

$$\begin{aligned} \log\cot A &= 60^\circ &&= 9.7614 \\ +\log\cot B &= 35^\circ\,16' &&= 10{,}1505 \\ \hline \log\cos c &= 35^\circ\,16' &&= 9.9119 \end{aligned}$$

Jpon the principle laid down in § 359, *c.*), the products $b + c$, be equal to the inclination of an edge to a plane of Model 117, ed over a solid angle. Now, $b + c$ = 19° 28′ + 35° 16′ = , and as the cross section of Model 107 is a triangle of which one s 70° 32′, and the other two angles are equal to each other, one of gles must be ½(180° — 70° 32′ = 109° 28′) = 54° 44′. Hence it ent that the tripolar normals terminate in the middle of each plane the middle of each solid angle of the tetrahedron.

Combinations containing the Tetrahedron.

[T, ½pmt. Model 118. This combination is described in §§ 269, It is an incomplete pyramid with a square equator, and the mine- ich it represents are quoted at page 122, Part II. in Class 6, Order ns 1. *The inclination* of a plane of ½PMT on a plane of ½pmt, is

109° 28′, which sufficiently shows the relation of this form to the octahedron and tetrahedron.

p, m, t. ½PMT.

P,M,T. ½pmt. Model 38. The first of these combinations would be represented by Model 37, if the twelve pentagonal faces were away. These combinations represent the planes of the cube in combination with those of the tetrahedron; in one of them, the cube predominates, and in the other the tetrahedron. They represent complete prisms combined with incomplete pyramids. Minerals: page 105, Part II. Class 3, Order 1, Genus 1. Groups, *a*, *d*.

Analysis. The inclination of the planes of the cube to those of the tetrahedron is $= \frac{70^\circ\ 32'}{2} + 90^\circ = 125^\circ\ 16'$. As a single plane of the tetrahedron is exactly similar to a single plane of the octahedron, the same analytical processes serve to discriminate both the tetrahedron and the octahedron. See §§ 364—368.

MT. PM, PT, ½pmt. Model 65, if four alternate triangular planes were away.

mt. pm, pt, ½PMT. Model 78. The rhombic dodecahedron and the tetrahedron. The first is an incomplete prism with a complete pyramid; the second an incomplete prism with an incomplete pyramid. Minerals: page 110 and 114, Part II.

Analysis. See § 366.

p,m,t, mt. pm, pt, ½PMT. Model 37.

P,M,T, mt. pm, pt, ½pmt. Model 36.

p,m,t, MT. PM, PT, ½pmt, Model 34, supposing four alternate triangular planes to be away.

Combinations containing the tetrahedron, the cube, and the rhombic dodecahedron, one of them predominant over the other two, in each crystal. All of them are complete prisms with incomplete pyramids. Minerals: pages 105, 106, Part II. Class 3, Order 1, Genus 1.

Analysis, § 368.

P,M,T, MT. PM, PT, ½PMT, ½pmt. Model 35. The cube, the rhombic dodecahedron, the right tetrahedron, and the left tetrahedron. A complete pyramid with an incomplete prism. Minerals, page 105, Part II. Class 3, Order 1, Genus 1.

Analysis, § 368.

The combinations of which there are models are further explained in the description of the models, Part II. page 123.

2. The Hemiicositessarahedron.

½P_MT, ½PM_T, ½PMT_: or ½(3P_MT).

Varieties of this Combination:—

½P½MT, ½PM½T, ½PMT½: or ½(3P½MT).

½P⅓MT, ½PM⅓T, ½PMT⅓: or ½(3P⅓MT).

Model 119 is ½(3P⅓MT).

437. This is the hemihedral variety of the combination 3P_MT, Model 22. See §§ 267—269. The varieties are discriminated by the incidence of their planes upon one another, which are exactly the same as the corresponding angles of the homohedral combinations.

Upon comparing Model 119 with Model 22, both being held in upright position, it will be seen that the planes of the *half form*, Model 119, are those which belong to four octants of the *whole form*, Model 22, namely, the octants—

Znw, Zse, Nne, Nsw.

This is what constitutes the right hemiicositessarahedron. But there is another variety of the half form, which is called the left hemiicositessarahedron, and which comprises the other four octants of the whole form, namely, the octants

Zne, Zsw, Nnw, Nse.

In combination together, these two half forms are discriminated by the symbol ½(3P_MT), ⅛(3p_mt), because they are necessarily always unequal though always similar. When one of them occurs alone, it is always to be considered as the right form, and denoted by ½(3P_MT) Z^2nw, or simply by ½(3P_MT); the last symbol is sufficiently precise, because it is a rule of common acceptance, as I have already stated, § 269, that every *single* tetrahedral form, is considered to be the *right* form, or that which possesses the Znw octant.

There are two kinds of edges on the hemiicositessarahedron. *Long* edges which pass through the poles Z N n e w s and connect the following tripolar normals, Zne Zsw Nnw Nse ; and *short* edges, which pass through the bipolar normals and connect the eight tripolar normals. The following are the angles of the two varieties :—

	Across a short edge.	Across a long edge.
½(3P½MT)	146° 27′	109° 28′
½(3P½MT)	129° 31′	129° 31′

The hemiicositessarahedron is an incomplete pyramid with a rhombic equator. Minerals: page 122, Part II. Class 6, Order 3.

438. *Analysis.*—The planes of ½(3P_MT) have the same form as the planes of 3P+MT. Compare Model 119 with Model 17. The planes of ½(3P+MT) have the same form as the planes of 3P_MT. Compare Model 18 with Model 22. I point out these facts, because they present a remarkable example of contrariness, which is apt to puzzle a young crystallographer.

The following are the chief points to be observed in relation to the measurements of the hemiicositessarahedron. The angle across a short edge of the hemihedral form, is the same as the angle across a short edge of the homohedral form. The angle across a long edge of the hemihedral form is equal to twice the inclination of a plane of the homohedral form to axis p^s. I shall give instructions for finding the angle across a long edge *when that across* a short edge is known, or the angle across

a short edge when that across a long edge is known. This, with problem § 370, is sufficient information for the derivation of the symbol of the combination.

439. PROBLEM. *Given,* Model 119, $\frac{1}{2}$(3P$\frac{3}{2}$MT), *with the angle across a long edge* = 129° 31′; *required, the angle across a short edge.*

a.) First find the inclination of a plane to the Znw *normal.* Model 119 resembles a tetrahedron having a low three-sided pyramid superimposed upon each face. Hence the angle across the long edge is equal to the angle across the edge of a tetrahedron, plus twice the inclination of one of the oblique planes to the face of the tetrahedron. Consequently, if you take from the angle across the edge of the form = 129° 31′, the angle across the edge of the tetrahedron = 70° 32′, the residue, divided by 2, will give the inclination of one of the oblique planes to the face of the tetrahedron. Say 129° 31′ — 70° 32′ = 58° 59′; and $\frac{58^\circ\ 59'}{2}$ = 29° 29$\frac{1}{2}$′. The complement of this angle = 60° 30$\frac{1}{2}$′, is the inclination of a plane to the tripolar normal, which normal is perpendicular to the plane of the enclosed tetrahedron.

b.) Another Method.—The unipolar normal meets the tripolar normal at an angle of 54° 44′, § 349. The unipolar normals meet the plane of $\frac{1}{2}$(3P$\frac{3}{2}$MT) at an angle of $\frac{129^\circ\ 31'}{2}$ = 64° 54$\frac{1}{2}$′. Therefore, the inclination of that plane to the tripolar normal is = 180° — (54° 44′ + 64° 45$\frac{1}{2}$′) = 60° 30$\frac{1}{2}$′.

c.) Having now the inclination of a plane to the Znw normal, we can calculate the angle across an edge proceeding from that normal, by dividing the three-faced pyramid of Model 119 into six right-angled solid triangles, on the principle explained in § 359. We take one of these triangles with pole Znw for its vertex, and in which the known quantities are C = 90° = inclination of an external face to a plane cutting it through the middle; A = 60° = an interior edge where two intersecting planes meet at the Znw normal; and *b* = 60° 30$\frac{1}{2}$′ = inclination of an external plane to the Znw normal. With these data, we have to find B, which is half the angle across a short edge of the model.

Given, A = 60°; *b* = 60° 30$\frac{1}{2}$′; C = 90°; *to find,* B.

Formula 8. log cos B = log cos *b* + log sin A — 10.

$$\begin{aligned} \log \cos b &= 60^\circ\ 30\tfrac{1}{2}' = 9.6922 \\ +\ \log \sin A &= 60^\circ \qquad\ \ = 9.9375 \\ \hline \log \cos B &= 64^\circ\ 46' \quad = 9.6297 \end{aligned}$$

Twice this product, or 64° 46′ × 2 = 129° 32′, is the required angle across a short edge of Model 119. This result is, however, 1′ too much, as the angle across a short edge of $\frac{1}{2}$(3P$\frac{3}{2}$MT) is the same as that across a long edge, 129° 31′.

440. PROBLEM. *Given, the combination,* $\frac{1}{2}$(3P$\frac{3}{2}$MT), *with the angle across a long edge* = 109° 28′; *required, the angle across a short edge.*

Find, by problem § 439, *a.*), the inclination of the planes to the tripolar normal. $\frac{1}{2}(109° 28' — 70° 32') = 19° 28'$, the supplement of which is 70° 32'. The rest of the solution of this problem is contained in § 377, *b.*), where the required angle is proved to be 146° 27'.

441. PROBLEM. *Given,* Model 119, $\frac{1}{2}(3P\frac{1}{3}MT)$, *with the angle across a short edge* = 129° 31'; *required, the angle across a long edge.*

a.) Find the inclination of a plane to the tripolar normal, § 439. Call this x. Then half the angle across a long edge will be $180° — (54° 44' + x)$. § 439, *b.*)

b.) Take pole Zuw for the vertex of a right-angled solid triangle, and one-sixth of the crystal to form this triangle, as described in § 439, *c.*) The known parts of the triangle are C = 90°; A = 60°; and B = 64° 45½'. With these data, you have to find b, which is the inclination of a plane to the tripolar normal.

Formula 5. $\log \cos b = \log \cos B + 10 — \log \sin A$.

$$\begin{array}{rll} 10 + \log \cos B = 64° 45\frac{1}{2}' & = & 19.6298 \\ — \log \sin A = 60° & = & 9.9375 \\ \hline \log \cos b = 60° 30\frac{1}{2}' & = & 9.6923 \end{array}$$

c.) Now, according to *a.*), half the angle across a long edge of the model is equal to $180° — (54° 44' + 60° 30\frac{1}{2}') = 64° 45\frac{1}{2}'$. Twice 64° 45½' is 129° 31'.

442. PROBLEM. *Given, the combination* $\frac{1}{2}(3P\frac{1}{2}MT)$, *with the angle across a short edge* = 146° 27'; *required, the angle across a long edge.*

Proceed as directed in § 441. To find x, or the inclination of a plane to the tripolar normal, say,

Given, A = 60°; B = 73° 13½'; C = 90°; *to find, b.*

Formula 5. $\log \cos b = \log \cos B + 10 — \log \sin A$.

$$\begin{array}{rll} 10 + \log \cos B = 73° 13\frac{1}{2}' & = & 19.4603 \\ — \log \sin A = 60° & = & 9.9375 \\ \hline \log \cos b = 70° 32' & = & 9.5228 \end{array}$$

Then, $180° — (54° 44' + 70° 32') = 54° 44'$. Twice this product, or $54° 44' \times 2 = 109° 28'$, is the required angle across a long edge of the given combination.

443. PROBLEM. *Given,* Model 119, $\frac{1}{2}(3P\frac{1}{3}MT)$, *with the angle across a short edge* = 129° 31'; *required, the plane angles of the external faces.*

Take the right-angled solid triangle employed in § 441, and with the same given parts, A = 60°; B = 64° 45½'; C = 90°; find a, which is half an *external obtuse plane* angle of Model 119.

Formula 4. log cos a = log cos A + 10 — log sin B.

$$10 + \log\cos A = 60^\circ \quad = 19.6990$$
$$-\log\sin B = 64^\circ\ 45\tfrac{1}{2}' = \ \ 9.9564$$
$$\log\cos a = 56^\circ\ 26' \quad = \ \ 9.7426$$

Twice this product, or 56° 26′ × 2 = 112° 52′, is the obtuse plane angle of the model. Each acute plane angle is half the difference between 112° 52′ and 180°, or $\frac{180^\circ - 112^\circ\ 52'}{2}$ = 33° 34′.

444. Problem. *Given, the symbol* ½(3P_MT) *with the angle across a short edge* = 93° 40′, *and the angle across a long edge* = 176° 30′; *required, the value of the index* _ *in the symbol.*

These are the angles ascribed by Phillips to a variety of the hemi-icositessarahedron, which approaches nearly to the form of a cube. See his *Mineralogy*, article Arseniate of Iron, page 235, figure 5.

a.) First examine the accuracy of these measurements, as follows :— To find x, § 442, say,

Given, A = 60°; B = 46° 50′ (= $\frac{93^\circ\ 40'}{2}$); *to find, b.*

Formula 5. log cos b = log cos B + 10 — log sin A, which gives log cos 37° 49′ = log cos 46° 50′ + 10 — log sin 60°.

Then, 180° — (54° 44′ + 37° 49′) = 87° 27′. Twice this product = 174° 54′, is the angle across a long edge of the given combination, which differs nearly 2° from Phillips's measurement of 176° 30′.

b.) To find the index of the symbol, take the right-angled solid triangle described in § 376, *b.*) The parts known in the present example are, A = interior angle of 45°, where the north and north-west meridians cross one another; b = 87° 27′ = inclination of an external plane to p^s; C = 90° = inclination of an external plane to the north-west meridian. Then c will be the inclination of a long edge of the icositessarahedron to axis p^s, the cotangent of which will give the required index of the symbol.

Given, A = 45°; b = 87° 27′; *to find, c.*

Formula 9. log tan c = log tan b + 10 — log cos A.

$$10 + \log\tan b = 87^\circ\ 27' = 21.3513$$
$$-\log\cos A = 45^\circ \quad = \ \ 9.8495$$
$$\log\tan c = 88^\circ\ 12' = 11.5018$$

The cotangent of 88° 12′ is = $\frac{1}{32}$, which gives for the given hemiicositessarahedron the symbol ½($P_{\frac{1}{32}}$MT). But if the calculation is made on the basis of Phillips's measurement of the long edge, the angle produced is 88° 46′, the cotangent of which is = $\frac{1}{45}$, affording the symbol ½($3P_{\frac{1}{45}}$MT).

445. Combinations containing the Hemiicositessarahedron.

½PMT, ½(3p½mt).

½pmt, ½(3P½MT).

Combinations of the tetrahedron and the hemiicositessarahedron, somewhat resembling Model 94, on the supposition that the twelve rhombic planes on the corners of that model were absent. Minerals: page 122, Part II.

mt. pm, pt, $\frac{1}{2}$PMT Znw, $\frac{1}{2}$(3p$\frac{1}{2}$mt) Znw. Model 94.
MT. PM, PT, $\frac{1}{2}$pmt Znw, $\frac{1}{2}$(3p$\frac{1}{2}$mt) Zne. Model 96.

Both these forms contain the rhombic dodecahedron, the right hemioctahedron, and the right hemiicositessarahedron. They are, nevertheless, very different in appearance from one another, and the difference is principally caused by the predominance in the one form of the dodecahedron, and in the other of the hemioctahedron. It commonly happens with the combinations that contain both homohedral and hemihedral forms, that the predominance of the one or the other makes a very striking difference in the general appearance of the combination.

The hemiicositessarahedron contained on Model 94 is that described by the symbol $\frac{1}{2}$(3p$\frac{1}{2}$mt). The one contained on Model 95 is $\frac{1}{2}$(3p$\frac{1}{2}$mt). The proof of this exists in the difference of the angle of incidence of the two planes which form an edge at the pole Z.

The hemihedral forms $\frac{1}{2}$PMT and $\frac{1}{2}$(3P_MT) contained on Model 94, both occupy the same set of octants; but those contained on Model 95, occupy different sets of octants. The simpler form $\frac{1}{2}$PMT is therefore ascribed to the Znw octant, and the more complex form $\frac{1}{2}$(3P_MT), to the left set. This occurrence of two sets of hemihedral forms in different positions upon one crystal, shows the grounds of the distinction necessary to be made between the right and left hemihedral forms.

mt. pm, pt, $\frac{1}{2}$PMT, $\frac{1}{2}$(3P$\frac{1}{2}$MT) Znw, $\frac{1}{2}$(3p$\frac{1}{2}$mt) Zne.
P,M,T, MT. PM, PT, $\frac{1}{2}$PMT Zne, $\frac{1}{2}$PMT Znw, $\frac{1}{2}$(3p$\frac{1}{2}$mt) Znw.
$\frac{1}{2}$PMT, $\frac{1}{2}$(3p$\frac{1}{2}$mt), $\frac{1}{2}$(3p$\frac{1}{2}$mt.)

446. *Analysis of these Combinations.*—A single plane of the hemiicositessarahedron $\frac{1}{2}$(3P_MT), has the same properties as a single plane of the icositessarahedron 3P_MT. Consequently, the instructions given for the analysis of combinations that contain the latter, apply equally well to the analysis of those which contain the former. See §§ 381—384.

The inclination of a plane of $\frac{1}{2}$PMT to a plane of $\frac{1}{2}$(3P_MT) is $90°+x$, in which formula x signifies the inclination of a plane of $\frac{1}{2}$(3P_MT) to the tripolar normal.

3. The Hemitriakisoctahedron.

$\frac{1}{2}P_{+}MT$, $\frac{1}{2}PM_{+}T$, $\frac{1}{2}PMT_{+}$: or $\frac{1}{2}(3P_{+}MT)$.

Varieties of this Combination?:

$\frac{1}{2}P\frac{3}{2}MT$, $\frac{1}{2}PM\frac{3}{2}T$, $\frac{1}{2}PMT\frac{3}{2}$: or $\frac{1}{2}(3P\frac{3}{2}MT)$.
$\frac{1}{2}P_2MT$, $\frac{1}{2}PM_2T$, $\frac{1}{2}PMT_2$: or $\frac{1}{2}(3P_2MT)$. Model 18.

447. This combination only occurs with other forms, and never in an isolated state. I shall, however, describe it as a complete crystal;

because calculations founded on measurements of its angles are made exactly in the same way as if the planes could form a separate crystal. It presents two kinds of edges, namely, *short* edges which meet three together at the tripolar normals Znw Zne Nne Nsw, exactly as on the homohedral crystal $3P\frac{3}{2}MT$, Model 17; also, *long* edges which meet three together at the tripolar normals Zne Zsw Nnw Nse. The solid angles produced by the meeting of three short edges are obtuse: those produced by the meeting of three long edges are acute. The long and short edges meet together at the unipolar normals, and produce four-faced angles having the character of rhombic pyramids. The lines drawn on Model 18 to connect the unipolar normals, or the similar plane angles, indicate the position of the long edges of the homohedral combination, $3P\frac{3}{2}MT$, Model 17. It follows from this description, that if the angle across a short edge of Model 18 be known, that across a long edge can be found; or if the angle across a long edge be known, that across a short edge can be found; or if either be known, the inclination of the planes to the three axes $p^a m^a t^a$ can be determined, and the index for the symbol be thence deduced.

Rose describes the only known variety of this combination as $\frac{1}{2}(3P\frac{3}{2}MT)$, with the angles as follow: across a long edge $= 82°\ 10'$, across a short edge $= 162°\ 39\frac{1}{2}'$. Miller and Von Kobell describe it as $\frac{1}{2}(3P_2MT)$, with the angles as follow: across a long edge $= 90°$; across a short edge $= 152°\ 44'$. Model 18 agrees with $\frac{1}{2}(3P_2MT)$.

The combination on which the hemitriakisoctahedron occurs in the mineral world, according to Rose, is Fahlerz from Dillenberg,

$$\text{MT.PM, PT. } \tfrac{1}{2}(3P\tfrac{1}{2}MT),\ \tfrac{1}{2}(3p\tfrac{3}{2}mt).$$

The predominant form in this combination is that represented by Model 119, $\frac{1}{2}(3P\_MT)$. Its four acute solid angles are replaced by the twelve rhombic planes of the dodecahedron, which rest on the short edges of the hemiicositessarahedron, as shown by Model 94. The planes of the hemitriakisoctahedron, Model 18, replace the shorter edges of Model 119, appearing as long narrow planes meeting at the tripolar normals. See Rose's *Krystallographie*, fig. 34.

448. Problem. *Given*, Model 18, $\frac{1}{2}(3P_+MT)$, *with the angle across a short edge* $= 152°\ 44'$; *required, the angle across a long edge, and the value of the index* $_+$ *in the symbol.*

Find the inclination of the short edge to the tripolar normal Znw; next its inclination to the unipolar normal Z; and then, by means of a right-angled triangle, with pole Z for its vertex, find the angle across the long edge.

a.) To find the inclination of the short edge to the unipolar and tripolar normals.—Form a right-angled solid triangle as directed in § 359, with pole Znw for vertex, and the following known parts:—$A = 60°$; $C = 90°$; $B = 76°\ 22'$ $(= \frac{152°\ 44'}{2})$. Then find c, which is the inclination of an edge to the tripolar normal.

Formula 6. log cos c = log cot A + log cot B — 10.

$$\begin{aligned} \log \cot A &= 60^\circ &&= 9.7614 \\ + \log \cot B &= 76^\circ\ 22' &&= 9.3848 \\ \hline \log \cos c &= 81^\circ\ 57' &&= 9.1462 \end{aligned}$$

This product, 81° 57′, is the inclination of a short edge to the tripolar normal. Then the inclination of the short edge to the unipolar normal is 180° — (54° 44′ + 81° 57′) = 43° 19′.

b.) To find the angle across a long edge.—Assume Model 18 to be divided by planes passing through the long and short edges, and intersecting at axis $\bar{p}^s$. Take one of the sections as a right-angled triangle with pole Z for vertex. Then you have C = 90° = interior edge of intersection; B = 76° 22′ = half the angle across a short edge; a = 43° 19′ = inclination of a short edge to p^s. With these data, find A, which is half the angle across a long edge.

Given, a = 43° 19′; B = 76° 22′; *to find*, A.

Formula 10. log cos A = log cos a + log sin B — 10.

$$\begin{aligned} \log \cos a &= 43^\circ\ 19' &&= 9.8619 \\ + \log \sin B &= 76^\circ\ 22' &&= 9.9876 \\ \hline \log \cos A &= 45^\circ &&= 9.8495 \end{aligned}$$

Twice this product, or 45° × 2 = 90°, is the required angle across a long edge of Model 18.

c.) To find the inclination of a plane to a bipolar normal.—First, find the inclination of a plane to the tripolar normal by means of the triangle described in *a*). Say, *Given*, A = 60°; B = 76° 22′; C = 90°; *to find*, b, which is the inclination required.

Formula 5. log cos b = log cos B + 10 — log sin A.

$$\begin{aligned} 10 + \log \cos B &= 76^\circ\ 22' &&= 19.3724 \\ - \log \sin A &= 60^\circ &&= 9.9375 \\ \hline \log \cos b &= 74^\circ\ 12\tfrac{1}{2}' &&= 9.4349 \end{aligned}$$

This product, 74° 12½′, is the inclination of a plane to the tripolar normal. Its inclination to a bipolar normal is 180° — (35° 16′ + 74° 12½′) = 70° 31½′.

d.) To find the value of the index $_+$ *in the symbol.*—Suppose Model 18 to be divided by the north and east meridian into quadrants. Take one of these as a right-angled solid triangle, with pole Z for its vertex, and the following given parts: C = 90°, intersection of the two meridians; A = 70° 31½′ = inclination of a plane of Model 18 to the Zn bipolar normal, or to the north meridian; b = 45° = quadrant of the north meridian. With these data, find a, which will be that side of the solid triangle whose cotangent shows the relative value of axes p^s and t^s of the *form under investigation.*

Given, $A = 70° \ 31\frac{1}{2}'$; $b = 45°$; *to find*, a.

Formula 7. $\log \tan a = \log \tan A + \log \sin b - 10.$

$$\begin{array}{rll} \log \tan A = 70° \ 31\frac{1}{2}' & = & 10.4515 \\ + \log \sin b = 45° & = & 9.8495 \\ \hline \log \tan a = 63° \ 26' & = & 10.3010 \end{array}$$

The cotangent of this product, 63° 26′, is .5000 = ½, which proves that the plane whose inclination to the north meridian is 70° 31½′ belongs to the form $P\frac{1}{2}M\frac{1}{2}T$, or PMT_2, which is one of the three forms that produce the combination, P_2MT, PM_2T, PMT_2, or $3P_2MT$. See § 386.

449. PROBLEM. *Given*, Model 18, ½ $(3P_2MT)$, *with the angle across a short edge* = 152° 44′, *and the angle across a long edge* = 90°; *required, the plane angles of the faces.*

Take the solid triangle described in § 448, *a*), with the same given parts, and find *a*, which will be half the angle at pole Znw.

Given, $A = 60°$; $B = 76° \ 22'$; *to find*, a.

Formula 4. $\log \cos a = \log \cos A + 10 - \log \sin B.$

$$\begin{array}{rll} 10 + \log \cos A = 60° & = & 19.6990 \\ - \log \sin B = 76° \ 22' & = & 9.9876 \\ \hline \log \cos a = 59° \ 2' & = & 9.7114 \end{array}$$

Twice this product, or 59° 2′ × 2 = 118° 4′, is the obtuse plane angle of Model 18 at pole Znw.

Form a similar equation, substituting half the angle across a long edge for half the angle across a short edge, which will give half the plane angle at pole Zne.

$$\begin{array}{rll} 10 + \log \cos A = 60° & = & 19.6990 \\ - \log \sin B = 45° & = & 9.8495 \\ \hline \log \cos a = 45° & = & 9.8495 \end{array}$$

Twice this product, or 45° × 2 = 90°, is the plane angle at pole Zne.

The angles at pole Z and N are each ½ {360° — (118° 4′ + 90°) = 151° 56′} = 75° 58′.

Control over this Calculation.—With the triangle described in § 448, *b*.), find the value of the angle at pole Z by a direct process.

Given, $A = 45°$ (half angle across a long edge); $B = 76° \ 22'$; *to find*, c.

Formula 6. $\log \cos c = \log \cot A + \log \cot B - 10.$

$$\begin{array}{rll} \log \cot A = 45° & = & 10.0000 \\ + \log \cot B = 76° \ 22' & = & 9.3848 \\ \hline \log \cos c = 75° \ 58' & = & 9.3848 \end{array}$$

The four plane angles of the face of Model 18 are therefore 75° 58′ + 118° 4′ + 75° 58′ + 90° = 360°.

Many other problems respecting this combination could be given; but as they would be principally variations or repetitions of those relating to the triakisoctahedron, and as the hemihedral form is not of much consequence, I pass them over.

4. The Hemihexakisoctahedron with inclined faces.

$$\tfrac{1}{2}P_{-}MT_{+},\ \tfrac{1}{2}P_{+}M_{-}T,\ \tfrac{1}{2}PM_{+}T_{-},\ \tfrac{1}{2}P_{-}M_{+}T,\ \tfrac{1}{2}PM_{-}T_{+},\ \tfrac{1}{2}P_{+}MT_{-}:$$
$$\text{or } \tfrac{1}{2}(6P_{-}MT_{+}).$$

Varieties of this Combination:

$$\tfrac{1}{2}P\tfrac{1}{3}M\tfrac{1}{2}T,\ \tfrac{1}{2}PM\tfrac{1}{3}T\tfrac{1}{2},\ \tfrac{1}{2}P\tfrac{1}{2}MT\tfrac{1}{3},\ \tfrac{1}{2}P\tfrac{1}{3}MT\tfrac{1}{2},\ \tfrac{1}{2}P\tfrac{1}{2}M\tfrac{1}{3}T,\ \tfrac{1}{2}PM\tfrac{1}{2}T\tfrac{1}{3}:$$
$$\text{or } \tfrac{1}{2}(6P\tfrac{1}{3}M\tfrac{1}{2}T).\quad \text{Model 24.}$$
$$\tfrac{1}{2}P\tfrac{1}{5}M\tfrac{1}{3}T,\ \tfrac{1}{2}PM\tfrac{1}{5}T\tfrac{1}{3},\ \tfrac{1}{2}P\tfrac{1}{3}MT\tfrac{1}{5},\ \tfrac{1}{2}P\tfrac{1}{5}MT\tfrac{1}{3},\ \tfrac{1}{2}P\tfrac{1}{3}M\tfrac{1}{5}T,\ \tfrac{1}{2}PM\tfrac{1}{3}T\tfrac{1}{5}:$$
$$\text{or } \tfrac{1}{2}(6P\tfrac{1}{5}M\ T).$$

450. The first of these combinations is the hemihedral variety of the hexakisoctahedron $6P\tfrac{1}{3}M\tfrac{1}{2}T$. The second is a hemihedral combination of which no corresponding homohedral variety has been discovered, and no hemihexakisoctahedrons have been found to correspond with the rest of the known hexakisoctahedrons, § 408. Some of the properties of this combination have been detailed in §§ 263—270, 282, 283.

There are three kinds of edges on the hemihexakisoctahedron. Twelve *short* edges which connect the unipolar with the tripolar normals Znw Zse Nne Nsw. These have the same positions as the short edges of the hemitriakisoctahedron, Model 18. Twelve *long* edges, which connect the eight tripolar normals. These have the same positions as the short edges of the hemiicositessarahedron, Model 119. Twelve *middle* edges, which connect the unipolar normals with the tripolar normals Zne Zsw Nnw Nse. These have the same positions as the twelve acute edges of the hemitriakisoctahedron, Model 18.

Both the known varieties of this combination have the remarkable property, that the measurements across the longest and shortest edges are alike. These measurements are as follow:—

Combination.	Long edge.	Middle edge.	Short edge.
$\tfrac{1}{2}(6P\tfrac{1}{3}M\tfrac{1}{2}T)$	158° 13′	110° 55′	158° 13′
$\tfrac{1}{2}(6P\tfrac{1}{5}M\tfrac{1}{3}T)$	152° 20′	122° 53′	152° 20′

451. Combinations containing the Hemihexakisoctahedron.

mt. pm, pt, $\tfrac{1}{2}$PMT, $\tfrac{1}{2}(3\text{P}\tfrac{1}{2}\text{MT})$, $\tfrac{1}{2}(6p\tfrac{1}{3}m\tfrac{1}{2}t)$.

A combination exactly similar to Model 94, with the addition of six small narrow planes replacing the edges between the rhombic planes of the dodecahedron and the rectangular planes of the hemiicositessarahedron. *Grey Copper.*

P,M,T, MT. PM, PT, $\tfrac{1}{2}$PMT, $\tfrac{1}{2}$pmt, $\tfrac{1}{2}(3p\tfrac{1}{2}mt)$ Zne, $\tfrac{1}{2}(6p\tfrac{1}{5}m\tfrac{1}{3}t)$ Znw

A combination resembling Model 35, with the addition of six small

planes replacing the angles where P,M,T meet $\frac{1}{3}$PMT, at the four corners Znw Zse Nne Nsw; and with three narrow planes replacing the edges between MT. PM, PT, at the four corners Zne Zsw Nnw Nse. The set of twenty-four small planes constitute the *right* hemihexakisoctahedron, Model 24. The twelve small planes constitute the *left* hemiicositessarahedron, Model 119. Altogether, there are sixty-two planes on this combination. *Boracite.*

452. *Analytical Processes.* As I have illustrated the hexakisoctahedron very fully, I think it needless to give many problems regarding the hemihedral form; which, moreover, is not of much importance, although capable of affording as many analytical processes as are given between §§ 408—431. I shall therefore merely notice a few leading problems, and refer the reader to similar equations, given in the preceding pages, for the details.

Let Model 24, $\frac{1}{2}$(6P$\frac{1}{3}$M$\frac{1}{2}$T) be the subject of inquiry. Put the long edge = *l*, the middle edge = *m*, and the short edge = *s*.

If *l* and *s* are given, you form a solid triangle containing a sixth of the flat six-faced pyramid at pole Znw. Then you find x = inclination of a short edge to the Znw normal, y = inclination of a long edge to the Znw normal, and z = plane angle of a face at the Znw normal.

If *l* and *m* are given, you form a solid triangle consisting of a sixth of the acute six-faced pyramid at pole Zne. Then you find, p = inclination of a middle edge to the Zne normal, q = inclination of a long edge to the Zne normal, and r = plane angle of a face at the Zne normal.

If *m* and *s* are given, you form a solid triangle, consisting of a fourth of the rhombic pyramid at pole Z. Then you find f = inclination of a short edge to axis p^a, g = inclination of a middle edge to axis p^a, and h = plane angle of a face at pole Z.

With these data, and with the inclinations of the normals to one another, § 349, the forms of the meridians, and the inclination of the external planes to the meridians, you have all the necessary data for every kind of calculation directed in the article on the hexakisoctahedron, page 191.

5. The Pentagonal Dodecahedron.

M$_-$T. P$_-$M, P$_+$T.

Varieties of this Combination:—

M$\frac{3}{4}$T. P$\frac{3}{4}$M, P$\frac{4}{3}$T.
M$\frac{2}{3}$T. P$\frac{2}{3}$M, P$\frac{3}{2}$T.
M$\frac{1}{3}$T. P$\frac{1}{2}$M, P$\frac{2}{1}$T. Model 91.

453. This combination is described in § 108, and again in § 281. It has two kinds of edges, namely, six long edges, situated two on the north meridian at n and s, two on the east meridian at Z and N, and two on the equator at e and w; also, twenty-four short edges so situated as to form a three-faced pyramid at the termination of each of the tripolar normals.

The following are the angles across the edges of the different varieties:—

	Long edges.	Short edges.
$M\frac{3}{4}T.\ P\frac{3}{4}M,\ P\frac{4}{3}T$	106° 16′	118° 41′
$M\frac{2}{3}T.\ P\frac{2}{3}M,\ P\frac{3}{2}T$	112° 37′	117° 29′
$M\frac{1}{2}T.\ P\frac{1}{2}M,\ P\frac{2}{1}T$	126° 52′	113° 35′

The variety represented by Model 91, $M\frac{1}{2}T.\ P\frac{1}{2}M,\ P\frac{2}{1}T$, occurs in the mineral world as a complete crystal, particularly beautiful in Cobalt Glance and Iron Pyrites. It is thence sometimes called the Pyritohedron. The other varieties only occur in combination with other forms. The Pentagonal Dodecahedron is an incomplete prism with an incomplete pyramid, and has a rhombo-quadratic equator. See page 117, Part II., where it occurs in Class 5, Order 4. Genus 1.

454. PROBLEM. *Given,* Model 91, $M_{-}T.\ P_{-}M,\ P_{+}T$, *with the angle across a long edge* = 126° 52′; *required, the value of the indices* $_{-}$ *and* $_{+}$ *in the symbol.*

Divide 126° 52′ by 2: the cotangent of the remainder is the index of the shorter axis of each form. $\frac{126° 52'}{2} = 63° 26'$. cot = $\frac{1}{2}$. That is $M_{-}T = M\frac{1}{2}T$, $P_{-}M = P\frac{1}{2}M$, and $P_{+}T = PT\frac{1}{2}$, or taking T for unity $= P\frac{2}{1}T$. Hence the complete symbol is $M\frac{1}{2}T.\ P\frac{1}{2}M,\ P\frac{2}{1}T$. In many parts of this work the symbol is written $MT_2.\ PM_2,\ P_2T$; but I think it better to keep all the indices between the two letters of the symbol, and always to make axis $t^a = 1$.

In the same manner, if the given angle across a long edge is 106° 16′, you say, $\frac{106° 16'}{2} = 53° 8'$. cot = $\frac{3}{4}$. Then the symbol is $M\frac{3}{4}T.\ P\frac{3}{4}M,\ P\frac{4}{3}T$. And if the angle is = 112° 37′, then, $\frac{112° 37'}{2} = 56° 18\frac{1}{2}'$. cot = $\frac{2}{3}$, which gives $M\frac{2}{3}T.\ P\frac{2}{3}M,\ P\frac{3}{2}T$.

455. PROBLEM. *Given,* Model 91, $M_{-}T.\ P_{-}M,\ P_{+}T$, *with the angle across a long edge* = 126° 52′; *required, the angle across a short edge.*

Suppose the model to be divided by the north meridian into two pieces. Take one half of it as a solid triangle, having for its vertex the solid angle where plane $P_{-}M$ Zn meets the two front planes of $M_{-}T$. Then the known parts of the triangle are, C = 90° = inclination of the north meridian on the plane $P_{-}M$; A = 63° 26′ = inclination of $M_{-}T$ nw on the north meridian; and b = 116° 34′, the inclination of $P_{-}M$ Zn to the front edge of the model. This last angle consists of the prismatic angle of 90° added to the complement of the inclination of plane $P_{-}M$ to p^a, namely, 90° — 63° 26′ = 26° 34′. With these data find B, which is the angle across the short edge between $P_{-}M$ Zn and $M_{-}T$ nw.

Given, A = 63° 26′; b = 116° 34′; C = 90°; to find B.

Formula 8. Log cos B = log cos b + log sin A — 10. Since angle b = 116° 34′ is not in the table, you substitute its supplement, 180° —

116° 34′ = 63° 26′, of course paying attention to any possible ambiguity that can arise from that substitution, § 330.

$$\begin{aligned} \log\cos b &= 63^\circ\,26' = 9.6505 \\ +\ \log\sin \mathrm{A} &= 63^\circ\,26' = 9.9515 \\ \hline \log\cos \mathrm{B} &= 66^\circ\,25' = 9.6020 \end{aligned}$$

The supplement of this product, or 180° — 66° 25′ = 113° 35′, is the required angle across a short edge of Model 91.

456. PROBLEM. *Given*, Model 91, M_T. P_M, P+T, *with the angle across a short edge* = 113° 35′, *and across a long edge* = 126° 52′; *required, the plane angles of the external faces of the model.*

The solid triangle employed in § 455, serves also for this problem.

a.) *To find the plane angle of* P_M *at* Zn.

Given, A = 63° 26′; *b* = 63° 26′; *to find, a.*

Formula 7. $\log\tan a = \log\tan \mathrm{A} + \log\sin b - 10.$

$$\begin{aligned} \log\tan \mathrm{A} &= 63^\circ\,26' = 10.3010 \\ +\ \log\sin b &= 63^\circ\,26' = 9.9515 \\ \hline \log\tan a &= 60^\circ\,48' = 10.2525 \end{aligned}$$

Twice this product, or 60° 48′ × 2 = 121° 36′, is the obtuse plane angle of P_M at Zn. The product is doubled, because, from the nature of the solid triangle, *a* is only *half* the required angle.

b.) *To find the plane angle of* M_T *at* Zn.

Given, A = 63° 26′; *b* = 63° 26′; *to find, c.*

Formula 9. $\log\tan c = \log\tan b + 10 - \log\cos \mathrm{A}.$

$$\begin{aligned} 10 + \log\tan b &= 63^\circ\,26' = 20.3010 \\ -\ \log\cos \mathrm{A} &= 63^\circ\,26' = 9.6505 \\ \hline \log\tan c &= 77^\circ\,24' = 10.6505 \end{aligned}$$

In equation *a*), the process afforded the correct angle, but in this case, we have the supplement of the correct angle, which is 102° 36′, as may be found by approximate measurement with the goniometer.

c.) There are in all five plane angles on each face of Model 91, of which one is found by *a*) to be 121° 36′, and two are found by *b*) to be each 102° 36′, or together 205° 12′. The remaining two are similar to one another. According to the principle in § 16, *t*, all the angles of a pentagon are equal to (180° × 5) — 360° = 540°. Now, if one of them is 121° 36′, and two others are 205° 12′, the two last must be together equal to 540° — 326° 48′ = 213° 12′, or separately, they must be 106° 36′. This last product is corroborated by problem, § 457. The five plane angles of each face of Model 91, are therefore 121° 36′ + 106° 36′ + 106° 36′ + 102° 36′ + 102° 36′ = 540°.

457. PROBLEM. *Given*, Model 91, M$\frac{1}{2}$T. P$\frac{1}{2}$M, P$\frac{2}{1}$T, *with the angle across a short edge* = 113° 35′; *required, the plane angle of the faces at* pole Znw.

Form a solid triangle on the principle explained in § 359, having pole Znw for its vertex, and for its given parts, C = 90°; A = 56° 47½′ (= $\frac{113° 35′}{2}$); B = 60°. With these data, find *b*, which will be half a plane angle at Znw.

Formula 5. log cos *b* = log cos B + 10 — log sin A.

$$\begin{array}{rl} 10 + \log\cos B = 60° & = 19.6990 \\ -\log\sin A = 56°\ 47\tfrac{1}{2}' = & \ \ 9.9226 \\ \hline \log\cos b = 53°\ 18' = & \ \ 9.7764 \end{array}$$

Twice this product, or 53° 18′ × 2 = 106° 36′, is the plane angle at the meeting of two short edges at pole Znw.

458. PROBLEM. *Given*, Model 91, M$\frac{1}{2}$T. P$\frac{1}{2}$M, P$\frac{2}{1}$T, *with the angle across a short edge* = 113° 35′, *and across a long edge* = 126° 52′; *required, the plane angle of* M$\frac{1}{2}$T *at* Zn.

Take Formula 6. log cos *c* = log cot A + log cot B — 10, in which A is the supplement of 113° 35′; B, the half of 126° 52′; and *c* the required plane angle of M$\frac{1}{2}$T at Zn.

$$\begin{array}{rl} \log\cot A = 66°\ 25' = & -\ 9.6400 \\ +\ \log\cot B = 63°\ 26' = & \ \ \ 9.6990 \\ \hline \log\cos c = 77°\ 24' = & -\ 9.3390 \end{array}$$

The supplement of this product is the required angle, 180° — 77° 24′ = 102° 36′, § 456, *b*).

459. PROBLEM. *Given*, Model 91, M$\frac{1}{2}$T. P$\frac{1}{2}$M, P$\frac{2}{1}$T, *with the angle across a short edge* = 113° 35′; *required, the inclination of the external planes to the tripolar normal* Znw.

Take the solid triangle used in § 457.

Given, A = 56° 47½′; B = 60°; *to find, a*.

Formula 4. log cos *a* = log cos A + 10 — log sin B.

$$\begin{array}{rl} 10 + \log\cos A = 56°\ 47\tfrac{1}{2}' = & 19.7385 \\ -\log\sin B = 60° & = \ \ 9.9375 \\ \hline \log\cos a = 50°\ 46' = & \ \ 9.8010 \end{array}$$

This product, 50° 46′, is the inclination of the external planes to the Znw normal.

460. COMBINATIONS CONTAINING THE PENTAGONAL DODECAHEDRON.

p,m,t, M$\frac{1}{2}$T. P$\frac{1}{2}$M, P$\frac{2}{1}$T. Model 47.
P,M,T, m$\frac{1}{2}$t. p$\frac{1}{2}$m, p$\frac{2}{1}$t. Rose, figure 53.

These combinations are complete prisms with incomplete pyramids. See page 107, Part II. Class 3, Order 4, Genus 1. Bright White Cobalt and Iron Pyrites from Elba, present these combinations.

$M\frac{1}{2}T.\ P\frac{1}{2}M,\ P\frac{2}{1}T$, pmt.

$M\frac{1}{2}T.\ P\frac{1}{2}M,\ P\frac{2}{1}T$, PMT. Model 92.

$m\frac{1}{2}t.\ p\frac{1}{2}m,\ p\frac{2}{1}t$, PMT. Model 93.

These combinations are incomplete prisms with incomplete pyramids, and fall into Class 5, Order 4, Genus 1, page 117, Part II. Model 92 is called the middle crystal between the cube and the pentagonal dodecahedron.

p,m,t, $M\frac{1}{2}T.\ P\frac{1}{2}M,\ P\frac{2}{1}T$, PMT. Model 48 or 49.

P,M,T, $m\frac{1}{2}t.\ p\frac{1}{2}m,\ p\frac{2}{1}t$, pmt. Rose, figure 54.

P,M,T, mt, $\text{M}\frac{1}{2}\text{T}$. pm, $\text{P}\frac{1}{2}\text{M}$, pt, $\text{P}\frac{2}{1}\text{T}$.

Complete prisms with incomplete pyramids. Class 3, Order 4, Genus 1, page 107, Part II.

461. *Analysis of these Combinations.*

a.) The planes of P,M,T, incline upon adjoining planes of $M\frac{1}{2}T.\ P\frac{1}{2}M,\ P\frac{2}{1}T$, at an angle of

$$90^\circ + 63^\circ\ 26' = 153^\circ\ 26',$$

in which formula, 63° 26′ represents the inclination of a plane of the pentagonal dodecahedron to a unipolar normal, § 454, to all of which normals, the planes of P,M,T, are perpendicular.

b.) The planes of PMT incline upon adjoining planes of $M\frac{1}{2}T.\ P\frac{1}{2}M,\ P\frac{2}{1}T$, at an angle of

$$90^\circ + 50^\circ\ 46' = 140^\circ\ 46',$$

in which formula, 50° 46′ represents the inclination of a plane of the pentagonal dodecahedron to a tripolar normal, to all of which normals, the planes of PMT are perpendicular.

c.) The planes of the rhombic dodecahedron incline upon adjoining planes of the pentagonal dodecahedron, that is to say, a plane of MT upon a plane of $M\frac{1}{2}T$, at an angle of

$$135^\circ + 26^\circ\ 34' = 161^\circ\ 34',$$

in which formula, 135° represents the inclination of MT upon M, and 26° 34′ represents the inclination of $M\frac{1}{2}T$ upon t^a, of which two angles it is that the inclination of MT upon $M\frac{1}{2}T$ is composed. Refer to the figure in § 396. If n E w is the required angle, then c n E is 63° 26′ and o E n is 26° 34′; while i E w is 45°, and o E w is 135°. Here, we assume o E = M, E w = MT, and n E = $M\frac{1}{2}T$.

6. The Hemihexakisoctahedron with Parallel Faces.

$$P_-MT_+,\ P_+M_-T,\ PM_+T_-:\ \text{or } 3P_-MT_+.$$

Varieties of this Combination :

$$P\tfrac{1}{3}M\tfrac{1}{2}T,\ PM\tfrac{1}{3}T\tfrac{1}{2},\ P\tfrac{1}{2}MT\tfrac{1}{3}:\ \text{or } 3P\tfrac{1}{3}M\tfrac{1}{2}T.$$

$$P\tfrac{1}{4}M\tfrac{1}{2}T,\ PM\tfrac{1}{4}T\tfrac{1}{2},\ P\tfrac{1}{2}MT\tfrac{1}{4}:\ \text{or } 3P\tfrac{1}{4}M\tfrac{1}{2}T.$$

$$P\tfrac{1}{5}M\tfrac{1}{3}T,\ PM\tfrac{1}{5}T\tfrac{1}{3},\ P\tfrac{1}{3}MT\tfrac{1}{5}:\ \text{or } 3P\tfrac{1}{5}M\tfrac{1}{3}T.$$

Model 25 is the first of these combinations, or $3P\frac{1}{3}M\frac{1}{2}T$.

462. This combination is described in §§ 177—193, and again in § 282. The first variety occurs in an isolated state, as Iron Pyrites from Piedmont. It is a complete pyramid with a rhombic equator. See Class 2, Order 3, Genus 1, page 102, Part II. The other varieties occur only in combination.

There are three kinds of edges on the hemihexakisoctahedron with parallel faces; namely, twelve *long edges* which meet in pairs at Z and N on the north meridian, at e and w on the east meridian, and at n and s on the equator; twelve *short edges* which also meet in pairs, at Z and N on the east meridian, at n and s on the north meridian, and at e and w on the equator; twenty-four *middle edges*, which meet three together so as to form a three-faced pyramid at the termination of every tripolar normal, and thence proceed to meet the junction of the long and short edges on the principal sections. It therefore presents three different kinds of solid angles or pyramids.

The angles across the different kinds of edges of the several varieties of this combination, are as follow:

	Long Edges.	Middle Edges.	Short Edges.
$3P\frac{1}{3}M\frac{1}{2}T$	149° 0′	141° 47′	115° 23′
$3P\frac{1}{4}M\frac{1}{2}T$	154° 47′	131° 49′	128° 15′
$3P\frac{1}{5}M\frac{1}{3}T$	160° 32′	131° 5′	118° 59′

463. Problem. *Given,* Model 25, $3P_{-}MT_{+}$, *with the angle across a long edge* = 149°, *and across a short edge* = 115° 23′; *required, a.) the inclination of the short edges to the axes, b.) the inclination of the long edges to the axes, c.) the plane angle of the faces at pole Z, d.) the value of the indices* $_{-}$ *and* $_{+}$, *and e.) the angles of the equator and the two principal meridians.*

a.) To find the inclination of the short edges of Model 25 *to the axes* $p^2\,m^2\,t^2$.—Assume the model to be divided into quadrants by the north and east meridian. Take the Znw quadrant as a right-angled solid triangle, with pole Z for its vertex. Then C = 90°, is the interior angle at p^2 formed by the intersection of the two meridians; $A = \frac{149^\circ}{2} =$ 74° 30′, is half the angle across a long edge; and $B = \frac{115^\circ\,23'}{2} = 57^\circ\,41\frac{1}{2}'$, is half the angle across a short edge of the model. With these data, you can find, *a.)* = inclination of a short edge to an axis, *b.)* = inclination of a long edge to an axis, and *c.)* = the plane angle of a face at pole Z.

Given, A = 74° 30′; B = 57° 41½′; *required, a.*

Formula 4. log cos a = log cos A + 10 — log sin B.

$$
\begin{array}{rcl}
10 + \log\cos A = 74^\circ\,30' & = & 19.4269 \\
-\log\sin B = 57^\circ\,41\tfrac{1}{2}' & = & 9.9269 \\
\hline
\log\cos a = 71^\circ\,34' & = & 9.5000
\end{array}
$$

This product, 71° 34′, is the required inclination of the short edges of Model 25 to the axes $p^2\,m^2\,t^2$.

b.) To find the inclination of the long edges of Model 25 *to the axes* $p^a\ m^a\ t^a$. Employ the same triangle and the same quantities as in *a.*)

Given, $A = 74°\ 30'$; $B = 57°\ 41\frac{1}{2}'$; *required*, *b*.

Formula 5. $\log \cos b = \log \cos B + 10 - \log \sin A$.

$$\begin{aligned} 10 + \log \cos B &= 57°\ 41\tfrac{1}{2}' = 19.7279 \\ -\log \sin A &= 74°\ 30' = 9.9839 \\ \hline \log \cos b &= 56°\ 18' = 9.7440 \end{aligned}$$

This product, 56° 18′, is the required inclination of the long edges of Model 25 to the axes.

c.) To find the plane angle at pole Z of the external faces of Model 25. Take the triangle employed in *a.*) and *b.*)

Given, $A = 74°\ 30'$; $B = 57°\ 41\frac{1}{2}'$; *required*, *c*.

Formula 6. $\log \cos c = \log \cot A + \log \cot B - 10$.

$$\begin{aligned} \log \cot A &= 74°\ 30' = 9.4430 \\ +\log \cot B &= 57°\ 41\tfrac{1}{2}' = 9.8010 \\ \hline \log \cos c &= 79°\ 54' = 9.2440 \end{aligned}$$

This product, 79° 54′, is the required plane angle of the faces at pole Z.

d.) To find the value of the indices $_-$ *and* $_+$ *in the symbol descriptive of* Model 25, $3P_-MT_+$.—The combination represented by Model 25, contains the three octahedral forms P_-MT_+, P_+M_-T, PM_+T_-. The first named of these forms is that whose planes meet at poles Z and N, and, therefore, those whose dissection affords the solid triangle used in the above calculations. Hence, the cotangent of the inclination of the long edge to an axis shows the relation of p^a to m^a, and the cotangent of the inclination of a short edge to an axis, shows the relation of p^a to t^a. These cotangents are as follow:

$$\cot 71°\ 34' = .3333 \text{ or } \tfrac{1}{3} = p^a_1\, t^a_3 = p^a_2\, t^a_6.$$
$$\cot 56°\ 18' = .6669 \text{ or } \tfrac{2}{3} = p^a_2\, m^a_3.$$

This relation is equal to $p^a_2\, m^a_3\, t^a_6$, which gives us the symbol $P_2M_3T_6$; but if we make t^a equal to unity, then m^a becomes $\frac{3}{6}$ or $\frac{1}{2}$, and p^a becomes $\frac{2}{6}$ or $\frac{1}{3}$, which reduces the symbol to the convenient expression, $P\frac{1}{3}M\frac{1}{2}T$. Hence, the symbol for Model 23 is, briefly, $3P\frac{1}{3}M\frac{1}{2}T$, or at length, $P\frac{1}{3}M\frac{1}{2}T$, $PM\frac{1}{3}T\frac{1}{2}$, $P\frac{1}{2}MT\frac{1}{3}$.

e.) To find the angles of the equator, and of the north and east meridians of Model 23.—These three sections are all alike, so that the examination of one of them serves for the whole.

The equator is an octagon with three kinds of angles. The angles at poles n and s are equal to twice the inclination of a long edge to an axis, or 56° 18′ × 2 = 112° 36′. The angles at poles e and w are equal to twice the inclination of a short edge to an axis, or 71° 34′ × 2 = 143° 8′. The value of the angles between the four poles n e s w is found as follows:

$$\begin{array}{ll} \text{Aggregate value of the angles of the equator} & = 1080^\circ \\ \left.\begin{array}{l}\text{Value of angles at n, s, } 112^\circ\ 36' \times 2 = 225^\circ\ 12' \\ \text{Value of angles at e, w, } 143^\circ\ 8' \times 2 = 286^\circ\ 16'\end{array}\right\} & = 511^\circ\ 28' \\ \hline \text{The other four angles are together} & = 568^\circ\ 32' \end{array}$$

Hence the value of each angle where a long edge meets a short edge is $\frac{568^\circ\ 32'}{4} = 142^\circ\ 8'$.

464. Problem. *Given*, Model 25, $3P\frac{1}{2}M\frac{1}{2}T$, *with the angle across a middle edge* $= 141^\circ\ 47'$; *required, a.) the plane angle of the faces at pole* Znw, *b.) the inclination of the planes to the tripolar normal, and c.) the inclination of the middle edges to the tripolar normal.*

Form a right-angled solid triangle, as directed in § 359, taking pole Znw for its vertex, and designating the given parts as follows: $A = 60^\circ$ = interior vertical edge of the triangle; $B = \frac{141^\circ\ 47'}{2} = 70^\circ\ 53\frac{1}{2}'$ = half the angle across a middle edge; $C = 90^\circ$ = inclination of an external face to a section. With these data, find the following parts: side a = half the plane angle at Znw; side b = inclination of a plane to the Znw normal; side c = inclination of a middle edge to the Znw normal.

a.) Given, $A = 60^\circ$; $B = 70^\circ\ 53\frac{1}{2}'$; *to find, a.*

Formula 4. $\log \cos a = \log \cos A + 10 - \log \sin B$.

$$\begin{array}{rll} 10 + \log \cos A = 60^\circ & = & 19.6990 \\ - \log \sin B = 70^\circ\ 53\frac{1}{2}' & = & 9.9754 \\ \hline \log \cos a = 58^\circ\ 3' & = & 9.7236 \end{array}$$

Twice this product, or $58^\circ\ 3' \times 2 = 116^\circ\ 6'$, is the required plane angle at pole Znw.

b.) Given, $A = 60^\circ$; $B = 70^\circ\ 53\frac{1}{2}'$; *to find, b.*

Formula 5. $\log \cos b = \log \cos B + 10 - \log \sin A$.

$$\begin{array}{rll} 10 + \log \cos B = 70^\circ\ 53\frac{1}{2}' & = & 19.5150 \\ - \log \sin A = 60^\circ & = & 9.9375 \\ \hline \log \cos b = 67^\circ\ 47' & = & 9.5775 \end{array}$$

This product, $67^\circ\ 47'$, is the required inclination of a plane to the Znw normal.

c.) Given, $A = 60^\circ$; $B = 70^\circ\ 53\frac{1}{2}'$; *to find, c.*

Formula 6. $\log \cos c = \log \cot A + \log \cot B - 10$.

$$\begin{array}{rll} \log \cot A = 60^\circ & = & 9.7614 \\ + \log \cot B = 70^\circ\ 53\frac{1}{2}' & = & 9.5396 \\ \hline \log \cos c = 78^\circ\ 28' & = & 9.3010 \end{array}$$

This product, $78^\circ\ 28'$, is the inclination of a middle edge to the Znw normal.

Check on the accuracy of this Calculation.—See § 362.

b.) Inclination of plane, 67° 47′ cot .4084.
c.) Inclination of edge, 78° 28′ cot .2046.

465. PROBLEM. *Given,* Model 25, $3P\frac{1}{2}M\frac{1}{2}T$, *with the angle across a long edge* = 149°, *across a short edge* = 115° 23′, *and across a middle edge* = 141° 47′; *required, the plane angles of the faces at the point where the three different edges meet, namely, a.*) *the angle formed by a middle edge and a short edge, and b.*) *the angle formed by a middle edge and a long edge.*

a.) Assume the model to be divided into octants, and take the Znw octant as a solid triangle, having for its vertex the point where the long, short, and middle edges all meet. The problem is then as follows:

Given, $A = \frac{149°}{2} = 74° 30'$; $B = \frac{115° 23'}{2} = 57° 41\frac{1}{2}'$; $C = 141° 47'$;
to find, a = angle formed by a middle edge and a short edge.

Formula 37. $\sin \frac{1}{2} a = \sqrt{\frac{-\cos S \cos (S-A)}{\sin B \sin C}}$, where $S = \frac{1}{2}(A+B+C)$.
Log $\sin \frac{1}{2} a =$
$\frac{1}{2}\{\log \cos S + \log \cos (S - A) + 20 - (\log \sin B + \log \sin C)\}$.

$$\begin{array}{llll}
A = 74° 30' & S = 136° 59\frac{1}{4}' & & 180° \\
B = 57° 41\frac{1}{2}' & A = 74° 30' & & S = 136° 59\frac{1}{4}' \\
C = 141° 47' & \overline{\hphantom{00000000}} & & \overline{\hphantom{00000000}} \\
\overline{\hphantom{00000000}} & S - A = 62° 29\frac{1}{4}' & \text{Suppt. of } S = & 43° \ 0\frac{3}{4}' \\
2)\ 273° 58\frac{1}{2}' & & & \\
\overline{\hphantom{00000000}} & & & \\
S = 136° 59\frac{1}{4}' & & &
\end{array}$$

$$\begin{array}{rr}
\log \cos S = 43° \ 0\frac{3}{4}' = & -\ 9.8640 \\
+ \log \cos (S - A) = 62° 29\frac{1}{4}' = & 9.6646 \\
& \overline{\hphantom{0000000}} \\
+ 20 = & 39.5286
\end{array}$$

$$-\left\{\begin{array}{l}
\log \sin B = 57° 41\frac{1}{2}' = \quad 9.9269 \\
+ \log \sin C = \left\{\begin{array}{c}180° \\ 141° 47' \\ \hline 38° 13'\end{array}\right\} = -\ 9.7914
\end{array}\right\} = -\ 19.7183$$

$$\begin{array}{rr}
& \overline{\hphantom{0000000}} \\
& 2)\ 19.8103 \\
& \overline{\hphantom{0000000}} \\
\log \sin \frac{1}{2} a = 53° 30' = & 9.90515
\end{array}$$

Twice this product, or 53° 30′ × 2 = 107°, is the plane angle formed by the meeting of a middle edge and a short edge.

b.) The faces of Model 25 have four angles, equal together to 360°. One of these angles was found by § 464 *a.*), to be 116° 6′; a second was found by § 463 *c.*), to be 79° 54′; and a third by § 465 *a.*), to be 107°. The fourth must consequently be 360° — (79° 54′ + 116° 6′ + 107°) = 57°.

466. PROBLEM. *Given, the symbol,* $3P\frac{1}{2}M\frac{1}{2}T$; *required, the angle across a long edge and across a short edge of the combination.*

This requires a reversal of the processes given in § 463, d and *a.*)

The angle of which $\frac{1}{3}$ is the cotangent, is the inclination of the short edges to p^a, because $\frac{1}{3}$ expresses the relation of p^a to t^a, and the short edges connect the poles of these two axes.

The angle of which $\frac{2}{3}$ is the cotangent, is the inclination of the long edges to p^a, because $\frac{2}{3}$ expresses the relation of p^a to m^a, and the long edges connect the poles of these two axes. The fraction $\frac{2}{3}$ expresses the relation of p^a to m^a, because t^a is 1, m^a is $\frac{1}{2}$ of 1, and p^a is $\frac{1}{3}$ of 1; or, multiplying all these axes by 6, because t^a is $= 6$, $m^a = 3$, and $p^a = 2$.

These two cotangents, $\frac{1}{3}$ and $\frac{2}{3}$, correspond to the angles 71° 34′ and 56° 18′. If these are taken as sides of a right-angled solid triangle, and called side a and side b, then angle A and angle B of the same triangle will be half of each of the two angles required by the problem.

Given, $a = 71° 34'$; $b = 56° 18'$; *to find*, A. *Formula* 13.
Given, $a = 71° 34'$; $b = 56° 18'$; *to find*, B. *Formula* 14.

On working these problems, the answers will be found to be the angles given in the problem § 463, namely, the angle across a long edge $= 149°$, across a short edge $= 115° 23'$.

467. PROBLEM. *Given*, Model 46; *required, the inclination of a plane of* P_-MT_+ *upon the plane* PZ, *to be calculated from the symbol*, p,m,t. $3P\frac{1}{3}M\frac{1}{2}T$. *Or, the inclination of a plane of* P_-MT_+ *upon the plane* PZ *and the angle across a short edge being given; required, the value of the indices of* P_-MT_+.

Model 46 represents one of the compounds of this form of frequent occurrence, and in which, the short edges are entirely replaced. A special process is therefore required to determine their value, without a knowledge of which we cannot calculate the value of the indices of the combination.

a.) From the symbol $P\frac{1}{3}M\frac{1}{2}T$, *to find the inclination of a plane of that form to the equator.* If the combination $3P\frac{1}{3}M\frac{1}{2}T$ contained no forms but $P\frac{1}{3}M\frac{1}{2}T$, its equator would resemble the rhombic plane PZ seen upon Model 46. Hence, the inclination of PZ to $P\frac{1}{3}M\frac{1}{2}T$, on Model 46, is the supplement of the inclination of PZ to the external plane of the small pyramidal portion of $P\frac{1}{3}M\frac{1}{2}T$ which is assumed to be cut off, or replaced.

Take a right angled solid triangle, consisting of one-fourth of the replaced pyramid, with pole n for its vertex. Then *angle* $C = 90°$ is the right angle between the equator and the north meridian; *side a* is the side bounded by axis m^a, axis p^a, and the Zn edge of the north meridian, and as p^a is 2 and m^a is 3, the value of this side is the angle of which $\frac{2}{3}$ or .6667 is the tangent $= 33° 42'$; *side b* is the side bounded by axis m^a axis t^a, and the nw edge of the equator, and as m^a is $\frac{1}{2}$ and t^a is 1, the value of this side is the angle of which a $\frac{1}{2}$ or .5000, is the cotangent $= 63° 56'$. We have therefore *given*, $a = 33° 42'$; $b = 63° 26'$; to find, A, which is the inclination of an external plane to the equator.

Formula 13. log tan A = log tan a + 10 — log sin b.

$$\begin{aligned} 10 + \log\tan A &= 33^\circ\ 42' = 19.8241 \\ -\log\sin B &= 26^\circ\ 34' = 9.9515 \\ \hline \log\tan A &= 36^\circ\ 43' = 9.8726 \end{aligned}$$

This product 36° 43′, is the inclination of $P\frac{1}{3}M\frac{1}{2}T$ to the equator. Its supplement, or 180° — 36° 43′ = 143° 17′, is the required inclination of a plane of P_-MT_+ to the plane PZ, Model 46, calculated from the symbol $P\frac{1}{3}M\frac{1}{2}T$.

b.) *With the foregoing data, to find the angle across a long edge of* $3P\frac{1}{3}M\frac{1}{2}T$. Use the same triangle as in *a*), to find angle B, which is half the angle across a long edge of $3P\frac{1}{3}M\frac{1}{2}T$.

Given, a = 33° 42′; b = 63° 26′; *to find*, B.

Formula 14. log tan B = log tan b + 10 — log sin a.

$$\begin{aligned} 10 + \log\tan b &= 63^\circ\ 26' = 20\ 3010 \\ -\log\sin a &= 33^\circ\ 42' = 9.7442 \\ \hline \log\tan B &= 74^\circ\ 30' = 10.5568 \end{aligned}$$

Twice this product, or 74° 30′ × 2 = 149°, is the angle across a long edge of $3P\frac{1}{3}M\frac{1}{2}T$.

c.) *With the foregoing data, to find the angle across a short edge of* $3P\frac{1}{3}M\frac{1}{2}T$. Take the same triangle as in *a*) and *b*), but alter the vertex to pole Z. Then you have C = 90° = intersection of the north and east meridians at p^a; B = 74° 30′ = half the angle across the long edge; a = 56° 18′ = inclination of a long edge to axis p^a. With these data, you have to find A, which is half the angle across a short edge.

Given, a = 56° 18′; B = 74° 30′; *to find*, a.

Formula 10. log cos A = log cos a + log sin B — 10.

$$\begin{aligned} \log\cos a &= 56^\circ\ 18' = 9.7442 \\ +\log\sin B &= 74^\circ\ 30' = 9\ 9839 \\ \hline \log\cos A &= 57^\circ\ 41' = 9.7281 \end{aligned}$$

Twice this product, or 57° 41′ × 2 = 115° 22′, is the required angle across a short edge of $3P\frac{1}{3}M\frac{1}{2}T$.

d.) *Given*, Model 46, p,m,t. $3P_-MT_+$, *with the inclination* of pZ *upon* P_-MT_+ = 143° 17′, *and the angle across a long edge* = 149°; *required, the inclination of the long edge to axis* p^a. Take the solid triangle described in *a*) with pole n for its vertex. Then you have given, C = 90° = inclination of the plane PZ to the east meridian of the given triangle; A = 74° 30′ = half the angle across the long edge, and B = 36° 43′ = supplement of the inclination of plane PZ to a plane of P_-MT_+ on Model 46, which supplement is the inclination of an external plane of the replaced pyramid to its equator or to plane PZ of the model. See *a*). With these data, you have to find *b*, the complement of which is the inclination of the long edge to axis p^a.

Given, A $= 74° \; 30'$; B $= 36° \; 43'$; *to find*, b.

Formula 5. $\log \cos b = \log \cos B + 10 - \log \sin A$.

$$\begin{aligned} 10 + \log \cos B &= 36° \; 43' = 19.9040 \\ - \log \sin A &= 74° \; 30' = 9.9839 \\ \hline \log \cos b &= 33° \; 42' = 9.9201 \end{aligned}$$

The complement of $33° \; 42'$ is $56° \; 18'$, which is the required inclination of a long edge to axis p^a. See § 463, *b*.

e.) It appears from this investigation, that when you have given, Model 46, and are required to find the angle across the replaced short edge or the inclination of the short edges to axis p^a, in order to deduce thence the value of the indices of the symbol, you have in the first place, to find the inclination of the long edge to axis p^a, by problem, § 467, *d*), then the angle across a short edge by § 467, *c*), and finally, the value of the indices by problem, § 463, *d*). The fundamental data of these calculations, are the angle across a long edge of Model 46, and the inclination of PZ to a plane of P_MT_+, both of which must be taken with the goniometer.

468. Combinations containing the Hemihexakisoctahedron with parallel faces.

p,m,t .$3P\frac{1}{3}M\frac{1}{2}T$. Model 46.
P,M,T. $3p\frac{1}{3}m\frac{1}{2}t$. Rose, fig. 53, without *o*.
P,M,T. pmt, $3p\frac{1}{3}m\frac{1}{2}t$. Rose, fig. 53[a].
$M\frac{1}{2}T. P\frac{1}{2}M, P\frac{2}{1}T, 3p\frac{1}{3}m\frac{1}{2}t$. Rose, fig. 51.
$M\frac{1}{2}T. P\frac{1}{2}M, P\frac{2}{1}T$, pmt, $3p\frac{1}{3}m\frac{1}{2}t$. Rose, fig. 51[a].
p,m,t, $M\frac{1}{2}T. P\frac{1}{2}M, P\frac{2}{1}T, 3P\frac{1}{3}M\frac{1}{2}T, 3p\frac{1}{4}m\frac{1}{2}t$. Rose, fig. 47[a].

These combinations present the cube, P,M,T; the octahedron, PMT; the Pentagonal Dodecahedron, $M\frac{1}{2}T. P\frac{1}{2}M, P\frac{2}{1}T$; and the Hemihexakisoctahedron, grouped in various methods. Iron Pyrites is the mineral which presents all the varieties. Those containing P,M,T, are complete prisms with incomplete pyramids, and the others are incomplete prisms with incomplete pyramids. See Part II. Class 3, Order 1, page 105; Class 3, Order 4, page 107; Class 5, Order 4, page 117.

469. *Analysis of these Combinations.*

The inclination of a plane of pmt upon a plane of $3P\_MT_+$ is $90° + x$, in which formula, x is the inclination of a plane of $3P\_MT_+$ to the tripolar normal.

The inclination of a plane of $3p\frac{1}{3}m\frac{1}{2}t$ to a plane of $M\frac{1}{2}T. P\frac{1}{2}M, P\frac{2}{1}T$ is $x + y$, in which, x is the inclination of a plane of $M\frac{1}{2}T. P\frac{1}{2}M, P\frac{2}{1}T$ to the tripolar normal, and y is the supplement of the inclination of a plane of $3p\frac{1}{3}m\frac{1}{2}t$ to the tripolar normal.

The inclination of a plane of p,m,t, upon a plane of $3P\frac{1}{3}M\frac{1}{2}T$ is found by the problem in § 467.

470. THE ASPECT OF COMPLEX CRYSTALS BELONGING TO THE OCTAHEDRAL SYSTEM OF CRYSTALLISATION, USEFUL AS A MEANS OF DISCRIMINATING THEIR COMPONENT FORMS.

As the 13 combinations quoted in § 341 are all that belong to the Octahedral System of Crystallisation, it follows that every single crystal of this class must either consist of one of these combinations, or of two or more of them combined together. In every example of such combinations, there will be *one combination predominant*, and all the co-existing combinations will appear upon it subordinately, replacing its angles or edges. Perhaps the combinations called "middle crystals," such as P,M,T. PMT, Model 29, and M_-T. P_-M, P_+T, PMT, Model 92, may be held to be exceptions from this rule; yet even in these, one of the combinations always predominates over the other.

In examining a crystal of this kind, you first fix your attention upon the combination which predominates, and then endeavour to discriminate the co-existing subordinate combinations. Since no combination can occur but such as are indicated in § 341, and since these can only occur upon one another in a certain regular order, there is not many forms to choose among, nor much difficulty to be found in discriminating them all from one another, even when several occur upon one crystal. The following table will, however, probably afford assistance in discriminating a variety of forms such as can co-exist on the same crystal.

The points attended to in the table are these: 1.) The *predominant combination,* 2.) the number and inclinations of the planes that replace the *edges* of the predominant form, and 3.) the number and inclination of the planes that replace the *angles* of the predominant form. Where the edges and angles are of different kinds, this difference is noted. No respect is paid to *hemihedral* subordinate forms. No *double replacements* are attended to, because it would make the table too long. The reader will therefore observe, that *the cube with* 3 *planes replacing each edge,* is P,M,T, mt, m_-t, m_+t. pm, p_-m, p_+m, pt, p_-t, p_+t, and that *the rhombic dodecahedron with* 3 *planes replacing each edge,* is MT.PM,PT, $3p_-mt$, $6p_-mt_+$.

The table contains only such combinations as are of frequent occurrence in the mineral kingdom.

"Among all the occurring forms of the octahedral system of crystallisation, homohedral and hemihedral, the most important are the following:

The Octahedron	= PMT.
The Cube	= P,M,T.
The Rhombic Dodecahedron	= MT. PM, PT.
The Icositessarahedron $\frac{1}{2}$............	= $3P\frac{1}{2}MT$.
The Tetrahedron	= $\frac{1}{2}PMT$.
The Pentagonal Dodecahedron $\frac{1}{2}$...	= $M\frac{1}{2}T$. $P\frac{1}{2}M$, $P\frac{2}{1}T$.

"These forms occur more frequently than any others; they very often form complete isolated crystals, and when they occur in combination, *it is their* faces which predominate. As this is not the case with the

other forms, they are of far less importance."—ROSE, *Elemente der Krystallographie, p.* 58.

THE OCTAHEDRON predominant. PMT.

Angles replaced by:

1 tangent plane = p,m,t.

4 planes, inclining on the edges = m_-t, m_+t. p_-m, p_+m, p_-t, p_+t.

4 planes, inclining on the planes = $3p_-mt$.

2 planes, inclining on the edges = m_-t. p_-m, p_+t.

8 planes, inclining obliquely, partly on the edges and partly on the planes = $6p_-mt_+$.

Edges replaced by:

1 tangent plane = mt. pm, pt.

2 planes, inclining on the planes = $3p_+mt$.

THE CUBE predominant. P,M,T.

Angles replaced by:

1 tangent plane = pmt.

3 planes, inclining on the planes = $3p_-mt$.

3 planes, inclining on the edges = $3p_+mt$.

3 planes, inclining obliquely, partly on the planes and partly on the edges= $3p_-mt_+$.

6 planes, inclining obliquely, partly on the planes and partly on the edges = $6p_-mt_+$.

Edges replaced by:

1 tangent plane = mt. pm, pt.

1 plane with different inclination at each side = m_-t. p_-m, p_+t.

2 planes = m_-t, m_+t. p_-m, p_+m, p_-t, p_+t.

THE RHOMBIC DODECAHEDRON predominant. MT. PM, PT.

Unipolar angles replaced by:

1 tangent plane = p,m,t.

4 planes, inclining on the planes = m_-t, m_+t. p_-m, p_+m, p_-t, p_+t.

4 planes, inclining on the edges = $3p_-mt$.

8 planes, inclining obliquely = $6p_-mt_+$.

Tripolar Angles replaced by:

1 tangent plane = pmt.

3 planes, inclining on the edges = $3p_-mt$.

3 planes, inclining on the planes = $3p_+mt$.

6 planes, inclining obliquely = $6p_-mt_+$.

Edges replaced by:

1 plane = $3p_-mt$.

2 planes = $6p_-mt_+$.

THE ICOSITESSARAHEDRON predominant. $3P_-MT$.

Unipolar angles replaced by:

1 tangent plane = p,m,t.

4 planes inclining on the edges = m_-t, m_+t. p_-m, p_+m, p_-t, p_+t.

Tripolar angles replaced by 1 plane = pmt.
Bipolar angles replaced by 1 plane = mt. pm, pt.
Long edges replaced by 1 plane = m_-t, m_+t. p_-m, p_+m, p_-t, p_+t.

THE TRIAKISOCTAHEDRON predominant. $3P_+MT$.

Tripolar angles replaced by 1 plane = pmt.
Long edges replaced by 1 plane = mt. pm, pt.
Unipolar angles replaced by 1 plane = p,m,t.

THE TETRAKISHEXAHEDRON predominant.
M_-T, M_+T. P_-M, P_+M, P_-T, P_+T.

Unipolar angles replaced by 1 plane = p,m,t.
Long edges replaced by 1 plane = mt. pm, pt.
Tripolar angles replaced by 1 plane = pmt.

THE HEXAKISOCTAHEDRON predominant. $6P_-MT_+$.

Unipolar angles replaced by 1 plane = p,m,t.
Bipolar angles replaced by 1 plane = mt. pm, pt.
Tripolar angles replaced by 1 plane = pmt.

THE TETRAHEDRON predominant. $\frac{1}{2}$PMT.

Edges replaced by:

1 tangent plane = p,m,t.
2 planes = $\frac{1}{2}(3p_-mt)$ Znw.

Angles replaced by:

1 plane = $\frac{1}{2}$pmt Zne.
3 planes, inclining on the planes = mt. pm, pt.
3 planes, inclining on the edges = $\frac{1}{2}(3p_-mt)$ Zne.
6 planes = $\frac{1}{2}(6p_-mt_+)$.

THE HEMICOSITESSARAHEDRON predominant. $\frac{1}{2}(3P_-MT)$.

Obtuse angles replaced by 1 plane = $\frac{1}{2}$pmt.
Acute angles replaced by:

3 planes, inclining on the short edges = mt. pm, pt.
6 planes, inclining on the planes = $\frac{1}{2}(6p_-mt_+)$.

Short edges replaced by 1 plane = $\frac{1}{2}(3p_+mt)$.

THE HEMIHEXAKISOCTAHEDRON WITH PARALLEL FACES predominant.
$3P_-MT_+$.

Unipolar angles replaced by 1 plane = p,m,t.

2 planes, inclining on the long edges = m_-t. p_-m, p_+t.
4 planes = $3p_-mt_+$.

THE PENTAGONAL DODECAHEDRON predominant. M_-T. P_-M, P_+T.

Long edges replaced by 1 plane = p,m,t.
Tripolar angles replaced by 1 plane = pmt.

3 planes = $3p_-mt_+$.

Bipolar angles replaced by 1 plane = mt. pm, pt.

471. I have paid no attention to hemihedral replacing forms in the above table, because their planes replace the angles and edges of other forms precisely as do the planes of the corresponding homohedral forms. The reader has only to remember that whenever a hemihedral subordinate form is present, the predominant combination is affected by its replacements only at four tripolar angles instead of eight, which four angles are invariably either those at the poles Znw Zse Nne Nsw, or at the poles Zne Zsw Nnw Nse. There are in all but four varieties of hemihedral combinations, namely, $\frac{1}{2}$pmt; $\frac{1}{2}$(3p$_-$mt); $\frac{1}{2}$(3p$_+$mt); $\frac{1}{2}$(6p$_-$mt$_+$), and these all contain hemioctahedrons, and, therefore, only affect alternate octants of a crystal. The pentagonal dodecahedron, and hemihexakisoctahedron with parallel faces, do not, as I have shown, § 281, properly belong to the hemihedral combinations; and they cannot, as I shall now proceed to show, cause the crystallographer any perplexity.

"Any number of holohedral forms may occur in combination with each other, and with any hemihedral forms with inclined faces, or with any hemihedral forms with parallel faces. IT IS SAID, *that hemihedral forms with inclined faces have never been observed in combination with hemihedral forms with parallel faces.*"—MILLER, *Treatise on Crystallography*, p. 24.

"The different hemihedral forms with inclined faces can occur in combination with one another, as can also those with parallel faces; and forms out of either class can occur in combination with homohedral forms. The combinations already cited contain many examples of this character. *But hemihedral forms with inclined faces have never been observed in combination with hemihedral forms with parallel faces, although* WE CAN SEE NO REASON WHY THIS SHOULD BE SO."—ROSE, *Elemente der Krystallographie*, p. 58.

If you refer to ROSE's classification of minerals, contained in Part II. of this work, you will find that "the minerals whose crystals present hemihedral forms with parallel faces are distinguished by one star *, and those which present hemihedral forms with inclined faces, by two stars **, page 2; and if, farther, you examine the crystals belonging to these minerals, pages 15 to 32, you will observe that every mineral which *sometimes* presents either M$_-$T. P$_-$M, P$_+$T, or 3P$_-$MT$_+$, *never* presents any one of the four hemihedral combinations, and *vice versa*, every mineral which *sometimes* presents any one of the four hemihedral combinations, *never* presents either M$_-$T. P$_-$M, P$_+$T, or 3P$_-$MT$_+$. In other words, the presence of either M$_-$T. P$_-$M, P$_+$T, or 3P$_-$MT$_+$ upon a mineral, proves the mineral to be one of those which presents *no kind* of hemihedral forms, while the presence of any of the four hemihedral combinations is a guarantee that the mineral does not include M$_-$T. P$_-$M, P$_+$T, or 3P$_-$MT$_+$ among its possible combinations. The question, *why* any given mineral produces only homohedral forms? belongs to the theory of crystallisation, and need not be discussed here. I have speculated sufficiently regarding it in SECTION XI. In the meantime, we know, *from observation*, that certain minerals present hemihedral

forms, and that certain others do not. This simple fact contains an answer to the doubt expressed or implied in the quotations from MILLER and ROSE. The *so-called hemihedral forms with parallel faces,* are in fact homohedral forms, and the minerals which present them are found by experience to be of that class which never present any hemihedral form whatever. Hence the reason why hemihedral forms with inclined faces never occur in combination with hemihedral forms with parallel faces, is, that the two kinds of forms characterise minerals of entirely different nature. It cannot be said, that I am taking advantage of merely new definitions of the words *form* and *combination,* in order to clear up this difficulty; for it must be remembered, that I have shown every one of the four kinds of hemihedral combinations to be produced by the *partial* intersection of *eidogens,* or by the intersection of *half rhombuses,* whereas the homohedral combinations, including $M_{-}T$. $P_{-}M$, $P_{+}T$, and $3P_{-}MT_{+}$ in the number, are all produced by the intersection of *eidogens* or *complete rhombic prisms.* This is a tangible ground for the distinction between hemihedral and homohedral combinations.

With the expression of these views, I dismiss the Octahedral System of Crystallisation, in the account of which, I think, I have clearly established the separate identity of nine homohedral and four hemihedral combinations, and shown how the combinations of these varieties with one another can be readily analysed, and by what symbols they can be intelligibly and conveniently described.

II. THE PYRAMIDAL SYSTEM OF CRYSTALLISATION.

472. The character of the Forms belonging to this system, as given by ROSE, is this:—They have three axes, which are all placed at right angles to one another, but of which two are equal and different from the third. Therefore, the AXES $= p_x^a\, m^a\, t^a$, including $p_-^a\, m^a\, t^a$ and $p_+^a\, m^a\, t^a$.

ROSE's enumeration of the Forms belonging to this system of crystallisation, is as follows:

A. Homohedral Forms:

1. The Quadratic Octahedron:
   first position, ... $= P_xMT$. Model 12.
   second position, ... $= P_xM, P_xT$. Model 13.
2. The Horizontal Planes, ... $= P$.
3. The Quadratic Prism:
   first position, ... $= MT$. Model 2. } Model 4.
   second position, ... $= M,T$. Model 3. }
4. The Eight-sided Pyramid, or Dioctahedron, ... $= P_xM_{-}T, P_xM_{+}T$. Model 22ª.*
5. The Eight-sided Prism .... $= M_{-}T. M_{+}T$.

* The Dioctahedron, or Eight-sided Pyramid, would be represented by Model 22, provided the upper pyramid $P_{-}MT$ was away, and the planes of $PM_{-}T$ and PMT_{-}, were continued from the equator in both directions, till they met in eight-faced solid angles at *the poles Z and N.*

B. Hemihedral Forms:

1. The Tetrahedron .... $= \frac{1}{2}P_xMT$.
2. The Hemi-dioctahedron, ... $= \frac{1}{2}(P_xM_-T,\ P_xM_+T)$.

Rose's Catalogue of the Minerals that belong to the Pyramidal System, is given in Part II., pages 3—12. A synopsis of the forms and principal combinations belonging to the system is given at pages 32, 33. A symbolic catalogue of the forms and combinations presented by the crystals of the minerals of this system, is given at pages 33—43.

The Axes of every natural crystal or combination belonging to this system, are $p^a_x m^a t^a$; that is to say, $p^a_- m^a t^a$, or $p^a_+ m^a t^a$, but not $p^a m^a t^a$. See § 340, No. 2), and Part II. page 32.

The Forms of which a crystal or combination is composed, may be either equiaxed or unequiaxed. The latter kinds generally prevail, but the former are not excluded, although they never produce an equiaxed combination.

Rose designates the axes of the crystals of this system as follows: p^a by c, m^a by a, t^a by a; whereas, in the octahedral system, all the three axes were designated by the term a, and in the prismatic system they are designated by c, a, b. Hence, p^a is sometimes a, and sometimes c, and t^a is sometimes a and sometimes b. *It appears to me, that much is lost and nothing gained by altering the names of the axes, either in this or any other system.* I shall, therefore, always call them $p^a\ m^a\ t^a$. This will enable me to describe the forms belonging to the pyramidal and the other four systems of crystallisation briefly and yet distinctly, but it will oblige me to pay little attention to Rose's descriptions and illustrations, many of which are rendered tedious by two injudicious principles of the German system of crystallography, one of which is the above-mentioned change in the designation of the axes in different systems; the other is the assumption of *fundamental forms*, and the consequent abandonment of a measure of unity for the value of the axes. See § 479. To point out the special evils which flow from these two injudicious principles, would take up too much room, and needlessly interfere with the description of the system.

A. Homohedral Forms of the Pyramidal System.

1. The Quadratic Octahedron.

P_xMT, Model 12. P_xM, P_xT, Model 13.

473. The quadratic octahedron is a pyramid with a square base, similar to the base of the regular octahedron, but the principal axis of which is either greater or less than the principal axis of the regular octahedron. When the principal axis is *greater* than that of the regular octahedron, the result is an acute *quadratic octahedron*, similar to Model 13; but when it is *less* than that of the regular octahedron, the result is an obtuse *quadratic octahedron*, similar to Model 12.

474. The quadratic octahedrons assume two different positions on the crystals of this system, which positions Rose calls the *first* position and *second* position. As the octahedrons which occupy these two different positions require different symbols, I shall treat of them separately.

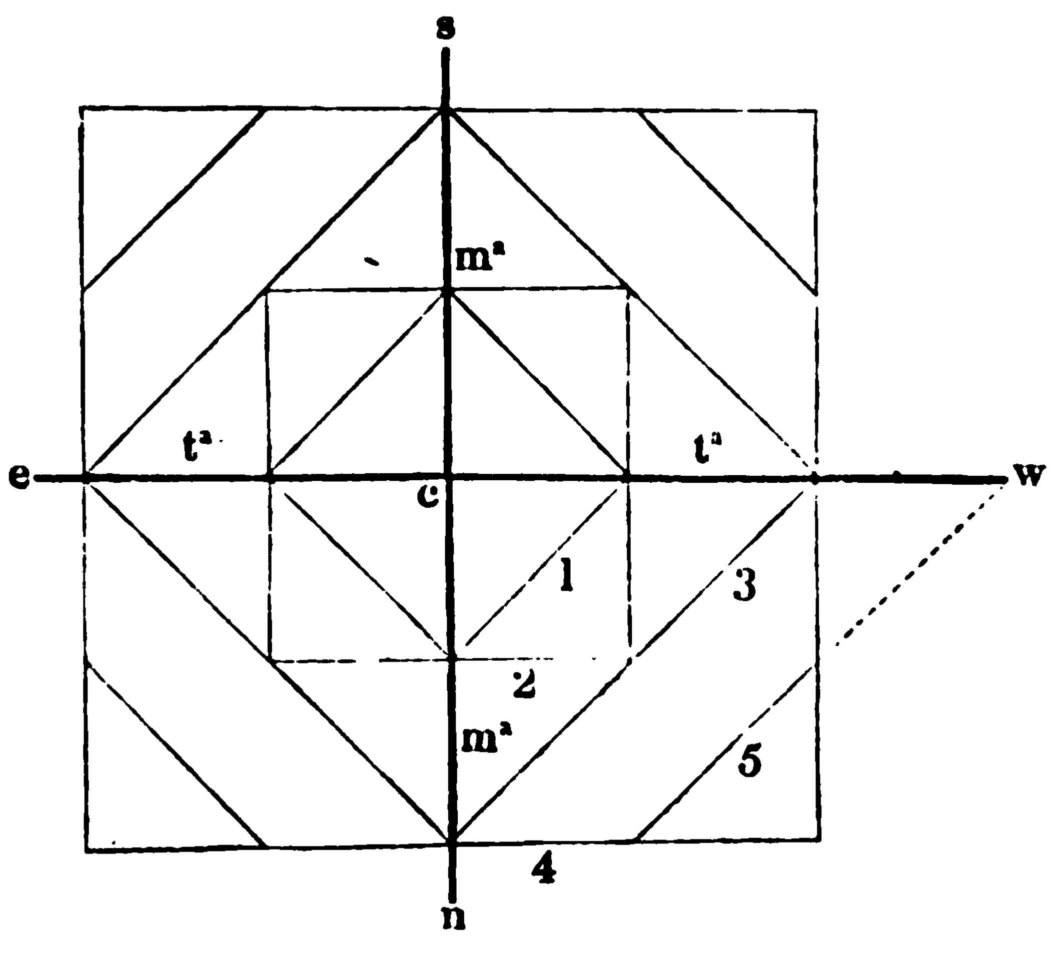
a.) Octahedrons of the first position.—These are the octahedrons whose planes occur in the north-east zone and north-west zone, and whose oblique terminal edges form the boundary of the north meridian

and the east meridian. See the coloured marks on Model 12. The octahedrons of this class require the symbol P_xMT; or if *acute*, the symbol P_+MT, and if *obtuse*, the symbol P_-MT.

b.) Octahedrons of the second position. These octahedrons have their planes on the north zone and the east zone, and their oblique terminal edges form the boundary of the north-east meridian and north-west meridian. See Model 13. They consist of a rhombic form of the east zone, and an equal and similar rhombic form of the north zone. Hence, they require the symbol P_xM, P_xT; or, if *acute*, the symbol P_+M, T_+T, and if *obtuse*, the symbol P_-M, P_-T.

475. *Simultaneous occurrence of Octahedrons of the two positions on one crystal.* These two kinds of octahedrons often occur together on one crystal, when they necessarily present planes on the north zone, the east zone, the north-east zone and the north-west zone. The planes upon the north and east zones, or those upon the other two zones, are either subordinate or predominant, according to the relative size of the two octahedrons, P_xMT and P_xM, P_xT.

The annexed diagram shows the relative positions of the equators of the octahedrons which belong to these two classes. The lines m^a and t^a represent the two equatorial axes, and the letters n e s w, the poles of these axes. Put the square marked No. 1, equal to the base of an octa-



hedron, whose axes p^a is equal to the diagonal of the square, or to twice the semi-axis c to m^a. The symbol for this octahedron will be PMT, equal to the regular octahedron. Let square 2 be an octahedron with the same axis p^a. This will require the symbol PM, PT. The axes are again all equal, but the planes in this case belong to the north and east zones, whereas in the first case they belonged to the octahedral zones.

With the same axis p^a, take square 3 for the base of the octahedron.

In this case, the equatorial axes are doubled in length, and the planes belong to the octahedral zones; the symbol is consequently PM_2T_2, or better expressed $P\frac{1}{2}MT$.

With the same axis p^a, take square 4 for the base of the octahedron. In this case, the equatorial axes are doubled in length as before, but the planes belong to the north zone and east zone; consequently, the symbol becomes PM_2, PT_2; or $P\frac{1}{2}M$, $P\frac{1}{2}T$.

With the same axis p^a, take square 5 for the base of an octahedron, having its planes on the octahedral zones. In this case the equatorial axes are trebled, and the form requires the symbol PM_3T_3, or $P\frac{1}{3}MT$.

In this manner you may produce an unlimited series of octahedral forms, containing an octahedron of the first position and of the second position alternately: the octahedrons of the two kinds differing essentially in this, that with the same axes, those which have the symbol P_xM, P_xT, contain twice the equatorial base of those which have the symbol P_xMT. Thus, the combination PM, PT, contains twice the base of the form PMT, although the axes of both of them are p^a m^a t^a, and the north and east meridians are precisely equal.

Example of another series of Octahedrons.

Let axis p^a be equal to a side of square 4, which is twice the length of an axis of square 1. Then proceed as before, and with the given axis produce the following series of octahedrons:—

With square 1,	you have	P_2MT
2,	—	P_2M, P_2T
3,	—	PMT
4,	—	PM, PT
5,	—	$P\frac{2}{3}MT$

As in this example the vertical axis p^a, assumed to be common to all the bases, is made equal to the axes of squares 3 and 4, the two smaller squares, 1 and 2, present examples of acute octahedrons. If we suppose the existence of a series of squares yet smaller than No. 1, we of course provide for so many octahedrons still more acute than P_2MT. The next smaller square, for example, would produce the octahedron P_4M, P_4T; the next P_4MT, the next P_8M, P_8T, the next P_8MT, and so on in regular order, provided you take no intermediate squares, such as that marked No. 5, but only such as are each half the size of the next in order above it.

It follows, from these considerations, that every octahedron of the *first position* can be denoted by the symbol P_xMT, and every octahedron of the *second position* by the symbol P_xM, P_xT; and that in both of these symbols, the algebraic index $_x$ of any given form can be replaced by an arithmetical index, when the relation is known of axis p^a to axis m^a or t^a.

476. *Combinations of Octahedrons which replace each other's edges.*— It is evident, on an examination of the diagram in § 475, that the octahedrons PMT and PM, PT, cannot appear on the same crystal, if the base

of the latter is precisely twice the size of the base of the former, because the planes of PM, PT, lye exactly in the same position, and at the same distance from the axes, as the edges of PMT; yet, we find by the combination of the regular octahedron PMT, with the rhombic dodecahedron MT. PM, PT, that the forms PMT and PM, PT, *do combine* with one another. The solution of this difficulty is found in the fact, that while *the axes of the two combining forms continue relatively the same, the forms differ absolutely in size.* Hence the form PMT can have its oblique edges replaced by the combination PM, PT if the base of the latter is ever so little smaller than twice the size of the base of the former; and the combination PM, PT, can have its oblique edges replaced by the form PMT, if the base of the latter is anything *more than half*, or anything *less than twice* the extent of the base of the former. Thus, we have Model 64, mt. pm, pt, PMT, in which the axes of mt. pm, pt are $p^a m^a t^a$, and those of PMT are the same, but in which the base of the octahedron, pm, pt, is more than half, but less than twice, the extent of the base of the co-existing octahedron PMT. A similar example is afforded by Model 77, containing the forms, p. pm, pt, PMT.

477. A word or two may be said respecting the *indices of the symbols* of the octahedrons that replace one another's edges. If the edges of the form P^3_2MT are replaced by tangent planes, these planes must have the same relations to $p^a m^a t^a$ as the planes of the form P^3_2MT. Thus, if square 3 is the base of P^3_2MT, then square 4 will be the base of the replacing octahedron, and will require the symbol P^3_2M, P^3_2T. But if square 2 is the base of the combination whose edges are replaced, and this combination bears the symbol P^3_2M, P^3_2T, then the form which replaces the edges of P^3_2M, P^3_2T, must have a base corresponding to square 3, the equatorial axes of which are twice the length of the axes of square 2, while the axes p^a of both is the same. Hence the symbol of the replacing octahedron having a base like square 3, must be P^3_4MT.

478. *Every Mineral of the Pyramidal System has belonging to it, a double series of Quadratic Octahedrons.*—As we cannot fix a limit to the number of forms which may occur in any given zone, we must admit that every different mineral may have belonging to it a series of octahedrons of each of the two positions, all the octahedrons of one position having the symbol P_xM, P_xT, and all those of the other position, the symbol P_xMT. The indices of these symbols may be either $+$ or $-$. The value changes with every mineral, and with every different octahedron of the same mineral. Yet the indices of every octahedron of the same mineral have a very simple relation to one another. See § 326. We cannot fix a theoretical limit to the possible variety of indices, yet practically, the variety is found to be very small, as I have shown at Part II. page 32.

479. *Rule according to which the Octahedrons of a given Mineral are ascribed to the north and east zones, or to the north-east and north-west zones.*

a.) "In order to examine the relations to one another of the octahedrons of a given mineral, they are all made to depend upon a chosen variety, which is called the *Fundamental Form* or *Principal Octahedron.*

b.) "Which, among all the octahedrons that occur, is chosen for this purpose, is quite immaterial; but we generally take that which occurs most frequently, or that whose planes commonly predominate in the combinations, or that to which all the other forms bear the simplest relations. No more definite rule than this can be given for the choice of the fundamental form.

c.) "The designation of the different quadratic octahedrons is then as follows:—

The fundamental form ($a : a : c$),

Forms of the first position ($a : a : mc$),

Forms of the second position ($a : \infty a : mc$), in which signs, m is always a simple, whole, or fractional number, [and c the name of the vertical axis]." Rose, *Elemente der Krystallographie*, p. 63.

480. It will be learnt from this quotation, that in the symbols given by Rose for the octahedrons of this system, there is *no measure of unity retained.* Hence, the symbol ($a : a : c$) serves equally to designate the forms represented by

Model 12. P$\frac{2}{3}$MT, the fundamental form of Zircon.
Model 13. P$\frac{5}{2}$M, P$\frac{5}{2}$T, the fundamental form of Anatase.

And the means by which the symbol ($a : a : c$) is made to discriminate one fundamental form from another, is the accompanying register of the measurements of the angles of each fundamental form. This is a capital defect, and which I think exists in all the various editions of the German system of crystallography. There is no way of showing the relation of the octahedrons to one another so easy and distinct, as to mark those of different zones by different symbols, and to state the relative lengths of their axes in figures.

Next, observe, that while the fundamental form, once chosen, is placed in the first position, and all the other octahedrons are made to depend upon it, Rose declares that no definite rule can be given for the choice of this fundamental form; and that, in fact, the choice is of no moment. I point out this declaration for the purpose of remarking, that it is a mistaken principle, and that on the contrary as much care must be taken in placing the forms of the pyramidal system as in placing those of the octahedral system; and this I think a point of sufficient importance to merit a full explanation.

If we take the combination P,M,T, mt. pm, pt, PMT, Model 64, and place the planes of PMT on the north and east zone, then, the planes of mt. pm, pt, fall of course into the octahedral zones, for the oblique planes of the octahedron and those of the dodecahedron, occupy respectively what are called the *first* and *second positions* of the octahedrons of the pyramidal system. *If, after thus* changing the position of Model 64, we

attempt to describe its forms in accurate symbols, we obtain the following results:

$$\text{m,t. } P\tfrac{10000}{7072}M,\ P\tfrac{10000}{7072}T,\ p\tfrac{10000}{14141}\text{mt.}$$

These are awkward indices, but if we shorten them to

$$\text{m,t. } P\tfrac{10}{7}M,\ P\tfrac{10}{7}T,\ p\tfrac{10}{14}\text{mt,}$$

the angle intimated across an edge of $P\frac{10}{7}M$ will be 110° 1′ instead of 109° 28′, which includes an error of half a degree. This error is avoided by turning the crystal 45° horizontally, and throwing the planes into their usual positions which require the symbols, mt. pm, pt, PMT.

Hence, the reason why the regular octahedron is called PMT, and not P_xM, P_xT, is simply, because the first is *very convenient*, and the last *very inconvenient*. The case is precisely the same with the two kinds of octahedrons of the pyramidal system. If you take all the octahedrons of any given mineral of this system, and give them the shortest and most convenient symbols that their measurements admit; and if you then throw their planes from the *first position* into the *second*, you will find that their symbols cannot be completed by indices equally simple with those that were used before. Hence, the positions of the octahedrons of any given mineral are to be regulated by the indices demanded to express the relation of their axes. This is a case in which the principle of expediency is in fact the soundest philosophy.

2. The Horizontal Planes. P.

481. It seems needless to say more of this form, than that it is the form P, consisting of two horizontal planes. Rose's symbol for it is $(\infty a : \infty a : c)$.

3. The Quadratic Prisms. MT and M,T.

482. The quadratic prisms are four-sided vertical prisms, having a square base. There are two kinds, one of them containing the forms M and T, the other, the form MT. Rose designates the latter $(a : a : \infty c)$, or the prism of the *first* position; the former, $(a : \infty a : \infty c)$, or the prism of the *second* position. The prism of the first position = MT, is that which replaces the horizontal edges of the pyramids of the first position = P_xMT. The prism of the second position = M,T, is that which replaces the horizontal edges of the pyramids of the second position = P_xM, P_xT.

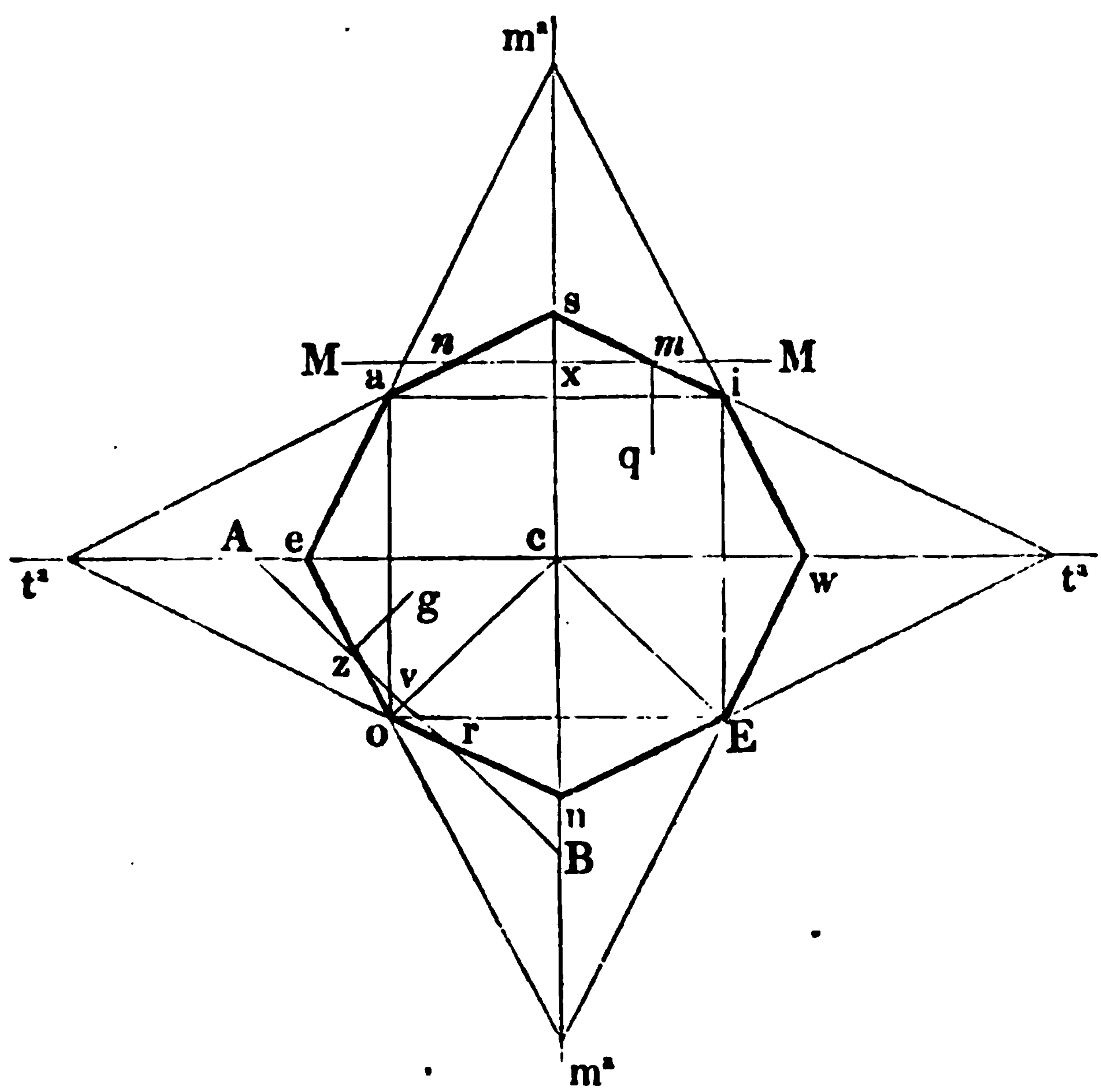
The first of these prisms is shown by Model 2, P_-,MT. The second by Model 3, P_+,M,T. The two prisms often occur together, as shown in Model 4, P_+,M,T, mt. or P_+,m,t, MT.

4. The Dioctahedron, or Eight-Sided Pyramid. P_xM_-T, P_xM_+T.

483. This combination would be represented by Model 22, P_-MT, *PM_-T, PMT_-*, provided the four upper and four lower planes consti-

tuting the form P_MT were away, and the planes of the other two octahedrons, PM_T, PMT_, were continued both ways from the equator till they met in eight-faced solid angles at the poles Z and N. The combination would then consist of sixteen scalene triangles. It is a combination that never produces a complete crystal, and only in one or two cases appears predominant; but it occurs pretty frequently in combination with the quadratic pyramids and quadratic prisms, exhibiting four pair of two planes on the zenith end of the crystal, and a similar number of planes on the nadir end.

484. The dioctahedron is a combination of two similar and equal octahedrons with rhombic bases, one of them having the equatorial relation of $m \underline{^a}\, t^a$, and the other the relation of $m \perp^a t^a$. The following diagram shows at once the equator of the dioctahedron, and of its two component pyramids.



Assume, as an example, the existence of the dioctahedron, $P_3M_1T_2$, $P_3M_2T_1$. Then the rhombus n t^a s t^a, will be the base of the octahedron $P_3M_1T_2$; and the rhombus e m^a w m^a will be the base of the octahedron $P_3M_2T_1$. These two forms cut one another at the points a i E o, and the base or equator of the resulting combination is the octagon n o e a s i w E. The eight faces of the pyramid which rests upon this equator, incline upon the eight edges n o, o e, n E, E w, &c.

The solid angles of the dioctahedron are therefore of three different kinds, and are situated as follows: one kind at the poles Z, N; another kind at *the poles n, e, w, s,* and a third kind at the poles nw, ne, se, sw.

The edges of the dioctahedron are also of three different kinds:—Eight edges of one kind occur round the equator; eight of a second kind on the north and east meridians; and eight of a third kind on the north-east and north-west meridians.

5. The Eight-Sided Prism. M_T, M+T.

Varieties of this Combination, according to Miller:

$M\frac{1}{2}T, M\frac{2}{1}T.$	$M\frac{1}{5}T, M\frac{5}{1}T.$
$M\frac{1}{3}T, M\frac{3}{1}T.$	$M\frac{3}{5}T, M\frac{5}{3}T.$
$M\frac{2}{3}T, M\frac{3}{2}T.$	$M\frac{1}{7}T, M\frac{7}{1}T.$
$M\frac{1}{4}T, M\frac{4}{1}T.$	

485. The equator of the eight-sided prism has the same characters as the equator of the right-sided pyramid or dioctahedron. Refer to the diagram, page 241. The rhombus $n\,t^a s\,t^a$ is the equator of the rhombic prism $M\frac{1}{2}T$. The rhombus $m^a e\,m^a w$ is the equator of the rhombic prism $M\frac{2}{1}T$. The combination of these two rhombic prisms produces an equiaxed eight-sided prism, the equator of which is the octagon, shown by the thick lines n o e a s i w E in the diagram. As the axes m^a and t^a of the two separate rhombic prisms differ, so do the external angles of the combination; but there are always four angles of one value, and four of another value: the angles at n e s w being all similar, and those at o a i E also similar. A single angle of one kind added to a single angle of the other kind, make together an angle of 270°. See § 399. Hence, if the angle across the north pole is called n; and that across the north-west pole is called E; then, E is 270° — n; and n is 270° — E.

The eight-sided prisms of most frequent occurrence are $M\frac{1}{2}T$, $M\frac{2}{1}T$. and $M\frac{1}{3}T$, $M\frac{3}{1}T$. In general, the eight-sided prisms occur subordinately, replacing the edges of the prisms M,T, or MT, or M,T, mt. They very rarely occur predominant, or without the square prisms.

486. It is easy to see in the diagram, page 241, the positions which the planes of M_T, M+T, occupy in reference to the two square prisms. Let the square o a i E be the equator of the prism M,T. Then if $m\frac{1}{2}t$, $m\frac{2}{1}t$, replace the edges of M,T, the planes of $m\frac{1}{2}t$ appear on the north and south sides of the combination, and near the bipolar normals; while the planes of $m\frac{2}{1}t$ appear on the east and west sides of the combination, yet also near the bipolar normals. Again, let the line AB represent a side of the square prism MT. Then, the planes of $m\frac{1}{2}t$ will appear attached to the north and south poles of the prism MT, and the planes of $m\frac{2}{1}t$ to the east and west poles. Finally, let the square prism mt replace the corners of M,T, and produce the combination M,T, mt, as is shown at the corner marked v. Then, the planes of $m\frac{1}{2}t$ replace the edges between mt and M, and those of $m\frac{2}{1}t$ the edges between mt and T, as will be perceived if the octagon in the diagram be supposed to become *small enough* to touch the angles near v and r.

B. Hemihedral Forms of the Pyramidal System.

1. THE TETRAHEDRON. $\frac{1}{2}P_xMT$.

487. This is a hemihedral form with inclined faces, bearing the same relation to the octahedron, P_xMT, that the regular tetrahedron, $\frac{1}{2}$PMT, bears to the regular octahedron, PMT. It differs from the regular tetrahedron in these particulars: its axes are $p_x^a m^a t^a$; its edges at Z and N have not the same angle as its vertical edges; its planes are isosceles triangles; its north meridian and east meridian are rhombuses, while its equator is a square.

The angle across the edges at Z and N is the complement of the angle across the equator of its corresponding homohedral octahedron. The angle across the lateral edges is the complement of the angle across the terminal edges of the corresponding homohedral octahedron.

2. THE HEMI-DIOCTAHEDRON.

488. The hemihedral variety of the dioctahedron is a parallel-faced hemihedral form, of the same character as the hemihedral forms described in § 272. Refer to the diagram in page 241. Conceive the axis of a rhombic cutting prism to cross the axes m^a t^a, in the direction of the line s t^a. The four planes of such a prism would meet at the lines marked s i and o n. Conceive the axis of another rhombic cutting prism to cross the axes m^a t^a in the direction of the line m^a w. The four planes of such a prism would meet at the lines e a and E a. The eight planes thus produced constitute the hemi-dioctahedron. This combination never constitutes a complete crystal, but only appears subordinately upon the angles of other combinations. There may be another hemi-dioctahedron in the alternate octants of the dioctahedron. The edges of the planes of this second variety would of course meet at the alternate equatorial lines i w, e o, a s, n e. The different kinds of the hemi-dioctahedron are easily distinguished by means of the signs denoting the polaric positions of their planes. At present, they are forms of rare occurrence and little importance.

489. ZONES OF THE PYRAMIDAL SYSTEM.

The equatorial zone embraces the forms M, M_-T, MT, M_+T, T, and therefore includes all the different prisms of this system.

The north zone embraces the form, P, P_-M, PM, P_+M, M. The east zone embraces the forms, P, P_-T, PT, P_+T, T. The two zones together, therefore, include all the square-based octahedrons of the second position, the horizontal planes P, and the square prism of the second position.

The north-east and north-west zones embrace the forms P, P_-MT, PMT, P_+MT, MT. They, therefore, include all the square-based octahedrons of the first position, the horizontal planes P, and the square prism of the *first position.*

The only form of the pyramidal system that is not crossed by the above zones, is the dioctahedron, the planes of all the varieties of which fall in the open triangular spaces situated betwixt the equator and the intersections of the four meridians. Hence, a plane found in the division Z^2n^2w will belong to the form $P_xM_{-}T$, while a plane found in the adjoining division Z^2nw^2 will belong to the form $P_xM_{+}T$.

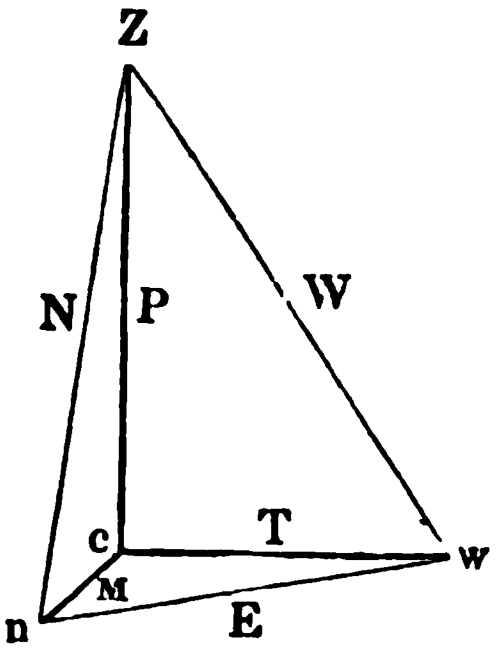
If a combination of the pyramidal system be divided into octants by the equator and the north and east meridians, and if the octants be four of one kind and four of another kind, the combination contains a tetrahedron. § 285.

If a combination of this system be divided into vertical octants by the intersection of the four meridians, and if the octants be four of one kind and four of another kind, the combination contains a hemi-dioctahedron.

Mathematical Properties of the Forms of the Pyramidal System.

QUADRATIC OCTAHEDRONS OF THE FIRST POSITION.

490. Let the annexed figure represent the octant of *a square-based octahedron of the first position*, having the properties which are described in § 300, &c., as being common to right-angled solid triangles so formed.



491. PROBLEM. *Given*, Model 12, P_MT, *with the angle across the equator, namely, the inclination of plane* Znw *on plane* Nnw = 84° 20′ (*Zircon*, ROSE); *required, the value of the index* _ *in the symbol.*

Take a right-angled solid triangle with pole n for its vertex. Then, half the angle across the equator $= \frac{84° \, 20'}{2} = 42° \, 10'$, will be angle A; the plane angle cnw = 45°, will be side *b*; and the angle across the edge M, will be angle C = 90°. With these data, you can find side *a* of the solid triangle, which is equal to the plane angle Znc in the diagram, the tangent of which side is the required value of axis p^s, when axis m^s = axis t^s, is unity.

Given, A = 42° 10′; *b* = 45°; *to find*, *a*.

Formula 7. log tan a = log tan A + log sin b — 10.

$$\begin{array}{rll} \log \tan A = & 42° \; 10' & = 9.9570 \\ + \log \sin b = & 45° & = 9.8495 \\ \hline \log \tan a = & 32° \; 38\tfrac{1}{2}' & = 9.8065 \end{array}$$

The natural tangent of this product is .6405, the nearest vulgar fraction to which is $\frac{16}{25}$. This is the value of the index _, and it produces the *symbol* $P\frac{16}{25}MT$. I have, however, called the combination $P\frac{2}{3}MT$, but this

is perhaps taking too much liberty in the abridgment of the sign, since P$\frac{2}{3}$MT would indicate the cotangent .6667, which differs 4 per cent. from that found by calculation.

HAUY's measurement across the equatorial edge of the octahedron is 83° 38′. Let us see what symbol this requires. 83° 38′ ÷ 2 = 41° 49′.

$$\begin{array}{rl} \log\tan 41^\circ\, 49' & = 9.9516 \\ +\ \log\sin 45^\circ & = \underline{9.8495} \\ \log\tan 32^\circ\, 19' & = 9.8011 \end{array}$$

The cotangent of 32° 19′ is .6326 or nearly $\frac{12}{19}$, which is nearer to $\frac{5}{8}$ than to $\frac{2}{3}$.

These statements show that we possess the power to describe these combinations either by short indices, which indicate their angles approximately, or else by larger indices, which show the precise angles quoted by any given authority.

492. PROBLEM. *Given,* Model 12, P_MT, *with the angle across the equator* = 84° 20′; *required, the angle across a terminal edge.*

ROSE and PHILLIPS both call the angle across the equatorial edge of this crystal of Zircon, 84° 20′, while the angle across a terminal edge, is called by ROSE, 123° 19′, and by PHILLIPS, 123° 15′.

If the same triangle and the same quantities are taken as in the last problem, then half the angle across a terminal edge of the model, will be angle B of the given solid triangle.

Given, A = 42° 10′; b = 45°; *to find,* B.

Formula 8. $\log\cos B = \log\cos b + \log\sin A - 10.$

$$\begin{array}{rl} \log\cos b = 45^\circ & = 9.8495 \\ +\ \log\sin A = 42^\circ\, 10' & = \underline{9.8269} \\ \log\cos B = 61^\circ\, 39\tfrac{1}{2}' & = 9.6764 \end{array}$$

Twice this product, or 61° 39½′ × 2 = 123° 19′, is the required angle across a terminal edge of the model, and this result agrees with the angle quoted by ROSE.

493. PROBLEM. *Given,* Model 12, P_MT, *with the angle across a terminal edge* = 123° 19′; *required, the angle across the equatorial edge.*

Take a right-angled triangle, consisting of an octant of the model, with pole n for its vertex. Then, with angle C = 90°; angle B = $\frac{123^\circ\, 19'}{2}$ = 61° 39½′; and side b = 45°, which is one-fourth of the square equator, seek for angle A, which is half the inclination across the equator, from plane Znw to plane Nnw.

Given, B = 61° 39½′; b = 45°; *to find,* A.

Formula 22. $\log\sin A = \log\cos B + 10 - \log\cos b.$

$$\begin{array}{rl} 10 + \log\cos B = 61^\circ\, 39\tfrac{1}{2}' & = 19.6764 \\ -\ \log\cos b = 45^\circ & = \underline{\ 9.8495} \\ \log\sin A = 42^\circ\, 10' & = \ 9.8269 \end{array}$$

Twice this product, or $42° 10' \times 2 = 84° 20'$, is the required angle across an equatorial edge of the model.

494. PROBLEM. *Given,* Model 12, *with the symbol* P$\frac{2}{3}$MT ; *required, the angle across a terminal edge of the model.*

Seek in the table of indices, page 139, for the cotangent equivalent to $\frac{2}{3}$, which is .6667. The angle corresponding to this cotangent is $56° 18\frac{1}{2}'$. This is the inclination of a terminal edge of the form to axis p^s. Now, take an octant of the form as a right-angled solid triangle, with pole Z for its vertex. You have then given, side $a = 56° 18\frac{1}{2}'$, and side $b = 56° 18\frac{1}{2}'$. Both these angles are alike, because the inclination of all the four oblique edges of a quadratic octahedron to axis p^s is the same. With these data, you have to find angle A or angle B, either of which is half the required inclination across a terminal edge of the form.

Given, $a = 56° 18\frac{1}{2}'$; $b = 56° 18\frac{1}{2}'$; *to find,* A.

Formula 13. $\log \tan A = \log \tan a + 10 - \log \sin b$.

$$
\begin{aligned}
10 + \log \tan a &= 56° 18\tfrac{1}{2}' = 20.1760 \\
- \log \sin b &= 56° 18\tfrac{1}{2}' = 9.9201 \\
\hline
\log \tan A &= 60° 59' \phantom{\tfrac{1}{2}} = 10.2559
\end{aligned}
$$

Twice this product, or $60° 59' \times 2 = 121° 58'$, is the required angle across a terminal edge of the form. This product, however, is $1° 21'$ too little, because the index $\frac{2}{3}$ does not express the relations of the axes with sufficient exactness.

495. PROBLEM. *Given,* Model 12, *with the symbol* P$\frac{1}{2}\frac{6}{3}$MT ; *required, the angle across a terminal edge of the model.*

Proceed exactly as in the last problem, but change the value of the quantities. The equivalent of $\frac{1}{2}\frac{6}{3}$ given in the Table of Indices, page 139, is .640. The corresponding angle is $57° 23'$. Then the equation is

Given, $a = 57° 23'$; $b = 57° 23'$; *to find,* A.

$$
\begin{aligned}
10 + \log \tan 57° 23' &= 20.1939 \\
- \log \sin 57° 23' &= 9.9255 \\
\hline
\log \tan 61° 40\tfrac{1}{2}' &= 10.2684
\end{aligned}
$$

Twice this product, or $61° 40\frac{1}{2}' \times 2 = 123° 21'$, is the required angle across a terminal edge of the model, and this product agrees with the measured angle within two minutes; consequently, the symbol P$\frac{1}{2}\frac{6}{3}$MT expresses very exactly the form described by the measurements of ROSE and PHILLIPS, while the symbol P$\frac{2}{3}$MT does so only approximately. Through the remainder of this section, I shall indicate Model 12 by the term P$\frac{1}{2}\frac{6}{3}$MT, in order that the reader may see clearly the nature of the calculations, and the relation of their products.

496. PROBLEM. *Given,* Model 12, P_MT, *with the inclination of a plane to axis* $p^s = 47° 50'$; *required, the angle across a terminal edge.*

Assume the zenith pyramid of Model 12 to be divided into eight portions by the intersection of the four meridians. Take one of those *eighths* as a solid triangle, with pole Z for its vertex. Then you have given, angle C = 90° = inclination of an external plane to the north-west meridian; A = 45° = an interior angle formed by the meeting of two adjacent meridians at p^a; and b = 47° 50′ = the given inclination of a plane to p^a. With these data you have to find B, which is half the angle across a terminal edge.

Given, A = 45°; b = 47° 50′; to find B.

Formula 8. log cos B = log cos b + log sin A — 10.

$$\begin{aligned} \log\cos b &= 47°\,50' &= 9.8269 \\ +\ \log\sin A &= 45° &= 9.8495 \\ \hline \log\cos B &= 61°\,39\tfrac{1}{2}' &= 9.6764 \end{aligned}$$

Twice this product, or 61° 39½′ × 2 = 123° 19′, is the required angle across a terminal edge of Model 12.

497. Problem. *Given,* Model 12, P_MT, *with the inclination of a terminal edge to axis* p^a = 57° 21′; *required, the angle across a terminal edge of the model.*

When you have the inclination of *one* terminal edge to the principal axis, you have the inclination of all four. Take the inclination of two edges to p^a as sides a and b of a solid triangle, each equal to 57° 21′, and then find angle A, as directed in problem § 495.

The following short Formula is useful in examining quadratic octahedrons. Let x be the inclination of a terminal edge to axis p^a, and $2y$ the angle across a terminal edge, then

$$\cot y = \cos x.$$

Example: $\cos x = 57°\,21' = 9.7320$

$\cot y = 61°\,39\tfrac{1}{2}' = 9.7320$

Twice 61° 39½′, or 123° 19′, is the required angle across a terminal edge of the model. §§ 495, 499.

498. Problem. *Given,* Model 12, P_MT, *with the angle across a terminal edge* = 123° 19′; *required, the inclination of a plane to axis* p^a *and to the equator.*

Take a solid triangle, consisting of an eighth part of the zenith pyramid of the model, divided in the manner described in § 496, and having pole Z for its vertex. Then, you have given, C = 90°; B = $\frac{123°\,19'}{2}$ = 61° 39½′; A = 45°; to find b, which is the required inclination of a plane to axis p^a.

Formula 5. log cos b = log cos B + 10 — log sin A.

$$\begin{aligned} 10 + \log\cos B &= 61°\,39\tfrac{1}{2}' &= 19.6764 \\ -\ \log\sin A &= 45° &= 9.8495 \\ \hline \log\cos B &= 47°\,50' &= 9.8269 \end{aligned}$$

This product, 47° 50′, is the inclination of a plane to axis p^a, and its complement = 42° 10′, is the inclination of a plane to the equator.

499. Problem. *Given*, Model 12, P_MT, *with the angle across a terminal edge* = 123° 19′; *required, the inclination of the terminal edge to axis* p^a.

Take an octant as a solid triangle, with pole Z for its vertex, and the following known parts: C = 90°; A = $\frac{123° \ 19'}{2}$ = 61° 39½′; B = 61° 39½′; and with these data, find a or b, either of which is the required inclination of the terminal edge to axis p^a. The solution of this problem requires Formula 4 modified by 104; namely, cos a = cot A.

$$\text{nat cot A} = 61° \ 39\tfrac{1}{2}' = 9.7319$$
$$\text{nat cos } a = 57° \ 21' = 9.7320$$

This product, 57° 21′, is the inclination of a terminal edge of Model 12 to axis p^a.

500. Problem. *Given*, Model 12, P$\frac{1}{2}\frac{2}{3}$MT, *with the angle across a terminal edge* = 123° 19′; *required, the plane angles of the external faces.*

a.) With the same solid triangle and the same given quantities as in problem, § 499, and with the help of Formula 6, find c.

Given, A = 61° 39½′; B = 61° 39½′; *to find*, c.

Formula 6. log cos c = log cot A + log cot B — 10.

$$\text{log cot A} = 61° \ 39\tfrac{1}{2}' = 9.7319$$
$$+ \text{ log cot B} = 61° \ 39\tfrac{1}{2}' = 9.7319$$
$$\text{log cos } c = 73° \ 5' = 9.4638$$

This product, 73° 5′, is the plane angle of a face at pole Z.

b.) Take the triangle described in § 496, with the following given quantities: A = 61° 39½′; B = 45°; C = 90°; to find b, which is half the obtuse angle of a face at pole Z.

Formula 5. log cos b = log cos B + 10 — log sin A.

$$10 + \text{log cos B} = 45° = 19.8495$$
$$- \text{ log sin A} = 61° \ 39\tfrac{1}{2}' = 9.9446$$
$$\text{log cos } b = 36° \ 32\tfrac{1}{2}' = 9.9049$$

Twice this product, or 36° 32½′ × 2 = 73° 5′, is the plane angle of the external faces at pole Z.

c.) If the obtuse plane angle at pole Z is called z, then the acute plane angles at the base of the pyramid will each be ½ (180° — z). Hence, 180° — 73° 5′ = 106° 55′. And $\frac{106° \ 55'}{2}$ = 53° 27½′. This last product, 53° 27½′, is the value of each of the plane angles of Model 12 at the poles n e w s.

d.) Check on this Calculation.—Take the solid triangle described in § 491, and with the quantities there given, find side c, which is the plane angle at pole n.

Given, $A = 42° \ 10'$; $b = 45°$; *to find,* c.

Formula 9. $\log \tan c = \log \tan b + 10 - \log \cos A$.

$$\begin{aligned} 10 + \log \tan b &= 45° &&= 20.0000 \\ - \log \cos A &= 42° \ 10' &&= \underline{\ 9.8699} \\ \log \tan c &= 53° \ 27\tfrac{1}{2}' &&= 10.1301 \end{aligned}$$

QUADRATIC OCTAHEDRONS OF THE SECOND POSITION.

Example: Model 13. $P\frac{5}{2}M$, $P\frac{5}{2}T$.

501. PROBLEM. *Given,* Model 13, *with the symbol* $P\frac{5}{2}M$, $P\frac{5}{2}T$; *required, the inclination of the planes to the equator and to axis* p^s.

a.) Look in the Table of Indices, page 139, for the decimal fraction equivalent to $\frac{5}{2}$. You find it to be 2.500. This number is the *tangent* of the inclination of the planes to the equator $= 68° \ 12'$, and the *cotangent* of their inclination to axis $p^s = 21° \ 48'$.

According to Haüy, the inclination of the planes to the equator of the crystal of Anatase, which this model represents, is $\frac{137° \ 10'}{2} = 68° \ 35'$: according to Mohs, it is $\frac{136° \ 22'}{2} = 68° \ 11'$. The latter quotation agrees very nearly with the index $\frac{5}{2}$.

502. PROBLEM. *Given,* Model 13, P_xM, P_xT, *with the angle across the equator* $= 136° \ 24'$; *required, the value of the index* $_x$ *in the symbol.*

Rule. The tangent of half the angle across the equator is the value of the index.

Example: $\frac{136° \ 24'}{2} = 68° \ 12'$. tan 2.5002 or $\frac{5}{2}$.

503. PROBLEM. *Given,* Model 13, $P\frac{5}{2}M$, $P\frac{5}{2}T$; *required, the angle across the terminal edge of the model.*

Find by problem § 501, the inclination of the planes to the axis p^s. Call this $a = 21° \ 48'$. Then take a solid triangle, containing an eighth part of the zenith pyramid of Model 13, with pole Z for its vertex. In this you have given, angle $C = 90°$ = inclination of a plane to the north meridian; $B = 45°$ = interior angle formed by the intersection of the north and north-west meridian; $a = 21° \ 48'$ = inclination of a plane to axis p^s. With these data, you can find A, which is half the desired angle across a terminal edge.

Given, $a = 21° \ 48'$; $B = 45°$; *to find,* A.

Formula 10. $\log \cos A = \log \cos a + \log \sin B - 10$.

$$\begin{aligned} \log \cos a &= 21° \ 48' &&= 9.9678 \\ + \log \sin B &= 45° &&= \underline{9.8495} \\ \log \cos A &= 48° \ 57\tfrac{1}{2}' &&= 9.8173 \end{aligned}$$

Twice this product, or $48° \ 57\frac{1}{2}' \times 2 = 97° \ 55'$, is the required angle across a terminal edge of Model 13.

504. PROBLEM. *Given,* Model 13, $P_{+}M$, $P_{+}T$, *with the angle across a terminal edge* $= 97° \ 55'$; *required, the inclination of a plane to the equator.*

Take an octant of the model as a solid triangle, with pole nw for its vertex. Then you have given, $A = \frac{97° \ 55'}{2} = 48° \ 57\frac{1}{2}' =$ half the angle across a terminal edge; $a = 45° =$ one-fourth of the equator, or the inclination of the north edge of the equator to the nw normal. With these data, you can find B, which is the required inclination of a plane of Model 13 to the equator.

$$\textit{Given, } A = 48° \ 57\tfrac{1}{2}'; \ a = 45°; \ \textit{to find, } B.$$

Formula 2. $\log \sin B = \log \cos A + 10 - \log \cos a.$

$$\begin{aligned} 10 + \log \cos A &= 48° \ 57\tfrac{1}{2}' = 19.8173 \\ -\log \cos a &= 45° \qquad\quad = \ \ 9.8495 \\ \hline \log \sin B &= 68° \ 12' \quad\ \ = \ \ 9.9678 \end{aligned}$$

This product, $68° \ 12'$, is the required inclination of the planes to the equation. Twice $68° \ 12' = 136° \ 24'$, is the inclination of a plane of the upper on a plane of the lower pyramid of the model across the equator.

505. PROBLEM. *To transpose a quadratic octahedron from the first position to the second position, or from the second position to the first position.*

a.) Before determining upon the index of a quadratic octahedron, it is proper to ascertain what index it gives, both when reckoned as P_xMT and as P_xM, P_xT. When the positions of a series of octahedrons of a given mineral have been previously fixed, we then perceive which member of the series we have in hand; and when such a series has not been determined, we can, after calculating two indices for a crystal, take that which consists of the simplest numbers.

b.) The resolution of the following problems gives the necessary information for this purpose:

$$\textit{Given, } A = x; \ B = 45°; \ C = 90°; \ \textit{to find, } a.$$
$$\textit{Given, } A = x; \ B = 45°; \ C = 90°; \ \textit{to find, } c.$$

To resolve these problems, you take a right-angled solid triangle consisting of an eighth part of the upper pyramid of a quadratic octahedron, formed by the intersection of the four meridians, and having pole Z for its vertex. § 496. Then A is half the angle across a terminal edge, which quantity varies with every octahedron; B is an interior angle formed by the meeting of the north with the north-west meridian at axis p^s; C is the angle where a meridian cuts a plane. Hence, a is the inclination of a plane to axis p^s, the cotangent of which inclination is the value of the index $_x$ in the symbol P_xM, P_xT; and c is the inclination of

a terminal edge to axis p^1, the cotangent of which inclination is the value of the index $_x$ in the symbol P_xMT.

The Formulæ to be employed in resolving these problems are as follow:

Formula 4. $\log \cos a = \log \cos A + 10 - \log \sin 45°$.
Formula 6. $\log \cos c = \log \cot A + \log \cot 45° - 10$.

c.) When you know the relation of the axes of an octahedron without calculation, as when the symbol $P\frac{5}{2}M, P\frac{5}{2}T$ is given, and you want to know what must be substituted in the symbol P_xMT in place of $\frac{5}{2}$ when the position of the octahedron is changed, you need only multiply the equatorial axes of P_xM, P_xT by the natural secant of $45° = 1.4142136$, to obtain the value of the equatorial axes of P_xMT. Thus, $1.4142 \times 2 = 2.8284$, shows that the symbol $P\frac{5}{2}M, P\frac{5}{2}T$, must be changed to $P\frac{5.0000}{2.8284}MT$.

On the other hand, if $P\frac{5.0000}{2.8284}MT$ is given, and the symbol is to be changed to P_xM, P_xT, you find the value of the equatorial axes of the latter by *dividing* the equatorial axes of the former by 1.4142. Thus, $\frac{2.8284}{1.4142} = 2$. Hence, $P\frac{5.0000}{2.8284}MT$ becomes $P\frac{5}{2}M, P\frac{5}{2}T$.

Again, change $P\frac{16}{25}MT$ into P_xM, P_xT. $\frac{25.0000}{1.4142} = 17.607$. Hence, $P\frac{16}{25}MT$ becomes $P\frac{160}{176}M, P\frac{160}{176}T$. But $\frac{160}{176} \div 4 = \frac{40}{44} = \frac{10}{11}$, which gives $P\frac{10}{11}M, P\frac{10}{11}T$. This seems as good a symbol for Model 12, as $P\frac{16}{25}MT$. But it is less accurate, since $\frac{10}{11}$ is the synonyme, see page 139, of .9091, which is the tangent of 42° 16½′. This gives the inclination of a Zenith to a Nadir plane = 84° 33′ instead of 84° 20′, § 491. Here a difference of 13′ in the equatorial angle arises, partly from the division of 25 by 1.4142 instead of 1.4142136, and partly from abridging the product 17.607 to 17.6.

Whenever the indices of the two symbols P_xM, P_xT, and $P_x^{\cdot}MT$, appear to be equally commodious, it is impossible to determine which is the preferable symbol, without examining a series of octahedrons of both positions belonging to the same mineral.

506. Combinations of Quadratic Octahedrons with one another.

a.) *In the same zones.*—The middle octahedron PMT, may combine with the form p_-mt, or with the form p_+mt. The first, p_-mt, replaces the apex of PMT, and produces four planes which incline on the planes of PMT, Model 14. The second, p_+mt, bevels the equatorial edges of PMT.

In the same way, PM, PT, is replaced at the summits by p_-m, p_-t, Model 14, and bevelled at the equatorial edges by p_+m, p_+t.

b.) *The predominant octahedron*, PMT, *with a subordinate octahedron*, p_xm, p_xt.

i. If the terminal edges of PMT are replaced by tangent planes, then the axes of p_xm, p_xt are unity, and the symbol is pm, pt.

ii. If the summits of PMT are replaced by four planes that incline on the terminal edges, these indicate the combination, p_-m, p_-t.

iii. If the equatorial angles of PMT are replaced by planes inclining on the terminal edges, these indicate the combination, p_+m, p_+t.

c.) The predominant octahedron, PM, PT, *with a subordinate octahedron*, $p_x$mt.

i. If the terminal edges of PM, PT, are replaced by tangent planes, then axis p^a of $p_x$mt, will be the same as axis p^a of PM, PT, while the two equatorial axes of $p_x$mt will be *twice as great* as the two equatorial axes of PM, PT.

ii. If the summits of PM, PT, are replaced by four planes that incline on the terminal edges, then axis p^a of the form $p_x$mt being unity, axes m^a and t^a of $p_x$mt will be *more than twice as great* as axes m^a and t^a of the combination PM, PT.

iii. If the equatorial angles of PM, PT, are replaced by planes that incline on the terminal edges, these indicate that if axis p^a of $p_x$mt is unity, then axes m^a and t^a are *less than twice as great* as axes m^a and t^a of PM, PT.

There are, consequently, eight varieties of combination between two quadratic octahedrons. It requires but *one measurement* either across an equatorial or a terminal edge of each octahedron, to afford information sufficient to lead to its symbol and index.

507. Combination of the Quadratic Octahedrons with the Horizontal Planes P.

P_-. $P\frac{4}{3}M$, $P\frac{4}{3}T$. Model 76. *Molybdate of Lead.*
p. pm, pt, PMT. Model 77. *Copper Pyrites.*

These are incomplete prisms with incomplete pyramids. Class 5, Order 1, Genus 2, page 115, Part II.

Analysis.—The inclination of P upon a plane of a quadratic octahedron, is $90° + x$, in which formula, x signifies the inclination of the plane of the octahedron to axis p^a.

508. Combination of the Quadratic Prisms with the Horizontal Planes P.

P_-,MT: or $P\frac{2}{3}$,MT. Model 2. *Rutile.*
P_+,M,T: or $P\frac{5}{4}$,M,T. Model 3. *Apophyllite.*
P_+,M,T, mt. Model 4. *Egeran.*

These combinations are complete prisms with a square equator. Class 1, Order 1, Genus 2, page 98, Part II.

Analysis.—The inclination of P, to M,T or MT is 90°. The inclination of M,T to the equatorial axes is 90°, to the equatorial bipolar normals, 45°. The inclination of MT to the equatorial axes is 45°, to the equatorial bipolar normals, 90°. Hence, M,T incline upon MT at an angle of $90° + 45° = 135°$.

ry short prisms of this class = P$\overline{-}$,M,T, were formerly termed , or *tabular crystals*.

9. COMBINATION OF THE QUADRATIC PRISMS WITH THE QUADRATIC OCTAHEDRONS.

MT. P$\frac{a}{3}$MT. Model 61. *Zircon.*
(M,T. P$\frac{a}{3}$M, P$\frac{a}{3}$T) $\times$ 2. Model 62. *Oxide of Tin.*
M,T, mt. P$\frac{2}{3}$M, P$\frac{2}{3}$T. Model 59. *Wernerite.*
M,T, mt. P$\frac{a}{3}$MT. Model 60. *Zircon.*

e combinations are incomplete prisms with complete pyramids. 4, Order 1, Genus 2, page 111, Part II.

alysis.—The inclination of a quadratic prism to a quadratic octan is 90° + x, in which formula, x signifies the inclination of a plane e quadratic octahedron to the equator, or half the angle across a ontal edge of the octahedron.

e prisms M,T and MT are transposed from the first position into econd, or from the second into the first, to suit the index of the edron. When the octahedron is measured and the index fixed, then ertical planes are named accordingly. Upon this principle, the ver- r prismatic planes of Models 59 and 60 may be called M,T, mt, t, MT, just as is most convenient to suit the terminating octahe- . This power of changing the description of the prism by a mere e in the kind of letter employed to designate it, is extremely con- nt.

). COMBINATION OF THE QUADRATIC PRISMS WITH QUADRATIC HEDRONS AND THE HORIZONTAL PLANES.

P$\frac{1}{4}$,M,T. p$\frac{1}{4}$mt. Model 41. *Apophyllite.*
p$_{+}$,m,t, MT. P$\frac{1}{4}$M, P$\frac{1}{4}$T. Model 42. *Idocrase.*

e combinations are complete prisms with incomplete pyramids. 3, Order 1, Genus 2, page 107, Part II.

alysis.—See §§ 508, 509.

THE DIOCTAHEDRON.

1. The symbol of the dioctahedron is P$_x$M$_-$T, P$_x$M$_+$T, the index $_x$ th of which forms is alike, since the dioctahedron contains two ic octahedrons in inverse positions as regards the equatorial axes. value of the indices $_-$ and $_+$ depends not only upon the relative sions of the two component octahedrons, but also upon the posi- ssigned to the dioctahedron as respects the axes of the combina- o which it may belong. Occurring only as a subordinate form, its on is generally determined by that of the quadratic octahedrons which it is found in combination; and as we have assumed the to give to all quadratic octahedrons an azimuthal change of posi- o the extent of 45° whenever we think fit to do so, it follows that like manner, assume the power to give an equal azimuthal remove

of 45° to the dioctahedrons by which the quadratic octahedrons may be modified. The consequences of this are easy to be seen, on examining the diagram in § 484. If the dioctahedron is placed as shown in the figure, the line cn represents the length of axis m^a; whereas, if the equator is moved 45° azimuthally, the line cE comes into the position of the line cn and becomes axis m^a; and since the lines cn and cE always differ in length, the index showing the relation of m^a to p^a or t^a also necessarily differs according to this change of position.

In order to be able to calculate the ratios of the axes of the dioctahedron, and to find its indices, we require the angle across two of the three kinds of edges which it presents externally, or if the measurement of only one edge can be procured, some other quantity must be had equivalent to the measurement of another edge, or to the value of one of the three axes of the form under investigation.

I shall proceed to show in what manner the axes of the component forms of the dioctahedron can be calculated from any two measurements across the external edges. The example which I shall take is the dioctahedron of the mineral Zircon, the measurements across the edges of which are given by Rose as follows:

Across the equator	= 127° 29′.	½ edge = e	= 63° 44½′.
Across the n meridian	= 147° 3′.	½ edge = n	= 73° 31½′.
Across the nw meridian	= 132° 43′.	½ edge = w	= 66° 21½′.

In each calculation, I shall show what the indices are to be on the two assumptions, that either the edge n or the edge w is ascribed to the north meridian. The instructions contained in these calculations, added to those already given respecting the dioctahedron, in §§ 409, 410, with other problems contained in the article on the hexakisoctahedron, will, I hope, sufficiently elucidate the nature of this combination.

512. Problem. *Given, a dioctahedron,* $P_x M_- T$, $P_x M_+ T$, *with an edge,* $e = 63° 44\frac{1}{2}'$, *and an edge,* n, $= 73° 31\frac{1}{2}'$; *required, the inclination of these two edges to axis* m^a, *and the value of the indices* $_x$, $_-$, $_+$, *in the symbol.*

a.) Take a right-angled solid triangle, with pole n for its vertex, and in which you have given, angle $A = 63° 44\frac{1}{2}'$, and angle $B = 73° 31\frac{1}{2}'$. Then angle $C = 90°$ will be the inclination of the north meridian to the equator; side a will be the inclination of the edge n to axis m^a, the cotangent of which angle will give the length of m^a when p^a is unity; and side b will be the inclination of the edge e to axis m^a, the cotangent of which will give the length of m^a when t^a is unity.

b.) *Given,* $A = 63° 44\frac{1}{2}'$; $B = 73° 31\frac{1}{2}'$; *to find, a.*

Formula 4. $\log \cos a = \log \cos A + 10 - \log \sin B$.

$$\begin{aligned} 10 + \log \cos A &= 63° 44\tfrac{1}{2}' = 16.6458 \\ - \log \sin B &= 73° 31\tfrac{1}{2}' = 9.9818 \\ \hline \log \cos a &= 62° 31\tfrac{1}{2}' = 9.6640 \end{aligned}$$

This product, 62° 21½′, is the inclination of the edge *n* to axis mª. cot 62° 31½′ = .5200 = $\frac{13}{25}$. That is to say, axis mª is to axis pª as 13 is to 25.

c.) Given, A = 63° 44½′; B = 73° 31½′; *to find, b.*

Formula 5. log cos *b* = log cos B + 10 — log sin A.

$$\begin{aligned} 10 + \log\cos B &= 73^\circ\ 31\tfrac{1}{2}' = 19.4527 \\ -\log\sin A &= 63^\circ\ 44\tfrac{1}{2}' = \underline{\ 9.9527} \\ \log\cos b &= 71^\circ\ 34' \ = \ 9.5000 \end{aligned}$$

This product, 71° 34′, is the inclination of the edge *e* to axis mª. cot 71° 34′ = .3333 = $\frac{1}{3}$. That is to say, axis mª is to axis tª as 1 is to 3, or if we make mª = 13, to bring the products of *b.*) and *c.*) into unison, the relation mª to tª is 13 to 39.

d.) Hence, the relations of the three axes are, pª = 25; mª = 13; tª = 39. This relation gives the symbol $P\frac{25}{39}M\frac{13}{39}T$. If we divide these fractions by 13, we obtain the symbol $P\frac{1.9231}{3}M\frac{1}{3}T$, which is very nearly $P\frac{2}{3}M\frac{1}{3}T$. But this symbol would only express the external angles of the form approximately, exactly as the external angles of the quadratic octahedron, Model 12, are expressed approximately by the symbol $P\frac{2}{3}MT$. See § 491.

If one form of the dioctahedron is $P_2M_1T_3$, or $P\frac{2}{3}M\frac{1}{3}T$, the co-existing form is $P_2M_3T_1$, or $P\frac{2}{1}M\frac{3}{1}T$.

513. PROBLEM. *Given, a dioctahedron,* P_xM_-T, P_xM_+T, *with an edge, e,* = 63° 44½′, *and an edge, n,* = 66° 21½′; *required, the inclination of these two edges to axis* mª, *and* the value of *the indices* $_x$, $_-$, $_+$, *in the symbol.*

a.) Proceed exactly as directed in § 512 *a.*) only changing the value of angle B from 73° 31½′ to 66° 21½′. The other quantities remain as they were.

b.) Given, A = 63° 44½′; B = 66° 21½′; *to find, a.*

Formula 4. log cos *a* = log cos A + 10 — log sin B.

$$\begin{aligned} 10 + \log\cos A &= 63^\circ\ 44\tfrac{1}{2}' = 19.6458 \\ -\log\sin B &= 66^\circ\ 21\tfrac{1}{2}' = \underline{\ 9.9619} \\ \log\cos a &= 61^\circ\ 7' \ = \ 9.6839 \end{aligned}$$

This product, 61° 7′, is the inclination of the edge *n* to axis mª. cot. 61° 7′ = .5517 = nearly $\frac{11}{20}$. That is to say, axis mª is to axis pª as 11 is to 20; or if we make mª = 10, to bring it into unison with the product of the following equation, we have the relation mª to pª = 10 to 1.8128 = $\frac{10}{18}$ = $\frac{5}{9}$.

c.) Given, A = 63° 44½′; B = 66° 21½′; *to find, b.*

Formula 5. log cos b = log cos B + 10 — log sin A.

$$\begin{aligned} 10 + \log \cos B &= 66° 21\tfrac{1}{2}' = 19.6032 \\ - \log \sin A &= 63° 44\tfrac{1}{2}' = 9.9527 \\ \hline \log \cos b &= 63° 26' \phantom{\tfrac{1}{2}} = 9.6505 \end{aligned}$$

This product, 63° 26′, is the inclination of the edge e to axis m^a. cot 63° 26′ = .5000 = $\frac{1}{2}$ = $\frac{10}{20}$. That is to say, axis m^a is to axis t^a as 10 is to 20 or 5 to 10.

d.) Hence, the relations of the three axes are, p^a = 9; m^a = 5; t^a = 10, which relations afford the symbol $P_{\frac{9}{10}}M_{\frac{5}{10}}T$; or, since $\frac{9}{10}$ is less than the true relations of p^a to t^a, the real indices of the form are very nearly $P_{\frac{10}{10}}M_{\frac{5}{10}}T$ or $PM_{\frac{1}{2}}T$; and those of the co-existing form of the dioctahedron are $P_{\frac{10}{5}}M_{\frac{10}{5}}T$, or P_2M_2T, or $PMT_{\frac{1}{2}}$.

514. Problem. *Given, a Dioctahedron,* P_xM_-T, P_xM_+T, *with an edge,* n = 73° 31½′, *and an edge,* w = 66° 21½′; *required, the inclination of the edge n, to axis* p^a, *and the value of the indices,* $_x$, $_-$, $_+$, *in the symbol.*

a.) *To find the inclination of the edge n to axis* p^a. The solution of this problem depends upon the principle explained in § 409, according to which the present problem is,

Given, A = 66° 21½′; B = 73° 31½′; C = 45°; *to find, a.*

Formula 37. $\sin \frac{1}{2} a = \sqrt{\frac{\cos S \cos (S - A)}{\sin B \sin C}}$, where $S = \frac{1}{2}(A + B + C)$.

Log $\sin \frac{1}{2} a$ =

$\frac{1}{2}$ {log cos S + log cos (S — A) + 20 — (log sin B + log sin C)}.

$$\begin{array}{lll} A = 66° 21\tfrac{1}{2}' & & S = 92° 26\tfrac{1}{2}' \\ B = 73° 31\tfrac{1}{2}' & & A = 66° 21\tfrac{1}{2}' \\ C = 45° & & \overline{} \\ \overline{} & & S - A = 26° \ 5' \\ 2\,)184° 53' & & \\ \overline{} & & \end{array}$$

$$\begin{aligned} S &= 92° 26\tfrac{1}{2}' \\ \text{Supplement of } S &= 87° 33\tfrac{1}{2}' \end{aligned}$$

$$\begin{aligned} \log \cos S = 87° 33\tfrac{1}{2}' &= - \ \ 8.6294 \\ + \log \cos (S - A) = 26° \ 5' &= \ \ 9.9534 \\ \hline + 20 &= \ \ 38.5828 \\ -\left\{ \begin{array}{l} \log \sin B = 73° 31\tfrac{1}{2}' = 9.9818 \\ + \log \sin C = 45° \phantom{31\tfrac{1}{2}'} = 9.8495 \end{array} \right\} &= \ \ 19.8313 \\ \hline 2)\ 18.7515 \\ \hline \sin \tfrac{1}{2} a = 13° 44\tfrac{1}{2}' &= \ \ 9.37575 \end{aligned}$$

Twice this product, or 13° 44½′ × 2 = 27° 29′, is the required inclination of the edge n to axis p^a. The tangent of this angle shows the relation of axis m^a to axis p^a of the form P_xM_-T. tan 27° 29′ = .5202. That is to say, axis p^a is to axis m^a as 1.0000 is to .5202, or nearly as 100 to 52, or 25 to 13.

b.) To find the value of the indices, $_x$, $_-$, $_+$, of the symbol.
Proceed according to the instructions given in problem, § 410.

Given, $A = 73° 31\frac{1}{2}'$; $b = 27° 29'$; $C = 90°$; *to find, a.*

Formula 7. $\log \tan a = \log \tan A + \log \sin b - 10.$

$$\begin{array}{rcl} \log \tan A = 73° 31\frac{1}{2}' & = & 10.5291 \\ + \log \sin b = 27° 29' & = & 9.6642 \\ \hline \log \tan a = 57° 21' & = & 10.1933 \end{array}$$

This product, 57° 21′, is the angle whose tangent gives the relation of axis t^a to axis p^a of the form P_xM_-T. tan 57° 21′ = 1.5607. That is to say, axis p^a is to axis t^a, as 1.0000 is to 1.5607, or nearly as 100 to 156, or 25 to 39.

c.) The relation of p^a to m^a was found by *a.*) to be nearly that of 25 to 13, and the relation of p^a to t^a was found by *b.*) to be nearly that of 25 to 39. Hence, the relations of the three axes are, $p^a = 25$; $m^a = 13$; $t^a = 39$. This affords the symbol $P\frac{25}{39}M\frac{13}{39}T$, or $P\frac{5}{3}M\frac{1}{3}T$, as deduced in § 512 *d.*)

515. PROBLEM. *Given, a dioctahedron,* P_xM_-T, P_xM_+T, *with an edge,* $n = 66° 21\frac{1}{2}'$, *and an edge,* $w = 73° 31\frac{1}{2}'$; *required, the inclination of the edge n to axis* p^a, *and the value of the indices,* $_x$, $_-$, $_+$, *in the symbol.*

a.) To find the inclination of the edge n to axis p^a. This is a repetition of the calculation in § 514, with a mere change in the quantities.

Given, $A = 73° 31\frac{1}{2}'$; $B = 66° 21\frac{1}{2}'$; $C = 45°$; *to find, a.*

The formula and preamble are given in § 514.

$$\begin{array}{rclcrcl} S & = & 92° 26\frac{1}{2}' & \quad & S & = & 92° 26\frac{1}{2}' \\ \text{Supplement of } S & = & 87° 33\frac{1}{2}' & & A & = & 73° 31\frac{1}{2}' \\ \cline{5-7} & & & & S - A & = & 18° 55' \end{array}$$

$$\begin{array}{rcl} \log \cos S = 87° 33\frac{1}{2}' & = - & 8.6294 \\ + \log \cos S - A = 18° 55' & = & 9.9759 \\ \hline + 20 & = & 38.6053 \\ -\left\{ \begin{array}{l} \log \sin B = 66° 21\frac{1}{2}' = 9.9619 \\ + \log \sin C = 45° \quad = 9.8495 \end{array} \right\} & = & 19.8114 \\ \hline & & 2)18.7939 \\ \hline \sin \frac{1}{2} a = 14° 26\frac{1}{2}' & = & 9.39695 \end{array}$$

Twice this product, or $14° 26\frac{1}{2}' \times 2 = 28° 53'$, is the required inclination of the edge n to axis p^a. The tangent of this angle shows the relation of axis m^a to axis p^a of the form under investigation. tan 28° 53′ = .5517; this makes the relation of m^a to p^a to be nearly as 11 to 20.

b.) To find the value of the indices, $_x$, $_-$, $_+$, *in the symbol*.—Proceed according to the instructions given in §§ 410 and 514, *b.*)

The present problem is:

Given, A $= 66° 21\frac{1}{2}'$; $b = 28° 53'$; C $= 90°$; *to find, a.*

Formula 7. log tan a = log tan A + log sin b — 10.

$$\begin{aligned} \log \tan A &= 66° 21\tfrac{1}{2}' = 10.3588 \\ + \log \sin b &= 28° 53' = 9.6840 \\ \hline \log \tan a &= 47° 49' = 10.0428 \end{aligned}$$

This product, 47° 49′, is the angle whose tangent shows the relation of axis t^a to axis p^a of the form under investigation. tan 47° 49′ = 1.1035. This makes the relation of t^a to p^a to be nearly as 11 to 10, or 22 to 20.

c.) The ratios of the three axes are consequently found by *a.*) and *b.*) to be as follow: $p^a = 20$; $m^a = 11$; $t^a = 22$. This affords the symbol $P\frac{20}{22}M\frac{11}{22}T$. If we divide these fractions by 11, we obtain $P\frac{1\cdot 818}{2}M\frac{1}{2}T$, or nearly $P\frac{2}{2}M\frac{1}{2}T = PM\frac{1}{2}T$, the symbol deduced in § 513 *c.*)

516. Problem. *Given, the symbol of a dioctahedron*, $P\frac{2}{3}M\frac{1}{3}T$, $P\frac{2}{1}M\frac{3}{1}T$; *required, the angles across the external edges of the combination.*

See Problem, § 411, article, *Hexakisoctahedron*.

III.

THE RHOMBOHEDRAL SYSTEM OF CRYSTALLISATION.

517. The character of the Forms belonging to this system, as given by Rose, is this:—They have four axes, three of which are equal to one another, and placed in one plane, crossing at angles of 60°; the fourth axis differs from the others in length, and is placed perpendicular to all of them.

Rose's enumeration of the Forms belonging to this system of crystallisation, is as follows:—

A. Homohedral Forms:

1. The six-sided Pyramid $= P_xT$, $P_xM\frac{1}{1}\frac{5}{3}T_2$. Model 26.
2. The Horizontal Planes $= P$.
3. The six-sided Prism $= T$, $M\frac{1}{1}\frac{5}{3}T_2$.
4. The twelve-sided Pyramid $= 3P_xM_yT_z$.
5. The twelve-sided Prism $= 3m_xt$.

B. Hemihedral Forms:

1. The Rhombohedron $= \frac{1}{2}P_xT$, $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$. Model 26a.
2. The Scalenohedron $= \frac{1}{2}(3P_xM\pm T\mp)$: or S. Model 26f.

518. I propose to designate all the Forms and Combinations belonging to the rhombohedral system, by symbols and indices which refer to the *same three* rectangular axes, p^a m^a t^a, that are referred to in all the other

systems of crystallisation. I shall, therefore, take no further notice of the set of four axes described in § 517, for the assumption of which I can perceive no use, since the set of rectangular axes answer the purposes of crystallography much better.

I have given a pretty full account of the Forms and Combinations peculiar to this system, in § 340, 3.), and in Part II. pages 43—45, which details the reader is requested to read as if they occurred in this paragraph.

ROSE's Catalogue of the Minerals that belong to the rhombohedral system, is given in Part II. pages 3—12. A symbolic Catalogue of the Forms and Combinations presented by the crystals of each of these Minerals, is given in Part II. pages 43—61.

A. Homohedral Forms of the Rhombohedral System.

1. THE SIX-SIDED PYRAMID, or *Hexagonal Dodecahedron.*

Model 26. P_xT, $P_xM\frac{15}{13}T_2$.

519. This combination has twelve faces which are isosceles triangles. Its equator is a regular hexagon. It has twelve similar oblique edges, which meet six together at poles Z and N, and produce two regular six-sided pyramids. It has six edges round the equator, between the zenith and nadir pyramids. The angles across the horizontal edges and terminal edges are different.

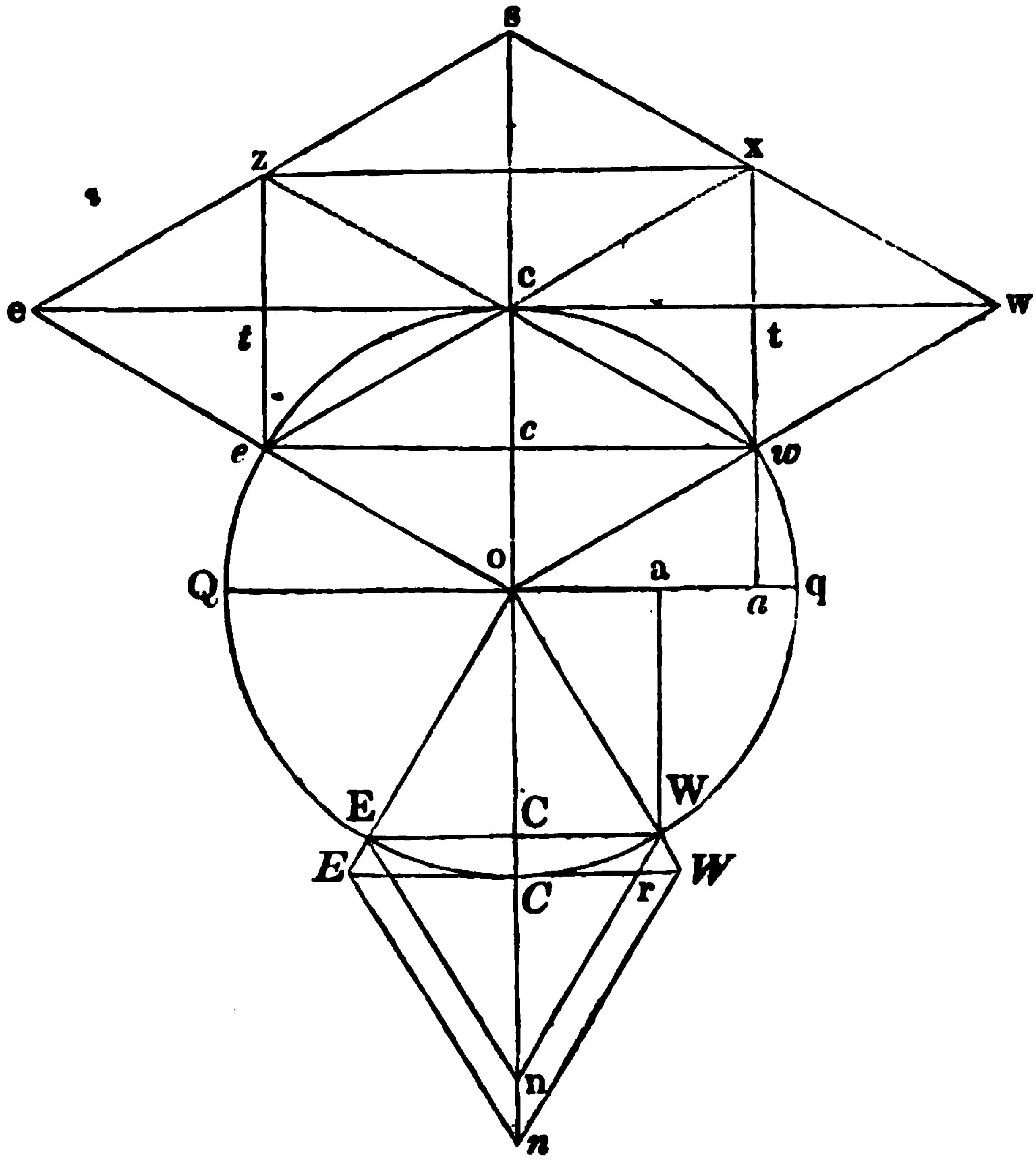
The "Forms" contained on this "Combination," and shown on the marked model, are the biaxial form, P_xT, and the triaxial form, $P_xM\frac{15}{13}T_2$. If the model is turned 45° horizontally, the "Forms" upon it will then be the biaxial form, P_xM, and the triaxial form, $P_xM_2T\frac{15}{13}$.

The index $_x$ in these symbols may be exchanged for the signs $_-$ or $_+$. Hence, we may have six-sided pyramids of two different positions, and we may also have two kinds of pyramids, *acute* and *obtuse*, resulting from the dimensions of the vertical axis.

The hexagonal dodecahedron is a complete pyramid with a rhombo-rectangular equator. It falls, therefore, into Class 2, Order 5. See Part II. page 103.

520. *Investigation of the equator of the Rhombohedral Forms.*—The diagram in the next page will serve to explain the properties of the equator of this and other Forms of the rhombohedral system. The rhombus e s w o measures 120° at o and s, and 60° at e and w. The diagonal e w bears to the diagonal o s, very nearly the relation of 26 to 15, and of course the line c w bears to the line c o the same relation of 26 to 15. An equation of this character could be expressed by the symbol $M\frac{15}{26}T$. But the equators of the crystals of this class are never complete rhombuses, being always cut by the planes or edges marked in the diagram *e*z and *w*x, which planes restrict the form of the equators to that of a regular hexagon. This is the figure of the base of nearly all the crystals of the rhombohedral system.

The relation of these six-sided equators to the equatorial axes m^a and p^a, is easy to be seen in the diagram, where the line o s = 30, holds the situation of axis m^a, and the line *t* t = 26, holds the situation of axis t^a.



Hence, from c o = 15, and c t = 13, we deduce the value of the equatorial axes, $m^a_{15} t^a_{13}$. The few cases in which this value is changed to $m^a_{14} t^a_{13}$, will be described hereafter. In the meantime, I proceed to show the relation of this six-sided equator to the pyramids and prisms of which it is the horizontal section.

521. Problem. *Given,* Model 26, *with the symbol* $P_x T, P_x M_y T_z$, *and with the angle across a horizontal edge* = 81° 50′; *required, a.) the inclination of the planes to axis* p^a, *b.) the inclination of the terminal edges to axis* p^a, *and c.) the value of the indices* $_x$, $_y$, $_z$ *in the symbol.*

a.) To find the inclination of the planes to axis p^a.—The inclination of the planes of Model 26 to axis p^a is the complement of their inclination to the equator. Hence, $\frac{81° 50'}{2}$ = 40° 55′ = inclination of the planes to the equator, and 90° — 40° 55′ = 49° 5′ = inclination of the planes to axis p^a.

b.) To find the inclination of the edges to axis p^a.—Suppose the zenith pyramid of Model 26 to be divided into twelve portions, by vertical sections passing through the six edges, and across the six planes, and meeting at axis p^a. Take one of these portions as a right-angled solid

triangle, with pole Z for its vertex. The given parts are then, angle C $= 90° =$ inclination of a plane to a section passing through it; angle B $= 30° (= \frac{360°}{12})$ = interior angle formed by the meeting of two vertical sections at axis p^a; side $a = 49° 5'$ = inclination of a plane to axis p^a. With these data, find side c, which is the required inclination of an edge to axis p^a.

Given, $a = 49° 5'$; $B = 30°$; *to find*, c.

Formula 12. $\log \tan c = \log \tan a + 10 - \log \cos B$.

$$10 + \log \tan a = 49° 5' = 20.0621$$
$$- \log \cos B = 30° = 9.9375$$
$$\log \tan c = 53° 6\tfrac{1}{2}' = 10.1246$$

This product, $53° 6\frac{1}{2}'$, is the inclination of an edge of Model 26, to axis p^a.

c.) To find the value of the indices of the symbols P_xT, $P_xM_yT_z$.—The four planes placed on the east zone of this combination, constitute the biaxial form P_xT. I have shown in *a.*) what are the dimensions of the triangle which is formed betwixt poles Z and w, and the centre of the crystal. The tangent of the angle formed at pole w; that is to say, the tangent of the inclination of a plane to the equator of the combination, is the index of the symbol P_xT. This angle is $40° 55'$, the tangent of which is $.8667 = \frac{13}{15} = p^a_{13}t^a_{15}$, which gives the symbol $P\frac{13}{15}T$.

I proceed next to deduce the value of the indices of the triaxial form $P_xM_yT_z$. I have shown in *a.*), that the inclination of a plane to axis p^a of Model 26, is $49° 5'$, the tangent of which is 1.1538; and in *b*), that the inclination of an edge to axis p^a, is $53° 6\frac{1}{2}'$, the tangent of which is 1.3323. Now the number 1.1538 represents the normal w or axis t^a, and the number 1.3323 represents the normal n or axis m^a, of Model 26. But $\frac{1.1538}{13}$, and $\frac{1.3323}{15}$, give the same product. Hence, the two tangents represent the two rectangular axes of a regular hexagon, § 521, which, as will be shown by several other methods, is the true shape of the equator of Model 26. But, if axis m^a of the form $P_xM_yT_z$ is 15; then, as I have shown in § 530, axis t^a must be 26. Hence the indices y and z may be displaced as follows:

$$P_xM_{15}T_{26}, \text{ or } P_xM\tfrac{15}{26}T.$$

We have still to find the index for axis p^a. The length of this axis, from the equator to the apex of the combination, has been found to be $\frac{13}{15}$ths of the length of the line, from the centre of the crystal to the middle of a horizontal edge, and this ratio is exactly indicated by the symbol $P\frac{13}{15}T$. Yet this index cannot be used as the equivalent of $_x$ in the symbol $P_xM\frac{15}{26}T$, because axis t^a of this form is precisely twice as long as axis t^a of the form $P\frac{13}{15}T$. If we endeavour to obviate this inconvenience, by doubling the divisor of the index, and writing the symbol $P\frac{13}{30}M\frac{15}{26}T$, we have a symbol in which two different values are ascribed to axis t^a, and in which axis p^a is apparently said to be $\frac{13}{15}$ of m^a, which is erroneous. To overcome the above, and some similar evils,

I propose the following standard of unity for the indices of the axes of rhombohedral forms.

Let the line ct, in the figure in page 260, be called 1, and the line cw, 2. Then the line co, which is $\frac{15}{26}$ of the line cw or 2, will be $\frac{15}{13}$ of the line ct or 1.

Agreement to this standard clears up all difficulties: the biaxial form $P\frac{13}{13}T$, will still be called $P\frac{13}{13}T$, the term 15 being readily understood to be the synonyme of the agreed value of axis t^a or unity.

The triaxial form $P_xM_yT_z$, is now called $P\frac{13}{13}M\frac{15}{13}T_2$, which indicates that axis p^a is $= \frac{13}{13}$; $m^a = \frac{15}{13}$; $t^a = \frac{2}{1}$ or $\frac{26}{13}$. All these indices refer to that standard of unity, which is a line drawn from the centre of a regular hexagon to the centre of one of its sides. This standard is called 13. Then t^a of the form P_xT is always the measure of this unity; t^a of the form $P_xM_yT_z$ is always twice this unity, which is best intimated by $_2$ following the letter T in the symbol. Then m^a of the form $P_xM_yT_2$ is always $\frac{15}{13}$ of the same unity; and p^a, which is the only axis of variable length, has always the same value as p^a of the form P_xT. This is evident from the fact, that the planes of $P_xM_yT_2$ are always equal and similar to those of P_xT, in the regular six-sided pyramids.

This agreement on a standard of unity, for the equatorial axes of these forms, gives us the following constant symbol to represent the regular six-sided pyramid.

$$P_xT,\ P_xM\tfrac{15}{13}T_2.$$

The value of the index $_x$, which is the only variable part of this symbol, is obtained by the easy process described in § 521 *c.*) Namely, *the index $_x$ is the tangent of the inclination of a plane of a regular six-sided pyramid, to the equator.* All other measurements or calculations that respect this combination, serve only to deduce or verify the angle that gives this tangent.

523. *The six-sided pyramid of the second position*, P_xM, $P_xM_2T\frac{15}{13}$. § 519.

If Model 26 is turned horizontally until the purple line, or a set of four planes, is on the north meridian and the blue line, or four terminal edges, are on the east meridian, the combination will represent $P\frac{13}{13}M$, $P\frac{13}{13}M_2T\frac{15}{13}$. In this case, the standard of unity is axis m^a. Accordingly, axis m^a of P_xM, is $\frac{15}{13}$ of axis p^a; axis m^a of the octahedral form is now $_2$; axis t^a is $\frac{15}{13}$ of the given unity; and axis p^a is $\frac{13}{13}$ of the same standard. It is therefore equally easy to describe a regular six-sided pyramid in either of these two positions.

524. In the article on the Rhombohedron, which is regarded as the hemihedral form of the six-sided pyramid, and which is denoted by the constant symbol $\frac{1}{2}P_xT$, $\frac{1}{2}P_xM\frac{15}{13}T_2$, in which the only variable part is the index $_x$, I have proposed to abridge the symbol of the rhombohedron to R_x. In like manner, I think that, in many cases, the symbol of the six-sided pyramid may be conveniently denoted by $2R_x$, which

means a *double rhombohedron.* The two positions of the six-sided pyramid can be easily marked by suitable polaric signs, thus:

$$2R\tfrac{1|3}{1|3}\ \text{Ze Zw.} \qquad\qquad 2R\tfrac{|5}{1|3}\ \text{Zn Zs.}$$

The first of these symbols indicates the combination $P\tfrac{1|3}{1|3}T$, $P\tfrac{1|3}{1|3}M\tfrac{1|5}{1|3}T_2$, and the second indicates the combination $P\tfrac{|3}{1|3}M$, $P\tfrac{1|5}{1|3}M_2T\tfrac{1|5}{1|3}$. A farther account of this proposal, with many illustrative examples, will be found in Part II.

525. PROBLEM. *Given,* Model 26. $2R_x$, *with the inclination of a plane to axis* p^a = 49° 5′; *required, the plane angles of the faces of the combination.*

a.) Employ the triangle and the given quantities described in § 521, *b.*) Seek side *b.*

Given, a = 49° 5′; B = 30°; *to find, b.*

Formula 11. $\log \tan b = \log \tan B + \log \sin a - 10.$

$$\begin{aligned} \log \tan B &= 30^\circ &&= 9.7614 \\ + \log \sin a &= 49^\circ\, 5' &&= 9.8783 \\ \hline \log \tan b &= 23^\circ\, 34' &&= 9.6397 \end{aligned}$$

Twice this product, or 23° 34′ × 2 = 47° 8′, is the acute plane angle at pole Z of an external face of Model 26.

b.) The other two angles are similar to one another. Therefore, each is ½ (180° — 47° 8′ = 132° 52′) = 66° 26′.

526. PROBLEM. *Given,* Model 26. $2R_x$, *with the angle across an equatorial edge* = 81° 50′; *required, the angle across a terminal edge.*

Employ the triangle and given quantities described in § 521, *a.*), *b.*) Seek angle A.

Given, a = 49° 5′; B = 30°; *to find,* A.

Formula 10. $\log \cos A = \log \cos a + \log \sin B - 10.$

$$\begin{aligned} \log \cos a &= 49^\circ\, 5' &&= 9.8162 \\ + \log \sin B &= 30^\circ &&= 9.6990 \\ \hline \log \cos A &= 70^\circ\, 53' &&= 9.5152 \end{aligned}$$

Twice this product, or 70° 53′ × 2 = 141° 46′, is the required angle across a terminal edge of Model 26.

527. PROBLEM. *Given,* Model 26, $2R_x$, with *the angle across a terminal edge* = 141° 46′; *required, the angle across the horizontal edge, and the value of the index* $_x$ *in the symbol.*

a.) Employ the triangle described in § 521, *b.*) You have given, A $= \frac{141^\circ\, 46'}{2} = 70^\circ\, 53'$; B = 30°; and have to find a, which is the inclination of a plane to axis p^a. The complement of a is the inclination of a plane to the equator, or half the required angle across the horizontal edge. The cotangent of a is the value of the index $_x$ in the symbol.

Given, A $= 70° 53'$; B $= 30°$; *to find*, *a*.

Formula 4. log cos a = log cos A + 10 — log sin B.

$$10 + \log \cos A = 70° 53' = 19.5152$$
$$- \log \sin B = 30° = 9.6990$$
$$\log \cos a = 49° 5' = 9.8162$$

The complement of 49° 5′ is 40° 55′. This, doubled, is 81° 50′, which is the required angle across a horizontal edge of Model 26.

b.) The cotangent of 49° 5 is .8667, or $\frac{13}{15}$, which is the value of the index $_x$ in the symbol $2\,R_x$ when the angle across a terminal edge is 141° 46′, or the angle across a horizontal edge 81° 50′. The index of any other regular six-sided pyramid may be found in the same manner.

528. Problem. *Given*, Model 26, $2\,R_x$, *with the angle across a terminal edge* = 141° 46′, *and the angle across a horizontal edge* = 81° 50′; *required, a.*) *the inclination of a terminal edge to axis* m^s; *b.*) *the inclination of a horizontal edge to axis* m^s; *and c.*) *the plane angle of a face at pole* n.

Take a right-angled solid triangle with pole n for its vertex, and having a portion of the equator for side a, and a portion of the north meridian for side b. Then C = 90° will be the edge between a and b; A = $\frac{141° 46'}{2} = 70° 53'$ will be the inclination of a plane to the north meridian; B = $\frac{81° 50'}{2} = 40° 55'$ will be the inclination of a plane to the equator; and c will be the external plane angle at pole n.

a.) *Given*, A = 70° 53′; B = 40° 55′; *to find*, *b*.

Formula 5. log cos b = log cos B + 10 — log sin A.

$$10 + \log \cos B = 40° 55' = 19.8783$$
$$- \log \sin A = 70° 53' = 9.9754$$
$$\log \cos b = 36° 53\tfrac{1}{2}' = 9.9029$$

This product, 36° 53½′, is the inclination of a terminal edge of Model 26 to the equator. The complement of this angle is the inclination of the terminal edge to axis p^s. Now, 90° — 36° 53½′ = 53° 6½′, which agrees with the product of problem § 521, *b.*)

b.) *Given*, A = 70° 53′; B = 40° 55′; *to find*, *a*.

Formula 4. log cos a = log cos A + 10 — log sin B.

$$10 + \log \cos A = 70° 53' = 19.5152$$
$$- \log \sin B = 40° 55' = 9.8162$$
$$\log \cos a = 60° = 9.6990$$

This product, 60°, is the inclination of a horizontal edge to axis m^s. Twice 60° is 120°, so that this equation *proves* the equator of Model 26 to be a regular hexagon, as has been several times *assumed* in the course *of this article*.

c.) *Given.* A = 70° 53′; B = 40° 55′; *to find, c.*

Formula 6. log cos c = log cot A + log cot B — 10.

$$
\begin{aligned}
\log\cot A &= 70^\circ\ 53' = 9.5398\\
+\ \log\cot B &= 40^\circ\ 55' = 10.0621\\
\hline
\log\cos c &= 66^\circ\ 26' = 9.6019
\end{aligned}
$$

This product, 66° 26′, is the plane angle of an external face at pole n. See § 525, *b.)*

2. The Horizontal Planes.

529. The form P.

530. *Combinations of six-sided pyramids with the horizontal planes.*

p_. P$\frac{1\,3}{1\,3}$T, P$\frac{1\,3}{1\,3}$M$\frac{1\,5}{1\,3}$$T_2$: or p_.2R$\frac{1\,3}{1\,3}$ Zw Ze. Model 96.

An incomplete prism with an incomplete pyramid. Minerals: Part II. page 118, Class 5, Order 5, Genus 1.

Analysis. The inclination of PZ to a plane of the six-sided pyramid is = 90° + x, in which formula, x signifies the inclination of a plane of the pyramid to axis p^s.

3. The Six-Sided Prism.

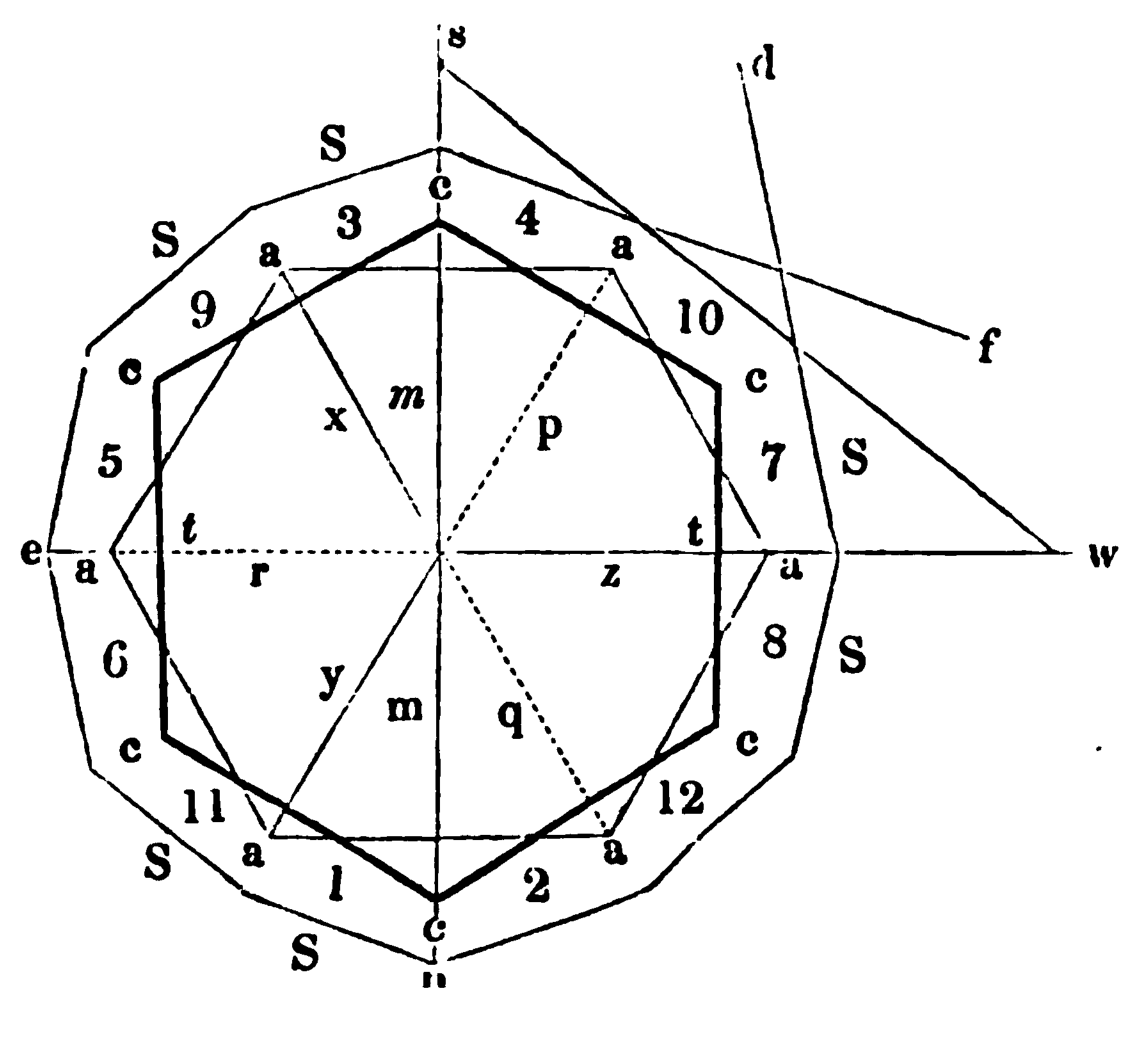
a.) *First Position.*

531. The regular six-sided prism, *first position,* is represented by the six vertical planes of Model 7, which contain the forms T, M$\frac{1\,5}{1\,3}$$T_2$. The angle across any vertical edge of a regular six-sided prism is 120°.

b.) *Second Position.*

When Model 7 is placed with two vertical edges in the east meridian, its six vertical planes represent the regular six-sided prism, *second position,* which contains the forms M, M_2T$\frac{1\,5}{1\,3}$.

The relative positions of the planes of these two varieties of the six-sided prism, are represented in the following diagram:



where the prism T, M$\frac{15}{13}$T$_2$, is represented by the hexagon whose corners are marked c, and the prism M, M$_2$T$\frac{15}{13}$, by the hexagon whose corners are marked a.

In the symbols of these two prisms, the axis of the form T or M is the standard of unity; then, axis t^a of the form M$\frac{15}{13}$T$_2$ is twice unity, and axis m^a of the same form is $\frac{15}{13}$ of unity; and in like manner, axis m^a of the form M$_2$T$\frac{15}{13}$ is twice unity, and axis t^a of the same form is $\frac{15}{13}$ of unity.

532. The six-sided prisms of the two positions occur in combination with one another, and produce the twelve-sided prism represented by the vertical planes of Model 10. The symbol for this combination is m,T, m$_2$t$\frac{15}{13}$, M$\frac{15}{13}$T$_2$; and the combination, in agreement with the indication of the symbol, exhibits six large planes and six small ones. This is invariably the case. The planes of the prism m, m$_2$t$\frac{15}{13}$, are always smaller than those of the prism T, M$\frac{15}{13}$T$_2$, and the second combination never occurs alone: in other words, when any regular six-sided prism is found, it is always to be ascribed to the first position, because no advantage can be obtained by ascribing such prisms now to one position and then to another.

It will be seen on examining the figure in § 531, that the planes of the second prism are tangent planes to the edges of the first prism. Hence, the inclination of a plane of m, m$_2$t$\frac{15}{13}$ on a plane of T,M$\frac{15}{13}$T$_2$ is $90° + \frac{120°}{2} = 150°$, as may be proved by measurement with the goniometer.

Axis m^a of the combination represented by Model 10, is always less than $\frac{15}{13}$ths of axis t^a of the combination; but since plane M is farther from the centre of the crystal than plane T, it follows, that axis m^a is never so little as $\frac{13}{13}$ths of axis t^a. On this account, I have called it $\frac{14}{13}$ths as a medium quantity, and I find that the axial relation of m^a_{14}, t^a_{13}, is quite sufficient to distinguish the twelve-sided prism, not only from the six-sided prism, but from all the forms of the prismatic system.

I have suggested in Part II., page 45, that in order to obtain short symbols to be used in describing complex combinations, we might denote the regular six-sided prism of the first position by the roman capital letter, V, (the initial of the word *vertical*), and the combination of the two six-sided prisms by the letters V, *v*.

533. *Combinations containing six-sided prisms and horizontal planes.*

P$_x$,T, M$\frac{15}{13}$T$_2$: or P$_x$,V. Model 7.
P$_x$,m,T, m$_2$t$\frac{15}{13}$, M$\frac{15}{13}$T$_2$: or P$_x$,V, *v*. Model 10.

These are complete prisms with a rhombo-rectangular equator. Minerals: Class 1, Order 5. Model 7 belongs to Genus 1; Model 10 to Genus 2. Part II. page 99. 33 minerals occur in the form of Model 7, and 11 minerals in the form of Model 10.

Analysis.—Inclination of P upon any vertical plane, 90°.

534. *Combinations containing six-sided prisms and six-sided pyramids.*

T, M$\frac{15}{13}$T$_2$. P$\frac{5}{4}$T, P$\frac{5}{4}$M$\frac{15}{13}$T$_2$: or V. 2R$\frac{5}{4}$ Zw Ze. Model 73.
T, M$\frac{15}{13}$T$_2$. P$\frac{13}{13}$T, P$\frac{13}{13}$M$\frac{15}{13}$T$_2$: or V. 2R$\frac{13}{13}$ Zw Ze. Model 74.

These combinations represent incomplete prisms with complete pyramids; and rhombo-rectangular equators. Minerals: Part II., page 113, Class 5, Order 4, Genus 1.

Analysis.—A plane of a regular six-sided prism inclines upon a plane of a regular six-sided pyramid at an angle of $90° + x$, in which formula, x signifies the inclination of a plane of the pyramid to the equator.

535. *Combinations containing regular six-sided prisms, six-sided pyramids, and horizontal planes.*

$P, T, M\frac{1\,5}{1\,3}T_2.\ p\frac{4}{3}t,\ p\frac{4}{3}m\frac{1\,5}{1\,3}t_2$: or P, V. $2r\frac{4}{3}$ Zw Ze. Model 58.

$P, T, M\frac{1\,5}{1\,3}T_2.\ pm,\ pm_2t\frac{1\,5}{1\,3}$: or P, V. $2r_1$ Zn Zs. Model 56.

$P, m, T,\ m_2t\frac{1\,5}{1\,3},\ M\frac{1\,5}{1\,3}T_2.\ pm,\ p\frac{1\,5}{2\,6}t,\ pm_2t\frac{1\,5}{1\,3},\ p\frac{1\,5}{2\,6}m\frac{1\,5}{1\,3}t_2$:

or P, V, *v*. $2r_1$ Zn Zs, $2r\frac{1\,5}{2\,6}$ Zw Ze. Model 52.

These combinations represent complete prisms with incomplete pyramids; and rhombo-rectangular equators. Minerals: Part II., page 109. The axes of the two first are $p^a_x m^a_{15} t^a_{13}$. They belong to Class 3, Order 5, Genus 1. The axes of the last are $p^a_x m^a_{14} t^a_{13}$. It belongs to Class 3, Order 5, Genus 2. The description of Model 52 affords an example of the benefit to be derived from the use of the abridged symbols.

Analysis.—See §§ 532—534.

4. The Twelve-Sided Pyramid.

536. The twelve-sided pyramid bears the same relation to the six-sided pyramid, that the dioctahedron bears to the quadratic pyramid. It occurs very seldom, and always subordinately. Generally, it produces twelve pair of small planes, replacing the solid angles between the planes of six-sided prisms and six-sided pyramids; as, for example, the acute solid angles of Model 58.

The base of this combination is shown by the twelve-sided external figure in the diagram in page 265, where the twelve sides are numbered 1 to 12. The combination consists of three rhombic octahedrons: 1st, the form $P_xM\sqsubset T$, whose equatorial edges are marked 1, 2, 3, 4, and the angle of whose base is shown by the line 4af; 2dly, the form P_xM_+T, whose equatorial edges are marked 5, 6, 7, 8, and the angle of whose equator is shown by the line 7d; 3dly, the form P_xM_T, whose equatorial edges are marked 9, 10, 11, 12, and the angle of whose equator is shown by the line sw. In many cases, this twenty-four-faced pyramid is sufficiently well indicated by the short symbol $3p_xm_yt_z$.

5. The Twelve-Sided Prism.

537. The twelve-sided prism bears the same relation to the two six-sided prisms, that the twelve-sided pyramid bears to the two six-sided pyramids. The shape of the equator and the positions of the twelve sides of this combination, are shown by the lines marked 1 to 12 in the diagram in § 531. The forms belonging to this combination are, therefore, $M\sqsubset T$, M_T, M_+T. I believe that this twelve-sided prism never occurs but in combination with the two six-sided prisms, or with the combination V, *v, the edges* of which it replaces, and forms a combina-

tion containing 24 vertical planes. I propose to distinguish this twelve-sided prism by the term $3m_x t$. There will then be three kinds of prisms peculiar to the rhombohedral system, namely:

The 6-sided prism $= V$ $= T, M\frac{1}{1}\frac{5}{3}T_2$.
The 12-sided prism $= V, v$ $= m, T, m_2 t\frac{1}{1}\frac{5}{3}, M\frac{1}{1}\frac{5}{3}T_2$.
The 24-sided prism $= V, v, 3m_x t = m, T, m_2 t\frac{1}{1}\frac{5}{3}, M\frac{1}{1}\frac{5}{3}T_2, m\sqsupset t, m\llcorner t, m_+ t$.

Of these, the six-sided prism is by far the most important, the one of most frequent occurrence, and the only one that occurs predominant upon any combination.

B. Hemihedral Forms of the Rhombohedral System.

1. THE RHOMBOHEDRON. $\frac{1}{2}P_x T, \frac{1}{2}P_x M\frac{1}{1}\frac{5}{3}T_2$: or R_x.

Examples:

$\frac{1}{2}PT, \frac{1}{2}PM\frac{1}{1}\frac{5}{3}T_2$: or R_1 Model 26^a.
$\frac{1}{2}P\frac{1}{2}T, \frac{1}{2}P\frac{1}{2}M\frac{1}{1}\frac{5}{3}T_2$: or $R\frac{1}{2}$ — 26^b.
$\frac{1}{2}P\frac{8}{3}T, \frac{1}{2}P\frac{8}{3}M\frac{1}{1}\frac{5}{3}T_2$: or $R\frac{8}{3}$ — 26^c.
$\frac{1}{2}P_2 T, \frac{1}{2}P_2 M\frac{1}{1}\frac{5}{3}T_2$: or R_2 — 26^d.
$R\frac{5}{4}$ Zw, $R\frac{5}{8}$ Ze, $r\frac{5}{2}$ Ze — 26^e.

Varieties of this Combination:

See the List of Indices, Part II., page 43.

The rhombohedron is a complete pyramid with a rhombo-rectangular equator, and falls, therefore, into Class 2, Order 5. See Part II., page 103, where the minerals which occur in this form are enumerated.

538. The rhombohedron is a solid bounded by six equal and similar rhombuses, arranged in three pair of parallel planes. It has twelve edges and eight solid angles.

539. *The Solid Angles.*—Every rhombohedron has two similar solid angles, different from the other six. One of these is situated at pole Z and the other at pole N. The other six solid angles, though different from the first two, are similar to one another. They are called the lateral solid angles. All the eight solid angles are three-faced: the faces which meet at poles Z and N are all alike, but those which meet at the lateral angles are different. The six lateral solid angles are divisible into two sets: the *upper lateral angles* are those at the lower ends of the upper terminal edges; the *lower lateral angles* are those at the upper ends of the lower terminal edges.

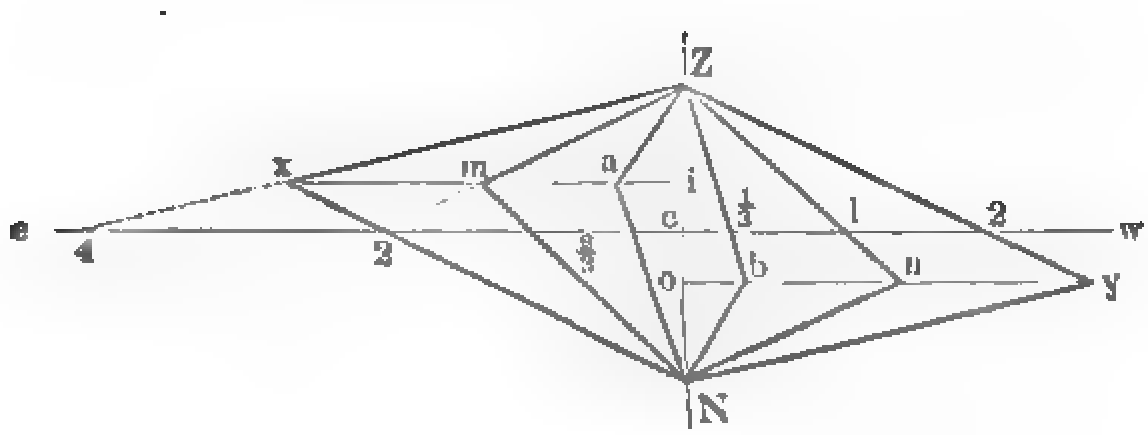
540. *The Edges.*—There are two kinds of edges on the rhombohedron; six terminal edges similar to one another, and six lateral edges, also similar to one another, but different from the terminal edges. The terminal edges meet three together at pole Z, forming a three-faced pyramid, and three at pole N, forming a second three-faced pyramid. Axis p^a connects the summits of these two pyramids. The terminal edges of the zenith and nadir pyramids do not meet together, nor do they any where touch the equator of the combination. They terminate in the lateral solid angles, which are connected by the lateral edges in a

zigzag line of six equal divisions. Hence, the lateral edges are not horizontal nor parallel to the equator.

541. *Oblique Sections.*—A section through a rhombohedron, across two terminal and two lateral edges, and parallel to an external plane, is a rhombus, whose angles are supplementary of one another. This is true of all rhombohedrons. Hence, if x denotes the angle across a terminal edge of a rhombohedron, and y the angle across a lateral edge, then the angle across x is $180° - y$, and the angle across y is $180° - x$. When the angle across the terminal edges is more than 90°, the rhombohedron is called *obtuse;* when the angle is less than 90°, the rhombohedron is called *acute.* The cube represents the rhombohedron, the angle across whose edge is 90°, § 363, and forms, therefore, the point of separation between the two kinds of rhombohedrons; but the cube is not considered to belong to the rhombohedrons, nor is the rhombohedron whose symbol has *unity for index,* the variety whose terminal edge measures 90°.

542. *Horizontal Sections.*—A horizontal section through the three upper or three lower lateral angles of a rhombohedron, produces an equilateral triangle. See the horizontal planes of Model 114, which represents a combination containing a rhombohedron with the terminal planes P. A horizontal section through a rhombohedron, any where betwixt the lateral angles and pole Z, produces an equilateral triangle. See Model 114ª. A horizontal section exactly through the *middle of the crystal,* and which is consequently *equal to the equator,* is a *regular hexagon,* or figure of six equal sides and six angles of 120°. See the brown lines drawn on Models 26ª, 26ᵇ, 26ᶜ, 26ᵈ, 114, and 114ª. The angles of this equator, or of the horizontal hexagonal section, are at the middle points of the six lateral edges. A horizontal section through a rhombohedron, any where between the equator and the lateral angles, is a hexagon whose angles are 120°, but whose sides are alternately long and short.

543. *Vertical Sections.*—A vertical section through a terminal edge of a rhombohedron, and, therefore, through the oblique diagonal of one of its planes, is a rhomboid. The direction of such a section is marked on Models 26ª, 26ᵇ, 26ᶜ, 26ᵈ, by the purple line which shows the east meridian. The *long* sides of this rhomboidal section correspond with the oblique diagonal of the planes of the crystal, and the *short* sides correspond with its terminal edges. No matter whether the rhombohedron is acute or obtuse: this relation invariably holds true. The following diagram serves to illustrate this point:



Let Z N be axis p^a, and e w be axis t^a of a series of rhombohedrons. Then, Z a N b will be a vertical section of a rhombohedron nearly similar to Model 26^c; Z m N n will be a vertical section of a rhombohedron similar to Model 26^a; and Z x N y, will be a vertical section of a rhombohedron similar to Model 26^b. These sections are all rhomboids, and in all of them the long sides are the oblique diagonals of the external planes of the crystals, and the short sides, the section through their terminal edges. Thus, Zb, Zn, and Zy are the oblique diagonals of the planes of the three Models, 26^c, 26^a, and 26^b, while Za, Zm, and Zx show the corresponding sections through their terminal edges.

544. The diagram shows another fact, which I shall presently demonstrate by a trigonometrical calculation, but which I may notice in the meantime, because it is obvious in the figure. This fact is, that *the distance from any point of axis* p^a *between pole* Z *and pole* N, *to a terminal edge of a rhombohedron, is twice as far as the distance to a plane, measured on the same horizontal line.* Thus, line c to $\frac{2}{3}$ = line c to $\frac{1}{3} \times 2$; line c to 2 = line c to 1 $\times$ 2; and line c to 4 = line c to 2 $\times$ 2. This, as I have shown in §§ 360—362, is an important and constant property of all rhombohedrons, and it enables us to establish the following very useful principles:

a.) *The cotangent of the inclination of the planes of a rhombohedron to axis* p^a, *is the length of axis* p^a, *when axis* t^a *of the form* P_xT, *to which the planes are assumed to belong, is unity.*

b.) *The tangent of the inclination of a terminal edge of a rhombohedron to axis* p^a, *is twice the tangent of the inclination of a plane to axis* p^a.

545. *The Crystallographic Forms that constitute a Rhombohedron.* Every rhombohedron contains a hemihedral biaxial form and a hemihedral triaxial form.

Place the models of the four rhombohedrons, 26^a, 26^b, 26^c, 26^d, in upright position, according to the coloured lines drawn upon them. The brown lines, indicating the equator, are to be horizontal. The blue line, indicating the north meridian, is to be on the north zone. The purple line, indicating the east meridian, is to be on the east zone. The plane marked T in ink, is to be exposed to the west. The letters stamped upon the models are to be disregarded.

a.) *The Hemihedral Biaxial Form,* $\frac{1}{2}P_xT$ Zw.—It will now be seen that each of the four models contains two planes on the east zone: one plane from Z to w, and another from e to N. These constitute the hemihedral biaxial form $\frac{1}{2}P_xT$ Zw Ne.

b.) *The Hemihedral Triaxial Form,* $\frac{1}{2}P_xM\frac{1}{1}\frac{x}{3}T_3$ Zne Znw.—It will also be seen, that each of the four models contains two zenith and two nadir octahedral planes, occupying the positions Zne Znw Nnw Nsw, and forming the hemioctahedron with parallel faces described in § 272, *d.*)

546. *The Rhombohedron is the Hemihedral Form of the Regular*

Six-sided Pyramid.—If lines are drawn from the terminal solid angle of a rhombohedron to the middle of every lateral edge, in the manner shown by the blue dotted lines on the under side of Model 26[b], these lines indicate the six terminal edges of the six-sided pyramid represented by Model 26. And if the terminal edges and lateral angles of Model 26[b] were replaced by sections made through the dotted lines drawn upon one side of the model, and the brown lines drawn upon the opposite side, the resulting solid would be a regular six-sided pyramid. The central portion of all the six planes of the original rhombohedron would be left upon the new form, but with the addition of six other planes precisely similar. If these six new planes were extended till they met one another and formed a complete figure by hiding the residual portions of the original form, the new figure would be a rhombohedron exactly similar to Model 26[b], but having a different position. The rhombohedron is consequently the hemihedral form of the regular six-sided pyramid.

547. *The Equator of the Rhombohedron is a regular Hexagon.*—It is evident that the replacement of the terminal edges and lateral angles of the rhombohedron, described in § 546, would have no effect on the shape of the equator of the combination, because all the sections are assumed simply to *meet at* and not to *cross* the equator. Hence, as a regular hexagon is the form of the equator of the six-sided pyramid, so is it also the form of the equator of the rhombohedron; and since the equatorial axes of the six-sided pyramid are described by the term $m^a_{15}t^a_{13}$, and its octahedral form by the symbol $P_xM\frac{1}{1}\frac{5}{3}T_2$, § 522, so may also the axes of the equator of the rhombohedron be described by the term $m^a_{15}t^a_{13}$, and its hemioctahedral form by the symbol $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$.

548. *Every Rhombohedron may assume four different positions on the equatorial Axes.*

1st. The *first* position is already described. Its symbol is,

$$\tfrac{1}{2}P_xT\ Zw\ Ne,\ \tfrac{1}{2}P_xM\tfrac{1}{1}\tfrac{5}{3}T_2\ Zne\ Znw\ Nnw\ Nsw.$$

2nd. Turn the Model horizontally 180°, so as to place a zenith plane towards the east, but still keep the blue line on the north meridian. The symbol of the combination will then be

$$\tfrac{1}{2}P_xT\ Ze\ Nw,\ \tfrac{1}{2}P\ M\tfrac{1}{1}\tfrac{5}{3}T_2\ Znw\ Zsw\ Nne\ Nse.$$

The above are therefore the two rhombohedrons that produce the six-sided pyramid of the *first position*, § 522.

3rd. Turn the Model horizontally 90°, so as to reverse the meridians, and place a zenith plane towards the north, and the blue line in the direction of the east meridian. The symbol of the combination is now,

$$\tfrac{1}{2}P_xM\ Zn\ Ns,\ \tfrac{1}{2}P_xM_2T\tfrac{1}{1}\tfrac{5}{3}\ Zse\ Zsw\ Nne\ Nnw.$$

4th. Turn the Model horizontally 180°, so as to place a zenith plane towards the south, but keeping the blue line on the east meridian. The symbol of the combination will then be

$$\tfrac{1}{2}P_xM\ Zs\ Nn,\ \tfrac{1}{2}P_xM_2T\tfrac{1}{1}\tfrac{5}{3}\ Zne\ Znw\ Nse\ Nsw.$$

The rhombohedrons of the third and fourth division, are those which constitute the regular six-sided pyramid of the *second position*, § 523.

These four positions of the rhombohedron, may be denoted briefly as follows, it being unnecessary to particularize the polaric position of every individual plane, because a knowledge of the position of a single zenith plane, leads to a knowledge of the whole:

1. $\frac{1}{2}P_xT\,Zw$, $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$	3. $\frac{1}{2}P_xM\,Zn$, $\frac{1}{2}P_xM_2T\frac{1}{1}\frac{5}{3}$.
2. $\frac{1}{2}P_xT\,Ze$, $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$	4. $\frac{1}{2}P_xM\,Zs$, $\frac{1}{2}P_xM_2T\frac{1}{1}\frac{5}{3}$.

549. *Abridged Symbol for the Rhombohedron.*—As the symbol for the rhombohedron is somewhat complex, and as all the terms of it are constant, except the index that relates to axis p^a, I have proposed, Part II. page 45, to abridge it to the single letter R, the initial of the word Rhombohedron, so as to be enabled to say,

R_xZw, instead of $\frac{1}{2}P_xT\,Zw$, $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$.

This greatly shortens the descriptions of complex combinations. The same index is in every case to be appended to R, that would be placed after P in the full symbol.

550. *The Index of the Rhombohedron.*—The biaxial form $\frac{1}{2}P_xT$, and the triaxial form $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$, of the rhombohedron, both take the same index, exactly as do the biaxial and triaxial forms which constitute the six-sided pyramid, § 522. If, therefore, you find the index for the form $\frac{1}{2}P_xT$, you find also the index for the form $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$, for it is the same. But the index of the form $\frac{1}{2}P_xT$, is the ratio of axis p^a to axis t^a. This ratio is the cotangent of the inclination of a plane of the rhombohedron to axis p^a, § 544. Hence, the finding of the index of a rhombohedron, is merely the finding of the inclination of a plane of the rhombohedron to its perpendicular axis.

551. PROBLEM. *Given,* Model 26^a, *with the angle across a terminal edge* $= 104°\ 28\frac{2}{3}'$; *required, the value of the index* $_x$ *in the symbol* $\frac{1}{2}P_xT$, $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$.

This is merely an example of the general problem described in § 359.

Take a sixth of Model 26^a, divided as described in § 359, as a right-angled solid triangle, with pole Z for its vertex. The known parts are $C = 90°$; $A = 60°$; $B = 104°\ 28\frac{2}{3}' \div 2 = 52°\ 14\frac{1}{3}'$, or half the angle across a terminal edge. With these given quantities, find *b*, which is the inclination of a plane of the rhombohedron to axis p^a; and *c*, which is the inclination of a terminal edge to axis p^a.

a.) *Given,* $A = 60°$; $B = 52°\ 14\frac{1}{3}'$; *to find, b.*

Formula 5. $\log\cos b = \log\cos B + 10 - \log\sin A$.

$$\begin{array}{rcl} 10 + \log\cos B = 52°\ 14\frac{1}{3}' & = & 19.7870 \\ -\log\sin A = 60° & = & 9.9375 \\ \hline \log\cos b = 45° & = & 9.8495 \end{array}$$

This product, 45°, is the inclination of a plane of Model 26ᵃ to axis pᵃ. The cotangent of 45° is 1.0000, or $\frac{1}{1}$, so that Model 26ᵃ requires the symbol $\frac{1}{2}PT$, $\frac{1}{2}PM\frac{1}{1}\frac{2}{3}T_2$, or R_1. It is the simplest form of the rhombohedron, and bears the same relation to the acute and obtuse rhombohedrons, that the cube bears to the short and long square prisms. Compare Models 26ᵇ, 26ᵃ, 26ᵈ; and Models 2, 1, 3.

b.) Given, A = 60°; B = 52° 14⅓′; *to find, c.*

Formula 6. log cos c = log cot A + log cot B — 10.

$$\begin{array}{lll} & \log\cot A = 60° & = 9.7614 \\ + & \log\cot B = 52°\ 14\frac{1}{3}' & = 9.8891 \\ \hline & \log\cos c = 63°\ 26' & = 9.6505 \end{array}$$

This product, 63° 26′, is the inclination of a terminal edge of the rhombohedron to axis pᵃ.

The tangent of 45° is 1.0000
The tangent of 63° 26′ is 2.0000

Hence, the tangent of the inclination of a terminal edge of a rhombohedron to axis pᵃ is twice the tangent of the inclination of a plane to the same axis, as was stated in § 544 *b.*) This determination justifies our ascribing to axis tᵃ of the triaxial form $\frac{1}{2}P_xM\frac{1}{1}\frac{2}{3}T_2$, twice the length that we ascribe to axis tᵃ of the biaxial form $\frac{1}{2}P_xT$.

552. PROBLEM. *Given*, Model 26ᵃ, *with the angle across a terminal edge* = 104° 28⅜′; *required, the plane angles of the faces.*

Take the same solid triangle and the same given quantities as in § 551, and find side *a.*

Given, A = 60°; B = 52° 14⅓′; *to find, a.*

Formula 4. log cos a = log cos A + 10 — log sin B.

$$\begin{array}{lll} 10 + \log\cos A = 60° & = 19.6990 \\ — \log\sin B = 52°\ 14\frac{1}{3}' & = 9.8979 \\ \hline \log\cos a = 50°\ 46' & = 9.8011 \end{array}$$

Twice this product, or 50° 46′ × 2 = 101° 32′, is the obtuse plane angle at pole Z. Its supplement, 180° — 101° 32′ = 78° 28′, is the value of the acute plane angle at a lateral solid angle of the model.

553. PROBLEM. *Given*, Model 26ᵃ, *with the symbol*, R_1; *required, the angle across the terminal edges and across the lateral edges.*

a.) The index of a rhombohedron is the cotangent of the inclination of a plane to axis pᵃ. The index 1 or 1.0000 is the cotangent of 45°, which is therefore the inclination of a plane of Model 26ᵃ to axis pᵃ.

Take a solid triangle, consisting of a sixth of the crystal, with pole Z for its vertex, and find B, equal to half the angle across a terminal edge of the crystal, *with the help* of the following known quantities: C = 90°;

$b = 45°$; $A = 60°$. See problem § 551, of which the present problem is the counterpart.

Formula 8. log cos B = log cos b + log sin A — 10.

$$\begin{array}{rl} \log\cos b = 45° & = 9.8495 \\ + \log\sin A = 60° & = 9.9375 \\ \hline \log\cos B = 52°\ 14\tfrac{1}{3}' & = 9.7870 \end{array}$$

Twice this product, or $52°\ 14\frac{1}{3}' \times 2 = 104°\ 28\frac{2}{3}'$, is the angle across a terminal edge of the rhombohedron R_1.

b.) The angle across a lateral edge is the supplement of the angle across a terminal edge, or $180° - 104°\ 28\frac{2}{3}' = 75°\ 31\frac{1}{3}'$.

554. *General Formulæ for calculating the relation between the Index and the Angles of a Rhombohedron.*

Put B = half the angle across a terminal edge,
and b = inclination of a plane to axis p^a.

Then,

a.) log cos b = log cos B + 10 — 9.9375 (log sin 60°).
b.) log cos B = log cos b + 9.9375 (log sin 60°) — 10.

See §§ 551 and 553 for the derivation and details of these formulæ.

Examples:

1.) *Given,* Model 26^b, R_x, B = 67° 13'; *required, b, and the value of the Index.*

$$\textit{Formula a.})\ 10 + \log\cos 67°\ 13' = \begin{array}{r} 19.5880 \\ -\ \ 9.9375 \\ \hline \end{array}$$

$$\log\cos 63°\ 26' = 9.6505 = b.$$

cot 63° 26' = .5000 = ½. Therefore, Model 26^b = R½. Haüy's *Chaux carbonatée équiaxe.*

2.) *Given,* Model 26^d, R_x, B = 39° 14'; *required, b, and the value of the Index.*

$$\textit{Formula a.})\ 10 + \log\cos 39°\ 14' = \begin{array}{r} 19.8891 \\ -\ \ 9.9375 \\ \hline \end{array}$$

$$\log\cos 26°\ 33' = 9.9516 = b.$$

cot 26° 33' = 2.000 = $\frac{2}{1}$. Therefore, Model 26^d = R$\frac{2}{1}$. Haüy's *Chaux carbonatée inverse.*

3.) *Given,* Model 26^c, R$\frac{8}{3}$; *required, b and* B.

By the Table of Indices, page 139, you find $\frac{8}{3}$ = 2.667 = cot 20° 33' = b.

$$\text{Then, }\textit{Formula b.})\ \log\cos 20°\ 33' = \begin{array}{r} 9.9714 \\ +\ 9.9375 \\ \hline \end{array}$$

$$\log\cos 35°\ 50' = 9.9089 = B.$$

Twice this product, or $35° 50' \times 2 = 71° 40'$, is the angle across a terminal edge of Model 26^c. Haüy's *Mercure sulfuré primitif.* He states the angle to be $71° 48'$.

555. *Combination of Rhombohedrons with one another.*—Several rhombohedrons are often found in combination upon one crystal. Model 26^e represents a combination of three different rhombohedrons. It will be useful to examine what are the possible varieties of combination that can take place among the rhombohedrons.

Rhombohedron of the First Position predominant $= R_x Zw$. § 548, 1st.

a.) It can combine with a rhombohedron of the same position, whose index is $_-$. The planes of r_- appear at pole Z, inclining on the planes of R^x. It can also combine with a rhombohedron of the same position, whose index is $_+$. The planes of r_+ appear at the lower lateral angles inclining on the planes of R_x.

b.) It can combine with a rhombohedron of the second position, $R_x Ze$, having any index whatever. If the index of the second rhombohedron is equal to the index of the first rhombohedron, and if the two rhombohedrons are similar in size as well as equal in their axial relations $=$ $R_x Zw$, $R_x Ze$, then the combination will be a regular six-sided pyramid; but if the rhombohedron of the second position is subordinate, $= r_x Ze$, then its planes replace the upper lateral angles of $R_x Zw$, and incline upon the terminal edges; and in this case, the edge of combination of the planes of the two rhombohedrons is parallel with the blue lines drawn on Models 26^a, 26^b, or with lines drawn from pole Z to the angles of the hexagonal equator.

c.) If the planes of the rhombohedron $r_x Ze$ are tangent planes to the edges of the rhombohedron $R_x Zw$, as they are represented by Model 26^e, then the index of $r_x Ze$ has a divisor twice as large as the index of $R_x Zw$, the dividend remaining the same. Thus, if $R_x Zw$ is $R\frac{5}{4}$, then $r_x Ze$ must be $r\frac{5}{8}$, because the equator of the second form has twice the diameter of the equator of the first form. This is evident from the circumstance, that the inclination to axis p^a of a *plane of the second form* is equal to the inclination to the same axis of an *edge of the first form*, while the inclination to that axis of an edge of the first form has a tangent of twice the length of the tangent of the inclination of the planes of the first form to the given axis. Of course, the edges of the second form bear the same relation to its planes, as the edges and planes of the first form bear to one another. The equatorial axes of the second form are therefore twice as long as the equatorial axes of the first form.

d.) If the rhombohedron of the second position, $r_x Ze$, has an index whose divisor is greater than twice the divisor of the index of the rhombohedron $R_x Zw$, then the second rhombohedron is obtuse, and its planes appear on $R_x Zw$ at pole Z, replacing the upper part of the terminal edges of $R_x Zw$.

e.) If the *rhombohedron* of the second position, $r_x Ze$, has an index

whose divisor is less than the divisor of the index of the rhombohedron R_x Zw, then the second rhombohedron is acute, and its planes replace the upper lateral angles of the rhombohedron R_x Zw, and incline on its terminal edges; but the edge of combination between the planes of R_x Zw and r_x Ze is not, as in case *b.*), parallel with the blue lines drawn on Model 26[b], but has a more vertical position.

f.) The rhombohedron of the first position does not combine with the rhombohedrons of the third and fourth positions.

The rhombohedron of the second position is never considered to be predominant, merely because there can be only one predominant rhombohedron on any given combination, and it is most convenient to follow the rule, always to place this in the first position.

556. *Rhombohedron of the Third Position predominant* $=$ R_xZn. § 548, 3rd.

The rhombohedron of the third position can combine with other rhombohedrons of the third position having the indices $_-$ or $_+$, and with rhombohedrons of the fourth position having any index whatever, exactly as rhombohedrons of the first position can combine with other rhombohedrons of the first position, and with all kinds of rhombohedrons of the second position. But the rhombohedrons of the third and fourth positions do not combine with rhombohedrons of the first and second positions, except when they are in the condition of homohedral six-sided pyramids.

557. ANALYSIS OF COMBINATIONS CONTAINING SEVERAL RHOMBOHEDRONS.

Take, as an example, Model 26[c]. $R\frac{5}{4}$ Zw, $\text{R}\frac{5}{8}$ Ze, $r\frac{5}{2}$ Ze. *Chabasite.*

a.) According to Phillips, the inclination of the large planes, or the planes of $R\frac{5}{4}$, to one another, measured over the tangent planes that replace the edges of $R\frac{5}{4}$, is 94° 46′. With this information, you can find the index of the predominant rhombohedron, by the method given in § 554.

$$\textit{Formula a.)}\ 10 + \log\cos\frac{94^\circ\,46'}{2} = 47^\circ\,23' = 19.8306$$
$$- \ 9.9375$$
$$\log\cos 38^\circ\,34' = 9.8931$$

cot 38° 34′ $=$ 1.2542 $=$ nearly $\frac{5}{4}$ (within 6′). This gives the symbol $R\frac{5}{4}$ Zw.

b.) The planes which replace the edges of $R\frac{5}{4}$ Zw require the symbol $\text{R}\frac{5}{8}$ Ze, according to the principle explained in § 555, *c.*)

c.) The planes which replace the lateral solid angles, and which constitute the rhombohedron r_+Ze, may be calculated from two different measurements: one upon the zenith planes of $\text{R}\frac{5}{8}$, which Phillips quotes at 143° 59′, the other upon the nadir planes of $R\frac{5}{4}$, which angle he quotes at 120° 5′.

d.) The angle of inclination $\text{R}\frac{5}{8}$ upon r_+ contains the following angles: The inclination of $\text{R}\frac{5}{8}$ to the equator $= x + 90^\circ +$ the inclination of

r_+ to axis $p^a = y$. These three quantities compose the edge of combination between the planes of any two rhombohedrons in the same zone and on the same pyramid. Consequently, if X is the angle across the edge of combination, then $y = X - (x + 90°)$

$$\text{and } x = X - (y + 90°).$$

Now angle x in the present case, or the inclination of a plane of R$\frac{5}{8}$ to the equator, is the complement of the inclination of the same plane to axis p^a. By the Table of Indices, page 139, you find $\frac{5}{8} = .6250$, which is the cotangent of 58°, or the tangent of 32°. The former is the inclination of a plane of R$\frac{5}{8}$ to axis p^a, the latter, its inclination to the equator. Therefore, $x = 32°$, and the equation $y = X - (x + 90°)$, becomes $y = 143° 59' - (32° + 90° = 122°) = 21° 59'$.

This product, 21° 59′, is therefore the inclination of r_+ to axis p^a. cot 21° 59′ = 2.4772 = nearly $\frac{5}{2}$ (within 10′).

e.) The angle of inclination of a zenith plane of R$\frac{5}{4}$ Zw upon a nadir plane of r_+ Ze contains two angles, which are, the inclination of a plane of R$\frac{5}{4}$ and of a plane of r_+ to the equator. Call the first x and the second y. These two quantities always compose the edge of combination between the planes of two different rhombohedrons that meet at the equator. Hence, if the edge of combination is called X, then $x = X - y$, and $y = X - x$.

Now, angle x, or the inclination of a plane of R$\frac{5}{4}$ to the equator, is the angle of which $\frac{5}{4} = 1.250$ is the tangent. This angle is 51° 20½′. Hence, as X is given at 120° 5′, the equation $y = X - x$, becomes $y = 120° 5' - 51° 20\frac{1}{2}' = 68° 44\frac{1}{2}'$. This product, 68° 44½′, is the inclination of a plane of r_+ Ze to the equator, the tangent of which angle is the index of the rhombohedron. Hence, tan 68° 44½′, = 2.570 = nearly $\frac{5}{2}$ (within 32′.)

f.) The index of r_+ Ze is found, by *d.*), to be 2.4772, and by *e.*), it is found to be 2.570. The index chosen as the correct relation is 2.5000, which stands nearly mid-way between the two determinations. These differences afford an example of what very frequently occurs in crystallography. The index of a symbol is often greater or smaller, according to the measurement after which it is calculated. It seldom happens that the measurements of two different edges of the same natural crystal are found to be mathematically accurate, or that the measurements of the same edge by different persons agree to a nicety. Hence, the necessity of correcting such measurements, by calculating the indices of symbols from as many different original measurements as possible, and thus making one measurement serve to check another. It is as unsafe to fix the index of a symbol from a calculation grounded on the measurement of a single angle, as it would be to determine the atomic weight of a chemical element from the evidence afforded by a single analysis.

From the result of the foregoing calculations, I conclude that the rhombohedrons contained on Model 26ᵉ are R$\frac{5}{4}$ Zw, R$\frac{5}{8}$ Ze, r$\frac{5}{2}$ Ze.

The minerals which occur in the shape of complex rhombohedrons, are enumerated in Part II. page 103, in Class 2, Order 5.

558. COMBINATIONS OF THE RHOMBOHEDRON WITH THE HORIZONTAL PLANES P.

$P_{-}.\frac{1}{2}P\frac{3}{2}T, \frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2$: or $P_{-}.R\frac{3}{2}$. Model 114. *Corundum.*
$p_{-}.\frac{1}{2}PT, \frac{1}{2}PM\frac{1}{1}\frac{5}{3}T_2$: or $p_{-}.R_1$. Model 114^a. *Calcareous Spar.*

These combinations are incomplete prisms with incomplete pyramids, having a rhombo-rectangular equator. Minerals: Class 5, Order 5, Genus 1, Part II., page 118.

Analysis.—The inclination of plane PZ to a plane of a rhombohedron is $90° + x$, in which formula, x signifies the inclination of the plane of the rhombohedron to axis p^a. If you have given, the combination $P.R_x$, with the inclination of PZ upon $P_xT = X$, then the index of R_x is the cotangent of $(X - 90°)$.

559. COMBINATIONS OF THE RHOMBOHEDRON WITH THE REGULAR SIX-SIDED PRISM.

$T, M\frac{1}{1}\frac{5}{3}T_2. \frac{1}{2}PM\ Zn, \frac{1}{2}PM_2T\frac{1}{1}\frac{5}{3}$: or $V.R_1\ Zn$. Model 71.
$T, M\frac{1}{1}\frac{5}{3}T_2. \frac{1}{2}PT\ Zw, \frac{1}{2}PM\frac{1}{1}\frac{5}{3}T_2$: or $V.R_1\ Zw$. Model 72.

These combinations are incomplete prisms with complete pyramids, having a rhombo-rectangular equator. Minerals: Class 4, Order 5, Genus 1, Part II. page 113.

Analysis.—The inclination of plane Tw to plane $P_xT\ Zw$ is $90° + x$, in which formula, x signifies the inclination of plane $P_xT\ Zw$ or of any plane of the given rhombohedron, to the equator.

The inclination of plane $M_{-}T$ to plane $P_xM\ Zn$, or of plane Tw to plane $\frac{1}{2}P_xM_2T\frac{1}{1}\frac{5}{3}\ Zsw$, is $90° + y$, in which formula, y signifies half the angle across the lateral edge of the given rhombohedron.

The common practice of crystallographers is to put the terminal planes of Model 71 into the same positions as the terminal planes of Model 72. In that case, the rhombohedron is the same in both combinations, but then, the prism of Model 71 must be denoted as the *prism of the second position.* Now, I think it is much better to consider the positions of the prisms to be *fixed,* and the positions of the pyramids to be variable. This has the great advantage of diminishing the estimated number of the prisms without increasing the estimated number of the pyramids, or adding any thing to the difficulty of naming and discriminating them. This, of course, was difficult to be done without the use of the polaric signs, Ze, Zn, &c., but *with* these signs, this arrangement can be readily carried into effect.

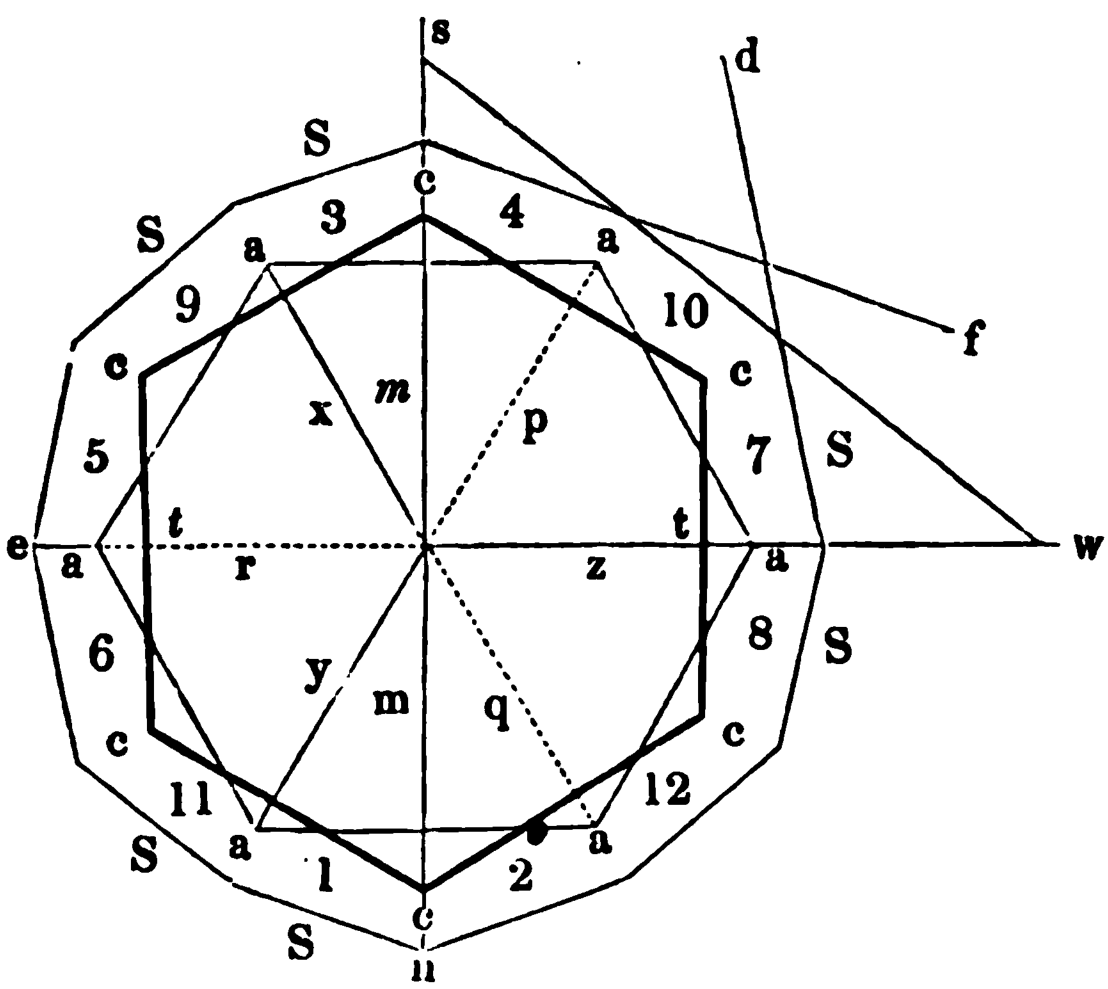
560. COMBINATION OF THE RHOMBOHEDRON, THE SIX-SIDED PRISM, AND THE HORIZONTAL PLANES.

$P, T, M\frac{1}{1}\frac{5}{3}T_2. \frac{1}{2}p\frac{3}{2}m\ Zn, \frac{1}{2}p\frac{3}{2}m_2t\frac{1}{1}\frac{5}{3}$: or $P.V.r\frac{3}{2}\ Zn$. Model 57. *Corundum.*

This is a complete prism with an incomplete pyramid, and a rhombo-rectangular equator. Minerals: Part II. page 108, Class 3, Order 5, Genus 1.

2. The Scalenohedron. Model 26f.

561. The scalenohedron is a six-sided pyramid with scalene triangular faces and a twelve-sided equator, similar to the twelve exterior edges of the following diagram:



There are three kinds of edges on the scalenohedron: six acute and short terminal edges, which have the positions of the terminal edges of a rhombohedron of the first position, or as the lines p, q, r, in the above diagram; six obtuse and long terminal edges, situated like the terminal edges of a rhombohedron of the second position, or like the lines x, y, z, in the above diagram; and six lateral edges which connect the terminal edges by a zigzag, situated like the zigzag lateral edges of a rhombohedron. Hence, the equator passes through the middle of every lateral edge, and divides three upper lateral angles from three lower lateral angles. The figure of the horizontal section made through this combination any where betwixt the lateral angles and the summit is six-sided, with three angles of one kind and three of another, placed alternately, as shown in the diagram in § 563. The vertical section through the terminal edges of the combination is a rhomboid, whose sides show the inclination of the two kinds of terminal edges to axis p^s.

562. The scalenohedron is the parallel-faced hemihedral form of the didodecahedron or twelve-sided pyramid. It occurs much more frequently than the homohedral form, is often predominant, and sometimes produces complete isolated crystals, uncombined with other forms. The planes of the twelve-sided pyramid which combine to produce the scalenohedron, are enumerated in § 274, and are marked S in the diagram in § 561. They are as follow:

$\frac{1}{2}P_xM_-T$, marked 9 and 11, and forming part of the octahedron, 9, 10, 11, 12.

$\frac{1}{2}P_xM_{\sqsubset}T$, marked 1 and 3, and forming part of the octahedron, 1, 2, 3, 4.

$\frac{1}{2}P_xM_+T$, marked 7 and 8, and forming part of the octahedron, 5, 6, 7, 8.

The complementary planes, marked 10, 12, 2, 4, 6, and 8, combine to produce a second or inverse scalenohedron.

In like manner, scalenohedrons are formed, which have the same relation to axis m^a, that the above two varieties have to axis t^a.

There are a great many varieties of the scalenohedron; probably as many or more varieties than of the rhombohedron. They are, however, all so very imperfectly described in the books on mineralogy, that I have found it impossible to give a clear account of them, and have, therefore, denoted them in the Tables of Minerals merely by a temporary proximate sign. The scalenohedron which is best known, because most abundant in a separate state, is the *metastatique* scalenohedron of calcareous spar, which is represented by Model 26^f. The axes of the three hemioctahedrons of which this combination is composed, are to be found by the following processes:

563. PROBLEM. *Given*, Model 26^f, *with the angle across an obtuse terminal edge* $= 144° \, 20\frac{1}{8}'$, *the angle across an acute terminal edge* $= 104° \, 28\frac{2}{3}'$, *and the angle across a lateral edge* $= 133° \, 26'$; *required, a.) the indices of the hemioctahedron whose planes meet at the obtuse terminal edge,* Zw; *b.) the indices of the hemioctahedron whose planes meet at the acute terminal edge,* Ze; *and c.) the indices of the hemioctahedron whose planes lye on the north meridian of the scalenohedron.*

Put $o = \frac{144° \, 20\frac{1}{2}'}{2} = 72° \, 10\frac{1}{4}'$ = half the angle across the obtuse edge.

$a = \frac{104° \, 28\frac{2}{3}'}{2} = 52° \, 14\frac{1}{3}'$ = half the angle across the acute edge.

$l = \frac{133° \, 26'}{2} = 66° \, 43'$ = half the angle across the lateral edge.

a.) The first calculation to be made is on the model of that described § 403, relating to six-faced pyramids.

Assume Model 26^f to be divided into six portions, by planes passing through the terminal edges. Take one of these portions as an oblique-angled solid triangle, with pole Z for its vertex. Then the known parts are, C = 60° = interior angle formed by the intersection of two planes at axis p^a; A $= 52° \, 14\frac{1}{3}'$ = half the angle across an acute edge; and B $= 72° \, 10\frac{1}{4}'$ = half the angle across an obtuse edge. With these data, find a = inclination of an obtuse edge of the scalenohedron to axis p^a.

Given, A $= 52° \, 14\frac{1}{3}'$; B $= 72° \, 10\frac{1}{4}'$; C = 60°; *to find, a.*

Formula 37. $\sin \frac{1}{2} a = \sqrt{\frac{-\cos S \cos (S-A)}{\sin B \sin C}}$, where $S = \frac{1}{2}(A+B+C)$

Log sin $\frac{1}{2} a$ ||

$\frac{1}{2}\{\log \cos S + \log \cos (S - A) + 20 - (\log \sin B + \log \sin C)\}$.

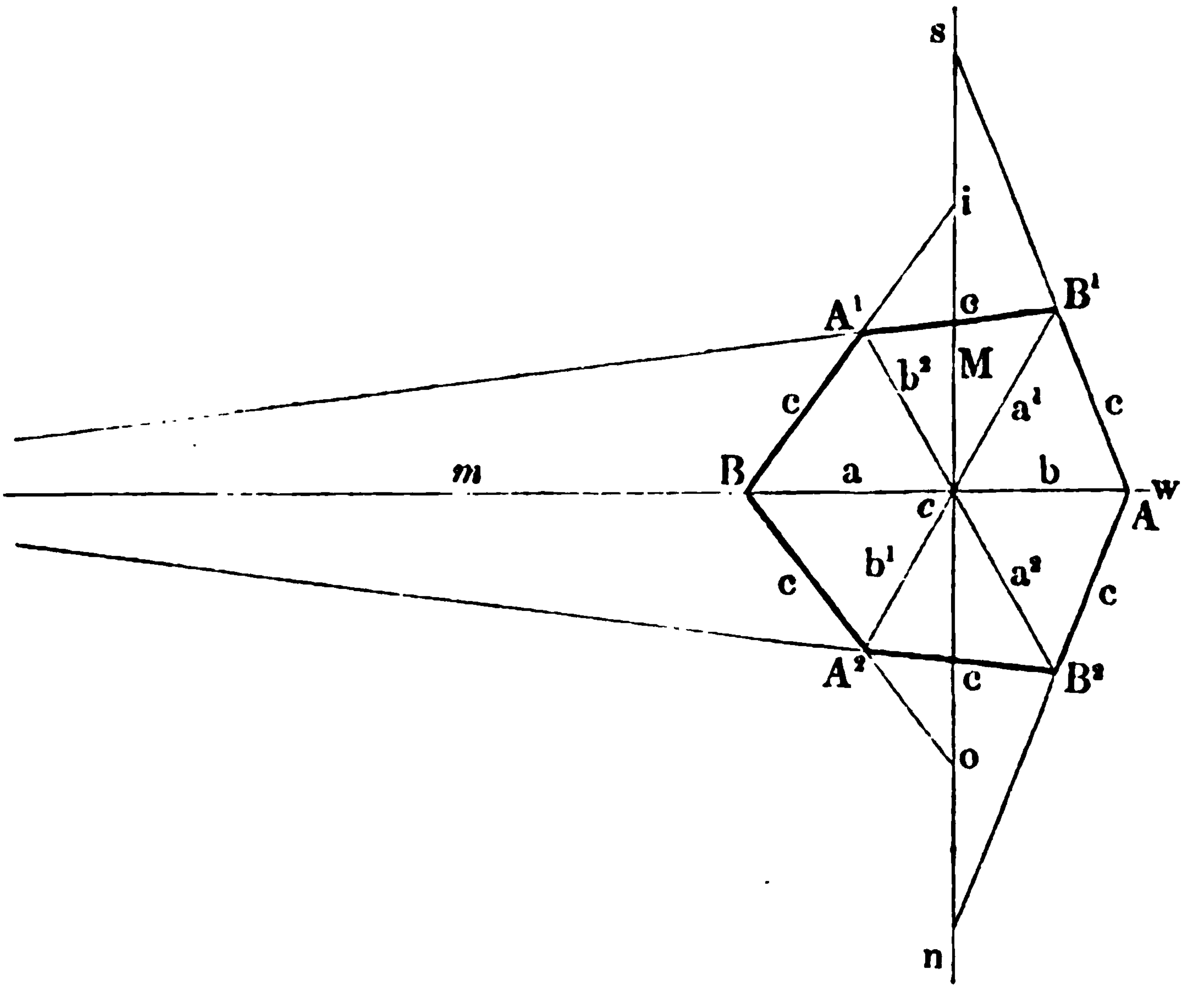
$$\begin{array}{ll}
A = 52° \, 14\frac{1}{3}' & S = 92° \, 12' \, 17\frac{1}{2}'' \\
B = 72° \, 10\frac{1}{4}' & A = 52° \, 14' \, 20'' \\
C = 60° & \overline{} \\
\overline{} & S - A = 39° \, 57' \, 57\frac{1}{2}'' \\
2)184° \, 24' \, 35'' & \\
\overline{} & \\
S = 92° \, 12' \, 17\frac{1}{2}'' & \\
\end{array}$$

$$\left.\begin{array}{r} S = 92° \, 12' \, 17\frac{1}{2}'' \\ \textit{Supplement of } S = 87° \, 47' \, 42\frac{1}{2}'' \end{array}\right\} = 180°$$

$$\log \cos S = 87^\circ\ 47\tfrac{2}{3}' = -\ 8.5852$$
$$+ \log \cos (S - A) = 39^\circ\ 58' = \quad 9.8844$$
$$+ 20 = \quad 38.4696$$
$$-\left\{\begin{array}{l} \log \sin B = 72^\circ\ 10\tfrac{1}{4}' = 9.9786 \\ + \log \sin C = 60^\circ \qquad = 9.9375 \end{array}\right\} = \quad 19.9161$$
$$2)18.5535$$
$$\log \sin \tfrac{1}{2}\, a = 10^\circ\ 54' = \quad 9.27675$$

;e this product, or 10° 54′ × 2 = 21° 48′, is the inclination of the se terminal edge of the scalenohedron to axis p^s. As this edge is on ast meridian, and proceeds from pole Z to pole w, the cotangent of nclination to p^s is the index of the octahedron whose two planes meet e Zw edge, in so far as respects axes p^s and t^s. cot 21° 48′ = 00 = $\frac{5}{2}$ = P$\frac{5}{2}$M$_x$T.

) You next proceed to investigate the value of the index $_x$ in the ol P$\frac{5}{2}$M$_x$T just quoted. The hexagon, A,B, in the following dia-ı, is a cross section through the zenith pyramid of the scalenohe-



. Line ns is axis m^s, and line ew is axis t^s. Line *c*w is axis t^s e hemioctahedron, found by *a.*) to be $\frac{5}{3}$ of the length of axis p^s. t you have now to determine, is the length of the line *c*s or *c*n, the ion of which to the line *c*w is the required value of $_x$ in the symbol $_x$T; for the triangle swn is half the equator of the given octahe-, and *c*w, *c*n, are its two equatorial axes. The lines B¹w and B²w sent *the width of so much* of the planes of P$\frac{5}{2}$M$_x$T as appear upon

the scalenohedron. The portions sB^1 and nB^2, and the entire other half of the octahedron, are replaced by the planes of the two co-existent hemioctahedrons, as is shown in the diagram.

To obtain the relation of line cn to line cw, you take an octant of the scalenohedron, Model 26^f, which is assumed to be divided for this purpose by the equator, the north meridian, and the east meridian. You employ this as a right-angled solid triangle with pole Z for its vertex. The base of this octant is shown by the lines ccB^2w; the base of the given triangle is shown by the lines cnw. The known parts of this right-angled triangle are $C = 90°$ = inclination of the east meridian on the north meridian, or of line cw on cn; $A = 72° \, 10\frac{1}{4}'$ = half the angle across an obtuse terminal edge of Model 26^f; $b = 21° \, 48'$ = inclination of the same obtuse terminal edge to axis p^a. With these given quantities, you can find a, which is the inclination of the north terminal edge of the given octahedron to axis p^a, the tangent of which angle is the required length of line cn in the diagram.

Given, $A = 72° \, 10\frac{1}{4}'$; $b = 21° \, 48'$; *to find*, a.

Formula 7. log tan a = log tan A + log sin b — 10.

$$\begin{aligned} \log \tan A &= 72° \, 10\tfrac{1}{4}' = 10.4926 \\ + \log \sin b &= 21° \, 48' \quad = \;\; 9.5698 \\ \hline \log \tan a &= 49° \;\; 6' \quad = 10.0624 \end{aligned}$$

The tangent of this product, $49° \, 6'$, is $1.1544 = \frac{15}{13}$, which gives $p^a_{13} m^a_{15}$. This is a ratio not reducible to a convenient vulgar fraction fit to replace the index $_x$ in the symbol $P_{\frac{5}{2}} M_x T$, at least, so long as the index $\frac{5}{2}$ follows the sign P. I think it better, therefore, to take axis p^a for unity, and to write the indices of this symbol as follows: $P_1 M_{\frac{15}{13}} T_{\frac{2}{3}}$. In writing the symbols of the associated hemioctahedrons of the scalenohedron, axis p^a must then always be taken for unity.

c.) You proceed now to investigate the hemioctahedron, whose two zenith planes meet at the acute terminal edge of the scalenohedron, which edge is marked B in the diagram, page 281. There are two calculations to make with this view. By the first you find the inclination of the acute terminal edge of the scalenohedron to axis p^a, the tangent of which inclination is the length of the line Bc; and then by employing this product in an equation similar to that given in *b.*), you obtain an angle whose tangent is the length of the line co or ci. The ratio of axis $m^a = co$, and of axis $t^a = cB$, to axis p^a, being thus determined, you are enabled to fix the indices of the symbol of the hemioctahedron, whose planes are marked A^1B and A^2B in the diagram.

You may find the inclination of the obtuse terminal edge of the scalenohedron to axis p^a by means of Formula 37, as shown in *a.*), merely changing the designations of the given quantities to $A = 72° \, 10\frac{1}{4}'$; $B = 52° \, 14\frac{1}{3}'$; $C = 60°$; with which to find a, which is the required angle. But the following method is shorter:

d.) *Take the* same solid triangle as was employed in *a.*), but use the

following data: A = 52° 14$\frac{1}{3}$′ = half the angle across an acute edge; a = 21° 48′ = inclination of an obtuse edge to axis p^a; B = 72° 10$\frac{1}{4}$′ = half the angle across an obtuse edge. With these data, find b = inclination of an acute edge of the scalenohedron to axis p^a.

Given, A = 52° 14$\frac{1}{3}$′; a = 21° 48′; B = 72° 10$\frac{1}{4}$′; *to find*, b.

Formula 31. log sin b = log sin B + log sin a — log sin A.

$$\begin{array}{rlcl} & \log \sin B & = 72° \; 10\tfrac{1}{4}' & = 9.9786 \\ + & \log \sin a & = 21° \; 48' & = 9.5698 \\ \hline & & & 19.5484 \\ - & \log \sin A & = 52° \; 14\tfrac{1}{3}' & = 9.8979 \\ \hline & \log \sin b & = 26° \; 34' & = 9.6505 \end{array}$$

This product, 26° 34′, is the inclination of the acute terminal edge of the scalenohedron to axis p^a. Its cotangent is 2.0, its tangent .5, which gives the ratio of $p^a_2 t^a$, or the symbol P_2M_xT, or $PM_xT\frac{1}{2}$.

e.) You next take a right-angled solid triangle, whose base is described by the triangle B*c*o in the diagram in page 281. Pole Z is its vertex, and you know the following parts, A = 52° 14$\frac{1}{3}$′ = half the angle across an acute edge of the scalenohedron; b = 26° 34′ = inclination of that edge to axis p^a. With these you have to find, a = inclination to axis p^a of a line or an edge from o to pole Z, the tangent of which angle gives the length of the line o*c*, or the ratio of axis m^a of the hemioctahedron to axis p^a.

Given, A = 52° 14$\frac{1}{3}$′; b = 26° 34′; *to find*, a.

Formula 7. log tan a = log tan A + log sin b — 10.

$$\begin{array}{rlcl} & \log \tan A & = 52° \; 14\tfrac{1}{3}' & = 10.1109 \\ + & \log \sin b & = 26° \; 34' & = 9.6505 \\ \hline & \log \tan a & = 30° & = 9.7614 \end{array}$$

The cotangent of 30° is 1.7532, or $\frac{26}{15}$; its tangent is .5774 or $\frac{15}{26}$, which gives the ratio of $p^a_{26} m^a_{15}$. If, therefore, we again call axis p^a of the symbol $P_xM_yT_z$ = 1, then the indices derived from *d*.) and *e*.) will be $P_1M\frac{15}{26}T\frac{1}{2}$.

This is the symbol of the hemioctahedron whose two zenith planes meet at the acute terminal edge Ze.

f.) You have now to examine the axial relations of the hemioctahedron whose planes lye on the north zone of the scalenohedron, and meet at the lateral edges marked $A^1 B^1$ and $A^2 B^2$ in the diagram. The angle across the lateral edge, from a plane of the upper on a plane of the lower pyramid, is given at 133° 26′. The supplement of this angle, or 180° — 133° 26′ = 46° 34′, is the inclination of plane $A^1 B^1$ upon plane $A^2 B^2$ measured across the upper pyramid. The inclined edge which this angle measures does not appear upon the scalenohedron, but its position is *from pole Z to the* extremity of the line c e, where the lines which

pass from $A^1 B^1$ and $A^2 B^2$, converge to a point at *e*. Let this inclined edge be called X, the equatorial axes of the given hemioctahedron, are the line *c* M c $=$ m^a, and the line *c m* e $=$ t^a. By means of an oblique-angled solid triangle, you first ascertain the inclination of the edge X to axis p^a, then the length of the line *c m* e, which is the tangent of the inclination of the edge to axis p^a; and finally, with these data, and by means of a right-angled solid triangle, you determine the length of the line *c* M c.

Let the triangle A^1, *c*, e, be the base of the oblique-angled solid triangle, which has pole Z for its vertex. Then angle B $= 72° \ 10\frac{1}{4}'$, is half the angle across an obtuse edge of the scalenohedron; angle A $= \frac{46° \ 34'}{2} = 23° \ 17'$, is half the inclination of plane $A^1 B^1$ on plane $A^2 B^2$; and side $a = 21° \ 48'$, is the inclination of an obtuse terminal edge to axis p^a, or that angle whose tangent is the line A^1 *c* in the diagram. With these data, you can find side *b*, which is the inclination of the edge X to axis p^a.

Given, $A = 23° \ 17'$; $B = 72° \ 10\frac{1}{4}'$; $a = 21° \ 48'$; *to find*, *b*.

Formula 31. $\log \sin b = \log \sin B + \log \sin a - \log \sin A$.

$$
\begin{array}{rll}
\log \sin B = 72° \ 10\frac{1}{4}' & = & 9.9786 \\
+ \log \sin a = 21° \ 48' & = & 9.5698 \\
\hline
 & & 19.5484 \\
- \log \sin A = 23° \ 17' & = & 9.5969 \\
\hline
\log \sin b = 63° \ 26' & = & 9.9515
\end{array}
$$

This product, 63° 26′, is the inclination of the edge X to axis p^a. Its cotangent is .5000; its tangent, 2.0000, which gives the ratio of $p^a \, t^a_2$, or the symbol $P\frac{1}{2}M_x T$, or $PM_x T_2$.

g.) Now form a right-angled solid triangle, with pole Z for its vertex, and whose base is shown by the triangle e *c* M in the diagram. The given parts of the triangle are as follow: $C = 90° =$ inclination of side *m* upon side M; $A = 23° \ 17' =$ half the angle across the edge X, or half the inclination of plane $A^1 B^1$ on plane $A^2 B^2$; $b = 63° \ 26' =$ inclination of the edge X to axis p^a. With these data, you have to find *a*, the tangent of which angle is the length of the line *c* M c, compared with axis p^a of the hemioctahedron under examination.

Given, $A = 23° \ 17'$; $b = 63° \ 26'$; *to find*, *a*.

Formula 7. $\log \tan a = \log \tan A + \log \sin b - 10$.

$$
\begin{array}{rl}
\log \tan A = 23° \ 17' & = 9.6338 \\
+ \log \sin b = 63° \ 26' & = 9.9515 \\
\hline
\log \tan a = 21° \ 3' & = 9.5853
\end{array}
$$

The cotangent of 21° 3′ is 2.5983, or $\frac{13}{5}$. Its tangent is .3849, or $\frac{5}{13}$. This gives the ratio of $p^a_{13} \, m^a_5$.

If, therefore, we again put axis p^a of the form $P \, M_y T_z = 1$, then the indices afforded by *f*.) and *g*.) become $P_1 M_{\frac{5}{13}} T_2$. This is the symbol of

the hemioctahedron whose planes lye on the north zone of the scalenohedron.

h.) The solution of the problem proposed at the beginning of this section, page 280, is as follows:

a.) is found by *a.*) and *b.*) to be $\frac{1}{2}\,\mathrm{P}\frac{1}{1}\mathrm{M}\frac{1}{1}\frac{5}{3}\mathrm{T}\frac{2}{3}$

b.) is found by *d.*) and *e.*) to be $\frac{1}{2}\,\mathrm{P}\frac{1}{1}\mathrm{M}\frac{1}{2}\frac{5}{6}\mathrm{T}\frac{1}{2}$

c.) is found by *f.*) and *g.*) to be $\frac{1}{2}\,\mathrm{P}\frac{1}{1}\mathrm{M}_{1}\frac{5}{3}\mathrm{T}\frac{2}{1}$

The full symbol is $\frac{1}{2}(\mathrm{PM}\frac{1}{1}\frac{5}{3}\mathrm{T}\frac{2}{3},\ \mathrm{PM}\frac{1}{2}\frac{5}{6}\mathrm{T}\frac{1}{2},\ \mathrm{PM}_{1}\frac{5}{3}\mathrm{T}\frac{2}{1})$.

Theory of the Scalenohedron.

564. The angle across the terminal edges of the rhombohedron, R_1, Model 26^a, is exactly the same as the angle across the acute terminal edges of the scalenohedron, Model 26^f. These edges of both combinations have the same positions, as have also the six lateral edges. But there is between the two combinations this important difference, that, whereas the terminal edges of the rhombohedron have the axial relations of $p^a_1\ t^a_2$, the acute terminal edges of the scalenohedron have the axial relations of $p^a_2\ t^a_1$; or, in other words, when the equatorial axis is considered the same for both combinations, axis p^a of the scalenohedron is four times as long as axis p^a of the rhombohedron.

If we consider the rhombohedron to be produced by a single eidogen cutting axis p^a at an angle of 63° 26′, then we must consider the scalenohedron to be composed of three equal and similar eidogens, which are placed at equal distances on the equator, and which cut axis p^a at the complementary angle of 26° 34′.

Whether the eidogen of 104° 28$\frac{2}{3}$′, which thus produces the rhombohedron R_1 and the metastatique scalenohedron, has the property of producing other scalenohedrons by cutting axis p^a at angles different from 63° 26′ and 26° 34′, or whether the eidogen which produces every different rhombohedron has the property of producing only one scalenohedron, by cutting axis p^a at an angle complementary to the angle which affords the rhombohedron, are questions which I am not prepared to determine. I have had no opportunity, either of seeing a sufficient number of crystals, or of deriving sufficient information from books, to enable me fully to comprehend the nature of the scalenohedron. It seems to be clear, however, that there is a relation between every rhombohedron and one particular scalenohedron, which is, that the terminal edge of the rhombohedron, and the acute terminal edge of the scalenohedron, cut axis p^a at angles which are the complement of one another.

Short Notation for the Scalenohedron.

565. The calculation of the axes of the octahedrons of the scalenohedron is very troublesome, and its symbol is of inconvenient length. I venture, therefore, to propose a method of dispensing with both; but I do so with considerable doubt as to its propriety, because I am not sufficiently acquainted with the varieties of the scalenohedron to be able to tell whether the following method can always be safely employed.

The proposed method is, to denote the scalenohedron, $\frac{1}{2}(PM\frac{1}{1}\frac{5}{3}T\frac{2}{3}$, $PM\frac{1}{2}\frac{5}{6}T\frac{1}{2}$, $PM\frac{1}{1}\frac{5}{3}T\frac{2}{1})$, simply by the letter S, and to fix its index on the following principle:—Let the index of the scalenohedron be the same as the index of that rhombohedron to which it bears the same relation that the metastatique scalenohedron bears to the cleavage rhombohedron of calcareous spar. What this relation is, I have explained in the Theory of the scalenohedron, § 564. It is, briefly, that *the angle across the acute terminal edge of the scalenohedron is the same as the angle across the terminal edge of the corresponding rhombohedron.*

To find the index of a scalenohedron on this principle, you require only the angle across its acute terminal edge. With this information you calculate the index of the corresponding rhombohedron, after the methods described in §§ 551, 554, and you take the same index for the index of the scalenohedron. Thus, let the acute terminal edge of the scalenohedron be given $= 104° \ 28\frac{2}{3}'$, then you say,

$$\begin{array}{lr} 10 + \log\cos 52° \ 14\frac{1}{3}' = & 19.7870 \\ & -\ 9.9375 \\ \hline \log\cos 45° \quad = & 9.8495 \end{array}$$

cot 45° = 1.0000. Hence, the scalenohedron is S_1, and is related to the rhombohedron R_1.

The chief inconvenience attached to this brief notation is, that when a symbol is given, only one external angle of the scalenohedron can be calculated from it, instead of three angles, which characterise the combination. We might however prepare a register of the known varieties of the scalenohedron, or a better mathematician than myself may be able to point out a method of finding the other angles when one is ascertained.

566. *Positions of the Scalenohedron.*—A scalenohedron is in position, when its acute terminal edges are placed in the same position as the terminal edges of the rhombohedron which has the same index. Hence, every scalenohedron may, like every rhombohedron, assume one of the following four positions:—

R_1 Zw	R_1 Ze	R_1 Zn	R_1 Zs
S_1 Zw	S_1 Ze	S_1 Zn	S_1 Zs

Model 26ᵃ, as marked in ink, is in the position R_1 Zw, and Model 26ᶠ, also as marked in ink, is in the position S_1 Zw. It follows that the obtuse terminal edge of the scalenohedron has always the position of the inclined diagonal of the faces of the rhombohedron.

The above method of denoting the indices and positions of the scalenohedron had not occurred to me when the second part of this work was printed, and these points were therefore not attended to in the chapter which contains the description of the combinations of the scalenohedron.

567. COMBINATIONS CONTAINING THE SCALENOHEDRON.

a.) Scalenohedrons with Rhombohedrons.

r_1 Zw, S_1 Zw. In this case, the rhombohedron appears on the summit of the combination, so that its terminal edges can be measured. All the edges of S_1 are visible.

R_- Zw, s_+ Zw. A scalenohedron bevelling the edges of a rhombohedron. All the edges of s_+ are visible.

R_- Zw, s_+ Ze. A scalenohedron replacing the lateral angles of the rhombohedron. The terminal edges of s_+ can be measured.

See Part II. page 53. Such combinations are too numerous for quotation here.

b.) Scalenohedrons with Vertical Prisms and Horizontal Planes.

t, $m\frac{1}{1}\frac{5}{3}t_2$. S_1 Zw. A scalenohedron with its lateral angles replaced by six vertical planes.

t, $m\frac{1}{1}\frac{5}{3}t_2$. S_1 Zn. A scalenohedron with its lateral edges replaced by six vertical planes. This is commonly called a combination of the scalenohedron with the six-sided prism of the second position, m, $m_2t\frac{1}{1}\frac{5}{3}$; but it is much better to consider the position of the prism as fixed, and that of the scalenohedron as variable. The signs, Zn and Zw, intimate the difference between the two combinations with sufficient precision, as will be seen if Model 26^f is held in the two positions.

P.S. A scalenohedron with its summits truncated.

c.) Scalenohedrons with Scalenohedrons.

In combinations of scalenohedrons with one another, the edges of the separate scalenohedrons are either all visible, or are replaced in such a manner as to be readily calculated.

568. MINERALS WHICH PRESENT THE SCALENOHEDRON.

The scalenohedrons alone, and their combinations with rhombohedrons, are complete prisms with rhombo-rectangular equators, and fall into Class 2, Order 5. See Part II. page 103. The combinations of scalenohedrons with vertical prisms, but without horizontal planes, are incomplete prisms with complete pyramids and rhombo-rectangular equators. See Class 4, Order 5, Part II. page 113. The combinations of scalenohedrons with the horizontal planes, but without vertical planes, are incomplete prisms with incomplete pyramids and rhombo-rectangular equators. See Class 5, Order 5, Part II. page 118.

569. ASPECT OF COMPLEX CRYSTALS BELONGING TO THE RHOMBOHEDRAL SYSTEM, WITH THE SYMBOLS OF FORMS THAT REPLACE THE EDGES AND ANGLES OF PREDOMINANT COMBINATIONS.

THE SIX-SIDED PYRAMID predominant. 2 R_x Zw Ze. Model 26.

Terminal solid angles replaced by:

1 horizontal plane = p. Model 96.

6 planes, inclining on the planes = $2r_-$ Zw Ze.

6 planes, *inclining on* the edges = $2r_-$Zn Zs.

Terminal edges replaced by:
  1 plane $= 2r_x$ Zn Zs.
Lateral edges replaced by:
  1 vertical plane $=$ V. Models 73, 74.
  2 planes $= 2r_+$ Zw Ze.
Lateral solid angles replaced by:
  1 plane $=$ m; $m_2 t \frac{1}{1}\frac{2}{3}$. But in this case, the position of the combination is altered to suit the symbol, V. 2 R_x Zn Zs.
  2 planes, inclining on the terminal edges $= 2r_+$Zn Zs.

THE SIX-SIDED PRISM predominant. P,V: or P,T,M$\frac{1}{1}\frac{2}{3}T_2$. Model 7.

Vertical edges replaced by:
  1 plane $=$ 12-sided prism $=$ P,V, *v*. Model 10.
  2 planes? $=$ 18-sided prism $=$ P,V, $3m_x t$.
  3 planes $=$ 24-sided prism $=$ P,V, *v*, $3m_x t$.
Terminal edges replaced by:
  1 plane $= 2r_x$ Zw Ze. Model 58.
  2 planes $= 2r_-$ Zw Ze, $2r_+$ Zw Ze.
  1 plane, to the extinction of P $=$ 2 R_x Zw Ze. Model 74.
  1 plane, to the extinction of V $=$ 2 R_x Zw Ze. Model 96.

The following series of Models shows the mutual relations of the Six-Sided Pyramid and Six-Sided Prism.

Model 26.		2 R_x Zw Ze.
96.	p.	2 R_x Zw Ze.
74.	v.	2 R_x Zw Ze.
73.	V.	2 R_x Zw Ze.
58.	P,V.	$2r_x$ Zw Ze.
7.	P,V.	

Alternate terminal edges replaced by:
  1 plane $= r_x$ Ze or r_x Zw.
  1 plane to the extinction of P $= R_x$ Zw. Model 72: or R_x Ze.
  1 plane to the extinction of V $= R_x$ Zw. Model 114: or R_x Ze.
Solid angles replaced by:
  1 plane $= 2r_x$ Zn Zs. Model 56.
  2 planes? $= 3p_x m_y t_x$.
Alternate solid angles replaced by:
  1 plane $= r_x$ Zn. Model 57.
  1 plane, to the extinction of P $= R_x$ Zn. Model 71.
  1 plane, to the extinction of V $= R_x$ Zn. Model 114.

THE RHOMBOHEDRON predominant. R_x Zw. Model 26[a].

Terminal solid angle replaced by:
  1 horizontal plane $=$ p. Models 114, 114[a].
  3 planes, inclining on the planes $= r_-$Zw.
  3 planes, inclining on the terminal edges $= r_-$Ze.
  6 planes, inclining unequally $= s_-$ Zw.

'erminal edges replaced by:

1 tangent plane $=$ r₋Ze. See § 555 *c*.)

2 planes $= s_{-}$ Zw.

ateral edges replaced by:

1 vertical plane $=$ m, $m_2 t\frac{1}{1}\frac{\xi}{3}$. But in this case, the whole combination is put into another position, to agree with the Symbol V. R_1 Zn.

2 planes $= s_{+}$ Zw.

ateral solid angles replaced by:

1 vertical plane $=$ t, $m\frac{1}{1}\frac{\xi}{3} t_2$: or V.

1 plane, inclining on the planes $= r_{+}$ Zw.

1 plane, inclining on the terminal edges $= r_{+}$ Ze.

2 planes, inclining on the lateral edges $= s_{+}$.

THE SCALENOHEDRON predominant. S_x Zw. Model 26^f.

erminal solid angle replaced by:

1 horizontal plane $=$ p.

3 planes, inclining on the obtuse terminal edges $= r_x$ Zw.

3 planes, inclining on the acute terminal edges $= r_x$ Ze.

6 planes, inclining on the planes $= s_{-}$ Zw.

btuse terminal edges replaced by:

1 plane $= r_x$ Zw.

2 planes $= s_x$ Zw.

cute terminal edges replaced by:

1 plane $= r_x$ Ze.

2 planes $= s_x$ Ze.

teral edges replaced by:

1 vertical plane $=$ m, $m_2 t \frac{1}{1}\frac{\xi}{3}$. But in this case, the entire combination would be changed in position, so as to suit the symbol $=$ V. S_x Zn.

2 planes $= s_{+}$ Zw.

teral solid angles replaced by:

1 vertical plane $=$ V.

1 plane, inclining on the obtuse terminal edge $= r_{+}$ Zw.

1 plane, inclining on the acute terminal edge $= r_{+}$ Ze.

ANALYSIS OF AN EXAMPLE. Model 52.

The predominant form of this combination is P,V. Its vertical edges e replaced by 1 plane $=$ *v*. Its terminal edges are replaced by 1 plane $=$ $2r_x^4$ Zw Ze. Its solid angles are replaced by 1 plane $= 2r_x$ Zn Zs. hese are all the modifications that are present. Hence the combinaon is

P,V, *v*. $2r_x$ Zw Ze, $2r_x$ Zn Zs.

ompare this Symbol with the description of Model 52, given in Part . page 128, which is, P,V, *v*. $2r_1$ Zn Zs, $2r\frac{1}{2}\frac{\xi}{8}$ Zw Ze. The value of e two indices $\frac{}{1}$ and $\frac{1}{2}\frac{\xi}{8}$, is determined, either by the process given in 530, or by that given in § 534.

A SECOND EXAMPLE. Model 26°.

The predominant form is R_x Zw. Its terminal edges are replaced by 1 broad tangent plane = R_- Ze. Its lateral solid angles are replaced by 1 small plane inclining on the terminal edge = r_+ Ze. The full symbol is therefore, R_x Zw, R_- Ze, r_+ Ze. Compare this with the description of Model 26°, given in Part II. page 126, where the symbol is $R\frac{1}{4}$ Zw, $R\frac{3}{8}$ Ze, $r\frac{3}{4}$ Ze. The derivation of the three indices is explained in § 557.

IV. THE PRISMATIC SYSTEM OF CRYSTALLISATION.

570. The character of the Forms belonging to this system, as given by ROSE, is this:—They have three Axes, which are placed at right angles to one another, but are all of different length.

ROSE's enumeration of the Forms belonging to this system of crystallisation is as follows:—

A. Homohedral Forms:

1. The Rhombic Octahedron = P_+M_-T. Model 21.
2. Rhombic Prisms:
   1. Vertical Prisms............... = M_xT.
   2. Length Prisms............... = P_xT.
   3. Transverse Prisms........... = P_xM.
3. Single Planes:
   1. Length Planes................ = T.
   2. Transverse Planes............ = M.
   3. Horizontal Planes .... = P.

B. Hemihedral Forms:

1. The Rhombic Tetrahedron = $\frac{1}{2}P_+M_-T$.

ROSE's Catalogue of the Minerals that belong to the Prismatic System, is given in Part II. pages 3—13. A symbolic catalogue of the Forms and Combinations presented by the crystals of each of these minerals is given in Part II. pages 61—77.

The characters of the Forms and Combinations belonging to the crystals of this system, are distinctly stated in § 340, 4.) and in Part II. page 61. The symbols of the Forms are added in the above Table to ROSE's names. The Forms appear in the following zones:

Zones:	Forms:
Prismatic,	M, M_-T, M_+T, T.
North,	P, P_-M, P_+M, M.
East,	P, P_-T, P_+T, T.
Octahedral,	$P_xM_yT_z$.

No Hemihedral Forms occur in any zone save the octahedral, and in that only occasionally. The Axes of the Combinations are $p^x_x\ m^y_y\ t^z_z$, and most commonly $p^1_+\ m^a_-\ t^a$.

A. Homohedral Forms of the Prismatic System.

571. Since the "Homohedral Forms" of the Prismatic system of crystallisation are merely the seven fundamental Forms of Crystallography, P,M,T, M_xT. P_xM, P_xT, $P_xM_yT_z$, which have already been fully described, it follows that we have now only to study the peculiarities of their several combinations.

572. TABLE OF CHARACTERISTIC COMBINATIONS BELONGING TO THE PRISMATIC SYSTEM OF CRYSTALLISATION.

Illustrated by Models of Natural Crystals.

P,	M,	T,	M_xT.	P_xM,	P_xT,	$P_xM_yT_z$.	Axes.	Models.
—	—	—	—	—	—	$P\frac{19}{10}M\frac{8}{10}T$.	$p^a_{19}m^a_8t^a_{10}$	21.
p_+,	—	—	—	—	—	$P\frac{19}{10}M\frac{8}{10}T$.	$p^a_+m^a_-t^a$	80.
—	m_-.	—	—	—	—	$P\frac{19}{10}M\frac{8}{10}T$.	$p^a_+m^a_-t^a$	70.
—	—	—	$M\frac{8}{10}T$.	—	—	$P\frac{19}{10}M\frac{8}{10}T$.	$p^a_+m^a_-t^a$	66.
—	—	—	—	—	$p\frac{19}{10}t$,	$P\frac{19}{10}M\frac{8^3}{10}T$.	$p^a_+m^a_-t^a$	120.
P_-,	—	—	$M\frac{4}{3}T$.	—	—	—	$p^a_-m^a_4t^a_5$	6.
P_-,	—	—	$M\frac{4}{3}T$.	$p\frac{4}{3}m$.	—	—	$p^a_-m^a_4t^a_5$	44.
P_-,	m,	t_+,	$M\frac{4}{3}T$.	$p\frac{4}{3}m$,	$p\frac{4}{3}t$.	—	$p^a_-m^at^a_+$	50.
—	M_-,	—	$M\frac{4}{3}T$.	$P\frac{4}{3}M$.	—	—	$p^am^a_-t^a_+$	100.
—	—	—	$M\frac{4}{3}T$.	—	$P\frac{5}{4}T$.	—	$p^a_{12}m^a_4t^a_5$	82.
$P\frac{18}{10}$,	$M\frac{9}{10}$.	T.	—	—	—	—	$p^a_{12}m^a_9t^a_{10}$	5.
—	—	—	$M\frac{6}{10}T$.	—	$P\frac{7}{10}T$,	—	$p^a_7m^a_6t^a_{10}$	82[a]
—	—	T_-,	$M\frac{6}{10}T$.	$p\frac{7}{4}m$,	$P\frac{10}{7}T$.	—	$p^a_+m^at^a_-$	110.
—	—	—	$M\frac{4}{3}T$.	—	$P\frac{5}{4}T$.	—	$p^a_{25}m^a_{10}t^a_{20}$	82[b]
—	M_-,	—	$M\frac{4}{3}T$.	—	$P\frac{5}{4}T$.	—	$p^a_+m^a_-t^a$	104.
—	M,	T.	—	$P\frac{4}{7}M$.	—	—	$p^a_+m^at^a_-$	79[a]
—	—	T_+,	$M\frac{5}{8}T$.	—	$P\frac{7}{10}T$.	—	$p^a_+m^a_-t^a$	111.
—	M,	T,	$M\frac{5}{8}T$.	—	$P\frac{7}{10}T$,	—	$p^a_+m^a_-t^a$	97.
P_+,	—	T,	$M\frac{8}{17}T$.	$p\frac{17}{12}m$.	—	—	$p^a_+m^a_-t^a$	55.
(P_+,	—	T,	$M\frac{8}{17}T$.)	× 2	—	—	$p^a_+m^a_-t^a$	9.
p_+,	M_-,	T.	—	—	—	$P\frac{4}{7}M\frac{6}{7}T$.	$p^a_+m^a_-t^a$	43.
p_+,	M_-,	T,	$M\frac{4}{9}T$.	$P\frac{5}{4}M$,	$P\frac{10}{9}T$,	$p\frac{5}{9}m\frac{4}{9}t$.	$p^a_+m^a_-t^a$	51.
—	—	—	$M\frac{19}{36}T$,	—	$P\frac{28}{29}T$,	$p\frac{33}{36}m\frac{19}{36}t$.	$p^a_+m^a_-t^a$	90.
—	—	—	$M\frac{10}{18}T$.					

It will be seen in the above Table, that each of the seven Forms may combine with any or with all the rest. The reader will be best able to form a clear idea of the aspect of the resulting solids, if he ranges the Models in the order in which they are described in the Table, and then compares the Models with the descriptions. If, after that, he examines the catalogue of Minerals contained in pages 61 to 77 of the Second

Part of this work, he will then perceive, not only the range of combination of the Minerals of this system collectively, but also of each particular Mineral.

The farther study of the Prismatic System of Crystallisation relates to the mathematical analysis of the combinations, the details of which are given below.

THE RHOMBIC OCTAHEDRON. P_+M_-T. Model 21.

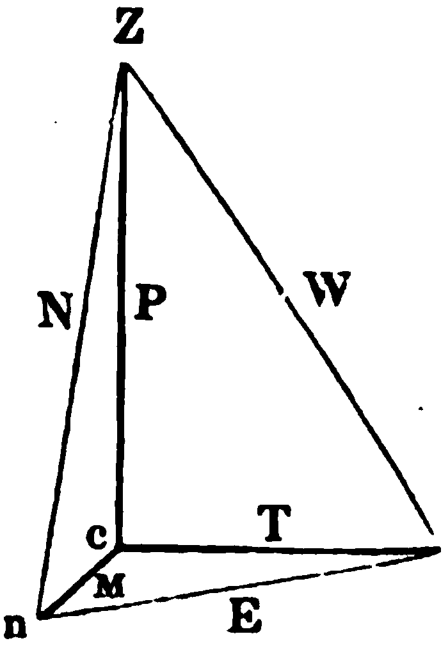
573. The rhombic octahedron contains eight planes, twelve edges, and six solid angles. The planes are scalene triangles, whose angles meet at the terminations of the three axes, $p^a\ m^a\ t^a$. The edges are of three kinds, which respectively bound the equator, the north meridian, and the east meridian. Hence the angles across any two of these sections differ from one another, and the equator, the north meridian, and the east meridian, are all rhombuses. The solid angles are also of three kinds, consisting of one pair of similar solid angles at poles Z and N, another pair at poles n and s, and a third pair at poles e and w.

A rhombic octahedron is in position when its longest axis is in the place of p^a, and its shortest axis in the place of m^a. The third axis, t^a, is then considered unity. A similar rule is observed in respect to the predominant octahedron of a combination.

The crystals of a given mineral of this system often present many varieties of the rhombic octahedron, differing in the relation of their axes. The value of those axes is found by calculation from the angles across the external edges of the Combinations.

The rhombic octahedron is a complete pyramid, with a rhombic base. Minerals, Class 2, Order 3, Part II. page 102.

574. PROBLEM. *Given,* Model 21. P_+M_-T, *with the angle across the obtuse terminal edge* $= 106° 33'$, *and the angle across the acute terminal edge* $= 84° 58'$; *required, a.) the inclination of the acute terminal edge to axes* p^a *and* t^a, *b.) the inclination of the obtuse terminal edge to axes* p^a *and* m^a, *c.) the angle across the equatorial edge, d.) the inclination of the equatorial edge to axes* m^a *and* t^a, *e.) the external plane angles at pole* Z, *pole* n, *and pole* w, *and f.) the value of the indices* $_+$ *and* $_-$ *in the symbols* P_+M_-T, $p^a_+m^a_-t^a$.



Let $\frac{106° 33'}{2} = 53° 16\frac{1}{2}' = N$;

and $\frac{84° 58'}{2} = 42° 29' = W$.

Take an octant of the rhombic octahedron, obtained by sections through the equator and the two meridians, as a right-angled solid triangle, with pole Z for its vertex. Then the given quantities of the triangle are the three angles, $C = 90°$; $A = 53° 16\frac{1}{2}'$; and $B = 42° 29'$.

a.) To find the inclination of the acute terminal edges to axes p^a *and* t^a.

Given, $A = 53^\circ\ 16\frac{1}{2}'$; $B = 42^\circ\ 29'$; *to find*, a.

Formula 4. $\log\cos a = \log\cos A + 10 - \log\sin B$.

$$10 + \log\cos A = 53^\circ\ 16\tfrac{1}{2}' = 19.7767$$
$$-\log\sin B = 42^\circ\ 29' = 9.8295$$
$$\log\cos a = 27^\circ\ 42' = 9.9472$$

This product, $27^\circ\ 42'$, is the inclination of the acute terminal edge to axis p^a. Its complement, $90^\circ - 27^\circ\ 42' = 62^\circ\ 18'$, is the inclination of the same edge to axis t^a.

b.) To find the inclination of the obtuse terminal edge to axes p^a *and* m^a.

Given, $A = 53^\circ\ 16\frac{1}{2}'$; $B = 42^\circ\ 29'$; *to find*, b.

Formula 5. $\log\cos b = \log\cos B + 10 - \log\sin A$

$$10 + \log\cos B = 42^\circ\ 29' = 19.8677$$
$$-\log\sin A = 53^\circ\ 16\tfrac{1}{2}' = 9.9039$$
$$\log\cos b = 23^\circ\ 4' = 9.9638$$

This product, $23^\circ\ 4'$, is the inclination of the obtuse terminal edge to axis p^a. Its complement, $90^\circ - 23^\circ\ 4' = 66^\circ\ 56'$ is the inclination of the same edge to axis m^a.

c.) To find the angle across the equatorial edge of the octahedron.

Take the same octant as a right-angled solid triangle with pole w for its vertex. The given parts are then $B = 42^\circ\ 29'$ = half the angle across the acute terminal edge; $a = 62^\circ\ 18'$ = inclination of that edge to axis t^a. See *a.*) With these data you have to find A, which is the inclination of a plane to the equator, or half the required angle across the equatorial edge of the Model.

Given, $a = 62^\circ\ 18'$; $B = 42^\circ\ 29'$; *to find*, A.

Formula 10. $\log\cos A = \log\cos a + \log\sin B - 10$.

$$\log\cos a = 62^\circ\ 18' = 9.6673$$
$$+\log\sin B = 42^\circ\ 29' = 9.8295$$
$$\log\cos A = 71^\circ\ 42' = 9.4968$$

This product, $71^\circ\ 42'$, is the inclination of the planes of Model 21 to the equator. Twice $71^\circ\ 42' = 143^\circ\ 24'$, is the angle across the equatorial or lateral edge.

d.) To find the inclination of the equatorial edge to axis t^a *and* m^a.

The inclination of the equatorial edge to axis t^a is equal to side b of the solid triangle employed in *c.*)

Given, $a = 62^\circ\ 18'$; $B = 42^\circ\ 29'$; *to find*, b.

Formula 11. $\log\tan b = \log\tan B +$ long $\sin a - 10$.

$$\log\tan B = 42^\circ\ 29' = 9.9618$$
$$+\log\sin a = 62^\circ\ 18' = 9.9471$$
$$\log\tan b = 39^\circ\ 2' = 9.9089$$

This product, 39° 2′, is the inclination of the equatorial edge to axis t^s. Its complement = 90° — 39° 2′ = 50° 58′, is the inclination of the same edge to axis m^s.

e.) To find the external plane angles at pole Z, *pole* n, *and pole* w.

Employ an octant of Model 21. Then, the external plane angle at pole Z is *side c* of a solid triangle having pole Z for its vertex, the angle at pole n is *side c* of a triangle having pole n for its vertex, and the angle at pole w is *side c* of a triangle having pole w for its vertex. The data with which the required sides can be found, are either the angle across the two edges which meet at each of the three poles, or the inclination of these edges to the three unipolar normals, all of which quantities are contained in the foregoing sections of this problem.

1st. To find the plane angle at pole Z.

Given, A = 53° 16½′; B = 42° 29′; *to find, c.*

Formula 6. log cos *c* = log cot A + log cot B — 10

$$\begin{array}{rl} \log\cot A = 53^\circ\ 16\tfrac{1}{2}' = & 9.8728 \\ +\ \log\cot B = 42^\circ\ 29' = & 10.0382 \\ \hline \log\cos c = 35^\circ\ 27' = & 9.9110 \end{array}$$

This product 35° 27′, is the plane angle of an external face at pole Z.

2nd. To find the plane angle at pole n.

Given, A = 53° 16½′; B = 71° 42′; *to find, c.*

Formula 6. log cos *c* = log cot A + log cot B — 10.

$$\begin{array}{rl} \log\cot A = 53^\circ\ 16\tfrac{1}{2}' = & 9.8728 \\ +\ \log\cot B = 71^\circ\ 42' = & 9.5195 \\ \hline \log\cos c = 75^\circ\ 43' = & 9.3923 \end{array}$$

This product, 75° 43′, is the external plane angle at pole n.

3rd. To find the plane angle at pole w.

Given, A = 42° 29′; B = 71° 42′; *to find, c.*

Formula 6. log cos *c* = log cot A + log cot B — 10.

$$\begin{array}{rl} \log\cot A = 42^\circ\ 29' = & 10.0382 \\ +\ \log\cot B = 71^\circ\ 42' = & 9.5195 \\ \hline \log\cos c = 68^\circ\ 50' = & 9.5577 \end{array}$$

This product, 68° 50′, is the external plane angle at pole w.

Check on the three calculations in e.)

$$\text{Plane angle at pole}\left\{\begin{array}{l} Z = 35^\circ\ 27' \\ n = 75^\circ\ 43' \\ w = 68^\circ\ 50' \end{array}\right\} = 180^\circ$$

f.) To find the value of the two indices + *and* — *in the symbols of Model* 21, P_+M_-T, $p_+^+m_-^+t^s$.

The value of the sign +, or the ratio of axis p^s to axis t^s, is the tangent of the inclination of the acute terminal edge to axis t^s.

The value of the sign —, or the ratio of axis m^a to axis t^a, is the tangent of the inclination of the equatorial edge to axis t^a.

The inclination of the acute terminal edge to axis t^a, see *a.*) is 62° 18′, tan = 1.9047. The inclination of the equatorial edge to axis t^a, see *d.*) is 39° 2′. tan = .8107. These relations give the indices $P_{1.9047}M_{0.8107}T$. $p^a_{1.9047}\ m^a_{0.8107}\ t^a_{1\cdot 0}$. For convenience sake, I have abridged these indices to $P\frac{19}{10}M\frac{8}{10}T$, $p^a_{19}\ m^a_{8}\ t^a_{10}$, although they do not then express the exact relations of the external angles. How far it may be prudent to abridge indices in this manner, I am not prepared to determine. It very seldom happens that the ratios of the axes of Forms belonging to Minerals of the Prismatic system can be indicated by indices so simple as those which serve to indicate the axes of Forms belonging to Minerals of the Octahedral and Pyramidal systems. See § 294, *r.*)

The next problem shows the amount of error introduced by this abridgement of the indices.

575. PROBLEM. *Given,* Model 21, *with the symbol* $P\frac{19}{10}M\frac{8}{10}T$, *required, the angle across its three external edges.*

a.) *To find the angle across the equatorial edge.* Take a right-angled solid triangle with pole w for its vertex. The given quantities are: *side a* or the inclination of the acute terminal edge to axis t^a, which is the angle whose tangent is $\frac{19}{10}$. By the Table of Indices, page 139, $\frac{19}{10}$ = 1.90. tan 62° 14½′, and *side b*, or the inclination of the equatorial edge to axis t^a, or the angle whose tangent is $\frac{8}{10}$ = .8 = 38° 40′. With these given quantities, you can find angle A, or the inclination of a plane to the equator, and angle B or the inclination of a plane to the east meridian.

Given, a = 62° 14½′; b = 38° 40′; *to find,* A.

Formula 13, log tan A = log tan a + 10 — log sin b.

10 + log tan a = 62° 14½′ = 20.2788
— log sin b = 38° 40′ = 9.7957

log tan A = 71° 48′ = 10.4831

Twice this product = 71° 48′ × 2 = 143° 36′, is the angle across the equatorial edge. This is 12′ more than the angle found in problem, § 574, *c.*) and quoted by Rose as the true angle, 143° 24′. The reader will perhaps consider this difference too much to be neglected. But then, we find Phillips quoting this angle at 143° 25′, Mohs at 143° 17′, Haüy at 143° 2′, and Kupffer at 143° 26.8′. Between the last two quotations there is a difference of nearly 25′. Hence the difference between the measurements of different authorities is greater than the difference indicated by short approximate indices, and by long indices which describe exactly the measurements of one selected authority. This case is not peculiar to the given crystal of Sulphur, but applies to many of the forms of the Prismatic system. We may object to approximate indices, and resolve to give exact indices, even though they be long. But when we attempt to put this resolution into practice, we find the angles

quoted by different authorities, nay, the angles across different edges of the same crystal, taken by the same person, to be frequently so irreconcilable with one another, that we are forced after all, to adopt approximate indices as a *pro tempore* expedient.

b.) *To find the angle across the acute terminal edge of* Model 21.

Given, $a = 62° 14\frac{1}{2}'$; $b = 38° 40'$; *to find*, B.

Formula 14. $\log \tan B = \log \tan b + 10 - \log \sin a$.

$$\begin{aligned} 10 + \log \tan b &= 38° 40' = 19.9032 \\ - \log \sin a &= 62° 14\tfrac{1}{2}' = 9.9469 \\ \hline \log \tan B &= 42° 7\tfrac{1}{2}' = 9.9563 \end{aligned}$$

Twice this product, or $42° 7\frac{1}{2}' \times 2 = 84° 15'$, is the angle across the acute terminal edge of the form. This, however, is 53′ too little, as Rose quotes this angle at 84° 58′. This difference is very considerable, but then Haüy quotes this angle at 84° 24′, and Phillips at 85° 5′, between which angles there is nearly an equal difference. But the two calculations serve to show that in replacing the ratios of p^a to m^a, namely, 1.9047 : 0.8107 by 1.9 : 0.8, we reduce the value of m^a too much.

c.) *To find the angle across the obtuse terminal edge of* Model 21.

Employ a solid triangle with pole n for its vertex. You have given, $A = 71° 48'$ = inclination of a plane to the equator, see *a.*); $b = 51° 20'$ = inclination of the equatorial edge to axis m^a. This angle is the complement of the inclination of the same edge to axis t^a, formed by *a.*) = 38° 40′. With these data, find B = inclination of a plane to the north meridian.

Given, $A = 71° 48'$; $b = 51° 20'$; *to find*, B.

Formula 8. $\log \cos B = \log \cos b + \log \sin A - 10$.

$$\begin{aligned} \log \cos b &= 51° 20' = 9.7957 \\ + \log \sin A &= 71° 48' = 9.9777 \\ \hline \log \cos B &= 53° 36' = 9.7734 \end{aligned}$$

Twice this product, or $53° 36' \times 2 = 107° 12'$, is the angle across the obtuse terminal edge of Model 21. This angle is quoted by Rose at 106° 33′, so that the calculation gives an excess of 39′, arising from the substitution of $\frac{8}{10}$ for 0.8107 as the length of axis m^a. The value of this angle as quoted by Haüy, is $107° 18\frac{2}{3}'$, by Phillips 106° 30′, by Kupffer 106° 16.5′, by Brooke 106° 20′, and by Mohs 106° 38′.

d.) The chief purpose of the present problem is to show the reader in what manner the angles of a rhombic octahedron may be calculated from the indices of its symbol, and that of the previous problem to show how the indices of the symbol can be found from the external angles of the crystal. The calculations are in both cases very easy, and it may seem strange that where this is the case, there should be more difficulty with the indices of the symbols than there is with the indices of other forms which require much more difficult calculations.

576. Indices of the Rhombic Prisms, M_xT, P_xM, P_xT.

M_xT. The index of the vertical prism M_xT is the cotangent of the inclination of its planes to axis m^a, or the tangent of their inclination to axis t^a. See §§ 322—327.

P_xM. The index of the form P_xM, is the cotangent of the inclination of its planes to axis p^a, or the tangent of their inclination to axis m^a.

P_xT. The index of the form P_xT, is the cotangent of the inclination of its planes to axis p^a, or the tangent of their inclination to axis t^a.

When the index of any one of these prisms is known, and the external angle is required, it may be found by the following rule:—

The angle across the edge of a rhombic prism is twice the angle whose cotangent is equal to the index of the prism.

Example: P_-, $M\frac{4}{5}T$. Model 6.

$\frac{4}{5} = \frac{8}{10} = .8.$ cot 51° 20′. Twice 51° 20′ = 102° 40′.

This product is the inclination of plane $M\frac{4}{5}T$ n w on plane $M\frac{4}{5}T$ n e.

577. Analysis of Combinations of the Prismatic System.

a.) P, M, T, or P_+, M_-, T. Model 5.

A rectangular prism. The angle across any edge is 90°. It contains 3 pair of rectangular planes. The length of the edges of the Model is as the numbers 12, 9, 10, affording the symbol $P\frac{12}{10}$,$M\frac{9}{10}$,T. It is a complete prism with a rectangular equator. Minerals ; Class 1, Order 2, Part II. page 98.

b.) P_-, $M\frac{4}{5}T$. Model 6.

A right rhombic prism. P on $M\frac{4}{5}T$ = 90°. $M\frac{4}{5}T$ n w on $M\frac{4}{5}T$ n e is twice the angle of which $\frac{4}{5}$ is the cotangent. Class 1, Order 3, Part II. page 98.

c.) p_+. $P\frac{12}{10}M\frac{8}{10}T$. Model 80.

A rhombic octahedron with its summits replaced by horizontal planes. The inclination of P on $P\frac{12}{10}M\frac{8}{10}T$, is the supplement of the inclination of $P\frac{12}{10}M\frac{8}{10}T$ to the equator. In the present case, this last angle is 71° 48′, see § 575, *c.*) Then, 180° — 71° 48′ = 108° 12′. This is the inclination of p_+ to $P\frac{12}{10}M\frac{8}{10}T$. An incomplete prism with an incomplete pyramid, having a rhombic equator. Class 5, Order 3, Part II. page 115.

d.) m —. $P\frac{12}{10}M\frac{8}{10}T$. Model 70.

A rhombic octahedron with the obtuse lateral solid angles replaced by one vertical plane. The inclination of m to $P\frac{12}{10}M\frac{8}{10}T$ is the supplement of the inclination of $P\frac{12}{10}M\frac{8}{10}T$ to the east meridian. 180° — 42° 29′ = 137° 31′. In like manner, the inclination of T to $P\frac{12}{10}M\frac{8}{10}T$, is the supplement of the inclination of $P\frac{12}{10}M\frac{8}{10}T$ to the north meridian. Model 70 is a complete pyramid with an incomplete prism, having a rhombo-rectangular equator. Class 4, Order 5, Part II. page 114.

e.) $M\frac{8}{10}T$. $P\frac{12}{10}M\frac{8}{10}T$. Model 66.

A rhombic octahedron with its equatorial edge replaced by a rhombic prism with *similar equatorial* axes. The inclination of $M\frac{8}{10}T$ to

$P\frac{10}{10}M\frac{8}{10}T$, is 90° + x, in which formula, x signifies the inclination of $P\frac{10}{10}M\frac{8}{10}T$ to the equator. A complete pyramid with an incomplete prism, and a rhombic equator. Class 4, Order 3, Part II. page 112.

f.) $p\frac{10}{8}t$, $P\frac{10}{8}M\frac{8}{10}T$. Model 120.

A rhombic octahedron with its acute terminal edges replaced by the form $p\frac{10}{8}t$, which has the same relation to axes p^a and t^a, as the form $P\frac{10}{8}M\frac{8}{10}T$. Hence the inclination of $p\frac{10}{10}t$ to $P\frac{10}{10}M\frac{8}{10}T$ is 90° + x, in which formula, x signifies the inclination of a plane of $P\frac{10}{10}M\frac{8}{10}T$ to the east meridian. An incomplete pyramid, with a rhombo-rectangular equator. Class 6, Order 5, Part II. page 122.

g.) P_-, $M\frac{4}{3}T$. $p\frac{4}{3}m$. Model 44.

The inclination of P_- upon $p\frac{4}{3}m$, is 90° + x, in which formula, x signifies the inclination of $p\frac{4}{3}m$ to axis p^a. A complete prism with an incomplete pyramid and a rhombic equator. Class 3, Order 3, Part II. page 107.

h.) P_-, m, t_x, $M\frac{4}{3}T$. $p\frac{4}{3}m$, $p\frac{4}{3}t$. Model 50.

The inclination of m upon $M\frac{4}{3}T$ is 90° + x, in which formula, x is the inclination of $M\frac{4}{3}T$ to axis m^a. For $p\frac{4}{3}m$ on P, see *g.*) The inclination of P upon $p\frac{4}{3}t$ is 90° + x, in which formula, x is the inclination of $p\frac{4}{3}t$ to axis p^a. The inclination of $p\frac{4}{3}m$ to m, or of $p\frac{4}{3}t$ to t, is 90° + x, in which formula, x signifies the inclination of $p\frac{4}{3}m$ or of $p\frac{4}{3}t$ to the equator. A complete prism with an incomplete pyramid and a rhombo-rectangular equator. Class 3, Order 5, Part II. page 109.

i.) M_-, $M\frac{4}{3}T$. $P\frac{4}{3}M$. Model 100.

Inclination of M on $M\frac{4}{3}T$, see *h.*) Inclination of M on $P\frac{4}{3}M$, see *h.*) If the angle is required across the edge that connects M with the solid angles on the east meridian, it may be found by the problem given in § 331; but you must know the inclination of $M\frac{4}{3}T$ and of $P\frac{4}{3}M$ to the east meridian. This rule applies to all combinations containing M_-T, P_xM. If, also, you know the angle across the oblique edge and one of the two other angles, you can with that information find the remaining angle, and determine the plane angles of the faces, by means of quadrantal solid triangles, because the east meridian of all combinations of M_xT, with P_xM is a side of 90°. See § 332. Model 100 is an incomplete prism with an incomplete pyramid, and a rhombo-rectangular equator. Class 5, Order 5, Part II. page 118.

j.) $M\frac{4}{3}T$. $P\frac{5}{4}T$. Model 82.

When the angles across the zenith edge and north edge are known, the angle across the inclined edge can be found by means of a quadrantal solid triangle, as shown in § 331. The inclination of $P\frac{5}{4}T$ to the west vertical edge is 90° + x, in which formula, x is the inclination of $P\frac{5}{4}T$ to the equator. An incomplete prism with an incomplete pyramid and a rhombic equator. Class 5, Order 3, Part II. page 115.

k.) $M\frac{6}{10}T$. $P\frac{7}{10}T$. Model 82*.

This combination was the subject of investigation in §§ 328—332, and serves as a model for the investigation of combinations formed by two rhombic prisms of different zones, which combinations produce octahe-

drons with a rectangular base, and afford quadrantal solid triangles when divided through the rectangular base. An incomplete prism with an incomplete pyramid and a rhombic equator. Class 5, Order 3, Part II. page 115.

l.) T_, M$\frac{6}{10}$T. p$\frac{7}{4}$m, P$\frac{10}{7}$T. Model 110.

The inclination of M$\frac{6}{10}$T to T is 90° + x, in which formula, x signifies the inclination of M$\frac{6}{10}$T to axis t^a. The inclination of P$\frac{10}{7}$T to T is 90° + x, in which formula, x signifies the inclination of P$\frac{10}{7}$T to axis t^a.

To find the inclination of p$\frac{7}{4}$m Zn *to axis* p^a. This is a very important calculation.

Assume Model 110 to be divided into two portions by the north meridian. This section will divide the small rhombic plane p$\frac{7}{4}$m Zn, exactly through the middle, and its inclination to that plane will be 90°. There are now two methods of proceeding; one to be used when you know the inclination of P_xT Zw to P_xT Ze across the zenith edge; another to be used when you know the inclination of M_xT n w to M_xT n e across the north vertical edge. A right-angled solid triangle is used in either case, but with the former angle, the vertex of the triangle is that solid angle where p_xm touches the zenith edge, and with the latter angle, the vertex of the triangle is that solid angle where p_xm touches the north vertical edge. Put the inclination of M_xT to axis m^a = B; the inclination of M_xT to p_xm = A, and the inclination of p_xm to the north meridian = C = 90°. Employing these data, and

Formula 4, $\log \cos a = \log \cos A + 10 - \log \sin B$,

you obtain in *side a* of the triangle, the inclination of plane p_xm Zn to the north vertical edge of the combination, the supplement of which angle is the required inclination of plane p_xm Zn to axis p^a.

By the other of the two methods of proceeding, above alluded to, you obtain in *side a,* the inclination of p_xm Zn to the zenith edge, which angle is 90ɔ more than the inclination of the same plane to axis p^a.

The great use of this method of calculation is to find the inclination of an oblique plane to a vertical or horizontal edge upon which it rests, and when no other planes occur in the same zone.

Model 110 is an incomplete prism with an incomplete pyramid and a rhombo-rectangular equator. Class 5, Order 5, Part II. page 119.

m.) M$\frac{4}{3}$T. P$\frac{5}{4}$T. Model 82[b].

The account of Model 82[a], see *k.*), applies equally to this combination.

n.) M_, M$\frac{4}{3}$T. P$\frac{5}{4}$T. Model 104.

Inclination of M to P$\frac{5}{4}$T = 90°: of M to M$\frac{4}{3}$T, see *h.*) An incomplete prism with an incomplete pyramid and a rhombo-rectangular equator. Class 5, Order 5, Part II. page 119.

o.) M, T. P$\frac{4}{7}$M. Model 79[a].

Inclination of T to all the other planes = 90°, of M to P$\frac{4}{7}$M, see *h.*) An incomplete prism with an incomplete pyramid and a rectangular equator. Class 5, Order 2, Part II. page 115.

p.) T+, M$\frac{5}{8}$T. P$\frac{7}{10}$T. Model 111.

M, T+, M$\frac{5}{8}$T. P$\frac{7}{10}$T. Model 97.

Inclination of T to $M\frac{5}{8}T$, and T to P_xT, see *l*.) The inclination of m to M_xT, see *h*.) The inclination of M_xT to P_xT, see *k*.) Both combinations are incomplete prisms with incomplete pyramids and rhombo-rectangular equators. Class 5, Order 5, Part II. page 119.

q.) P_+, T, $M\frac{8}{17}T$. $p\frac{17}{12}m$. Model 55.

Inclination of P to p_xm, see *g*.) Inclination of T to M_xT, see *l*.) Inclination of M_xT to p_xm, see *i*.) Inclination of p_xm to the north vertical edge, see *l*.) A complete prism with an incomplete pyramid and a rhombo-rectangular equator. Class 3, Order 5, Part II. page 109.

r.) (P_+, T, $M\frac{8}{17}T$) × 2. Model 9.

A twin crystal, each individual of which is a complete prism with a rhombo-rectangular equator. Class 1, Order 5, Part II. page 100.

s.) p_+, M_-, T. $P\frac{4}{7}M\frac{6}{7}T$. Model 43.

The value of the axes of the pyramid is determined from measurements across the Zn and Zw edges of the Form, by the method given in § 574. Inclination of p_+ to $P\frac{4}{7}M\frac{6}{7}T$, see *c*.) Of m to $P\frac{4}{7}M\frac{6}{7}T$, see *d*.) When you know the inclination of M to $P\frac{4}{7}M\frac{6}{7}T$, and of $P\frac{4}{7}M\frac{6}{7}T$ Znw to $P\frac{4}{7}M\frac{6}{7}T$ Zne, then you can determine the inclination of the Zn edge of the pyramid to axis p^2 by the process given in *l*.) In like manner, you can determine the inclination of the Zw edge of the pyramid to axis p^2, when you know the inclination of $P\frac{4}{7}M\frac{6}{7}T$ Znw to Tw and to $P\frac{4}{7}M\frac{6}{7}T$ Zsw. Model 43 is a complete prism with an incomplete pyramid and a rectangular equator. Class 3, Order 2, Part II. page 107.

t). p_+, M_-, T, $M\frac{4}{9}T$. $P\frac{5}{4}M$, $P\frac{10}{9}T$, $p\frac{5}{9}m\frac{4}{9}t$. Model 51.

Inclination of p_+ to $P\frac{5}{4}M$, see *g*.) to $P\frac{10}{9}T$, see *h*.) to $p\frac{5}{9}m\frac{4}{9}t$, see *c*.) Inclination of M to $M\frac{4}{9}T$, see *h*.) to $P\frac{5}{4}M$, see *h*.) to $p\frac{5}{9}m\frac{4}{9}t$, see *d*.) Inclination of T to $M\frac{4}{9}T$, see *l*.) to $P\frac{10}{9}T$, see *h*.) to $p\frac{5}{9}m\frac{4}{9}t$, see *d*.) Inclination of $M\frac{4}{9}T$ to $p\frac{5}{9}m\frac{4}{9}t$, see *e*.) Model 51 is a complete prism with an incomplete pyramid and a rhombo-rectangular equator. Class 3, Order 5, Part II. page 109.

u.) $M\frac{19}{36}T$, $M\frac{19}{18}T$. $P\frac{28}{29}T$, $p\frac{33}{36}m\frac{19}{36}t$. Model 90.

The inclination of $M\frac{19}{36}T$ n^2w to $M\frac{19}{18}T$ nw^2, is composed of three quantities, namely, $x + 90° + y$, in which formula, x signifies the inclination of $M\frac{19}{36}T$ to axis t^2, and y the inclination of $m\frac{19}{18}t$ to axis m^2. In the present example, x is the angle of which $\frac{19}{36}$ is the tangent, and y is the angle of which $\frac{19}{18}$ is the cotangent. These two angles added to 90° produce the interfacial angle of $M\frac{19}{36}T$ on $m\frac{19}{18}t$.

Model 90 is an incomplete prism with an incomplete pyramid and a rhombic equator. Class 5, Order 3, Part II. page 115.

B. *Hemihedral Forms of the Prismatic System.*

578. The only Hemihedral Form of the Prismatic System which is noticed by Rose, is the Rhombic Tetrahedron, the hemihedral form of the rhombic octahedron, a hemihedral form of rare occurrence and little importance.

But the Biaxial Forms M_xT. P_xM, P_xT, which are of so much importance in the Prismatic System, are all subject to become hemihedral,

and to produce the Forms $\frac{1}{2}M_xT$. $\frac{1}{2}P_xM$, $\frac{1}{2}P_xT$. *How are these disposed of?* They are classed together, to form the Fifth System of crystallisation. That is the reason why no hemihedral forms, save the rhombic tetrahedron, occur in the Prismatic System; while they are so very abundant in the Fifth or Oblique Prismatic System.

579. Aspect of Complex Crystals belonging to the Prismatic System of Crystallisation.

To belong to the Prismatic System of Crystallisation, a crystal must have the following characters:

1.) Its axes must be $p^a_x m_y t^a_z$; that is to say, three diameters, at right angles to one another, must be all different. 2.) When put into position, it must exhibit no hemihedral forms on the meridional zones. 3.) Its equator may be rectangular, rhombic, or rhombo-rectangular; but cannot be quadratic, rhombo-quadratic, or hexagonal with angles of 120°. All the combinations enumerated in § 572, and every crystal of all the minerals enumerated in pages 61 to 77 of Part II. possess these three distinctive characters, by which, indeed, any crystal of the Prismatic System can be readily distinguished from one belonging to either of the three preceding systems. Among themselves, however, the crystals of the Prismatic System differ so considerably, that it is difficult to effect any definite subordinate classification. But in fact, the discrimination of the seven Forms of this system is so extremely easy, that a very elaborate classification of combinations is unnecessary.

The Rhombic Octahedron predominant. $P_{+}M_{-}T$. Model 21.

Angle at pole Z replaced by:

- 1 horizontal plane = p.
- 2 planes, inclining on the east zone = $p_{-}t$.
- 2 planes, inclining on the north zone = $p_{-}m$.
- 4 planes, inclining on the planes, with edges of combination parallel to the equator = $p_{-}m_{-}t$, in which the equatorial axes of both octahedrons are similar.
- 4 planes, inclining partly on the planes and partly on the north meridian = $p_{-}m_{-}t_{+}$, in which axis t^a of the upper pyramid is longer than axis t^a of the predominant pyramid, axis m^a remaining the same.
- 4 planes, inclining partly on the planes, and partly on the east meridian = $p_{-}m_{+}t$, in which axis m^a of the upper pyramid is longer than axis m^a of the predominant pyramid, while axis t^a remains similar.

Angle at pole n replaced by:

- 1 vertical plane = m.
- 2 planes, inclining on the north zone = $p_{+}m$.
- 2 planes, inclining on the equator = $m_{-}t$.
- 4 planes, inclining on the planes, with edges of combination parallel to the east meridian = $p_{+}m_{\overline{-}}t$, in which axis m^a of

the subordinate pyramid is shorter than axis m^a of the predominant pyramid, while axis p^a and t^a remain similar.

4 planes, inclining partly on the planes and partly on the north meridian $= p_+m_{\sqsupset}t_+$, in which axis t^a of the subordinate pyramid is longer than axis t^a of the predominant pyramid, axis p^a remaining the same.

4 planes, inclining partly on the planes and partly on the equator $= p_{\ddagger}m_{\sqsupset}t$, in which axis p^a of the subordinate pyramid is longer, and axis m^a shorter, than the same axes of the predominant pyramid.

Angle at pole w replaced by:

1 vertical plane $= t$.

2 planes, inclining on the east zone $= p_+t$.

2 planes, inclining on the equator $= m_+t$.

4 planes, inclining on the planes, with edges of combination parallel to the north meridian $= p_+m_-t_{\lrcorner}$, in which axis t^a of the subordinate pyramid is shorter than axis t^a of the predominant pyramid, while axes p^a and m^a remain the same.

4 planes, inclining partly on the planes and partly on the east meridian $= p_+m_+t_{\lrcorner}$, in which axis m^a of the subordinate pyramid is longer than axis m^a of the predominant pyramid, axis p^a remaining the same.

4 planes, inclining partly on the planes and partly on the equator $= p_{\ddagger}m_-t_{\lrcorner}$, in which axis p^a of the subordinate pyramid is longer, and axis t^a shorter than the same axes of the predominant pyramid.

Hence the solid angles of every predominant rhombic octahedron may be affected by subordinate octahedrons in nine different ways. In all of these, however, the subordinate octahedrons shows *two different edges*, from measurements across which, the axial relations of the Forms can be calculated by the method given in § 574.

Equatorial edges replaced by:

1 tangent plane $= m_-t$, where the index has the same value as the index of the equatorial axes of the predominant octahedron.

2 planes $= p_{\ddagger}m_-t$, in which the subordinate octahedron has the same equatorial axes as the predominant form, but a longer vertical axis.

Edges of the north meridian replaced by:

1 tangent plane $= p_+m$, in which the index expresses the same value as the relation of p^a to m^a of the predominant octahedron.

2 planes $= p_+m_-t_+$, in which the subordinate octahedron agrees

with the predominant octahedron in the relation of p^a to m^a, but has t^a longer.

Edges of the east meridian replaced by:

1 tangent plane $= p_{+}t$, in which the index represents the relation of p^a to t^a in the predominant octahedron.

2 planes $= p_{+}m_{+}t$, in which the subordinate octahedron agrees with the predominant octahedron in the relation of p^a to t^a, but has m^a longer.

Hence the edges of every predominant octahedron may be affected by subordinate octahedrons in three different ways. In all of these subordinate Forms, the relation of two axes is the same as that of the corresponding two axes of the predominant form, and the value of the third axis is found by taking the angle across the new bevelled edge, and using it as an equation with one of the known quantities proper to the predominant octahedron.

THE RECTANGULAR PRISM predominant. P_{+}, $M_{\llcorner}$, T. Model 5.

Solid angles replaced by 1 plane $= p_x m_y t_z$.
Vertical edges replaced by 1 plane $= m_x t$.
Edges across the north meridian replaced by 1 plane $= p_x m$.
Edges across the east meridian replaced by 1 plane $= p_x t$.

THE RHOMBIC PRISM predominant. P, M_T. Model 6.

Obtuse solid angles replaced by:
1 plane $= p_x m$.
2 planes $= p_x m_y t_z$.
Acute solid angles replaced by:
1 plane $= p_x t$.
2 planes $= p_x m_y t_z$.
Terminal edges replaced by 1 plane $= p_x m_y t_z$.
Obtuse vertical edges replaced by:
1 plane $= m$.
2 planes $= m\sqsupset t$.
Acute vertical edges replaced by:
1 plane $= t$.
2 planes $= m_{+}t$.

THE COMBINATION M_T. $P_x T$, (*commonly called the* RECTANGULAR OCTAHEDRON) predominant. Model 82^a.

The position is supposed to be $= M\_T. P_x T$.

Solid angles at e and w replaced by:
1 plane $= t$.
2 planes, in the equatorial zone $= m_x t$.
2 planes, in the east zone $= p_x t$.
4 planes, inclining on the edges $= p_x m_y t_z$.

In this case, the indices of the octahedron $p_x m_y t_z$ are found from measurements across the two new edges produced.

Horizontal terminal edges replaced by:

1 plane $=$ p.

2 planes $=$ p_t.

Obtuse vertical edges replaced by:

1 plane $=$ m.

2 planes $=$ m_t.

Solid angles on the north meridian replaced by:

1 plane $=$ $p_x m$.

2 planes, inclining on the oblique edges $= p_x m_y t_z$.

In this case, only one edge of the resulting subordinate rhombic octahedron can be measured, but with that angle the inclination of that edge to axis p^a can be calculated by means of an oblique-angled solid triangle, in which you have three given quantities: Angle B $=$ half of $M_x T$ n e on n w; angle C $=$ half of $p_x m_y t_z$ upon $p_x m_y t_z$; angle A $=$ inclination of $M_x T$ upon $p_x m_y t_z$. Taking these as the three angles of an oblique-angled solid triangle, then, *side a* of that triangle will be the supplement of the inclination of the edge between the two planes of $p_x m_y t_z$ to axis p^a. See § 328. When this is found, the rest of the calculation is made after the methods given in §§ 574, 575. The formula to be used is No. 7. Given, A, *b*; to find, *a*; where A $=$ half the inclination of plane $p_x m_y t_z$ Znw on plane $p_x m_y t_z$ Zne; and *b* $=$ inclination of the Zn edge between these two planes to axis p^a, and consequently the angle whose cotangent shows the relation of p^a to m^a of the given octahedron. Hence, side *a* of the triangle is the inclination of the Zw edge of the same octahedron to axis p^a, or that angle whose cotangent gives the relation of p^a to t^a, desired to complete our knowledge of the relations of the three axes p^a m^a t^a.

V. THE OBLIQUE PRISMATIC SYSTEM OF CRYSTALLISATION.

580. The character of the Forms belonging to this system, as given by Rose, is this:—They have three axes, which are all unequal, and of which two cut one another obliquely, and are perpendicular to the third.

The distinction between Homohedral and Hemihedral Forms is dropped in this system. See § 578.

Rose's enumeration of the Forms belonging to it is as follows:—

A. Rhombic Prisms:

1. Vertical Prisms $= M_x T$.
2. Oblique Prisms:
   - *a.* Basic $= \frac{1}{2} P_x M_y T_z$ either $\begin{cases} Z^2nw\ Z^2ne. \\ Z^2nw\ Z^2sw. \end{cases}$
   - *b.* Front $= \frac{1}{2} P_+ M_y T_z$ either $\begin{cases} Zn^2w\ Zn^2e. \\ Znw^2\ Zsw^2. \end{cases}$
   - *c.* Rear $= \frac{1}{2} P_x M_y T_z$ either $\begin{cases} Zs^2w\ Zs^2e. \\ Zne^2\ Zse^2. \end{cases}$

B. Single Planes:

1. Vertical Planes:
   - *a.* Length Planes $=$ T.
   - *b.* Transverse Planes $=$ M.
2. Oblique Planes:
   - *a.* Basic $=$ either $\begin{cases} \frac{1}{2} P_x M\ Z^2n. \\ \frac{1}{2} P_x T\ Z^2w. \end{cases}$
   - *b.* Front $=$ either $\begin{cases} \frac{1}{2} P_+ M\ Zn^2. \\ \frac{1}{2} P_+ T\ Zw^2. \end{cases}$
   - *c.* Rear $=$ either $\begin{cases} \frac{1}{2} P_x M\ Zs. \\ \frac{1}{2} P_x T\ Ze. \end{cases}$

581. The Models numbered 79, 84, and 87, exhibit three combinations of the oblique prismatic system of crystallisation. If, in these combinations, two of the three axes be considered to lye at right angles to one another, in a plane parallel to the larger terminal plane of each combination, and if the third axis be considered to lye in the direction of the four shortest edges of each combination, then the three axes will have the properties ascribed to them by Rose; that is to say, two of the axes will cut one another obliquely, and be perpendicular to the third axis. All the descriptions which Rose gives of the Forms and Combinations of the oblique prismatic system of crystallisation, are founded upon this hypothesis. But it appears to me, that the Forms and Combinations of this system can be much more conveniently described by reference to $p^a\ m^a\ t^a$, the same three rectangular axes that are employed in all the other systems of crystallisation, than by reference to any system of oblique axes peculiar to itself. I do not intend, however, to institute here a comparison between the two methods, but shall adopt the system of rectangular axes, and merely describe my own method of notation.

582. The peculiarities of the Forms and Combination of the Oblique Prismatic System of Crystallisation, considered in reference to the new method of notation, are pretty fully described in § 340, 5), page 151; and in Part II. pages 77, 78. The chief peculiarities are as follow:

a.) First, as to Axes:

The axes are $p^a_x\ m^a_y\ t^a_z$, or *all three unequal.* No single crystal of any mineral which belongs to this system ever possesses the axial relations of $p^a\ m^a\ t^a$, or $p^a_x\ m^a\ t^a$, or $p^a_x\ m^a_{15}\ t^a_{13}$.

b.) Secondly, as to Forms:

Except in one or two rare, perhaps doubtful, cases, the horizontal planes P are never present.

All the vertical prismatic planes may be present, namely, M, M_xT, T.

The homohedral forms, P_xM, P_xT, and $P_xM_yT_z$, are of rare, perhaps doubtful, occurrence.

The biaxial hemihedral Forms $\frac{1}{2}M_xT$, $\frac{1}{2}P_xM$, $\frac{1}{2}P_xT$, and the triaxial Form $\frac{1}{2}P_xM_yT_z$, are abundant, and characteristic of this system.

c.) Thirdly, as to Combinations:

The minerals whose crystals belong to this system fall into two groups, whose combinations are as essentially different from one another, as the whole of them are collectively different from the crystals of the prismatic system.

North Combinations:

The crystals of one of these groups are characterised by hemihedral biaxial Forms belonging to the north zone, and hemihedral triaxial Forms, consisting of a single eidogen, whose axis lies in the plane of the north meridian. See §§ 271, 272, *a. b.*), and 294 *p.*)

East Combinations:

The crystals of the other group are characterised by hemihedral biaxial Forms belonging to the east zone, and hemihedral triaxial Forms, consisting of a single eidogen, whose axis lies in the plane of the east meridian. See §§ 271, 272, *c. d.*), and 294 *p.*)

The minerals which belong to one of these groups never present crystals that belong to the other group. Hence, we may divide the Forms which ROSE ascribes to the oblique prismatic system of crystallisation, into two separate systems, as follows:

Forms of North Combinations.	*Forms of East Combinations.*
M.	M.
T.	T.
M_xT.	M_xT.
$\frac{1}{2}P_xM$ Z^2n.	$\frac{1}{2}P_xT$ Z^2w.
$\frac{1}{2}P_+M$ Zn^2.	$\frac{1}{4}P_+T$ Zw^2.
$\frac{1}{2}P_xM$ Zs.	$\frac{1}{2}P_xT$ Ze.
$\frac{1}{2}P_xM_yT_z$ Z^2nw Z^2ne.	$\frac{1}{2}P_xM$ T_z $Z'nw$ Z^2sw.
$\frac{1}{2}P_+M_yT_z$ Zn^2w Zn^2e.	$\frac{1}{2}P_+M_yT_z$ Znw^2 Zsw^2.
$\frac{1}{2}P_xM_yT_z$ Zs^2w Zs^2e.	$\frac{1}{2}P_xM_yT_z$ Zne^2 Zse^2.

583. The *prismatic* portions of the crystals of these two groups have nothing peculiar, the equatorial planes of both being drawn from the series M, M_T, M_+T, T. But this is not the case with the *pyramidal* planes. Here the two groups differ essentially, presenting two different systems of pyramidal terminations, totally irreconcileable with one another. Both of these combinations, as I have said at page 78, Part II., are commonly called ***Oblique Prisms;*** and it is said of the NORTH COMBINATIONS, that the ***terminal planes,*** namely, the Form $\frac{1}{2}P_xM$ Zn, is set on the ***obtuse lateral***

edge of the prism, while of the EAST COMBINATIONS it is said, that the *terminal planes*, namely, the Form $\frac{1}{2}P_xT$ Zw, is set on the *acute lateral edge of the prism*. The additional fact, that the hemioctahedrons of the North Combinations have always the positions

Znw Zne Nsw Nse, or Zse Zsw Nne Nnw,

and the hemioctahedrons of the East Combinations, always the positions

Znw Zsw Nne Nse, or Zne Zse Nnw Nsw,

and the farther peculiarity, that the cleavage of a mineral of either group is generally typical of the inclination of its terminal planes, are circumstances which afford very decisive characteristics for the two groups of combinations, which collectively compose the crystals of this system.

584. *To find whether a crystal belongs to the group entitled North Combinations, or to that entitled East Combinations.*

Put the prismatic planes into position, so as to suit the symbol M_T or M_, T. Then observe whether the terminal planes are biaxial or triaxial; that is to say, whether the terminal planes are *crossed* by the north or east meridian, or are merely *separated from one another* by these meridians. If the Forms are biaxial, and are on the north zone, the crystal belongs to the North Combinations. If the biaxial Forms are on the east zone, the crystal belongs to the East Combinations. If the Forms are triaxial, and the pair of zenith planes lye on the same side of the east meridian, the crystal is a North Combination. If the pair of zenith planes lye on the same side of the north meridian, the crystal is an East Combination.

ROSE's catalogue of the minerals that belong to the oblique prismatic system, is given in Part II. pages 5—13. A symbolic catalogue of the combinations presented by the crystals of each of these minerals, is given in Part II. pages 77—91.

585. EXAMPLES OF NORTH COMBINATIONS.

Model 79. M_, T. $\frac{1}{2}P\frac{3}{7}M$ Zn. *Gypsum.*

115. T_, $M\frac{9}{13}T$. $\frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw. *Gypsum.*

75. T_, $M\frac{9}{13}T$. $\frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw, $\frac{1}{2}P\frac{6}{13}M\frac{10}{13}T$ Zse Zsw. *Gypsum.*

103. m, $M\frac{6}{7}T$. $\frac{1}{2}P\frac{5}{14}M$ Zn, $\frac{1}{2}P_{+}M\_T$ Zn^2e^2 Zn^2w^2, $\frac{1}{2}p_x m_y t_z$ Zne^2 Znw^2. *Azure Copper Ore.*

84. $M\frac{10}{19}T$. $\frac{1}{2}P\frac{4}{15}M$ Zn. *Hornblende.*

112. T, $M\frac{10}{19}T$. $\frac{1}{2}P\frac{4}{15}M$ Zn, $\frac{1}{2}P_xM_yT_z$ Zse Zsw. *Hornblende.*

113. (T, $M\frac{10}{19}T$. $\frac{1}{2}P\frac{4}{15}M$ Zn, $\frac{1}{2}P_xM_yT_z$ Zse Zsw) × 2. *Hornblende.*

67. $M\frac{40}{30}T$. $\frac{1}{2}P\_\ M\frac{40}{30}T$ Zne Znw, $\frac{1}{2}P\_\ M\frac{40}{30}T$ Zse Zsw. *Mesotype.*

79ᵇ. M_, T. $\frac{1}{2}P\frac{1}{2}M$ Zn. *Epidote.*

101. M_. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{2}\frac{1}{4}m$ Zs, $\frac{1}{2}P_{+}M\_T$ Zne^2 Znw^2. *Epidote.*

101[a]. M_{-}, t, $m{}^{1}_{1}{}^{8}_{1}t$. $\frac{1}{2}P\frac{1}{3}M$ Zn, $\frac{1}{2}p\frac{3}{2}m$ Zs, $\frac{1}{2}P_{+}M_{-}T$ Zne^{2} Znw^{2}. *Epidote.*

81. $M{}^{1}_{2}{}^{5}_{6}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P{}^{7}_{15}M$ Zs. *Felspar.*

109. T, $M{}^{1}_{2}{}^{5}_{6}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P{}^{7}_{13}M$ Zs. *Felspar.*

105. T, $\frac{1}{2}M{}^{1}_{2}{}^{5}_{6}T$ ne sw. $\frac{1}{2}P\frac{1}{2}M$ Zn Ns. *Felspar.*

81[a]. $\frac{1}{2}M{}^{1}_{2}{}^{5}_{6}T$ ne, $\frac{1}{2}M{}^{1}_{2}{}^{5}_{6}T$ nw. $\frac{1}{2}P\frac{1}{2}M$ Zn. *Felspar.*

586. EXAMPLES OF EAST COMBINATIONS.

Model 87. $M{}^{2}_{2}{}^{0}_{1}T$. $\frac{1}{2}P{}_{2}{}^{6}_{1}T$ Zw. *Augite.*

98. M, T, $M{}^{2}_{2}{}^{0}_{1}T$. $\frac{1}{2}P{}_{2}{}^{6}_{1}M{}^{1}_{2}{}^{0}_{1}T$ Znw Zsw. *Augite.*

99. (M, T, $M{}^{2}_{2}{}^{0}_{1}T$. $\frac{1}{2}P{}_{2}{}^{6}_{1}M{}^{1}_{2}{}^{0}_{1}T$ Znw Zsw) × 2. *Augite.*

53. p_{+}, M, T, $M{}^{2}_{2}{}^{0}_{1}T$. $\frac{1}{2}P{}_{2}{}^{6}_{1}M{}^{1}_{2}{}^{0}_{1}T$ Znw Zsw. *Augite.*

587. CLASSIFICATION OF CRYSTALS BELONGING TO THE OBLIQUE PRISMATIC SYSTEM.

As respects the Classes.

The occasional, perhaps doubtful, occurrence of the Form P, gives rise to a few complete prisms, Class 1.

No homohedral octahedrons belong to this system. Therefore no complete pyramids occur.

The presence of P gives rise to one or two complete prisms combined with incomplete pyramids, Class 3.

The simultaneous presence of two hemioctahedrons, very much alike, in combination with a prism, presents occasional combinations resembling incomplete prisms combined with complete pyramids, Class 4.

With these few exceptions, every combination of this system is an incomplete prism combined with an incomplete pyramid, Class 5.

As respects the Orders.

In consequence of the constant inequality of the equatorial axes m^{1} and t^{a}, only three kinds of equators, and therefore only three Orders, can possibly occur among the crystals of this system, namely, the rectangular, the rhombic, and the rhombo-rectangular.

As respects the Genera.

All the crystals of this system belong to the genus denominated $p^{a}_{x}m^{a}_{,}t^{a}_{,}$.

Hence the crystals of the oblique prismatic system, although very numerous, can only be referred to the following few Genera, and indeed are mostly comprised in the two last, namely:—Class 5, Order 3, Genus 1, and Class 5, Order 5, Genus 2.

Class 1.	Order 3.	Genus 1.	Part II. page 98.
— 3.	— 5.	— 3.	— II. — 109.
— 4.	— 3.	— 3.	— II. — 112.
— 5.	— 2.	— 1.	— II. — 115.
— 5.	— 3.	— 1*	— II. — 115.
— 5.	— 5.	— 2†	— II. — 118.

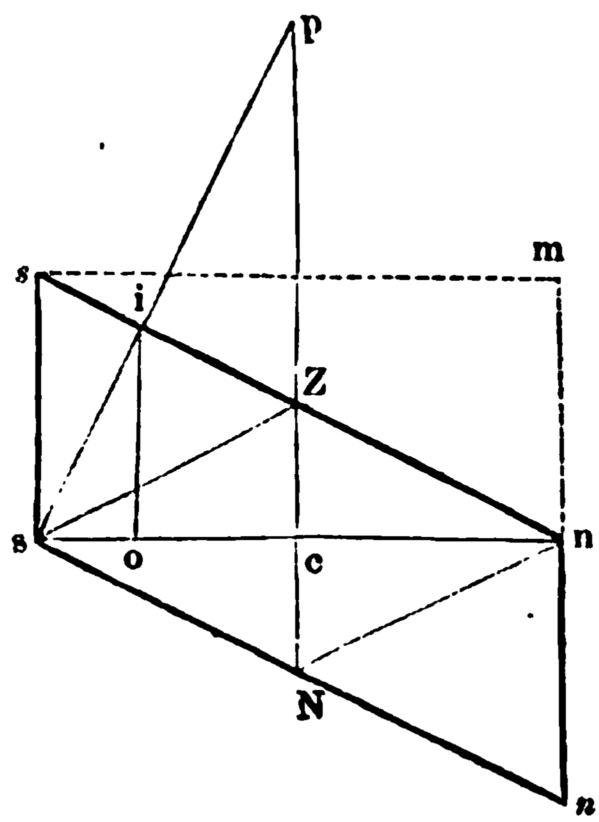
* *Groups c* and *g*, North Combinations.
Groups e and *h*, East Combinations.

† *Groups l, m, n, o, q, s, u,* and *x*, North Combinations.
Groups n, p, r, t, v, w, and *y*, East Combinations.

Mathematical Analysis of the Combinations of the Oblique Prismatic System.

588. Problem. *Given,* Model 79[b], *with the symbol* M_-, T. $\frac{1}{2}P_xM$ Zn; *required, the value of* $_x$ *in the symbol.*

Figure *s* n *n* s in the margin represents the north meridian of Model 79[b], or rather of Model 79, in so far as regards the length of the prism, but of Model 79[b], in so far as regards the angle of inclination of the pyramidal planes on the prismatic planes. The length of the prism is of no moment, as regards the following calculations.



Line *s* n is the diagonal of plane $\frac{1}{2}P_xM$ Zn, and line s *n*, the diagonal of plane $\frac{1}{2}P_xM$ Ns.

If we take line s n for axis m^s of the combination, and line Z N for axis p^s, then it is evident, that the value of the index $_x$ in the symbol, is the relation of line Z c to line c n, and it is also evident, that this relation is expressed by the tangent of the angle Z n c. But the angle which can be measured on the crystal is not the angle Z u c, but the angle Z n *n*, which is equal to the angle Z n c, added to the angle c n *n*, the prismatic angle of 90°.

Hence, if we put angle Z n *n*, or the inclination of $\frac{1}{2}P_xM$ Zn on M n $= X$, then,

$$\tan(X - 90^\circ) = \text{value of }_x \text{ in } \tfrac{1}{2}P_xM \text{ Zn.}$$

Put $X = 116^\circ\ 34'$. Then, $X - 90^\circ = 26^\circ\ 34'$. tan $= .5$, or $_x = \frac{1}{2}$. Hence the symbol is $\frac{1}{2}P\frac{1}{2}M$ Zn.

But if Model 79[b] is taken to represent a crystal of Epidote, then, according to Phillips, X is $115^\circ\ 41'$. Then $X - 90^\circ = 25^\circ\ 41'$. tan $=$ .4809, or $_x = \frac{11}{23}$. Here the symbol is $\frac{1}{2}P\frac{11}{23}M$ Zn.

According to Haüy, X is $114^\circ\ 37'$. Then $X - 90^\circ = 24^\circ\ 37'$. tan $= .4582$, or $_x = \frac{11}{24}$. Here the symbol is $\frac{1}{2}P\frac{11}{24}M$ Zn.

According to Mohs, X is $116^\circ\ 17'$. Then $X - 90^\circ = 26^\circ\ 17'$. tan $=$ .4939, or $_x =$ nearly $\frac{1}{2}$ (within 17′). This gives the symbol $\frac{1}{2}P\frac{1}{2}M$ Zn.

These examples show that we have in the oblique prismatic, as in the prismatic, system, very different measurements given by different authorities, and which cannot be expressed by the same symbol. We have

therefore either to *choose* a leader among these authorities, or else to adopt simple approximate numbers for indices, whenever we are uncertain which authority it is most proper to follow.

589. The indices of the symbols of Forms belonging to the east zone, as $\frac{1}{2}P_xT$ Zw, or $\frac{1}{2}p_xt$ Ze, are of course found in the same manner as the foregoing, from the inclination of $\frac{1}{2}P_xT$ Zw on T w, or of $\frac{1}{2}p_xt$ Ze on T e.

590. Sometimes several hemihedral biaxial forms occur on one crystal, as is shown by Models 101 and 101ª, the former of which exhibits the Forms $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{2}\frac{1}{4}m$ Zs, and the latter the Forms $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{3}{2}m$ Zs. The nature of these Forms is explained in the diagram in § 588, where line i n represents the zenith plane of the Form $\frac{1}{2}P\frac{1}{2}M$ Zn, and line i s the zenith plane of the Form $\frac{1}{2}p\frac{2}{1}m$ Zs. The value of the indices $\frac{1}{2}$ and $\frac{2}{1}$ is recognised at once on comparing the relation of the lines Z c to c n, and of p c to c s, since p c is four times as long as Z c. Upon the crystal, the angles that can be measured, are *first*, the angle Z n *n* already explained in § 588, *secondly*, the angle i s *n*, or more frequently the angle of plane i s upon a vertical plane, as in Models 101 and 101ª, and *thirdly*, the angle s i n.

When the angle measured is that of plane i s upon a vertical plane, the index of plane i s is found precisely in the same manner as the index of plane s n, § 588; because the abstraction of 90° from the measured angle, leaves the residual angle i s o, the tangent of which is the relation of i o to o s, or of p c to c s.

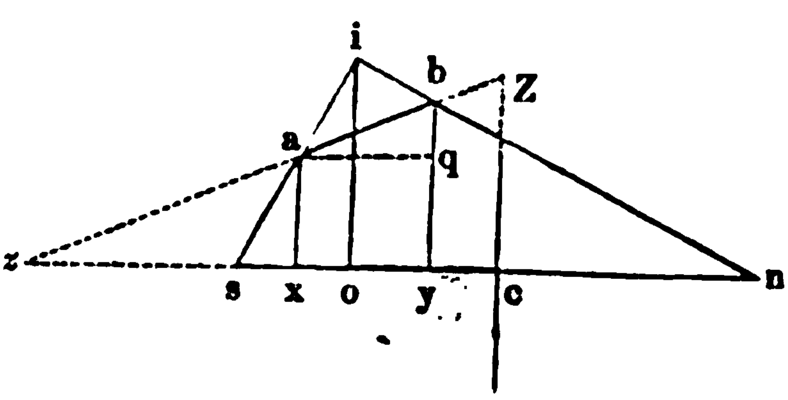
But when the angle measured is i s *n*, then it contains the angle i s o, and the angle o s *n*. The latter is equal to the angle Znc, the abstraction of which, when known, from i s *n*, leaves the angle i s o.

When the measured angle is that at the summit, s i n, it contains two angles, namely, the angle n i o, which is the complement of Z n c, supposed to be known, and the angle s i o, or the angle whose cotangent gives the index required for the plane s i.

Take the case of the combination $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{2}{1}m$ Zs, represented in the diagram, and examine the angles according to the above principles. Angle i n o is 26° 34′, because i n *n* is 116° 34′. Then, angle n i o is 90° — 26° 34′ = 63° 26′. Next, as s p c = *s* n s, it follows that s i o = 26° 34′. Therefore, s i o + o i n = 26° 34′ + 63° 26′ = 90°.

For the same reasons, angle i s *n* = 90°, because i s o = 63° 26′ and o s *n* = 26° 34′.

591. Sometimes the zenith pyramid of a combination of this kind, contains three, four, or more planes in one zone, the indices of all of which have to be separately calculated. The figure in the margin represents an example of this sort; where s a, a b, and b n, represent the three zenith planes of the biaxial forms $\frac{1}{2}p_{+}m$ Zs², $\frac{1}{2}p_xm$ Z²s, $\frac{1}{2}P_xM$ Zn. Line s n is



put = axis m^a and line Z c = axis p^a. A prismatic plane = M n is supposed to fall perpendicularly from n, and a similar plane = M s, from s. The mechanical measurements of the combination are assumed to be as follow:

b n on M n	b a on a s
b a on b n	a s on M s

The index of plane b n is derived from angle b n on M n, on the principle explained in § 588. The index of the plane a s is found from angle a s on M s in the same manner. When by these means, we know the angles n b y and s a x, or either of them, it is easy to find the index of the plane a b. If, for example, we know angles s a b and s a x, and wish to find q a b, then from angle s a b, we first deduct s a x, and then 90° = x a q. The residue is = q a b, which is the angle whose tangent is the index of plane a b.

If it is angle n b y that we know, then from a b n, we deduct n b y, and obtain as a residue a b q, the cotangent of which angle is the required index of plane a b.

In analysing combinations of this kind, the only difficulty experienced is in finding the inclination of any *one* of the oblique planes to axis p^a or to that equatorial axes to which the plane inclines. When this is done correctly, the indices of all the associated oblique planes of the same zone, however numerous, are found with ease.

592. The last diagram presents us with an explanation of the three different kinds of oblique planes which Rose has termed *Basic, Front,* and *Rear,* § 580.

The plane marked a b is *arbitrarily assumed* to be the *base* of the prism peculiar to a given Mineral. Then the inclination of this plane to the vertical edge of the prism, or, what is equivalent, the inclination of the line b *z* to a line perpendicular to the line *z* n, is the fundamental or characteristic angle of the series of Forms belonging to that mineral. If now we assume the plane a b to be directed northwards, then, the plane s a, will be a *front oblique plane* of the series, and the plane b n will be a *rear oblique plane* of the series. There must be only one *basic plane* for a given mineral, but there may be any number of *front* or *rear oblique planes.* Which among all the oblique planes that occur in the series of forms belonging to a given mineral, is the one that is to be chosen for the basic plane, is a thing needful enough to be known, but which it seems cannot be taught. "The different *front* and *rear* terminal planes have the same properties as the *basic* planes, and we are at liberty to take whichever of these oblique planes we please for the basic planes, and to determine the fundamental Form accordingly. No other rule for choosing the basic planes can be laid down, than that already given for the choice of fundamental Forms in general, [§ 479.] But the great variety of single planes and prisms that belong to the oblique prismatic system, often render the choice of the base of a mineral of this system much more difficult than is the choice of the fundamental Form of a mineral belong-

ing to any other system." ROSE. [He then cites a combination which has three oblique terminal planes, any one of which may be denoted as the basic plane, or all of which may be denoted as front or rear oblique planes, occurring independently of basic planes.]

It is evident upon a review of this matter, that by adhering to the system of rectangular axes, and considering all these oblique planes as hemihedral Forms of P_xM or P_xT, we can readily give every plane an exact symbol to indicate its obliquity to the equator, and by this means get rid of all doubt, difficulty, and confusion.

593. The explanation just given of the front, rear, and basic oblique planes, leads to the explanation of ROSE's *front*, *rear*, and *basic Oblique Prisms*, § 580. These oblique prisms are all, as already explained, hemi-octahedrons with parallel planes; they differ among themselves in their axial relations, and consequently in their polaric positions.

a.) *The Basic Oblique Prism* is that which has the same relation to axes p^a and m^a, or to axes p^a and t^a, as the *basic oblique planes*. An example is shown in Model 103, where the large upper seven-sided plane is a *basic plane*, and the two scalene triangles near the poles Ze and Zw are the zenith planes of the *basic oblique prism*. Again, in Model 101, the four similar five-sided planes which are crossed by the east meridian, are the *basic oblique prism*, and the upper and lower planes where the north and east meridians cross, are the *basic planes*. Finally, the two terminal planes of Model 87 are the *basic planes* of the prism of Augite, while the four terminal planes of Model 98, are the *basic oblique prism* of the same mineral. The relation of the basic planes to the basic oblique prism is shown in the fact that the oblique diagonal of the basic plane has the same obliquity to the equator of the combination, as the oblique edge which separates the planes of the basic oblique prism. Hence, the edges of combination between the basic planes and the basic prism, are always parallel to one another. See Models 101, 101^a, and 103.

b.) *The Front Oblique Prism* is that which, on a north combination, occurs to the north of the basic oblique prism, and which, on an east combination, occurs to the west of the basic oblique prism. In other words, the front oblique prism is the hemihedral form of an octahedron that is *more acute* than the octahedron of the basic oblique prism, but which occurs in the same octants. Model 103 exhibits a front oblique prism in combination with a basic oblique prism, the former predominant.

c.) *The Rear Oblique Prism* is merely the oblique prism of the inverse octants, and has, on a north combination, the positions Z s w Z s e, and on an east combination, the positions Z n e Z s e. Model 112 exhibits a basic plane and a rear oblique prism. Model 67, a front oblique prism, and a rear oblique prism. Model 75, the same.

d.) This classification and nomenclature of planes and prisms is of very little use, because the indices of the symbols of the different forms, and the notation descriptive of their polaric positions, give similar information in a more precise manner.

594. PROBLEM. *Given,* Model 79, *a crystal of* Gypsum, *with the symbol* M_{-}, T. $\frac{1}{2}P\frac{3}{7}M$ Zn; *required, the inclination of* $\frac{1}{2}P\frac{3}{7}M$ Zn *on* M n *and on* M s.

a.) The inclination of $P\frac{3}{7}M$ Zn on M n is that angle whose tangent is $\frac{3}{7}$, added to 90°. By the Table of Indicés, page 139, you find $\frac{3}{7} = .4286$ $=$ tan 23° 12′. Then 23° 12′ + 90° = 113° 12′. Upon applying the goniometer to Model 79, this will be found to be nearly correct. According to Haüy, the inclination in question is 113° 8′.

b.) Upon referring to the figure in § 588, it will be seen that the inclination of plane $\frac{1}{2}P_xM$ Zn to plane M s is the supplement of its inclination to plane M n, for as the four angles *s* s n *n* are together equal to 360°, so *s* and n, or s and *n* are together equal to 180°. Hence, if the inclination of $\frac{1}{2}P\frac{3}{7}M$ Zn on M n is 113° 12′, then, $\frac{1}{2}P\frac{3}{7}M$ Zn on M s is 180° — 113° 12′ = 66° 48′, which agrees with measurement by the goniometer.

595. PROBLEM. *Given,* Model 84, *a crystal of* Hornblende, *with the symbol* $M_xT.$ $\frac{1}{2}P_xM$ Zn, *and the angles* M_xT nw *on* ne = 124° 34′, *and* $\frac{1}{2}P_xM$ Zn *on* M_xT nw = 103° 13′; *required, the value of the Indices in the symbol.*

a.) The index of M_xT is the cotangent of $\frac{124° 34'}{2}$. Cot 62° 17′ = .5254, (within 3′ of .5265) = $\frac{10}{19}$. This gives the symbol $M\frac{10}{19}T$.

b.) Suppose Model 84 to be divided into two halves by a section through the north meridian. Take the west half as a right-angled solid triangle with pole Zn for its vertex. You have then the following given parts: C = 90° = inclination of $\frac{1}{2}P_xM$ to the north meridian; A = 76° 47′ = supplement of 103° 13′ the inclination of $\frac{1}{2}P_xM$ Zn on $M\frac{10}{19}T$ nw; and B = 62° 17′ = inclination of $M\frac{10}{19}T$ nw on the north meridian. With these data, you have to find *a* = inclination of $\frac{1}{2}P_xM$Zn to the north vertical edge between planes $M\frac{10}{19}T$ nw and $M\frac{10}{19}T$ ne. This is the angle which is named *X* in § 588, and the rule respecting which is again applicable here, namely:

$$\tan (X - 90°) = \text{value of }_x \text{ in } \tfrac{1}{2}P_xM \text{ Zn}.$$

Given, A, 76° 47′; B = 62° 17′; *to find, a.*

Formula 4. log cos *a* = log cos A + 10 — log sin B.

10 + log cos A = 76° 47′	= —	19.3591
— log sin B = 62° 17′	=	9.9471
log cos *a* = 75° 2′	= —	9.4120

In all calculations of this kind, one of the measured angles is *always greater than* 90°, namely, the inclination of $\frac{1}{2}P_xM$ or of $\frac{1}{2}P_xT$ on M_xT. Hence, the *supplement* and not the measured angle, must be taken into the calculation, and hence also the *product* of the calculation is not the required angle but its supplement. See § 330.

In the present calculation, therefore, the *negative* product 75° 2′, must be changed for its *supplement,* 104° 58′, which is the required inclination of $\frac{1}{2}P_xM$ Zn on the north vertical edge. Deducting the constant quantity

90°, we have a residue of 14° 58′, the tangent of which is .2673, which differs only 2′ from .2667 = $\frac{4}{15}$, which fraction I have therefore chosen to indicate the required value of $_x$. Hence the symbol for Model 84, is M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{15}$M Zn.

596. PROBLEM. *Given,* Model 87, *a crystal of* Augite, *with the symbol* M$\frac{8}{8}\frac{0}{1}$T. $\frac{1}{2}$P$\frac{6}{21}$T Zw; *required, the inclination of* M$\frac{8}{8}\frac{0}{1}$T nw *on* ne, *and the inclination of* $\frac{1}{2}$P$\frac{6}{21}$T Zw *on* M$\frac{8}{8}\frac{0}{1}$T nw.

a.) The inclination of M$\frac{8}{8}\frac{0}{1}$T nw on ne, is twice the angle whose cotangent is $\frac{8}{8}\frac{0}{1}$. By the Table of Indices, page 139, the index $\frac{8}{8}\frac{0}{1}$ = .952, and the corresponding angle is 46° 24′. Then twice 46° 24′ = 92° 48′, which is the required inclination of plane ne on plane nw.

b.) To find the inclination of plane $\frac{1}{2}$P$\frac{6}{21}$T Zw on plane M$\frac{8}{8}\frac{0}{1}$T nw, across the Znw edge, proceed as follows:—Assume Model 87 to be divided into two halves by the east meridian, and take the north half as a right-angled solid triangle with the solid angle near pole Zw for its vertex. Then, angle C of this solid triangle, = 90°, will be the inclination of plane $\frac{1}{2}$P$\frac{6}{21}$T Zw to the east meridian. Angle A will be the inclination of plane M$\frac{8}{8}\frac{0}{1}$T nw to the east meridian. This is the complement of the inclination of the same plane to the north meridian, found by *a.*) to be 46° 24′, and the complement of which is therefore 43° 36′. Side *b* of the triangle will be the inclination of plane $\frac{1}{2}$P$\frac{6}{21}$T Zw to the west vertical edge between the prismatic planes nw and sw. This angle is equal to the prismatic angle of 90° added to that angle whose tangent is $\frac{6}{21}$. By the Table of Indices, page 139, you find $\frac{6}{21}$ = .2857, which is the tangent of 15° 57′. Then, 90° + 15° 57′ = 105° 57′ is the value of *b*. But since this angle is greater than 90′, you cannot take it into the calculation, but must employ its supplement, which is 180° — 105° 57′ = 74° 3′. With these given quantities, you have to find angle B, which is the required angle across the Znw edge of Model 87.

Given, A = 43° 36′; *b* = 74° 3′; *to find,* B.

Formula 8. log cos B = log cos *b* + log sin A — 10.

$$\begin{aligned} \log\cos b &= 74^\circ\ 3' = -\,9.4390 \\ +\ \log\sin A &= 43^\circ\ 36' = 9.8386 \\ \hline \log\cos B &= 79^\circ\ 4\tfrac{1}{2}' = -\,9.2776 \end{aligned}$$

Then, 180° — 79° 4½′ = 100° 55½′. This product, 100° 55½′, is the required inclination of $\frac{1}{2}$P$\frac{6}{21}$T Zw on M$\frac{8}{8}\frac{0}{1}$T nw.

c.) *Investigation of the agreement of these calculated angles with the measured angles of natural crystals of Augite.*

According to Haüy, the inclination of M$\frac{8}{8}\frac{0}{1}$T nw on ne, is 92° 18′, which is 30′ different from that expressed by M$\frac{8}{8}\frac{0}{1}$T, and would require the symbol M$\frac{2}{2}\frac{4}{5}$T. According to Phillips, the same angle is 92° 55′, which, although differing but 7′ from 92° 48′, could be better expressed by the symbol M$\frac{1}{1}\frac{1}{0}$T. Hence the approximate symbol which I have adopted, M$\frac{8}{8}\frac{0}{1}$T, indicates an angle betwixt the two quotations of Haüy and Phillips.

The inclination of the terminal to the prismatic plane of Model 87, calculated from the symbols, $M\frac{8}{2}\frac{0}{1}T.\ \frac{1}{2}P\frac{6}{2\,1}$ T Zw, to be $= 100° 55\frac{1}{2}'$, is according to Haüy, 100° 5′, and according to Phillips, 100° 10′. These quotations are rather wide of the mark, but I have retained the index $\frac{6}{2\,1}$, because it appeared to agree better than any other, with the measurements and calculations of other forms of this mineral.

597. PROBLEM. *Given*, Model 112, *a crystal of* Hornblende, *with the symbol* T, $M_xT.\ \frac{1}{2}P_xM$ Zn, $\frac{1}{2}P_xM_yT_z$ Zse Zsw ; *with the angles* nw *on* ne = 124° 34′; $\frac{1}{2}P_xM$ Zn *on* M_xT nw = 103° 13′; $\frac{1}{2}P_xM_yT_z$ Zse *on* Zsw = 149° 38′; and $\frac{1}{2}P_xM_yT_z$ Zsw *on* M_xT sw = 110° 2′ ; *required, the value of the indices in the symbol.*

a.) M_xT is $M\frac{1}{1}\frac{0}{9}T$. See 595, *a.*)

b.). $\frac{1}{2}P_xM$ Zn is $\frac{1}{2}P\frac{}{1}\frac{4}{3}M$ Zn. See 595, *b.*)

c.) *To find the Indices of the Hemioctahedron* $\frac{1}{2}P_xM_yT_z$ Zse Zsw.

Assume Model 112 to be divided into two halves by the north meridian, and take the west half as an oblique-angled solid triangle, having for its vertex the solid angle near the pole Zs, where the two zenith planes of the hemioctahedron meet the two south planes of the prism. The given parts of this oblique angled solid triangle are as follow:—Angle A = supplement of the inclination of $\frac{1}{2}P_xM_yT_z$ Zsw on M_xT sw, or 180° — 110° 2′ = 69° 58′. Angle B = half the inclination of plane Zse on plane Zsw, or $\frac{149° 38'}{2} = 74° 49'$. Angle C = half the inclination of plane sw on plane se, or $\frac{124° 34'}{2} = 62° 17'$. With these data, you have to find *side a*, which is the inclination of the Zs oblique edge between the octahedral planes to the s vertical edge between the prismatic planes. The use to be made of the auxiliary angle denoted by *a* will be explained afterwards.

Given, A = 69° 58′; B = 74° 49′ ; C = 62° 17′; *to find, a.*

Formula 37. $\sin \frac{1}{2} a = \sqrt{\frac{-\cos S \cos (S - A)}{\sin B \sin C}}$, where $S = \frac{1}{2}(A + B + C)$.

$\log \sin \frac{1}{2} a =$

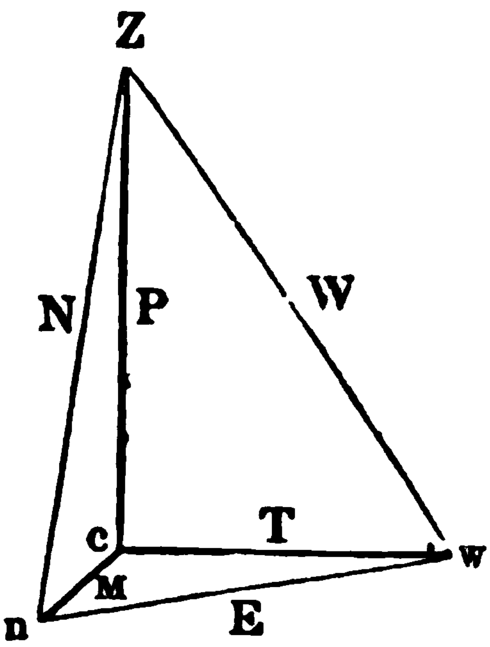
$$\tfrac{1}{2}\{\log \cos S + \log \cos (S - A) + 20 - (\log \sin B + \log \sin C)\}$$

$$\begin{array}{rlcrl}
A = & 69° 58' & & & \\
B = & 74° 49' & & S = & 103° 32' \\
C = & 62° 17' & & A = & 69° 58' \\
\hline
& 2)207° \ 4' & & S - A = & 33° 34' \\
\hline
S = & 103° 32' & & & \\
\text{Supplement of } S = & 76° 28' & & &
\end{array}$$

$$\begin{array}{rcr}
\log \cos S = 76° 28' & = - & 9.3692 \\
+ \log \cos (S - A) = 33° 34' & = & 9.9208 \\
\hline
+ 20 & = & 39.2900 \\
-\left\{\begin{array}{l} \log \sin B = 74° 49' = 9.9846 \\ + \log \sin C = 62° 17' = 9.9471 \end{array}\right\} & = - & 19.9317 \\
\hline
& & 2)19.3583 \\
\hline
\sin \tfrac{1}{2} a = 28° 32' & = & 9.6791
\end{array}$$

Twice this product, or 28° 32′ × 2 = 57° 4′, is the supplement of *side a* of the given oblique-angled solid triangle. See § 330. The auxiliary angle is therefore 180° — 57° 4′ = 122° 56′. This is the Zs^2 angle of the north meridian of Model 112. Hence, its supplement = 57° 4′ is the inclination of the edge betwixt the two octahedral planes to axis p^a, the cotangent of which angle shows the relation of the given hemioctahedron to axes p^a and m^a. Cot of 57° 4′ = .6478. tan 57° 4′ = 1.5438.

d.) To find now the relation of axes p^a to t^a of the same two octahedral planes, you need to employ a right-angled solid triangle with pole Z for its vertex, and the given quantities in which are as follow: angle A = inclination of plane $P_xM_yT_zZsw$ to the north meridian = 74° 49′, and side b = 57° 4′ = inclination of the oblique edge between the octahedral planes to axis p^a. With these given quantities, you can find side a of the right-angled solid triangle, which angle is the inclination to axis p^a of an edge of the same octahedron assumed to rise from axis t^a. This edge is not visible on the combination, but the cotangent of the angle thus found is the required relation of axis p^a to axis t^a of the given hemioctahedron.



Given, A = 74° 49′; b = 57° 4′; *to find*, a.

Formula 7. log tan a = log tan A + log sin b — 10.

$$\begin{aligned} \log \tan A &= 74° \, 49' = 10.5664 \\ + \log \sin b &= 57° \;\; 4' = \;\; 9.9239 \\ \hline \log \tan a &= 72° \;\; 5' = 10.4903 \end{aligned}$$

Cot 72° 5′ = .3233 Tan 72° 5′ = 3.0930.

e.) By the first operation, you find the relation of p^a to m^a to be as 1.0000 to 1.5438. By the second operation, the relation of p^a to t^a to be as 1.0000 to 3.0930. This gives the symbol $P_{1.0000}M_{1.5438}T_{3.0930}$. Here it is easy to see that axis m^a is exactly half of axis t^a. But the length of t^a has been taken at 19 for the associated prism $M\frac{19}{19}T$, and it is convenient to denote the transverse axis of the octahedron in reference to that standard. The relation of 1.5438 to 3.0930 is that of 9½ to 19 or of 19 to 38. The equivalent number for p^a is found by the proportion

$$3093 \; : \; 1000 \; :: \; 38 \; : \; 12$$

Hence the indices for $\frac{1}{2}P_xM_yT_z$ Zse Zsw, are $\frac{12}{38}$ and $\frac{19}{38}$, affording the symbol $\frac{1}{2}P\frac{12}{38}M\frac{19}{38}T$ Zse Zsw, and the complete symbol for Model 112 is T, $M\frac{19}{19}T$. $\frac{1}{2}P\frac{4}{13}M$ Zn, $\frac{1}{2}P\frac{12}{38}M\frac{19}{38}T$ Zse Zsw. It is given erroneously in page 136, Part II.

598. *Miscellaneous Remarks on Calculations peculiar to the Forms of the Oblique Prismatic System.*

a.) The symbols of all the hemioctahedrons of the oblique prismatic system may be found by the method described in § 597. The calculations are tedious, but unavoidable. There must always be two equations resolved; one to determine the inclination of the single oblique edge between the two zenith planes of the hemioctahedron to axis p^a, and the other to determine the inclination of the absent oblique edge belonging to the transverse meridian.

b.) *Calculation of* $\frac{1}{2}P_xM_yT_z$ Znw Zsw. When the oblique terminal edge of a hemioctahedron falls upon a *vertical* PLANE instead of a *vertical* EDGE, as it does on Model 98, then the calculations are much easier. In this case, we measure the inclination of $\frac{1}{2}P_xM_yT_z$ Znw, first on Zsw and then on T w, and call the first A and the second B. We divide the Model into two halves by the east meridian, and take the north half as a right-angled solid triangle, with the solid angle at pole Zw for its vertex. Then, with the above given quantities, we find side *b* of the solid triangle, which is the inclination of the oblique edge to plane T w. The supplement of this angle is the inclination of the oblique edge to axis p^a. When this is known, a second calculation is made on the model of that just given in § 597, *d*), in which the quantities used are the inclination of the oblique edge to axis p^a, and half the angle across that edge, and by which we find the relation of axis p^a to the third axis of the octahedron. This completes the calculation.

c.) Model 101. *To find the indices of* $\frac{1}{2}P_xM_yT_z$ Zne^2 Znw^2. The inclination of one of these planes to plane $\frac{1}{2}P\frac{1}{2}M$ Zn, is 90° more than its inclination to the north meridian. According to Haüy, the given inclination is 124° 57′ = 90° + 34° 57′. The inclination of plane $\frac{1}{2}P\frac{1}{2}M$ Zn to axis p^a (= 63° 26′, see § 588), is equal to the inclination to that axis of the oblique edge between the planes Zne^2 and Znw^2 of the hemioctahedron, which edge is replaced by the plane $\frac{1}{2}P\frac{1}{2}M$ Zn.

Take an octant of a rhombic octahedron as a right-angled solid triangle with pole Z for its vertex, and the following given quantities:

Given, A = 34° 57′, *b* = 63° 26′; *to find*, *a*.

Formula 7. log tan *a* = log tan A + log sin *b* — 10.

$$\begin{aligned} \log \tan A &= 34° \, 57' = 9.8444 \\ + \log \sin b &= 63° \, 26' = 9.9515 \\ \hline \log \tan a &= 32° \, 0\tfrac{1}{2}' = 9.7959 \end{aligned}$$

This product 32° 0½′ corresponds to cot 1. 6 = $\frac{8}{5}$.

The relation of the three axes of the Form under investigation are therefore p^a to t^a = 8 to 5, and p^a to m^a = 1 to 2 = 8 to 16. Hence the symbol for the hemioctahedron of Model 101 is $\frac{1}{2}P\frac{8}{5}M\frac{16}{5}T$ Zne Znw.

d.) Model 109. T, $M\frac{1}{2}\frac{2}{6}$T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{13}M$ Zs. Felspar. The index of $\frac{1}{2}P\frac{1}{2}M$ Zn, is found from a right-angled solid triangle, consisting

of the west half of the model, with the solid angle at pole Zn for its vertex, and with these given quantities: A = inclination of $\frac{1}{2}$P$\frac{1}{2}$M Zn on M$\frac{1}{2}\frac{5}{6}$T nw. B = inclination of M$\frac{1}{2}\frac{5}{6}$T nw on the north meridian. Then, a = supplement of the inclination of $\frac{1}{2}$P$\frac{1}{2}$M Zn to axes p^a. See § 595, *b*). Next, the index of $\frac{1}{2}$P$\frac{7}{13}$M Zs is found, either by a similar process, or else by the method described in § 590.

e.) Model 103. m, M$\frac{6}{7}$T. $\frac{1}{2}$P$\frac{5}{14}$M Zn, $\frac{1}{2}$P$_+$M$_-$T Zn^2e^2 Zn^2w^2, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zne^2 Znw^2. Blue Carbonate of Copper.

To find the indices of $\frac{1}{2}$P$_+$M$_-$T Zn^2e^2 Zn^2w^2. Take the angle across the edge between Zn^2e^2 and Zn^2w^2. Call the half of this B. Take the inclination of plane Zn^2w^2 on $\frac{1}{2}$P$\frac{5}{14}$M Zn. Call the supplement of this angle, A. Assume the model to be divided into two halves by the north meridian, and take the west half as a right-angled solid triangle, with the solid angle at pole Zn for its vertex. Then, with the aforesaid given quantities, A, B, and *Formula* 4, find side a, which is the inclination of plane $\frac{1}{2}$P$\frac{5}{14}$M Zn on the oblique edge between planes Zn^2e^2 and Zn^2w^2. From this auxiliary angle, first deduct the angle whose tangent is $\frac{3}{14}$, and then deduct 90°. The residue is the inclination of the oblique edge to axis p^a. When this is known, a second equation, on the model of that given in § 597, *d*), gives the rest of the information required for completing our knowledge of the length of the three axes of $\frac{1}{2}$P$_+$M$_-$T Zn^2e^2 Zn^2w^2.

According to Haüy, the angle of plane Zn^2e^2 on plane Zn^2w^2 is 107° 34′. The half of it = B = 53° 47′. And the angle of plane Zn on plane Zn^2w^2 is 116° 36′. Its supplement = A = 63° 24′.

Formula 4. log cos a = log cos A + 10 — log sin B.

$$\begin{array}{rl} 10 + \log\cos A = 63^\circ\ 24' = - & 19.6510 \\ -\log\sin B = 53^\circ\ 47' = & 9.9068 \\ \hline \log\cos a = 56^\circ\ 18' = - & 9.7442 \end{array}$$

This being a negative cosine, we have to take its supplement = 123° 42′, as the value of the inclination of plane Zn on the front oblique edge. This will be found to agree with the measurement of the Model.

The angle whose tangent is $\frac{3}{14}$ = .2143, is 12° 6′. This added to 90° is 102° 6′. Deducting this joint sum from 123° 42′, we have 21° 36′ for the inclination of the oblique edge at pole Zn to axis p^a. The tangent of 21° 36′ is .3959, which is the value of m^a when p^a is 1.0000.

We next use the Formula given in § 597, *d*) with these given quantities: A = 53° 47′; b = 21° 36′.

Formula 7. log tan a = log tan A + log sin b — 10.

$$\begin{array}{rl} \log\tan A = 53^\circ\ 47' = & 10.1353 \\ +\log\sin b = 21^\circ\ 36' = & 9.5660 \\ \hline \log\tan a = 26^\circ\ 41' = & 9.7013 \end{array}$$

The tangent of 26° 41′ is .5026, which is the value of t^a when p^a is 1.0000.

If we take $p^a = 10$, then $m^a = .3959$, is nearly 4, and $t^a = .5026$, is nearly 5. This gives the symbol $\frac{1}{2}P\frac{10}{3}M\frac{4}{3}T\ Zn^2e^2\ Zn^2w^2$.

The indices of the hemioctahedron $\frac{1}{2}p_x m_y t_z\ Zne^2\ Znw^2$, also contained on Model 103, are found as follows. The inclination of the plane Znw^2 on the plane $\frac{1}{2}P\_M$ Zn is 90° more than its inclination to the north meridian. The inclination to the equator of the edge across which this angle is taken, is equal to the inclination to the equator of the plane $\frac{1}{2}P\_M$ Zn. From these quantities we deduce the inclination of p^a to m^a of the axes of the form $\frac{1}{2}p_x m_y t_z$. Then by a second calculation on the model of that given in § 597, *d*), the operation is completed.

f.) Model 75. $T\_$, $M\frac{9}{13}T$. $\frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw, $\frac{1}{2}P\frac{6}{13}M\frac{10}{13}T$ Zse Zsw, Gypsum.

Model 67. $M\frac{40}{30}T$. $\frac{1}{2}P\_M\frac{40}{30}T$ Zne Znw, $\frac{1}{2}P\_M\frac{40}{30}T$ Zse Zsw. Mesotype.

In combinations of this sort, the indices of the hemioctahedron that produces the two planes Zne Znw, and of that which produces the two planes Zse Zsw, are found by separate calculations, both when the different hemioctahedrons are quite unlike, and when they are so similar and equal as almost to constitute a homohedral octahedron. The angles required in all cases are six, namely, 1.) Prismatic plane nw on ne. 2.) Octahedral plane Znw on Zne. 3.) Octahedral plane Znw on prismatic plane nw. These are for the *front* hemioctahedron. The *rear* hemioctahedron also requires three angles: 1.) sw on se. 2.) Zse on Zsw. 3.) Zsw on sw. The six angles being found by measurement, the calculations are made according to the examples given in § 597 *c* and *d*). The ratios of the axes being thus determined, the indices are easily found for the symbols.

599. The combinations of this system are so exceedingly numerous and diversified, that it is impossible to reduce the instructions for calculating the indices of their symbols to so orderly an arrangement as was contrived for the instructions given in the foregoing systems. I have endeavoured, however, to give such a collection of examples, as will serve to convey a variety of information relative to the calculations best suited to particular cases. I have also endeavoured, by grouping the Forms and Combinations of the system in several different modes, to show their mutual relations and dependencies in as striking a manner as possible. See §§ 580, 582, 587; and Part II. pages 115 and 118.

The calculations might have been carried into much greater detail, but not without frequent repetitions of calculations given in preceding sections, which did not appear to me to be expedient. I hope the *principles* upon which the different calculations are to be made, are explained so fully as to enable the reader to supply the details easily.

VI. THE DOUBLY OBLIQUE PRISMATIC SYSTEM OF CRYSTALLISATION.

600. The character of the Forms belonging to this system, as given by ROSE, is this:—They have three Axes, which are all unequal, and which cut one another obliquely.

ROSE'S enumeration of the Forms belonging to this system of Crystallisation, is as follows:—

The planes that belong to the Forms of this system all occur in sets of two, and they are of three kinds:

1. Planes that cut three Axes $= \frac{1}{4} P_x M_y T_z$.
2. Planes that cut two Axes $= \frac{1}{2} M_x T$.
3. Planes that cut one Axis $=$ M or T.

In referring crystals of this class to a system of three *rectangular* axes, we have merely to give each pair of planes a specific denomination. But according to both systems of notation, every pair of planes must have a different name, and as it is as easy to name them in reference to rectangular axes as to oblique axes, it does not appear that any advantage is gained by the adoption of the doubly oblique system of axes.

The properties of the Forms and Combinations belonging to this system, are described in § 340, 6), and in Part II. page 91. The only homohedral forms which commonly occur are M and T. In Part II. I have admitted $M_x T$ to be an occurring homohedral form, especially on Axinite; but I have had no opportunity of examining good minerals of this class, and having founded this opinion merely on the figures given in books, I am doubtful of its correctness. The forms of most frequent occurrence on the minerals of this system are $\frac{1}{2} M_x T$ and $\frac{1}{4} P_x M_y T_z$. These forms consist invariably of a pair of parallel planes. Every doubly oblique combination must contain at least three of such pairs of planes, of which two pair must belong to the prismatic zone, and one pair to the octahedral zone, and none of which must meet at a right angle. A single combination sometimes contains as many as 12 pair of planes, all belonging to the series M, T, $\frac{1}{2} M_x T$. $\frac{1}{4} P_x M_y T_z$. There are never present any planes of the Forms P, $P_x M$, $P_x T$.

The Axes of all the Combinations belonging to this system are $p_x^z m_y^x t_z^y$.

Rose's Catalogue of the minerals that belong to the doubly oblique prismatic system, is given in Part II. pages 7—13. A symbolic catalogue of the Forms and Combinations presented by the crystals of each of these minerals, is given in Part II. pages 91—94. Every crystal of this system is an incomplete prism, combined with an incomplete pyramid. They all fall therefore into Class 5. The equator of every combination is either rhombic, or, when M or T is present, rhombo-rectangular. Hence all the minerals embraced by this system, belong either to Class 5, Order 3, Genus 1, Group *i*, Part II. page 116, or to Class 5, Order 5, Genus 2, Group *z*, Part II. page 119.

601. Mathematical Analysis of the Combinations of this System.

Model 81^b. M$\frac{2}{3}$T. $\frac{1}{4}$P$_x$M$_y$T$_z$ Z^{2}nw, $\frac{1}{4}$P$_x$M$_y$T$_z$ Zn^{2}e, $\frac{1}{2}$p$_x$m$_y$t$_z$ Z^{2}ne^2.

a.) The index $\frac{2}{3}$ of M$\frac{2}{3}$T is determined from the obtuse angle of the equator at the north pole. When a combination contains several pair of prismatic planes, you begin by determining the obliquity of any one pair to axis m^a or axis t^a, and then estimate the obliquity of the rest according to their inclination to the known pair of planes.

b.) The Model presents an oblique-angled solid triangle at pole Zw, formed by the meeting of M$\frac{2}{3}$T with $\frac{1}{4}$P$_x$M$_y$T$_z$ Z^{2}nw. You take the three angles across the edges which meet here, and by means of an equation with Formula 37, you determine the plane angle at pole Zw of the plane M$\frac{2}{3}$T nw. Secondly, you assume the Model to be divided by the east meridian into two halves, and you take the north half as an oblique-angled solid triangle, with pole Zw for its vertex. The known quantities are now: A = inclination of $\frac{1}{4}$P$_x$M$_y$T$_z$ Z^{2}nw on plane M$\frac{2}{3}$T nw; B = inclination of M$\frac{2}{3}$T nw on the east meridian; c = plane angle of M$\frac{2}{3}$T nw at pole Zw. With these data, you can, by two separate processes, determine the inclination of plane $\frac{1}{2}$P$_x$M$_y$T$_z$ Z^{2}nw to the east meridian, and the inclination of the Z^{2}w oblique edge of the east meridian to the west vertical edge. Thirdly, by means of the last two products, employed in an equation with a right-angled solid triangle, you can determine the inclination of axis p^a to the edge of intersection between plane $\frac{1}{4}$P$_x$M$_y$T$_z$ Z^{2}nw and the north meridian. By these calculations you find the inclination of axis p^a to the purple line drawn on the Model from Z towards w, and to the blue line drawn from Z towards n. The tangents of these two angles are the indices of the Form $\frac{1}{2}$P$_x$M$_y$T$_z$ Z^{2}nw. The calculations are made exactly in the manner of the calculations described in the last section. See §§ 597, 598. But as the determination of the indices of one pair of planes requires three measurements to begin with, and these to be employed in three oblique-angled and one right-angled solid triangle, it follows that the investigation of Forms belonging to this system is particularly tedious. Hence it follows that they have been very little attended to by Crystallographers, and that the descriptions we meet with of crystallised Minerals belonging to the doubly oblique prismatic system, are not much to be relied on for correctness.

c.) By four similar calculations made on the solid triangle exhibited by Model 81^b at pole Zn^2, and formed by the meeting of planes M$\frac{2}{3}$T ne, M$\frac{2}{3}$T nw, and P$_x$M$_y$T$_z$ Zn^{2}e, it is possible to find the indices of the Form $\frac{1}{2}$P$_x$M$_y$T$_z$ Zn^{2}e.

d.) And by similar calculations made on the solid triangle formed by the meeting of planes M$\frac{2}{3}$T ne, M$\frac{2}{3}$T se, and p$_x$m$_y$t$_z$ Z^{2}ne^2, near pole Ze^2 of Model 81^b, the indices of the Form $\frac{1}{4}$p$_x$m$_y$t$_z$ Z^{2}ne^2 may be determined.

e.) Hence, for the investigation of the Combination represented by Model 81^b, and which contains only five pair of planes, you require the measurements across nine edges, and must solve nine equations with oblique-angled solid triangles, and three equations with right-angled solid triangles.

As the Minerals which belong to this system are not very important, I have considered it unnecessary to undertake these laborious calculations, but have indicated the hemioctahedrons of each Mineral by approximate symbols, which represent chiefly the comparative size of their planes, and their respective polaric positions.

SECTION XIV. MR. BROOKE'S POPULAR SYSTEM OF CRYSTALLOGRAPHY.

Resolution of Mr. H. J. Brooke's Primary Forms and their Modifications, or Secondary Forms, into the Forms P_x, M_x, T_x, M_xT. P_xM, P_xT, $P_xM_yT_z$, *or their Hemihedral or Tetartohedral Varieties.*

602. I have stated, in § 244, that Mr. BROOKE's symbols for Combinations are of the same character as HAUY's. This, however, alludes to MR. BROOKE's symbols for expressing "the character of the modifying planes of crystals, and their geometrical relations to the primary Form, as connected with the theory of decrements;" according to which theory, "the secondary Forms of Minerals consist of Modifications of the primary, occasioned by decrements on some of their edges or angles;" and the symbols alluded to constitute "a system of notation connected with the same theory, and capable of expressing the figure of any secondary Form."—H. J. BROOKE's *Familiar Introduction to Crystallography*, 8vo, London, 1823.

The book just quoted, contains, however, another and different system of Crystallographic notation, which, being independent of mathematical calculations, and comprehensible without much thought, has been oftener adopted and referred to by English writers, than any of the more scientific systems of Crystallography. This method consists in giving figures of a certain number of Forms, under the name of "primary Forms;" and then representing all the different angles and edges of these primary Forms as being replaced by one or more modifying planes, which are labelled *a*, *b*, *c*, and so on to *x*, *y*, *z*. Mr. Brooke gives 176 figures and descriptions of these primary Forms, their Modifications, and complete secondary Forms which they produce. The number of particular modifications thus classified, is one hundred and fifty. The number of primary Forms is fifteen. The number of complete secondary Forms is twenty-three. Now it will be easy to show that the whole of these Forms, simple and compound, consist merely of varieties of the seven Forms, P_x, M_x, T_x, M_xT. P_xM, P_xT, $P_xM_yT_z$, or of their Hemihedral or Tetartohedral Varieties; and that consequently the symbols of these Forms can be used to replace the unsystematic notation employed by Mr. Brooke.

Catalogue of Mr. Brooke's *Primary Forms and their Modifications.*

Every symbol expresses the primary Form as well as the Modification. In some cases, the same symbol expresses several of Mr. Brooke's Modifications. See *the Rectangular Octahedron, h, i, k, l,* and *the Rhombic Octahedron, d, e, f.* But when the algebraic indices x, y, z, are replaced by numerical indices, 1, 2, 3, &c., every symbol acquires a specific meaning.

The Cube, P, M, T.

a. P, M, T. pmt.
b. P, M, T. $3\,p_-mt$.
c. P, M, T. $3\,p_+mt$.
d. P, M, T. $6p_-mt_+$.
e. P, M, T, mt. pm, pt.
f. P, M, T, m_-t, m_+t. p_-m, p_+m, p_-t, p_+t.
g. P, M, T. ½pmt.
h. P, M, T. ½ ($3\,p_-mt$).
i. P, M, T. $3\,p_-mt_+$.
k. p, m, t, M_-T. P_-M, P_+T.

The Tetrahedron, ½PMT.

a. ½PMT, ½pmt.
b. mt. pm, pt, ½PMT.
½PMT, ½ ($3p_+mt$) Znw.
c. ½PMT Znw, ½($3\,p_-mt$) Zne.
d. ½PMT Znw, ½ ($6\,p_-mt_+$) Zne.
m_-t, m_+t. p_-m, p_+m, p_-t, p_+t, ½PMT.
m_-t. p_-m, p_+t, ½PMT.
e. p, m, t. ½PMT.
f. ½PMT, ½ ($3\,p_-mt$).

The Octahedron, PMT.

a. p, m, t. PMT.
b. PMT, $3p_-mt$.
c. m_-t, m_+t. p_-m, p_+m, p_-t, p_+t, PMT.
d. PMT, $6\,p_-mt_+$.
e. mt. pm, pt, PMT.
f. PMT, $3\,p_+mt$.

The Right Rectangular Prism.

P_+, M_-, T.

[Mr. Brooke places this Form so as to make it = P_x, M_-, T_+.—J. J. G.]

a. P_+, M_-, T. $p_xm_yt_z$.
b. P_+, M_-, T, m_-t.
c. P_+, M_-, T. p_xt.
d. P_+, M_-, T. p_xm.

The Octahedron with a Rhombic Base.

P_+M_-T: or $P_xM_yT_z$.

a. p_+. P_+M_-T.
b. p_-m, P_+M_-T.
c. p_-t, P_+M_-T.
d. P_+M_-T, $p_xm_yt_z$ Z^2nw.
e. P_+M_-T, $p_xm_yt_z$ Z^2nw.
f. P_+M_-T, $p_xm_yt_z$ Z^2nw.
g. m. P_+M_-T.
h. p_+m, P_+M_-T.
i. m_-t. P_+M_-T.
k. P_+M_-T, $p_xm_yt_z$ Zn^2w.
l. P_+M_-T, $p_xm_yt_z$ Zn^2w.
m. P_+M_-T, $p_xm_yt_z$ Zn^2w.
n. t. P_+M_-T.
o. p_+t, P_+M_-T.
p. m_+t. P_+M_-T.
q. P_+M_-T, $p_xm_yt_z$ Znw^2.
r. P_+M_-T, $p_xm_yt_z$ Znw^2.
s. P_+M_-T, $p_xm_yt_z$ Znw^2.
t. p_+m, P_+M_-T.
u. P_+M_-T, $p_xm_yt_z$ Z^2n^2w.
v. p_+t, P_+M_-T.
x. P_+M_-T, $p_xm_yt_z$ Z^2nw^2.
y. m_-t. P_+M_-T.
z. P_+M_-T, $p_xm_yt_z$ Zn^2w^2.

The Right Rhombic Prism.

P_+, M_-T.

a. P_x, M_-T. p_xm.
b. P_x, M_-T. $p_xm_yt_z$ Zn^2w Zn^2e.
c. P_x, M_-T. p_xt.
d. P_x, M_-T. $p_xm_yt_z$ Zne^2 Znw^2.
e. P_x, M_-T. $p_xm_yt_z$ Zn^2e^2 Zn^2w^2.
f. P_x, m, M_-T.
g. P_x, $m_{\mp}t$, M_-T.
h. P_x, t, M_-T.
i. P_x, M_-T, m_+t.

The Octahedron with a Square Base.

P_xM, P_xT: or P_xMT.

[Either of these symbols may be taken to represent a square-based octahedron. The choice is regulated by the value of the index x. See § 480. Mr. Brooke's Modifications, however, refer to the latter symbol.—J. J. G.]

a. p. P_xMT.
b. P_xMT, p_mt.
c. p_m, p_t, P_xMT.
d. P_xMT, $p_xm_yt_z$, $p_xm_zt_y$.
e. m, t. P_xMT.
f. p+m, p+t, P_xMT.
g. m_t, m+t. P_xMT.
h. P_xMT, $p_xm_yt_z$, $p_xm_zt_y$.
i. P_xMT, $p_+m_yt_z$, $p_+m_zt_y$.
k. P_xMT, $p_+m_yt_z$, $p_+m_zt_y$.
l. p_xm, p_xt, P_xMT.
m. P_xMT, $p_xm_yt_z$, $p_xm_zt_y$.
n. mt. P_xMT.
o. P_xMT, p+mt.

The Octahedron with a Rectangular Base. P_xM, P_xT.

[I consider these Combinations to be Modifications of the Rhombic Prism, and alter the positions accordingly.—J. J. G.]

M_T. P_xT.

a. t, M_T. P_xT.
b. M_T, m+t. P_xT.
c. M_T. P_xT, p+t.
d. M_T. P_xT, $p_+m_yt_z$.
e. M_T. p_xm, P_xT.
f. M_T. p+m, P_xT.
g. M_T. p_m, P_xT.
h. M_T. P_xT, $p_xm_yt_z$.
i. M_T. P_xT, $p_xm_yt_z$.
k. M_T. P_xT, $p_xm_yt_z$.
l. M_T. P_xT, $p_xm_yt_z$.
m. m, M_T. P_xT.
n. m=t, M_T. P_xT.
o. p, M_T. P_xT.
p. M_T. p_t, P_xT.

The Right-Square Prism.

P_x,M,T: or P_x, MT.

a. P_x,M,T. $p_x$mt.
b. P_x,M,T. $p_xm_yt_z$, $p_xm_zt_y$.
c. P_x,M,T. p_xm, p_xt.
d. P_x,M,T, mt.
e. P_x,M,T, m_t, m+t.

The Oblique Rhombic Prism.

Either M_T. $\frac{1}{2}P_xM$ Zn:
or M_T. $\frac{1}{2}P_xT$ Zw.

a. M_T. $\frac{1}{2}P_xM$ Z^2n, $\frac{1}{2}p_+m$ Zn^2.
b. M_T. $\frac{1}{2}P_xM$ Zn, $\frac{1}{2}p_xm_yt_z$ Zne Znw.
c. M_T. $\frac{1}{2}P_xM$ Zn, $\frac{1}{2}p_xm$ Zs.
d. M_T. $\frac{1}{2}P_xM$ Zn, $\frac{1}{2}p_xm_yt_z$ Zse Zsw.
e. M_T. $\frac{1}{2}P_xM$ Zn, $\frac{1}{2}p_xm_yt_z$ Zne^2 Znw^2.
f. M_T. $\frac{1}{2}P_xM$ Zn, $\frac{1}{2}p_xm_yt_z$ Zn^2e^2 Zn^2w^2.
g. M_T. $\frac{1}{2}P_xM$ Zn, $\frac{1}{2}p_xm_yt_z$ Zs^2e^2 Zs^2w^2.
h. m, M_T. $\frac{1}{2}P_xM$ Zn.
i. m=t, M_T. $\frac{1}{2}P_xM$ Zn.
k. t, M_T. $\frac{1}{2}P_xM$ Zn.
l. M_T, m+t. $\frac{1}{2}P_xM$ Zn.

The Rhombic Dodecahedron.

MT. PM, PT.

a. p, m, t, MT. PM, PT.
b. MT, m_t, m+t. PM, p_m, p+m, PT, p_t, p+t.
c. MT. PM, PT, 3 p_mt.
d. MT. PM, PT, 6 p_mt+.
e. MT. PM, PT, pmt.
f. MT. PM, PT, 3 p+mt.
g. MT. PM, PT, 3 p_mt.
h. MT. PM, PT, 6 p_mt+.
i. MT. PM, PT, 3 P_MT.
k. MT. PM, PT, 6 P_MT+.

The Right Oblique-angled Prism.

Either M_, T. $\frac{1}{2}P_xT$ Zw:
or M_x, T. $\frac{1}{2}P_xM$ Zn.

[Placed by Mr. Brooke in the position of P_, $\frac{1}{2}M_xT$ nw, $\frac{1}{2}M_xT$ ne.—J. J. G.]

a. M_,T. $\frac{1}{2}P_xT$ Zw, $\frac{1}{2}p_xm_yt_z$ Znw Zsw.
b. M_,T. $\frac{1}{2}P_xT$ Zw, $\frac{1}{2}p_xm_yt_z$ Zne Zse.
c. M_, T, m_t. $\frac{1}{2}P_xT$ Zw.
d. M_, T. $\frac{1}{2}P_xT$ Zw, $\frac{1}{2}p_xm_yt_z$ Zn^2w^2 Zs^2w^2.
e. M_, T. $\frac{1}{2}P_xT$ Zw, $\frac{1}{2}p_+t$ Zw^2.
f. M_, T. $\frac{1}{2}P_xT$ Zw, $\frac{1}{2}p_+t$ Ze.

THE REGULAR HEXAGONAL PRISM.

$P_x, T, M\{\frac{1}{1}\frac{5}{3}T_2$: or P_x, V.

a. P_x,V. $2r_x$ Zn Zs.
b. P_x,V. $3p_x m_y t_z$.
c. P_x,V. $2R_x$ Zw Ze.
d. P_x,V, *v*.
e. P_x, $3m_x t$.

THE RHOMBOID.

$\frac{1}{2}P_x T$, $\frac{1}{2}P_x M\{\frac{1}{1}\frac{5}{3}T_2$, or R_x.
[R_x is assumed to be R_1.—J. J. G.]

a. p. R_1.
b. R_1 Zw, r_- Zw.
c. R_1 Zw, r_- Ze.
d. R_1 Zw, s_- Zw.
e. v. R_1 Zw.
f. $3m_x t$. R_1 Zw.
g. R_1 Zw, r_+ Ze.
h. R_1 Zw, s_x Zw.
i. R_1 Zw, s_+ Zw.
k. R_1 Zw, r_+ Zw.
l. R_1 Zw, s_+ Zw.
m. R_1 Zw, R_2 Ze.
n. R_1 Zw, s_- Ze.
o. V. R_1 Zn.
p. R_1 Zw, s_+ Zw.

THE DOUBLY OBLIQUE PRISM.

$\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne. $\frac{1}{4}P_x M_y T_z$ Zne.

a. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne. $\frac{1}{4}P_x M_y T_z$ Zne, $\frac{1}{4}p_x m_y t_z$ Zn^2e.
b. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne. $\frac{1}{4}P_x M_y T_z$ Zne, $\frac{1}{4}p_x m_y t_z$ Zs^2e.
c. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne. $\frac{1}{4}P_x M_y T_z$ Zne, $\frac{1}{4}p_x m_y t_z$ Zse^2.
d. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne. $\frac{1}{4}P_x M_y T_z$ Zne, $\frac{1}{4}p_x m_y t_z$ Znw^2.
e. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M$ T ne. $\frac{1}{4}P_x M_y T_z$ Zne, $\frac{1}{4}p_x m_y t_z$ Zs^2e^2.
f. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne. $\frac{1}{4}P_x M$ T_z Zne, $\frac{1}{4}p_x m_y t_z$ Zs^2w^2
g. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne. $\frac{1}{4}P_x M_y T$ Zne, $\frac{1}{4}p_x m_y t_z$ Zn^2e^2.
h. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne. $\frac{1}{4}P_x M_y T_z$ Zne, $\frac{1}{4}p_x m_y t_z$ Zn^2w^2.
i. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ ne^2, $\frac{1}{2}m_x t$ n^2e. $\frac{1}{4}P_x M_y T_z$ Zne.
k. $\frac{1}{2}M_x T$ nw, $\frac{1}{2}M_x T$ n^2e, $\frac{1}{2}m_x t$ nw^2. $\frac{1}{4}P_x M_y T_z$ Zne.

MR. BROOKE'S *Table of Secondary Forms*. "It may be observed in the preceding Tables of Modifications that many of the secondary Forms of crystals, are similar to some of the classes of the primary. And it may also be remarked, in many instances, that the secondary Forms when complete, or the new figures, as they are termed, are different from all the primary Forms. The following table exhibits the relations of both these descriptions of secondary Forms to the several classes of the primary from which they might be produced; and it may thus be regarded as a kind of index to the Tables of Modifications." BROOKE, p. 214.

1. SECONDARY FORMS, CONTAINED WITHIN FOUR PLANES:

The Tetrahedron = $\frac{1}{2}$PMT.
Cube, *g*.

2. CONTAINED WITHIN SIX PLANES:

The Cube. = P,M,T.
Tetrahedron, *e*.
Octahedron, *a*.
Dodecahedron, *a*.

The Right-Square Prism = P_x,MT, or P_x,M,T.
Right-Square Prism, *d*.
Quadratic Octahedron, *a* with *e*, or *a* with *n*.

The Right Rectangular Prism. = P_x,M_-,T.
Rectangular Octahedron, *a*, *m*, and *o*.
Rhombic Octahedron, *a*, *g*, and *n*.
Right Rhombic Prism, *f* and *h*, with two primary planes

The Right Rhombic Prism.
$= P_x, M\_T.$

Rectangular Octahedron,
- c and m,
- p and m,
- a and e,
- a and f,
- a and g,
- b and o,
- n and o.

Rhombic Octahedron,
- a and i,
- a and p,
- a and y,
- b and n,
- h and n,
- t and n,
- c and g,
- o and g,
- v and g.

Right Rectangular Prism, d, b, c — with one pair of planes of the primary at right angles to the modification.

Right Rhombic Prism, g, or i.

The Right Oblique-angled Prism.
$= M_x, T.\frac{1}{2}P\frac{1}{2}T$ Zw:
or $M_x, T.\frac{1}{2}P\frac{1}{2}M$ Zn.
Oblique Rhombic Prism, h and k, with two primary planes remaining.

The Rhomboid. $= R_x$.
Rhomboid, modifications b, c, g, k, or m.

3. Contained within eight Planes:

The Regular Octahedron. = PMT.
Cube, a.
Tetrahedron, *with* a.
R. Dodecahedron, e.

The Quadratic Octahedron. =
P_xMT: or P_xM, P_xT.
Quadratic Octahedron, b, c, f, l, or o.
Right square Prism, a or c.

The Octahedron with a Rectangular base: M_T. P_xT:
or M_T. P_xM.
Right Rectangular Prism, b and c.
Right Rhombic Prism, a or c, with four primary planes added.

The Octahedron with a Rhombic base. $= P_x M_y T_z$.
Rectangular Octahedron, h, i, k, or l.
Rhombic Octahedron, d, e, f, k, l, m, q, r, s, u, x, or z.
Right Rectangular Prism, a.
Right Rhombic Prism, b, d, or e.

The Hexagonal Prism =
$P_x, T, M\frac{1\,5}{1\,3}T_2$: or P_x,V.
Rhomboid, a and e, or a and o.
Hexagonal Prism, d.
Right Rhombic Prism of 120°, h.

4. Contained within twelve Planes:

a. The planes being isosceles triangles.

(*Figure without name*). =
$\frac{1}{2}$(3P_MT) Zne.
The left Hemiicositessarahedron.
Tetrahedron, c.
Cube, h.

(*Figure without name*). =
$\frac{1}{2}$(3P_MT) Znw.
The right Hemiicositessarahedron.
Tetrahedron, f.

(*Figure without name*). =
$2R_+$, or P_+T, $P_+M\frac{1\,5}{1\,3}T_2$.
An Acute six-sided Pyramid.
Hexagonal Prism, a or c.
Rhomboid, particular planes of d, h, i, l, or n.

[This appears to be a mistake. I do not think it possible for particular planes of the Scalenohedron to produce regular isosceles six-sided pyramids, with lateral edges situated in the plane of the equator.—J. J. G.]

b. The planes being scalene triangles.
(*Figure without name*). $= S_x$.
The Scalenohedron.
Rhomboid, d, h, i, l, n, or p.

c. The Planes being Rhombs.

The Rhombic Dodecahedron. = MT. PM. PT.
Cube, *e.*
Tetrahedron, *b.*
Octahedron, *e.*

d. The planes being trapezoids.

(*Figure without name*). = $\frac{1}{2}(3P_{+}MT)$.
The Hemitriakisoctahedron.
Tetrahedron, *b.*

e. The planes being pentagons.

(*Figure without name*). = $M_{-}T$. $P_{-}M$, $P_{+}T$.
The Pentagonal Dodecahedron.
Cube, *k.*
Tetrahedron, *d?*

5. Contained within sixteen Triangular Planes.

(*Figure without name*). = $P_xM_yT_x$, $P_xM_xT_y$. *An eight-sided Pyramid of the Pyramidal System.*
Octahedron with square base, *d*, *h*, *i*, *k*, or *m*.
Right square Prism, *b.*

6. Contained within twenty-four Planes.

a. The planes being isosceles triangles.

(*Figure without name*). = $M_{-}T$, $M_{+}T$. $P_{-}M$, $P_{+}M$, $P_{-}T$, $P_{+}T$.
The Tetrakishexahedron.
Cube, *f.*
Tetrahedron, *d?*
Octahedron, *c.*
R. Dodecahedron, *b.*

(*Figure without name*). = $3P_{+}MT$. *The Triakisoctahedron.*
Cube, *c.*
Octahedron, *f.*
R. Dodecahedron, *f.*

b. The planes being equal trapezoids.

(*Figure without name*). = $3P_{-}MT$. *The Icositessarahedron.*
Cube, *b.*
Octahedron, *b.*
R. Dodecahedron, *c*, *g*, or *i.*

7. Contained within forty-eight Triangular Planes.

(*Figure without name*). = $6P_{-}MT_{+}$. *A Hexakisoctahedron.*
Cube, *d.*
Octahedron, *d.*
R. Dodecahedron, *d*, *h*, or *k.*

" *On the Application of the Tables of Modifications.*—The preceding Tables of Modifications are adapted principally to two purposes. The first is, by the remarks they contain upon the comparative characters of the secondary Forms belonging to the different classes of the primary, to assist the mineralogist in determining the primary Form of any Mineral from an examination of its secondary Form. And the second is to enable him to describe any secondary crystal, whose primary Form is known. An attempt is thus made to supply a language, by means of which the secondary Forms of crystals may be described independently of the theory of decrements, and without the assistance of mathematical calculation."—Brooke, *Introduction*, p. 223.

The "remarks" alluded to by Mr. Brooke, describe the Forms and Combinations which I have represented in symbols. In Mr. Brooke's book, these remarks, and the figures to which they relate, occupy 129

octavo pages, and yet convey little more information than is given by the present brief abstract of six pages. I mention this circumstance as a proof that the new symbols possess great descriptive powers, and may therefore well replace the notation of MR. BROOKE, which is so inartificially constructed that no memory can retain it.

The system presented by Mr. Brooke is, indeed, one of apparent ease but real difficulty, as a single investigation must satisfy every mineralogist. Suppose the student to have for examination the crystal represented by Model 32, containing the Forms, P, M, T, mt. pm, pt, PMT, and exhibiting a cube truncated on the edges and angles, or a combination of the cube, the rhombic dodecahedron, and the regular octahedron. Well, this is a *secondary Form*, and the student wants to know the *primary*, and the reason of his wanting to know the primary is, that in MR. BROOKE's Catalogue of Minerals, the *primary Form* of each mineral *alone* is named, so that the most perfect knowledge of the secondary Form, or natural crystal, is not sufficient to give the name of the Mineral, even if only a single Mineral should be known to exist in the shape of that particular secondary Form, and the same Mineral be totally unknown in the shape of its presumed primary Form. The student therefore looks in the Table of secondary Forms for P, M, T, the Cube, where he finds that this secondary Form indicates as its primary,

The Tetrahedron, Modification *e*.
The Regular Octahedron, Modification *a*.
The Rhombic Dodecahedron, Modification *a*.

He next looks in the Table of secondary Forms for MT. PM, PT, the Rhombic Dodecahedron, which Form is also upon his crystal. This indicates as its primary,

The Cube, Modification *e*.
The Tetrahedron, Modification *b*.
The Octahedron, Modification *e*.

He finally looks in the same Table for the Form PMT, the Octahedron, which also forms part of his secondary crystal, and finds that this indicates as its primary,

The Cube, Modification *a*.
The Tetrahedron, with Modification *a*.
The Rhombic Dodecahedron, Modification *e*.

He has therefore A CHOICE of no less than *four primary Forms*, namely,

The Cube,	The Rhombic Dodecahedron,
The Octahedron,	The Tetrahedron.

And here ends the power of the Table to guide him in finding the required primary Form. What is the remedy for this difficulty? *First*, he may destroy the crystal to find the *cleavage*. But this is a method always expensive, and often impossible. *Secondly*, he may *guess* at the *primary* from the *predominant* Form, which in this case is the *cube*. But suppose that Model 33, P, M, T, mt. pm, pt, PMT, had been the secondary crystal under examination, the Tables would then lead to the same result as in the above case, while the *predominant* Form would induce the student to

choose the *octahedron* as the required primary. And if Model 34, p, m, t, MT. PM, PT, pmt, had been the given secondary Form, the student would be induced by the same routine of examination, to choose the *Rhombic Dodecahedron* as its primary Form.

And after all this examination, the chances are exactly two to one that the examiner comes to a wrong conclusion. For, suppose him to have a Mineral in the Form of Model 32, P, M, T. mt. pm, pt, PMT, and that he chooses the predominant Form, or the cube, to be the primary, and consistently with this choice names his crystal, *the Cube with Modifications a and e*. Then, if his Mineral happens to be Grey Copper, he will be wrong, because Mr. Brooke says the primary Form of that Mineral is the tetrahedron, in which case Model 32 must be described as *the Tetrahedron with Modifications a, e, and b*. If the given Mineral was *Fluorspar*, the student would be equally in the wrong, because the primary Form of that Mineral is assumed to be the regular octahedron, in which case, Model 32 is described as the *Octahedron with Modifications a and e*. Finally, if the Mineral was Galena, the student would be right, because the primary Form of this Mineral is the cube, and Model 32 has then to be labelled *the Cube with Modifications a and e*.

The conclusion which I draw from this investigation is, that the doctrine of primary or primitive Forms, is worthless and mischievous, when made the basis of a system of Crystallographic notation. The search for "primary Forms" with the assistance of MR. BROOKE's Tables, or with any other help, is a search for a nonentity—a mere waste of time.—WEISS's discovery of the relations of the axes of crystals to their planes, has abolished primary Forms for ever.

SECTION XV. ON THE UTMOST POSSIBLE ABRIDGMENT OF EXACT CRYSTALLOGRAPHIC NOTATION.

603. Brevity is a desirable qualification in Crystallographic Notation, but it is of much less importance than accuracy and intelligibleness. We must never therefore displace a symbol which is accurate and intelligible, by a sign, however short, which gives an ambiguous indication, or which is liable to be forgotten or to be mistaken for something different.

In constructing the notation employed in this work, I have therefore not struggled to attain the utmost possible limit of shortness. I think indeed that brevity may in this matter be carried to an absurd and injurious extent, and it has been a rule with me to carry out my principle of notation, consistently and faithfully, to its full extent, rather than to take alarm at the occasional occurrence of a long symbol, and for the sake of avoiding that long symbol, to cut and carve at the notation, and break down the unity of the system, and produce, not a useful working notation, but a collection of enigmas.

The notation which I have adopted conveys information on four principal points, namely:—

1. The names of the axes that are cut by each Form or set of planes.
2. The lengths measured from the centre of the crystal, at which these axes are cut.
3. The comparative magnitude of each set of planes on a combination.
4. The polaric positions of the planes of hemihedral Forms.

None of this information can be prudently suppressed. Therefore, notation is inadmissible which is unable to convey all these particulars. On the other hand, that system of notation which indicates all these relations of planes most completely and most conveniently, is the best. By these principles the value of any notation may be estimated.

To name the axes that are cut by each Form, I denote the three axes by the letters P, M, T. To denote the lengths of the axes, I employ the vulgar fractions $\frac{1}{2}$, $\frac{1}{3}$, &c., as Indices. When a plane cuts one axis, the index is written after the letter, as $P\frac{1}{3}$. When it cuts two axes, the index is written between the two letters, as $M\frac{2}{3}T$, where it indicates the relation of the first-named axis to the second. When it cuts three axes, the relation of the first to the third is expressed after P, and the relation of the second to the third is expressed after M, as $P\frac{1}{3}M\frac{1}{2}T$. There must always be *two* indices to the symbol of a triaxial form, and *one* index to the symbol of a biaxial form, but the indices of axes that are *unity* may be suppressed. Thus, $M\frac{1}{1}T$ is written MT, and $P\frac{1}{1}M\frac{1}{1}T$ is PMT.

Professor MILLER has given the following method of shortening the symbols of the biaxial and triaxial forms. Instead of $M\frac{2}{3}T$, he writes $\{0 2 3\}$, and instead of $P\frac{1}{3}M\frac{1}{2}T$ he writes $\{2 3 6\}$. In the first example, 2 means M_2 and 3 means T_3. In the second example, 2 means $P = \frac{1}{3}$ of T, 3 means $M = \frac{1}{2}$ of T, and 6 means T = unity. But, in reality, very little advantage is gained by this abridgment, for $\{0 2 3\}$ and $\{2 3 6\}$, brackets included, are as long as $M\frac{2}{3}T$ and $P\frac{1}{3}M\frac{1}{2}T$, while they are less explicit, and have the great defect of being unable to denote the comparative magnitude of different Forms in the same combination, which information is of considerable utility, but can never be given by symbols that consist solely of figures. For this reason, I think that letters ought not to be dispensed with, even in a single symbol.

It is still more evident that it is impossible to remove the figures which denote the lengths of the axes of the different Forms, for these, being the only marks of the individuality of the Forms, are an indispensable portion of their names. Hence, the liberty of abridging crystallographic notation is placed under considerable restrictions. I would almost say, that the restrictions are such as render abridgment inexpedient. They do not, however, render it impossible, and as I know that many persons are peculiarly alive to the merits of a short notation, I shall proceed to show what can be done with safety in the way of abridgment.

The best plan, then, of constructing short symbols, is to take the "Forms" and fundamental "Combinations" of each of the six systems of crystal-

lisation, and ARBITRARILY *replace* ALL *the letters of each symbol by* ONE *letter*, preserving the characteristic indices and signs of position. Thus,

C may indicate the cube = P, M, T;

$O\frac{1}{2}$, the icositessarahedron = $P\frac{1}{2}MT$, $PM\frac{1}{2}T$, $PMT\frac{1}{2}$;

$K\frac{1}{2}$, the tetrakishexahedron = $M\frac{1}{2}T$, $M\frac{2}{1}T$. $P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$;

C, $p\frac{1}{2}$, $p\frac{2}{3}$, o, $o\frac{1}{2}$, $o\frac{2}{1}$, $\frac{1}{3}h\frac{1}{2}$, $\frac{1}{4}h\frac{1}{2}$ = Haüy's Parallélique or maximum crystal of Iron Pyrites, which contains 134 faces. See his symbol at page 22, Part II.

And so on. I shall give a Table of such abridged signs, and add a few examples and observations.

604. TABLE OF ABRIDGED NOTATION.

1. OCTAHEDRAL SYSTEM OF CRYSTALLISATION. Axes: $p^a\, m^a\, t^a$.

O = PMT = octahedron.

C = P,M,T = cube.

D = MT. PM, PT = rhombic dodecahedron.

O_- = $3P_-MT$ = icositessarahedron.

O_+ = $3P_+MT$ = triakisoctahedron.

${}_xO_y$ = $6P_-MT_+$ = hexakisoctahedron.

K_x = M_-T, M_+T. P_-M, P_+M, P_-T, P_+T = tetrakishexahedron.

P_x = M_-T. P_-M, P_+T = pentagonal dodecahedron.

${}_xH_y$ = $3P_-M\,T_+$ = hemihexakisoctahedron with parallel faces.

T = $\frac{1}{2}PMT$ = tetrahedron.

I_- = $\frac{1}{2}(3P_-MT)$ = hemiicositessarahedron.

S_+ = $\frac{1}{2}(3P_+MT)$ = hemitriakisoctahedron.

${}_xX_y$ = $\frac{1}{2}(6P_-MT_+)$ = hemihexakisoctahedron with inclined faces.

2. PYRAMIDAL SYSTEM. Axes: $p^a_x\, m^a\, t^a$.

O_x = P_xMT = quadratic octahedron of the ne and nw zones.

N_x = P_xM, P_xT = quadratic octahedron of the n and e zones.

P = P = horizontal planes.

Q = M,T = quadratic prism of the n and e zones.

Q = MT = quadratic prism of the ne and nw zones.

${}_xD_y$ = $P_xM_yT_z$. $P_xM_zT_y$ = dioctahedron, or eight-sided pyramid.

${}_xH_y$ = $\frac{1}{2}(P_xM_yT_z, P_xM_zT)$ = hemidioctahedron.

V_x = M_-T, M_+T = Eight-sided prism.

3. RHOMBOHEDRAL SYSTEM. Axes: $p^a_x\, m^a_{15}\, t^a_{13}$, or $p^a_x\, m^a_{14}\, t^a_{13}$.

$2R_x$ = P_xT, $P_xM\frac{1}{1}\frac{5}{3}T_2$ = six-sided pyramid.

P = P = horizontal planes.

V = T, $M\frac{1}{1}\frac{5}{3}T_2$ = six-sided prism, first position.

V = M, $M_2T\frac{1}{1}\frac{5}{3}$ = six-sided prism, second position.

${}^3_v x:{}_y:{}_z$ = or simply ${}^3_v$ = $3m_xt$ = subordinate twelve-sided prism, with alternate similar angles.

${}^3_o x:{}_y:{}_z$ = or simply ${}^3_o$ = $3p_xm_yt_z$ = subordinate twelve-sided pyramid.

R_x = $\frac{1}{2}P_xT$, $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$ = rhombohedron.

S_x = $\frac{1}{2}(3P_xM^{\pm}T_{\mp})$ = scalenohedron.

V, *v* = six-sided prism with vertical edges replaced.

V, *v*, ${}^3_v$ = twenty-four-sided prism.

4. PRISMATIC SYSTEM. Axes: $p_x^{:}\ m_y^{:}\ t_z^{:}$.

$_xO_y = P_xM_yT_z$ = rhombic octahedron.
P = P = horizontal planes.
M = M = north and south vertical planes.
T = T = east and west vertical planes.
$V_x = M_xT$ = vertical rhombic prism.
$N_x = P_xM$ = rhombic Form of the north zone.
$E_x = P_xT$ = rhombic Form of the east zone.

5. OBLIQUE PRISMATIC SYSTEM. Axes: $p_x^{:}\ m_y^{:}\ t_z^{:}$.

M = M = north and south vertical planes.
T = T = east and west vertical planes,
$V_x = M_xT$ = vertical rhombic prism.

$$\left.\begin{array}{l} N_x\ ^{n} = \frac{1}{2}P_xM\ Zn^2 \\ N_x\ \substack{z\\n} = \frac{1}{2}P_xM\ Z^2n \\ N_x\ ^{s} = \frac{1}{2}P_xM\ Zs^2 \\ N_x\ \substack{z\\s} = \frac{1}{2}P_xM\ Z^2s \end{array}\right\} = \text{hemihedral biaxial Forms of the north zone.}$$

$$\left.\begin{array}{l} E_x\ ^{w} = \frac{1}{2}P_xT\ Zw^2 \\ E_x\ \substack{z\\w} = \frac{1}{2}P_xT\ Z^2w \\ E_x\ ^{e} = \frac{1}{2}P_xT\ Ze^2 \\ E_x\ \substack{z\\e} = \frac{1}{2}P_xT\ Z^2e \end{array}\right\} = \text{hemihedral biaxial Forms of the east zone.}$$

$$\left.\begin{array}{l} _xO_y\ \substack{z\\n} = \frac{1}{2}P_xM_yT_z\ Z^2ne\ Z^2nw \\ _xO_y\ ^{n} = \frac{1}{2}P_xM_yT_z\ Zn^2e\ Zn^2w \\ _xO^y\ \substack{w\\e} = \frac{1}{2}P_xM_yT_z\ Zne^2\ Znw^2 \\ _xO_y\ \substack{s\\w} = \frac{1}{2}P_xM_yT_z\ Zs^2e^2\ Zs^2w^2 \end{array}\right\} = \begin{array}{c}\text{hemioctahedrons belonging to} \\ \text{north combinations.}\end{array}$$

$$\left.\begin{array}{l} _xO_y\ \substack{z\\w} = \frac{1}{2}P_xM_yT_z\ Z^2nw\ Z^2sw \\ _xO_y\ \substack{n\\s} = \frac{1}{2}P_xM_yT_z\ Zn^2w\ Zs^2w \\ _xO_y\ ^{w} = \frac{1}{2}P_xM_yT_z\ Znw^2\ Zsw^2 \\ _xO^y\ ^{e} = \frac{1}{2}P_xM_yT_z\ Zne^2\ Zse^2 \end{array}\right\} = \begin{array}{c}\text{hemioctahedrons belonging to} \\ \text{east combinations.}\end{array}$$

6. DOUBLY OBLIQUE PRISMATIC SYSTEM. Axes: $p_x^{:}\ m_y\ t_z^{:}$.

M = M = north and south vertical planes.
T = T = east and west vertical planes.

$$\left.\begin{array}{l} V_xw = \frac{1}{2}M_xT\ nw = \\ V_xe = \frac{1}{2}M_xT\ ne = \end{array}\right\} \text{hemihedral prismatic Forms.}$$

$_xO_y\,Znw = \frac{1}{4}P_xM_yT_z\ Znw$ = tetarto-octahedron.

Examples from the Octahedral System of Crystallisation:

Model 36. P, M, T, mt. pm, pt, ½pmt = C, d, t.
Model 37. p, m, t, mt. pm, pt, ½PMT = c, d, T.
Model 32. P, M, T, mt. pm, pt, PMT = C, d, o.
Model 33. P, M, T, mt. pm, pt, PMT = c, d, O.
Model 34. p, m, t, MT. PM, PT, pmt = c, D, o.
Model 45. P, M, T, $m\frac{1}{3}t$, $m\frac{3}{1}t$. $p\frac{1}{3}m$, $p\frac{3}{1}m$, $p\frac{1}{3}t$, $p\frac{3}{1}t$ = C, $k\frac{1}{3}$.
Model 46. p, m, t. $3P\frac{1}{3}M\frac{1}{2}T$ = c, $\frac{1}{3}H\frac{1}{2}$.
Model 48. p, m, t, $M\frac{1}{2}T$. $P\frac{1}{2}M$, $P\frac{2}{1}T$, PMT = c, $P\frac{1}{2}$, O.

Examples from the Pyramidal System:

Model 4. P_{+}, M, T, mt = P_{+}, Q, *q*.
Model 42. p_{+}, \textsc{m}, \textsc{t}, MT. P$\frac{3}{4}$M, P$\frac{3}{4}$T = p_{+}, \textsc{q}, *Q*. N$\frac{3}{4}$.
Model 59. M, T, mt. P$\frac{3}{5}$M, P$\frac{3}{5}$T = Q, *q*. N$\frac{3}{5}$.
Model 77. p. pm, pt, PMT = p. n, O.

Examples from the Rhombohedral System:
See Part II. pages 45—60.

Examples from the Prismatic System:

Model 80. p_{+}. P$\frac{10}{10}$M$\frac{8}{10}$T = p_{+}. $\frac{10}{10}$O$\frac{8}{10}$.
Model 50. P_{-}, m, t_{+}, M$\frac{4}{3}$T. p$\frac{4}{3}$m, p$\frac{4}{3}$t = P_{-}, m, t_{+}, V$\frac{4}{3}$. n$\frac{4}{3}$, e$\frac{4}{3}$.
Model 100. M_{-}, M$\frac{4}{3}$T. P$\frac{4}{3}$M = M_{-}, V$\frac{4}{3}$. N$\frac{4}{3}$.
Model 51. p_{+}, M_{-}, T, \textsc{m}$\frac{4}{9}$\textsc{t}. P$\frac{5}{4}$M, P$\frac{10}{9}$T, p$\frac{5}{9}$m$\frac{4}{9}$t = p_{+}, M_{-}, T, v$\frac{4}{9}$. N$\frac{5}{4}$, E$\frac{10}{9}$, $\frac{5}{9}$o$\frac{4}{9}$.
Model 97. \textsc{m}, \textsc{t}, M$\frac{5}{8}$T. P$\frac{7}{10}$T = \textsc{m}, \textsc{t}, V$\frac{5}{8}$. E$\frac{7}{10}$.
Model 79ᵃ. M, T. P$\frac{4}{7}$M = M, T. N$\frac{4}{7}$.
Model 90. M$\frac{10}{36}$T, \textsc{m}$\frac{10}{18}$\textsc{t}. P$\frac{28}{29}$T, p$\frac{33}{36}$m$\frac{10}{36}$t. = V$\frac{10}{36}$, v$\frac{10}{18}$. E$\frac{28}{29}$, $\frac{33}{36}$o$\frac{10}{36}$.

Examples from the Oblique Prismatic System:

Model 84. M$\frac{10}{19}$T. $\frac{1}{2}$P$\frac{4}{15}$M Zn = V$\frac{10}{19}$. N$\frac{4}{15}{}^{n}$.
Model 87. M$\frac{20}{21}$T. $\frac{1}{2}$P$\frac{6}{21}$T Zw = V$\frac{20}{21}$. E$\frac{6}{21}{}^{w}$.
Model 79ᵇ. M_{-}, T. $\frac{1}{2}$P$\frac{1}{2}$M Zn = M_{-}, T. N$\frac{1}{2}{}^{n}$.
Model 112. T, M$\frac{10}{19}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$P$\frac{10}{38}$M$\frac{10}{38}$T Zse Zsw } = T, V$\frac{10}{19}$. N$\frac{4}{13}{}^{n}$, $\frac{10}{38}$O$\frac{10}{38}{}^{s}$.
Model 105. T, $\frac{1}{2}$M$\frac{15}{26}$T ne. $\frac{1}{2}$P$\frac{1}{2}$M Zn = T, $\frac{1}{2}$V$\frac{15}{26}$e. N$\frac{1}{2}{}^{n}$.
Model 115. T_{-}, M$\frac{9}{13}$T. $\frac{1}{2}$P$\frac{5}{13}$M$\frac{9}{13}$T Z^{2}ne Z^{2}nw = T_{-}, V$\frac{9}{13}$. $\frac{5}{13}$O$\frac{9}{13}{}^{n}$.
Model 98. \textsc{m}, T, \textsc{m}$\frac{20}{21}$\textsc{t}. $\frac{1}{2}$P$\frac{6}{21}$M$\frac{20}{21}$T Znw Zsw = \textsc{m}, T, v$\frac{20}{21}$. $\frac{6}{21}$*O*$\frac{20}{21}$.

Examples from the Doubly-oblique Prismatic System:

Model 81ᵇ. M$\frac{2}{3}$T. $\frac{1}{4}P_{x}M_{y}T_{z}$ Z^{2}nw, $\frac{1}{4}$\textsc{p}$_{x}$\textsc{m}$_{y}$\textsc{t}$_{z}$ Zn^{2}e, $\frac{1}{4}p_{x}m_{y}t_{z}$ $Z^{2}ne^{2}$ } = V$\frac{2}{3}$w, v$\frac{2}{3}$e. ${}_{x}O_{y}{}^{w}$, ${}_{x}o_{y}{}^{n}$, ${}_{x}o_{y}{}^{e}$.
Model 107. M, $\frac{1}{2}$m$\frac{7}{10}$t n^{2}w, $\frac{1}{2}$M$\frac{7}{2}$Tnw^{2}, $\frac{1}{2}$m$\frac{7}{8}$t ne. $\frac{1}{4}P_{-}M_{+}T$ Z^{2}ne } = M, v$\frac{7}{10}$n, V$\frac{7}{2}$w, v$\frac{7}{8}$e. ${}_{x}O_{y}{}^{ne}$.

605. The reader will perceive from the foregoing Table and Examples, that it is by no means a difficult task to provide a short notation, when we possess a complete systematic notation upon which to found it. A short notation is only difficult of construction when we are without a complete systematic notation. The reason of this is, that when the arbitrary signs which constitute the short notation are merely synonymes of another more complete and fully-described notation, we are freed from the necessity of giving those exact definitions and detailed descriptions of the short notation, which would in other circumstances be indispensable.

I believe that the notation just described, is shorter, and yet as exact as any that has hitherto been proposed. It has one advantage over any that I am acquainted with, inasmuch as the single letters belonging to each symbol can be varied in size to indicate the comparative magnitude of

different Forms present on any one combination. But the notation, nevertheless, possesses several defects, from which no notation of this kind can be free. In the first place, we burden the memory with a long catalogue of synoymes; and in the second place, we lose the opportunity of looking over the zones of complex combinations, which are all represented in an orderly manner by the notation employed in Part II. of this work, but which are put out of sight when the short notation is employed. Thus, when a combination of the cube and the rhombic dodecahedron is represented by P,M,T, MT. PM, PT, we see at a glance the number of planes in the three zones, but when the same combination is represented by C, D, that convenience is lost, and in very complex combinations this is a considerable disadvantage.

I attach no particular value to the letters which I have chosen for the short synonymes of the regular symbols of the Crystallographic Forms. Any other set of letters would answer as well as those which are given in the Table. I would lay it down as a rule, however, that no letters should be taken for a purpose of this kind except such as can be printed with the types to be found in a common printing office; because the adoption of *strange types* produces a sort of unknown tongue, which no ordinary printer can print. Thus, Dalton invented a set of symbols for chemical notation, which could not be printed; and more recently, Berzelius spoiled his chemical notation, by inventing *crossed letters*, which no printer could print without founding new types expressly for that purpose. The consequence of the introduction of these crossed letters is, that Berzelius's notation will never come into common use. Even in Germany, it is being displaced by the modified notation proposed by Liebig and Poggendorff, in which the crossed letters are omitted.

A second rule to be observed, in constructing a useful notation, is to have no characters that occupy more than one line of print, because characters that run up and down into two or three lines are perplexing to the eye, troublesome and expensive to print, and, while they pretend to be short, are in reality long, since they occupy a great deal of space in a book. Of this character are the symbols employed by Haüy, see page 81, Part I. and page 22, Part II. of this work; and so also are the following symbols, taken from Mohs:

$$\frac{r}{l} \frac{\left(\frac{\sqrt[2]{2}}{3} P - 2\right)^3}{2}$$

The form represented by the above symbol is a hemihedral form of a dioctahedron of the pyramidal system $= \frac{1}{2}(P_x M_y T_z, P_x M_z T_y)$ or ${}_xH_y$, $p_x^a m^a t^a$. It is clear that Mohs's symbol must occupy two or three lines of print in a book, so that it is really not a short symbol.

$$\frac{\breve{P}r}{2}. \qquad\qquad \frac{\bar{P}r}{2}.$$

The first of these symbols represents a hemihedral biaxial form of the east zone, belonging to the oblique prismatic system $= \frac{1}{2}P_x T$ Zw or $E_x{}^w$.

The second represents a similar form belonging to the north zone = $\frac{1}{2}P_x M\,Zn$ or N_x^n. Both of Mohs's symbols appear short, but as each occupies two or three lines of print, they are in reality not short.

A third rule to follow is, to avoid too many brackets, braces, and similar arbitrary signs, such as are contained in the following symbols:

$$-\frac{(\breve{P})^4}{2}. \qquad 2(P)^{\frac{5}{3}}). \qquad \frac{[(P+\infty)^5]}{2}. \qquad \frac{1}{r}\ \frac{2\big((P)^{\frac{5}{3}}\big)}{2}$$

It is perhaps a matter of taste, but it appears to me that the longest symbol contained in the second Part of this work is much more convenient, and much easier to follow, than a symbol which presents so many abstractions as the shortest of those which are here quoted.

The symbol of a Form or Combination expressed in this short notation can never be printed alone, but must always be accompanied by the characteristic of the system to which it belongs; because the same letter is used in different systems to indicate different things, and this cannot be otherwise, so long as the letters of our alphabet are fewer in number than the Forms and fundamental Combinations of all the six systems of Crystallisation collectively. Thus S_x indicates a Hemitriakisoctahedron of the first system, and a Scalenohedron of the third system. Hence, to be perspicuous, we must say, S_x, $p^a\ m^a\ t^a$, and S_x, $p^a_x\ m^a_{14}\ t^a_{13}$. The symbols which refer directly to the seven fundamental Forms, P, M, T, MT, PM, PT, PMT, are free from this serious defect, and therefore, though longer, are in most cases preferable.

SECTION XVI. TABLE OF SINES AND TANGENTS.

606. In § 49, I promised to give a short Table of Sines and Tangents, which is accordingly here appended. It is not adapted to replace the common Tables of logarithms, which are more extensive, and therefore more useful; but it will nevertheless often save the reader of this work the trouble of seeking his book of logarithms when he wishes to find either the angle indicated by a particular index, or the index required to denote a particular angle. During the composition of the present treatise, I have been in the habit of marking in my logarithmic tables the angles that I had occasion to refer to, and as the whole of the angles thus marked are printed in the present Table, the reader will find it more frequently useful than from its limited extent he might imagine to be possible.

Another motive which prompted me to give this Table, was the wish to present the reader with a system of tangents and cotangents in vulgar fractions, adapted for use as indices of the seven crystallographic Forms. This is accordingly done, and serving as a counterpart to the Table of Indices given in page 139, will save the reader much calculation, and consequently much time.

The Table contains natural tangents and cotangents, and logarithmic tangents, cotangents, sines and cosines. I have not introduced any natural sines and cosines, because the system of calculation which I have adopted does not require them. The words sine and cosine appear, indeed, in the formulæ which relate to solid triangles, but the calculations are always made by means of the logarithmic equations. On the other hand, the natural tangents and cotangents are constantly employed in the construction and interpretation of the indices of the symbols.

The indices in vulgar fractions are placed against those decimal tangents and cotangents, which are their nearest synonymes.

Any term not contained in the Table may be found by taking the proportion of difference between the two terms nearest related to it in the Table.

EXAMPLE: *To find the tangent of* 26° 24′. The Table contains

$$\begin{aligned} \tan 26^\circ\, 34' &= .5000 \\ \tan 26^\circ\, 11' &= .4917 \\ \hline \text{difference} = \quad 23' &= .0083 \\ \text{23rd part of } .0083 &= .00036 \end{aligned}$$

This last product, .00036, is the tangent of 1′ or the 23rd part of the difference between 26° 34′ and 26° 11′. The number of minutes in 26° 24′ more than in 26° 11′ is 13′. Therefore, the tangent of 26° 24′ is to be found by adding 13 times the tangent of 1′ to the tangent of 26° 11′. Hence,

$$\begin{aligned} \tan 26^\circ\, 11' &= .4917 \\ .00036 \times 13 &= .00468 \\ \hline \tan 26^\circ\, 24' &= .49638 \end{aligned}$$

The large Table gives these tangents as follow:—

26° 11′	26° 24′	26° 34′
4916997	4964043	5000352

This shows how far this method of finding intermediate terms is to be depended upon. The approximation of the numbers is however often closer, though sometimes less so, than in this example. For calculations of importance, the larger table of sines and tangents should be invariably referred to, but for the calculations which occur in the course of studying the principles of the science, the numbers given in the present Table will generally suffice.

TABLE OF SINES AND TANGENTS.

Angles.	N. tan.	T.	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
° ′									° ′
00.00	0000			infinite.	inf. neg.	10.000	inf. neg.	infinite.	90.00
00.01	0003			3437.7	6.4637	10.000	6.4637	13.5363	89.59
00.30	0087			114.59	7.9408	10.000	7.9409	12.0591	89.30
00.38	0111	$\frac{1}{90}$	$\frac{90}{1}$	90.463	8.0435	10.000	8.0435	11.9565	89.22
00.57	0166	$\frac{1}{60}$	$\frac{60}{1}$	60.306	8.2196	9.9999	8.2196	11.7804	89.03
1.00	0175			57.290	8.2419	9.9999	8.2419	11.7581	89.00
1.09	0201	$\frac{1}{50}$	$\frac{50}{1}$	49.816	8.3025	9.9999	8.3026	11.6974	88.51
1.14	0215			46.449	8.3329	9.9999	8.3330	11.6670	88.46
1.26	0250	$\frac{1}{40}$	$\frac{40}{1}$	39.965	8.3982	9.9999	8.3983	11.6017	88.34
1.30	0262	$\frac{1}{38}$	$\frac{38}{1}$	38.188	8.4179	9.9999	8.4181	11.5819	88.30
1.47	0311			32.118	8.4930	9.9998	8.4933	11.5067	88.13
1.48	0314	$\frac{1}{32}$	$\frac{32}{1}$	31.821	8.4971	9.9998	8.4973	11.5027	88.12
1.55	0335	$\frac{1}{30}$	$\frac{30}{1}$	29.882	8.5243	9.9998	8.5246	11.4754	88.05
2.00	0349			28.636	8.5428	9.9997	8.5431	11.4569	88.00
2.07	0370	$\frac{1}{27}$	$\frac{27}{1}$	27.057	8.5674	9.9997	8.5677	11.4323	87.53
2.17	0399	$\frac{1}{25}$	$\frac{25}{1}$	25.080	8.6003	9.9997	8.6007	11.3993	87.43
2.23	0416	$\frac{1}{24}$	$\frac{24}{1}$	24.026	8.6189	9.9996	8.6193	11.3807	87.37
2.52	0501	$\frac{1}{20}$	$\frac{20}{1}$	19.970	8.6991	9.9995	8.6996	11.3004	87.08
3.00	0524			19.081	8.7188	9.9994	8.7194	11.2806	87.00
3.01	0527	$\frac{1}{19}$	$\frac{19}{1}$	18.976	8.7212	9.9994	8.7218	11.2782	86.59
3.11	0556	$\frac{1}{18}$	$\frac{18}{1}$	17.980	8.7445	9.9993	8.7452	11.2548	86.49
3.22	0588	$\frac{1}{17}$	$\frac{17}{1}$	16.999	8.7688	9.9992	8.7696	11.2304	86.38
3.35	0626	$\frac{1}{16}$	$\frac{16}{1}$	15.969	8.7959	9.9992	8.7967	11.2033	86.25
3.49	0667	$\frac{1}{15}$	$\frac{15}{1}$	14.990	8.8232	9.9990	8.8242	11.1758	86.11
3.57	0690	$\frac{2}{29}$	$\frac{29}{2}$	14.482	8.8381	9.9990	8.8392	11.1608	86.03
4.00	0699			14.301	8.8436	9.9989	8.8446	11.1554	86.00
4.05	0714	$\frac{1}{14}$	$\frac{14}{1}$	14.008	8.8525	9.9989	8.8536	11.1464	85.55
4.14	0740	$\frac{2}{27}$	$\frac{27}{2}$	13.510	8.8682	9.9988	8.8694	11.1306	85.46
4.24	0769	$\frac{1}{13}$	$\frac{13}{1}$	12.996	8.8849	9.9987	8.8862	11.1138	85.36
4.34	0799	$\frac{2}{25}$	$\frac{25}{2}$	12.520	8.9010	9.9986	8.9024	11.0976	85.26
4.46	0834	$\frac{1}{12}$	$\frac{12}{1}$	11.992	8.9196	9.9985	8.9211	11.0789	85.14
4.54	0857	$\frac{3}{35}$	$\frac{35}{3}$	11.664	8.9315	9.9984	8.9331	11.0669	85.06
4.58	0869	$\frac{2}{23}$	$\frac{23}{2}$	11.507	8.9374	9.9984	8.9390	11.0610	85.02
5.00	0875			11.430	8.9403	9.9983	8.9420	11.0580	85.00
5.12	0910	$\frac{1}{11}$	$\frac{11}{1}$	10.988	8.9573	9.9982	8.9591	11.0409	84.48
5.21	0936	$\frac{3}{32}$	$\frac{32}{3}$	10.678	8.9696	9.9981	8.9715	11.0285	84.39
5.26	0951	$\frac{2}{21}$	$\frac{21}{2}$	10.514	8.9763	9.9980	8.9782	11.0218	84.34
5.32	0969	$\frac{3}{31}$	$\frac{31}{3}$	10.322	8.9842	9.9980	8.9862	11.0138	84.28
5.42	0998	$\frac{1}{10}$	$\frac{10}{1}$	10.019	8.9970	9.9978	8.9992	11.0008	84.18
5.54	1033	$\frac{3}{29}$	$\frac{29}{3}$	9.6768	9.0120	9.9977	9.0143	10.9857	84.06
6.00	1051			9.5144	9.0192	9.9976	9.0216	10.9784	84.00
6.01	1054	$\frac{2}{19}$	$\frac{19}{2}$	9.4878	9.0204	9.9976	9.0228	10.9772	83.59
6.20	1110	$\frac{1}{9}$	$\frac{9}{1}$	9.0098	9.0426	9.9973	9.0453	10.9547	83.40
6.31	1142	$\frac{4}{35}$	$\frac{35}{4}$	8.7542	9.0550	9.9972	9.0578	10.9422	83.29
6.43	1178	$\frac{3}{17}$	$\frac{17}{3}$	8.4913	9.0680	9.9970	9.0710	10.9290	83.17
6.51	1201	$\frac{3}{25}$	$\frac{25}{3}$	8.3245	9.0765	9.9969	9.0796	10.9204	83.09
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Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles.	N. tan.	T	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
7.00	1228			8.1443	9.0859	9.9968	9.0891	10.9109	83.00
7.07	1249	[illegible]	[illegible]	8.0095	9.0930	9.9966	9.0964	10.9036	82.53
7.21	1290	[illegible]	[illegible]	7.7525	9.1070	9.9964	9.1106	10.8894	82.39
7.26	1305	[illegible]	[illegible]	7.6647	9.1118	9.9963	9.1155	10.8845	82.34
7.36	1334	[illegible]	[illegible]	7.4947	9.1214	9.9962	9.1252	10.8748	82.24
7.46	1363	[illegible]	[illegible]	7.3319	9.1308	9.9960	9.1348	10.8652	82.14
8.00	1405			7.1154	9.1436	9.9958	9.1478	10.8522	82.00
8.03	1414			7.0706	9.1462	9.9957	9.1505	10.8495	81.57
8.08	1429	[illegible]	[illegible]	6.9972	9.1507	9.9956	9.1551	10.8449	81.52
8.18	1459	[illegible]	[illegible]	6.8548	9.1594	9.9954	9.1640	10.8360	81.42
8.26	1483	[illegible]	[illegible]	6.7448	9.1663	9.9953	9.1710	10.8290	81.34
8.32	1500	[illegible]	[illegible]	6.6646	9.1714	9.9952	9.1762	10.8238	81.28
8.45	1539	[illegible]	[illegible]	6.4971	9.1822	9.9949	9.1873	10.8127	81.15
8.58	1578	[illegible]	[illegible]	6.3376	9.1927	9.9947	9.1981	10.8019	81.02
9.00	1584			6.3138	9.1943	9.9946	9.1997	10.8003	81.00
9.06	1602	[illegible]	[illegible]	6.2432	9.1991	9.9945	9.2046	10.7954	80.54
9.13	1623	[illegible]	[illegible]	6.1628	9.2046	9.9944	9.2102	10.7898	80.47
9.28	1667	[illegible]	[illegible]	5.9972	9.2161	9.9940	9.2221	10.7779	80.32
9.41	1706	[illegible]	[illegible]	5.8605	9.2258	9.9938	9.2321	10.7679	80.19
9.52	1739	[illegible]	[illegible]	5.7495	9.2339	9.9935	9.2404	10.7596	80.08
10.00	1763	[illegible]	[illegible]	5.6713	9.2397	9.9934	9.2463	10.7537	80.00
10.01	1766			5.6617	9.2404	9.9933	9.2471	10.7529	79.59
10.02	1769			5.6521	9.2411	9.9933	9.2478	10.7522	79.58
10.06	1781			5.6140	9.2439	9.9932	9.2507	10.7493	79.54
10.12	1799	[illegible]	[illegible]	5.5578	9.2482	9.9931	9.2551	10.7449	79.48
10.18	1817	[illegible]	[illegible]	5.5026	9.2524	9.9929	9.2594	10.7406	79.42
10.26	1841	[illegible]	[illegible]	5.4308	9.2579	9.9928	9.2651	10.7349	79.34
10.36	1871			5.3435	9.2647	9.9925	9.2722	10.7278	79.24
10.37	1874	[illegible]	[illegible]	5.3349	9.2654	9.9925	9.2729	10.7271	79.23
10.47	1905	[illegible]	[illegible]	5.2505	9.2721	9.9923	9.2798	10.7202	79.13
10.53	1923	[illegible]	[illegible]	5.2011	9.2760	9.9921	9.2839	10.7161	79.07
11.00	1944	[illegible]	[illegible]	5.1446	9.2806	9.9919	9.2887	10.7113	79.00
11.10	1974			5.0658	9.2870	9.9917	9.2953	10.7047	78.50
11.18	1998			5.0045	9.2921	9.9915	9.3006	10.6994	78.42
11.19	2001	[illegible]	[illegible]	4.9969	9.2928	9.9915	9.3013	10.6987	78.41
11.32	2041			4.9006	9.3009	9.9911	9.3098	10.6902	78.28
11.34	2047	[illegible]	[illegible]	4.8860	9.3021	9.9911	9.3110	10.6890	78.26
11.36	2053	[illegible]	[illegible]	4.8716	9.3034	9.9910	9.3123	10.6877	78.24
11.46	2083	[illegible]	[illegible]	4.8007	9.3095	9.9908	9.3187	10.6813	78.14
11.53	2104	[illegible]	[illegible]	4.7522	9.3137	9.9906	9.3231	10.6769	78.07
12.00	2126			4.7046	9.3179	9.9904	9.3275	10.6725	78.00
12.06	2144	[illegible]	[illegible]	4.6646	9.3214	9.9902	9.3312	10.6688	77.54
12.16	2174	[illegible]	[illegible]	4.5993	9.3273	9.9900	9.3373	10.6627	77.44
12.23	2196	[illegible]	[illegible]	4.5546	9.3313	9.9898	9.3416	10.6584	77.37
12.32	2223	[illegible]	[illegible]	4.4983	9.3365	9.9895	9.3469	10.6531	77.28
12.36	2235			4.4737	9.3387	9.9894	9.3493	10.6507	77.24
12.41	2251	[illegible]	[illegible]	4.4434	9.3416	9.9893	9.3523	10.6477	77.19
12.48	2272	[illegible]	[illegible]	4.4015	9.3455	9.9891	9.3564	10.6436	77.12
13.00	2309	[illegible]	[illegible]	4.3315	9.3521	9.9887	9.3634	10.6366	77.00
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles.	N. tan.	T.	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
13.08	2333	$\frac{7}{30}$	$\frac{30}{7}$	4.2859	9.3564	9.9885	9.3680	10.6320	76.52
13.14	2352	$\frac{4}{17}$	$\frac{17}{4}$	4.2524	9.3597	9.9883	9.3714	10.6286	76.46
13.24	2382	$\frac{5}{21}$	$\frac{21}{5}$	4.1976	9.3650	9.9880	9.3770	10.6230	76.36
13.27	2392			4.1814	9.3666	9.9879	9.3787	10.6213	76.33
13.28	2395			4.1760	9.3671	9.9879	9.3792	10.6208	76.32
13.30	2401	$\frac{6}{25}$	$\frac{25}{6}$	4.1653	9.3682	9.9878	9.3804	10.6196	76.30
13.38	2425			4.1230	9.3724	9.9876	9.3848	10.6152	76.22
13.43	2441	$\frac{10}{41}$	$\frac{41}{10}$	4.0970	9.3750	9.9874	9.3875	10.6125	76.17
13.53	2472			4.0459	9.3801	9.9871	9.3930	10.6070	76.07
13.54	2475			4.0408	9.3806	9.9871	9.3935	10.6065	76.06
14.00	2493			4.0108	9.3837	9.9869	9.3968	10.6032	76.00
14.01	2496			4.0058	9.3842	9.9869	9.3973	10.6027	75.59
14.02	2499	$\frac{1}{4}$	$\frac{4}{1}$	4.0009	9.3847	9.9868	9.3978	10.6022	75.58
14.17	2546			3.9279	9.3922	9.9864	9.4058	10.5942	75.43
14.23	2564	$\frac{10}{39}$	$\frac{39}{10}$	3.8995	9.3952	9.9862	9.4090	10.5910	75.37
14.30	2586			3.8667	9.3986	9.9859	9.4127	10.5873	75.30
14.37	2608	$\frac{6}{23}$	$\frac{23}{6}$	3.8345	9.4020	9.9857	9.4163	10.5837	75.23
14.45	2633	$\frac{5}{19}$	$\frac{19}{5}$	3.7983	9.4059	9.9854	9.4204	10.5796	75.15
14.56	2667	$\frac{4}{15}$	$\frac{15}{4}$	3.7495	9.4111	9.9851	9.4260	10.5740	75.04
15.00	2679			3.7321	9.4130	9.9849	9.4281	10.5719	75.00
15.01	2683	$\frac{11}{41}$	$\frac{41}{11}$	3.7277	9.4135	9.9849	9.4286	10.5714	74.59
15.15	2726	$\frac{3}{11}$	$\frac{11}{3}$	3.6680	9.4200	9.9844	9.4356	10.5644	74.45
15.30	2773			3.6059	9.4269	9.9839	9.4430	10.5570	74.30
15.31	2776			3.6018	9.4274	9.9839	9.4435	10.5565	74.29
15.32	2780	$\frac{5}{18}$	$\frac{18}{5}$	3.5978	9.4278	9.9838	9.4440	10.5560	74.28
15.39	2801	$\frac{7}{25}$	$\frac{25}{7}$	3.5696	9.4310	9.9836	9.4474	10.5526	74.21
15.47	2827			3.5379	9.4346	9.9833	9.4513	10.5487	74.13
15.48	2830			3.5339	9.4350	9.9833	9.4517	10.5483	74.12
15.57	2858	$\frac{2}{7}$	$\frac{7}{2}$	3.4989	9.4390	9.9830	9.4561	10.5439	74.03
16.00	2867			3.4874	9.4403	9.9828	9.4575	10.5425	74.00
16.09	2896	$\frac{11}{38}$	$\frac{38}{11}$	3.4533	9.4443	9.9825	9.4618	10.5382	73.51
16.16	2918	$\frac{7}{24}$	$\frac{24}{7}$	3.4271	9.4473	9.9823	9.4651	10.5349	73.44
16.23	2940	$\frac{5}{17}$	$\frac{17}{5}$	3.4014	9.4503	9.9820	9.4683	10.5317	73.37
16.30	2962	$\frac{8}{27}$	$\frac{27}{8}$	3.3759	9.4533	9.9817	9.4716	10.5284	73.30
16.42	3000	$\frac{3}{10}$	$\frac{10}{3}$	3.3332	9.4584	9.9813	9.4771	10.5229	73.18
16.46	3013			3.3191	9.4601	9.9811	9.4790	10.5210	73.14
16.47	3016			3.3156	9.4605	9.9811	9.4794	10.5206	73.13
16.56	3045	$\frac{7}{23}$	$\frac{23}{7}$	3.2845	9.4643	9.9808	9.4835	10.5165	73.04
17.00	3057			3.2709	9.4659	9.9806	9.4853	10.5147	73.00
17.06	3076	$\frac{4}{13}$	$\frac{13}{4}$	3.2506	9.4684	9.9804	9.4880	10.5120	72.54
17.10	3089			3.2371	9.4700	9.9802	9.4898	10.5102	72.50
17.21	3124	$\frac{5}{16}$	$\frac{16}{5}$	3.2008	9.4745	9.9798	9.4947	10.5053	72.39
17.32	3159	$\frac{6}{19}$	$\frac{19}{6}$	3.1652	9.4789	9.9793	9.4996	10.5004	72.28
17.39	3182	$\frac{7}{22}$	$\frac{22}{7}$	3.1429	9.4817	9.9791	9.5027	10.4973	72.21
17.45	3201	$\frac{8}{25}$	$\frac{25}{8}$	3.1240	9.4841	9.9788	9.5053	10.4947	72.15
17.53	3227	$\frac{10}{31}$	$\frac{31}{10}$	3.0991	9.4873	9.9785	9.5088	10.4912	72.07
17.56	3236			3.0899	9.4884	9.9784	9.5101	10.4899	72.04
18.00	3249	$\frac{13}{40}$	$\frac{40}{13}$	3.0777	9.4900	9.9782	9.5118	10.4882	72.00
18.12	3288			3.0415	9.4946	9.9777	9.5169	10.4831	71.48
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles.	N. tan.	T.	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles
18.16	3301			3.0296	9.4962	9.9775	9.5186	10.4814	71.44
18.18	3307			3.0237	9.4969	9.9775	9.5195	10.4805	71.42
18.26	3333	$\frac{1}{3}$	$\frac{3}{1}$	3.0003	9.5000	9.9771	9.5228	10.4772	71.34
18.41	3382			2.9572	9.5056	9.9765	9.5291	10.4709	71.19
18.56	3430			2.9152	9.5112	9.9758	9.5353	10.4647	71.04
18.58	3437	$\frac{11}{32}$	$\frac{32}{11}$	2.9097	9.5119	9.9758	9.5362	10.4638	71.02
19.00	3443			2.9042	9.5126	9.9757	9.5370	10.4630	71.00
19.01	3447	$\frac{10}{29}$	$\frac{29}{10}$	2.9015	9.5130	9.9756	9.5374	10.4626	70.59
19.02	3450			2.8987	9.5134	9.9756	9.5378	10.4622	70.58
19.06	3463			2.8878	9.5148	9.9754	9.5394	10.4606	70.54
19.07	3466			2.8851	9.5152	9.9754	9.5398	10.4602	70.53
19.11	3479	$\frac{8}{23}$	$\frac{23}{8}$	2.8743	9.5167	9.9752	9.5415	10.4585	70.49
19.17	3499	$\frac{7}{20}$	$\frac{20}{7}$	2.8582	9.5188	9.9749	9.5439	10.4561	70.43
19.26	3528	$\frac{6}{17}$	$\frac{17}{6}$	2.8344	9.5221	9.9745	9.5475	10.4525	70.34
19.28	3535	$\frac{29}{82}$	$\frac{82}{29}$	2.8291	9.5228	9.9744	9.5483	10.4517	70.32
19.29	3538			2.8265	9.5231	9.9744	9.5487	10.4513	70.31
19.32	3548	$\frac{11}{31}$	$\frac{31}{11}$	2.8187	9.5242	9.9743	9.5500	10.4500	70.28
19.39	3571	$\frac{5}{14}$	$\frac{14}{5}$	2.8006	9.5267	9.9739	9.5528	10.4472	70.21
19.48	3600	$\frac{9}{25}$	$\frac{25}{9}$	2.7776	9.5299	9.9735	9.5563	10.4437	70.12
19.59	3636	$\frac{4}{11}$	$\frac{11}{4}$	2.7500	9.5337	9.9730	9.5607	10.4393	70.01
20.00	3640			2.7475	9.5341	9.9730	9.5611	10.4389	70.00
20.08	3666	$\frac{11}{30}$	$\frac{30}{11}$	2.7277	9.5368	9.9726	9.5642	10.4358	69.52
20.13	3683	$\frac{7}{19}$	$\frac{19}{7}$	2.7155	9.5385	9.9724	9.5662	10.4338	69.47
20.19	3702	$\frac{10}{27}$	$\frac{27}{10}$	2.7009	9.5406	9.9721	9.5685	10.4315	69.41
20.30	3739			2.6746	9.5443	9.9716	9.5727	10.4273	69.30
20.33	3749	$\frac{3}{8}$	$\frac{8}{3}$	2.6675	9.5453	9.9714	9.5739	10.4261	69.27
20.51	3809	$\frac{8}{21}$	$\frac{21}{8}$	2.6256	9.5514	9.9706	9.5808	10.4192	69.09
20.56	3825			2.6142	9.5530	9.9703	9.5827	10.4173	69.04
20.57	3829			2.6119	9.5533	9.9703	9.5830	10.4170	69.03
21.00	3839			2.6051	9.5543	9.9702	9.5842	10.4158	69.00
21.02	3845	$\frac{5}{13}$	$\frac{13}{5}$	2.6006	9.5550	9.9701	9.5849	10.4151	68.58
21.10	3872			2.5826	9.5576	9.9697	9.5879	10.4121	68.50
21.15	3889	$\frac{7}{18}$	$\frac{18}{7}$	2.5715	9.5592	9.9694	9.5898	10.4102	68.45
21.20	3906			2.5605	9.5609	9.9692	9.5917	10.4083	68.40
21.22	3912	$\frac{9}{23}$	$\frac{23}{9}$	2.5561	9.5615	9.9691	9.5924	10.4076	68.38
21.30	3939			2.5386	9.5641	9.9687	9.5954	10.4046	68.30
21.36	3959			2.5257	9.5660	9.9684	9.5976	10.4024	68.24
21.37	3963			2.5236	9.5663	9.9683	9.5980	10.4020	68.23
21.46	3993			2.5044	9.5692	9.9679	9.6013	10.3987	68.14
21.47	3996			2.5023	9.5695	9.9678	9.6017	10.3983	68.13
21.48	4000	$\frac{2}{5}$	$\frac{5}{2}$	2.5002	9.5698	9.9678	9.6020	10.3980	68.12
22.00	4040			2.4751	9.5736	9.9672	9.6064	10.3936	68.00
22.01	4044			2.4730	9.5739	9.9671	9.6068	10.3932	67.59
22.10	4074	$\frac{11}{27}$	$\frac{27}{11}$	2.4545	9.5767	9.9667	9.6100	10.3900	67.50
22.13	4084			2.4484	9.5776	9.9665	9.6111	10.3889	67.47
22.15	4091	$\frac{9}{22}$	$\frac{22}{9}$	2.4443	9.5782	9.9664	9.6118	10.3882	67.45
22.23	4118	$\frac{7}{17}$	$\frac{17}{7}$	2.4282	9.5807	9.9660	9.6147	10.3853	67.37
22.30	4142			2.4142	9.5828	9.9656	9.6172	10.3828	67.30
22.37	4166	$\frac{5}{12}$	$\frac{12}{5}$	2.4004	9.5850	9.9652	9.6197	10.3803	67.23
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles	N. tan.	T.	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
22.45	4193			2.3847	9.5874	9.9648	9.6226	10.3774	67.15
22.50	4210	$\frac{8}{19}$	$\frac{19}{8}$	2.3750	9.5889	9.9646	9.6243	10.3757	67.10
22.56	4231	$\frac{11}{26}$	$\frac{26}{11}$	2.3635	9.5907	9.9642	9.6264	10.3736	67.04
23.00	4245			2.3559	9.5919	9.9640	9.6279	10.3721	67.00
23.04	4258			2.3483	9.5931	9.9638	9.6293	10.3707	66.56
23.05	4262			2.3464	9.5934	9.9638	9.6296	10.3704	66.55
23.08	4272			2.3407	9.5943	9.9636	9.6307	10.3693	66.52
23.12	4286	$\frac{3}{7}$	$\frac{7}{3}$	2.3332	9.5954	9.9634	9.6321	10.3679	66.48
23.30	4348	$\frac{10}{23}$	$\frac{23}{10}$	2.2998	9.6007	9.9624	9.6383	10.3617	66.30
23.35	4365			2.2907	9.6021	9.9621	9.6400	10.3600	66.25
23.38	4376	$\frac{7}{16}$	$\frac{16}{7}$	2.2853	9.6030	9.9620	9.6411	10.3589	66.22
23.45	4400	$\frac{11}{25}$	$\frac{25}{11}$	2.2727	9.6050	9.9616	9.6435	10.3565	66.15
23.58	4445	$\frac{4}{9}$	$\frac{9}{4}$	2.2496	9.6087	9.9608	9.6479	10.3521	66.02
24.00	4452			2.2460	9.6093	9.9607	9.6486	10.3514	66.00
24.03	4463			2.2408	9.6102	9.9606	9.6496	10.3504	65.57
24.04	4466			2.2390	9.6104	9.9605	9.6499	10.3501	65.56
24.05	4470			2.2373	9.6107	9.9604	9.6503	10.3497	65.55
24.06	4473			2.2355	9.6110	9.9604	9.6506	10.3494	65.54
24.14	4501	$\frac{9}{20}$	$\frac{20}{9}$	2.2216	9.6133	9.9599	9.6533	10.3467	65.46
24.27	4547	$\frac{5}{11}$	$\frac{11}{5}$	2.1994	9.6169	9.9592	9.6577	10.3423	65.33
24.30	4557			2.1943	9.6177	9.9590	9.6587	10.3413	65.30
24.37	4582	$\frac{11}{24}$	$\frac{24}{11}$	2.1825	9.6197	9.9586	9.6610	10.3390	65.23
24.42	4599	$\frac{23}{50}$	$\frac{50}{23}$	2.1742	9.6210	9.9583	9.6627	10.3373	65.18
24.46	4614	$\frac{6}{13}$	$\frac{13}{6}$	2.1675	9.6221	9.9581	9.6640	10.3360	65.14
25.00	4663			2.1445	9.6259	9.9573	9.6687	10.3313	65.00
25.01	4667	$\frac{7}{15}$	$\frac{15}{7}$	2.1429	9.6262	9.9572	9.6690	10.3310	64.59
25.12	4706	$\frac{8}{17}$	$\frac{17}{8}$	2.1251	9.6292	9.9566	9.6726	10.3274	64.48
25.14	4713			2.1219	9.6297	9.9564	9.6733	10.3267	64.46
25.15	4716			2.1203	9.6300	9.9564	9.6736	10.3264	64.45
25.21	4738	$\frac{9}{19}$	$\frac{19}{9}$	2.1107	9.6316	9.9560	9.6756	10.3244	64.39
25.28	4763	$\frac{10}{21}$	$\frac{21}{10}$	2.0997	9.6335	9.9556	9.6778	10.3222	64.32
25.34	4784	$\frac{11}{23}$	$\frac{23}{11}$	2.0903	9.6350	9.9552	9.6798	10.3202	64.26
25.39	4802	$\frac{12}{25}$	$\frac{25}{12}$	2.0825	9.6364	9.9549	9.6814	10.3186	64.21
25.47	4831	$\frac{14}{29}$	$\frac{29}{14}$	2.0701	9.6385	9.9545	9.6840	10.3160	64.13
25.48	4834			2.0686	9.6387	9.9544	9.6843	10.3157	64.12
26.00	4877			2.0503	9.6418	9.9537	9.6882	10.3118	64.00
26.11	4917			2.0338	9.6447	9.9530	9.6917	10.3083	63.49
26.20	4950			2.0204	9.6470	9.9524	9.6946	10.3054	63.40
26.30	4986			2.0057	9.6495	9.9518	9.6977	10.3023	63.30
26.34	5000	$\frac{1}{2}$	$\frac{2}{1}$	1.9999	9.6505	9.9515	9 6990	10.3010	63.26
26.40	5022			1.9912	9.6521	9.9512	9.7009	10.2991	63.20
26.50	5059			1.9768	9.6546	9.9505	9.7040	10.2960	63.10
27.00	5095			1.9626	9.6570	9.9499	9.7072	10.2928	63.00
27.13	5143			1.9444	9.6603	9.9490	9.7112	10.2888	62.47
27.21	5172	$\frac{29}{56}$	$\frac{56}{29}$	1.9333	9.6622	9.9485	9.7137	10.2863	62.39
27.22	5176			1.9319	9.6625	9.9485	9.7140	10.2860	62.38
27.28	5198	$\frac{13}{25}$	$\frac{25}{13}$	1.9237	9.6639	9.9481	9.7159	10.2841	62.32
27.30	5206			1.9210	9.6644	9.9479	9.7165	10.2835	62.30
27.31	5209			1.9196	9.6646	9.9479	9.7168	10.2832	62.29
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles.	N. tan.	T.	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
27.33	5217	12/23	23/12	1.9169	9.6651	9.9477	9.7174	10.2826	62.27
27.39	5239	11/21	21/11	1.9088	9.6666	9.9473	9.7192	10.2808	62.21
27.41	5246			1.9061	9.6671	9.9472	9.7199	10.2801	62.19
27.42	5250			1.9047	9.6673	9.9471	9.7202	10.2798	62.18
27.43	5254			1.9034	9.6675	9.9471	9.7205	10.2795	62.17
27.46	5265	10/19	19/10	1.8993	9.6683	9.9469	9.7214	10.2786	62.14
27.54	5295	9/17	17/9	1.8887	9.6702	9.9463	9.7238	10.2762	62.06
28.00	5317			1.8807	9.6716	9.9459	9.7257	10.2743	62.00
28.04	5332	8/15	15/8	1.8755	9.6726	9.9457	9.7269	10.2731	61.56
28.18	5384	7/13	13/7	1.8572	9.6759	9.9447	9.7311	10.2689	61.42
28.20	5392			1.8546	9.6763	9.9446	9.7317	10.2683	61.40
28.21	5396			1.8533	9.6766	9.9445	9.7320	10.2680	61.39
28.27	5418	13/24	24/13	1.8456	9.6780	9.9441	9.7339	10.2661	61.33
28.35	5448			1.8354	9.6798	9.9436	9.7363	10.2637	61.25
28.37	5456	6/11	11/6	1.8329	9.6803	9.9434	9.7369	10.2631	61.23
28.38	5460			1.8316	9.6805	9.9433	9.7372	10.2628	61.22
28.39	5464			1.8303	9.6808	9.9433	9.7375	10.2625	61.21
28.49	5501	11/20	20/11	1.8177	9.6831	9.9426	9.7405	10.2595	61.11
29.00	5543			1.8040	9.6856	9.9418	9.7438	10.2562	61.00
29.03	5555	5/9	9/5	1.8003	9.6863	9.9416	9.7446	10.2554	60.57
29.12	5589			1.7893	9.6883	9.9410	9.7473	10.2527	60.48
29.15	5600	14/25	25/14	1.7856	9.6890	9.9408	9.7482	10.2518	60.45
29.22	5627	9/16	16/9	1.7771	9.6905	9.9403	9.7503	10.2497	60.38
29.29	5654	13/23	23/13	1.7687	9.6921	9.9398	9.7523	10.2477	60.31
29.30	5658			1.7675	9.6923	9.9397	9.7526	10.2474	60.30
29.40	5696			1.7556	9.6946	9.9390	9.7556	10.2444	60.20
29.45	5715	4/7	7/4	1.7496	9.6957	9.9386	9.7571	10.2429	60.15
29.59	5770	15/26	26/15	1.7332	9.6988	9.9376	9.7611	10.2389	60.01
30.00	5774			1.7321	9.6990	9.9375	9.7614	10.2386	60.00
30.04	5789	11/19	19/11	1.7274	9.6998	9.9372	9.7626	10.2374	59.56
30.10	5812			1.7205	9.7012	9.9368	9.7644	10.2356	59.50
30.15	5832	7/12	12/7	1.7147	9.7022	9.9364	9.7658	10.2342	59.45
30.28	5883	10/17	17/10	1.6999	9.7050	9.9355	9.7696	10.2304	59.32
30.35	5910	13/22	22/13	1.6920	9.7065	9.9349	9.7716	10.2284	59.25
30.41	5934			1.6853	9.7078	9.9345	9.7733	10.2267	59.19
30.50	5969			1.6753	9.7097	9.9338	9.7759	10.2241	59.10
30.58	6001	3/5	5/3	1.6665	9.7114	9.9332	9.7782	10.2218	59.02
30.59	6005			1.6654	9.7116	9.9331	9.7785	10.2215	59.01
31.00	6009			1.6643	9.7118	9.9331	9.7788	10.2212	59.00
31.10	6048			1.6534	9.7139	9.9323	9.7816	10.2184	58.50
31.20	6088	14/23	23/14	1.6426	9.7160	9.9315	9.7845	10.2155	58.40
31.23	6100			1.6393	9.7166	9.9313	9.7853	10.2147	58.37
31.24	6104			1.6383	9.7168	9.9312	9.7856	10.2144	58.36
31.26	6112	11/18	18/11	1.6361	9.7173	9.9311	9.7862	10.2138	58.34
31.29	6124			1.6329	9.7179	9.9308	9.7870	10.2130	58.31
31.35	6148			1.6265	9.7191	9.9304	9.7887	10.2113	58.25
31.36	6152	8/13	13/8	1.6255	9.7193	9.9303	9.7890	10.2110	58.24
31.45	6188	13/21	21/13	1.6160	9.7212	9.9296	9.7916	10.2084	58.15
31.52	6216			1.6087	9.7226	9.9291	9.7935	10.2065	58.08
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles.	N. tan.	T.	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
31.57	6237			1.6034	9.7236	9.9287	9.7949	10.2051	58.03
32.00	6249	5/8	8/5	1.6003	9.7242	9.9284	9.7958	10.2042	58.00
32.01	6253			1.5993	9.7244	9.9283	9.7961	10.2039	57.59
32.10	6289			1.5900	9.7262	9.9276	9.7986	10.2014	57.50
32.17	6318	12/19	19/12	1.5829	9.7276	9.9271	9.8006	10.1994	57.43
32.18	6322			1.5818	9.7278	9.9270	9.8008	10.1992	57.42
32.19	6326			1.5808	9.7280	9.9269	9.8011	10.1989	57.41
32.28	6363	7/11	11/7	1.5717	9.7298	9.9262	9.8036	10.1964	57.32
32.37	6399	16/25	25/16	1.5627	9.7316	9.9255	9.8061	10.1939	57.23
32.38	6403			1.5617	9.7318	9.9254	9.8064	10.1936	57.22
32.39	6408			1.5607	9.7320	9.9253	9.8067	10.1933	57.21
32.44	6428	9/14	14/9	1.5557	9.7330	9.9249	9.8081	10.1919	57.16
32.50	6453			1.5497	9.7342	9.9244	9.8097	10.1903	57.10
32.53	6465			1.5468	9.7347	9.9242	9.8106	10.1894	57.07
32.55	6473	11/17	17/11	1.5448	9.7351	9.9240	9.8111	10.1889	57.05
32.59	6490			1.5408	9.7359	9.9237	9.8122	10.1878	57.01
33.00	6494			1.5399	9.7361	9.9236	9.8125	10.1875	57.00
33.01	6498	13/20	20/13	1.5389	9.7363	9.9235	9.8128	10.1872	56.59
33.07	6523	15/23	23/15	1.5330	9.7375	9.9230	9.8145	10.1855	56.53
33.12	6544			1.5282	9.7384	9.9226	9.8158	10.1842	56.48
33.13	6548			1.5272	9.7386	9.9225	9.8161	10.1839	56.47
33.24	6594			1.5166	9.7407	9.9216	9.8191	10.1809	56.36
33.34	6636			1.5070	9.7427	9.9208	9.8219	10.1781	56.26
33.41	6665	2/3	3/2	1.5004	9.7440	9.9202	9.8238	10.1762	56.19
33.42	6669			1.4994	9.7442	9.9201	9.8241	10.1759	56.18
33.50	6703			1.4919	9.7457	9.9194	9.8263	10.1737	56.10
33.59	6741			1.4835	9.7474	9.9187	9.8287	10.1713	56.01
34.00	6745			1.4826	9.7476	9.9186	9.8290	10.1710	56.00
34.13	6800	17/25	25/17	1.4705	9.7500	9.9175	9.8325	10.1675	55.47
34.17	6817	15/22	22/15	1.4669	9.7507	9.9171	9.8336	10.1664	55.43
34.22	6839			1.4623	9.7517	9.9167	9.8350	10.1650	55.38
34.23	6843	13/19	19/13	1.4614	9.7518	9.9166	9.8352	10.1648	55.37
34.31	6877	11/16	16/11	1.4541	9.7533	9.9159	9.8374	10.1626	55.29
34.42	6924	9/13	13/9	1.4442	9.7553	9.9149	9.8404	10.1596	55.18
34.49	6954	16/23	23/16	1.4379	9.7566	9.9143	9.8423	10.1577	55.11
34.59	6998			1.4290	9.7584	9.9135	9.8450	10.1550	55.01
35.00	7002	7/10	10/7	1.4281	9.7586	9.9134	9.8452	10.1548	55.00
35.13	7059	12/17	17/12	1.4167	9.7609	9.9122	9.8487	10.1513	54.47
35.16	7072	29/41	41/29	1.4141	9.7615	9.9119	9.8495	10.1505	54.44
35.19	7085	17/24	24/17	1.4115	9.7620	9.9117	9.8503	10.1497	54.41
35.22	7098			1.4089	9.7625	9.9114	9.8511	10.1489	54.38
35.30	7133			1.4019	9.7640	9.9107	9.8533	10.1467	54.30
35.32	7142	5/7	7/5	1.4002	9.7643	9.9105	9.8538	10.1462	54.28
35.45	7199	18/25	25/18	1.3891	9.7666	9.9093	9.8573	10.1427	54.15
35.50	7221	13/18	18/13	1.3848	9.7675	9.9089	9.8586	10.1414	54.10
36.00	7265			1.3764	9.7692	9.9080	9.8613	10.1387	54.00
36.02	7274	8/11	11/8	1.3747	9.7696	9.9078	9.8618	10.1382	53.58
36.15	7332	11/15	15/11	1.3638	9.7718	9.9066	9.8652	10.1348	53.45
36.23	7368	14/19	19/14	1.3572	9.7732	9.9058	9.8674	10.1326	53.37
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles.	N. tan.	T.	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
36.24	7373			1.3564	9.7734	9.9057	9.8676	10.1324	53.36
36.28	7391	17/23	23/17	1.3531	9.7740	9.9054	9.8687	10.1313	53.32
36.42	7454			1.3416	9.7764	9.9041	9.8724	10.1276	53.18
36.43	7458			1.3408	9.7766	9.9040	9.8726	10.1274	53.17
36.44	7463			1.3400	9.7768	9.9039	9.8729	10.1271	[illegible]
36.52	7499	3/4	4/3	1.3335	9.7781	9.9031	9.8750	10.1250	53.08
36.53	7504			1.3327	9.7783	9.9030	9.8753	10.1247	53.07
37.00	7536			1.3270	9.7795	9.9023	9.8771	10.1229	53.00
37.14	7600	19/25	25/19	1.3159	9.7818	9.9010	[illegible]	10.1192	52.46
37.18	7618	16/21	21/16	1.3127	9.7825	9.9006	9.8818	10.1182	52.42
37.24	7646	13/17	17/13	1.3079	9.7835	[illegible]	[illegible]	10.1166	52.36
37.28	7664			1.3048	9.7841	9.8997	[illegible]	10.1155	52.32
37.34	7692	10/13	13/10	1.3001	9.7851	9.8991	9.8860	10.1140	52.26
37.38	7710			1.2970	9.7858	9.8987	9.8871	10.1129	[illegible]
37.42	7729	17/22	22/17	1.2938	9.7864	9.8983	[illegible]	10.1119	52.18
37.45	7743			1.2915	9.7869	9.8980	9.8889	10.1111	52.15
37.46	7747			1.2907	9.7671	9.8979	9.8892	10.1108	52.14
37.49	7761			1.2884	9.7876	9.8976	9.8899	10.1101	52.11
37.52	7775	7/9	9/7	1.2861	9.7880	9.8973	9.8907	10.1093	[illegible]
38.00	7813			1.2799	9.7893	9.8965	[illegible]	10.1072	[illegible]
38.03	7827	18/23	23/18	1.2776	9.7898	9.8962	9.8936	10.1064	51.57
38.09	7855	11/14	14/11	1.2731	9.7908	9.8956	9.8952	10.1048	51.51
38.18	7898	15/19	19/15	1.2662	9.7922	9.8947	9.8975	10.1025	51.42
38.22	7916	19/24	24/19	1.2632	9.7929	9.8943	9.8985	10.1015	51.38
38.30	7954			1.2572	9.7941	9.8935	9.9006	10.0994	51.30
38.39	7997			1.2504	9.7956	9.8926	9.9029	10.0971	51.21
38.40	8002	4/5	5/4	1.2497	9.7957	9.8925	9.9032	10.0968	51.20
38.50	8050			1.2423	9.7973	9.8915	9.9058	10.0942	51.10
38.59	8093	17/21	21/17	1.2356	9.7987	9.8906	9.9081	10.0919	51.01
39.00	8098			1.2349	9.7989	9.8905	9.9084	10.0916	51.00
39.02	8107			1.2334	9.7992	9.8903	9.9089	10.0911	50.58
39.06	8127	13/16	16/13	1.2305	9.7998	9.8899	9.9099	10.0901	50.54
39.14	8165			1.2247	9.8010	9.8891	9.9120	10.0880	50.46
39.18	8185	9/11	11/9	1.2218	9.8017	9.8887	9.9130	10.0870	50.42
39.28	8234	14/17	17/14	1.2145	9.8032	9.8876	9.9156	10.0844	50.32
39.34	8263	19/23	23/19	1.2102	9.8041	9.8870	9.9171	10.0829	50.26
39.48	8332	5/6	6/5	1.2002	9.8063	9.8855	9.9207	10.0793	50.12
39.58	8381			1.1932	9.8078	9.8845	9.9233	10.0767	50.02
40.00	8391			1.1918	9.8081	9.8843	9.9238	10.0762	[illegible]
40.02	8401	21/25	25/21	1.1903	9.8084	9.8840	9.9243	10.0757	49.58
40.06	8421	16/19	19/16	1.1875	9.8090	9.8836	9.9254	10.0746	49.54
40.14	8461	11/13	13/11	1.1819	9.8102	9.8828	9.9274	10.0726	49.46
40.22	8501	17/20	20/17	1.1764	9.8114	9.8819	9.9295	10.0705	49.38
40.26	8521			1.1736	9.8120	9.8815	9.9305	10.0695	49.34
40.36	8571	6/7	7/6	1.1667	9.8134	9.8804	9.9330	10.0670	49.24
40.49	8637	19/22	22/19	1.1578	9.8153	9.8790	9.9364	10.0636	49.11
40.55	8667	13/15	15/13	1.1538	9.8162	9.8783	9.9379	10.0621	49.05
41.00	8693			1.1504	9.8169	9.8778	9.9392	10.0608	49.00
41.01	8698	20/23	23/20	1.1497	9.8171	9.8777	9.9394	10.0605	48.59
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles.	N. tan.	T	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
41.11	8749	7/8	8/7	1.1430	9.8185	9.8766	9.9420	10.0580	48.49
41.21	[redacted]			1.1363	9.8200	9.8755	9.9445	10.0555	48.39
41.23	8811			1.1349	9.8203	9.8752	9.9450	10.0550	48.37
41.24	8816			1.1343	9.8204	9.8751	9.9453	10.0547	48.36
41.25	8821	15/17	17/15	1.1336	9.8205	9.8750	9.9455	10.0545	48.35
41.34	[redacted]			1.1276	9.8218	9.8740	9.9478	10.0522	48.26
41.38	8889	8/9	9/8	1.1250	9.8224	9.8736	9.9488	10.0512	48.22
41.48	8941	17/19	19/17	1.1184	9.8238	9.8724	9.9514	10.0486	48.12
41.49	8946			1.1178	9.8240	9.8723	9.9516	10.0484	48.11
41.59	8999	9/10	10/9	1.1113	9.8254	9.8712	9.9542	10.0458	48.01
42.00	9004			1.1106	9.8255	9.8711	9.9544	10.0456	48.00
42.08	9046	19/21	21/19	1.1054	9.8266	9.8702	9.9565	10.0435	47.52
42.10	9057			1.1041	9.8269	9.8699	9.9570	10.0430	47.50
42.16	9089	10/11	11/10	1.1003	9.8277	9.8692	9.9585	10.0415	47.44
42.17	9094			1.0996	9.8279	9.8691	9.9588	10.0412	47.43
42.24	9131	21/23	23/21	1.0951	9.8289	9.8683	9.9605	10.0395	47.36
42.29	9158			1.0919	9.8295	9.8677	9.9618	10.0382	47.31
42.31	9169	11/12	12/11	1.0907	9.8298	9.8675	9.9623	10.0377	47.29
42.36	9195			1.0875	9.8305	9.8669	9.9636	10.0364	47.24
42.37	9201	23/25	25/23	1.0869	9.8306	9.8668	9.9638	10.0362	47.23
42.42	9228	12/13	13/12	1.0837	9.8313	9.8662	9.9651	10.0349	47.18
42.49	9266			1.0793	9.8323	9.8654	9.9669	10.0331	47.11
42.53	[redacted]	13/14	14/13	1.0768	9.8328	9.8650	9.9679	10.0321	47.07
43.00	9325			1.0724	[redacted]	9.8641	9.9697	10.0303	47.00
43.01	9331	14/15	15/14	1.0717	9.8339	9.8640	9.9699	10.0301	46.59
43.09	9374	15/16	16/15	1.0668	9.8350	9.8631	9.9719	10.0281	46.51
43.10	9380			1.0661	9.8351	9.8629	9.9722	10.0278	46.50
43.14	9402	47/50	50/47	1.0637	9.8357	9.8625	9.9732	10.0268	46.46
43.16	9413	16/17	17/16	1.0624	9.8359	9.8622	9.9737	10.0263	46.44
43.18	9424			1.0612	9.8362	9.8620	9.9742	10.0258	46.42
43.19	9429			1.0606	9.8363	9.8619	9.9745	10.0255	46.41
43.22	9446	17/18	18/17	1.0587	9.8367	9.8615	9.9752	10.0248	46.38
43.27	9473	18/19	19/18	1.0556	9.8374	9.8609	9.9765	10.0235	46.33
43.32	9501	19/20	20/19	1.0526	9.8381	9.8603	9.9778	10.0222	46.28
43.36	9523	20/21	21/20	1.0501	9.8386	9.8598	9.9788	10.0212	46.24
43.40	9545	21/22	22/21	1.0477	9.8391	9.8594	9.9798	10.0202	46.20
43.44	9567	22/23	23/22	1.0452	9.8397	9.8589	9.9808	10.0192	46.16
43.47	9584	23/24	24/23	1.0434	9.8401	9.8585	9.9816	10.0184	46.13
43.50	9601	24/25	25/24	1.0416	9.8405	9.8582	9.9823	10.0177	46.10
43.51	9606			1.0410	9.8406	9.8580	9.9826	10.0174	46.09
43.53	9618	25/26	26/25	1.0398	9.8409	9.8578	9.9831	10.0169	46.07
43.54	9623			1.0392	9.8410	9.8577	9.9833	10.0167	46.06
43.55	9629	26/27	27/26	1.0385	9.8411	9.8575	9.9836	10.0164	46.05
43.57	9640	27/28	28/27	1.0373	9.8414	9.8573	9.9841	10.0159	46.03
44.00	9657	28/29	29/28	1.0355	9.8418	9.8569	9.9848	10.0152	46.00
44.01	9663	29/30	30/29	1.0349	9.8419	9.8568	9.9851	10.0149	45.59
44.02	9668			1.0343	9.8420	9.8567	9.9853	10.0147	45.58
44.04	9679	30/31	31/30	1.0331	9.8423	9.8564	9.9858	10.0142	45.56
44.11	9719	35/36	36/35	1.0289	9.8432	9.8556	9.9876	10.0124	45.49
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

Angles.	N. tan.	T.	C.	N. cot.	L. sin.	L. cos.	L. tan.	L. cot.	Angles.
44.18	9759	40/41	41/40	1.0247	9.8441	9.8547	9.9894	10.0106	45.42
44.22	9781	44/45	45/44	1.0224	9.8446	9.8542	9.9904	10.0096	45.38
44.25	9798	49/50	50/49	1.0206	9.8450	9.8539	9.9912	10.0088	45.35
44.30	9827			1.0176	9.8457	9.8532	9.9924	10.0076	45.30
44.35	9856			1.0147	9.8463	9.8526	9.9937	10.0063	45.25
44.40	9884			1.0117	9.8469	9.8520	9.9949	10.0051	45.20
44.43	9902			1.0099	9.8473	9.8516	9.9957	10.0043	45.17
44.50	9942			1.0058	9.8482	9.8507	9.9975	10.0025	45.10
44.55	9971			1.0029	9.8489	9.8501	9.9987	10.0013	45.05
44.56	9977			1.0023	9.8490	9.8500	9.9990	10.0010	45.04
44.57	9983			1.0017	9.8491	9.8499	9.9992	10.0008	45.03
44.58	9988			1.0012	9.8492	9.8497	9.9995	10.0005	45.02
44.59	9994			1.0006	9.8494	9.8496	9.9997	10.0003	45.01
45.00	1.00			1.0000	9.8495	9.8495	10.000	10.0000	45.00
° ′									° ′
Angles.	N. cot.	C.	T.	N. tan.	L. cos.	L. sin.	L. cot.	L. tan.	Angles.

PART II.

APPLICATION OF CRYSTALLOGRAPHY

TO

MINERALOGY.

PART II.

APPLICATION OF CRYSTALLOGRAPHY

TO

MINERALOGY.

SECTION I.—A TABULAR ARRANGEMENT OF MINERALS, ACCORDING TO SIX SYSTEMS OF CRYSTALLISATION.

THE following Table is copied from the "*Elemente der Krystallographie von* GUSTAV ROSE, Zweite Auflage, Berlin, 1838," in which he gives us this explanation of its composition: —

"In the following Table, minerals are classed in six divisions, according to the *systems of crystallisation* to which their forms belong; and in these divisions they are further arranged, according to differences in their chemical composition, into Classes, Orders, Genera, and Species.

The CLASS is determined by the number of chemical simple bodies which the Mineral contains. This principle gives rise to seven classes, namely:—

I. *Simple Bodies.*

II. *Binary Compounds*, or those which contain two simple bodies, as Specular Iron, and Chloride of Sodium.

III. *Double Binary Compounds*, or *Salts;* namely, substances that contain two binary compounds, as Carbonate of Lime, Spinel, and Red Silver.

IV. *Treble Binary Compounds*, or such as contain a double and a single binary compound, as Gypsum, and Apatite.

V. *Quadruple Binary Compounds*, or *Double Salts;* such, namely, as contain two binary compounds, as Felspar and Bournonite.

VI. *Quintuple Binary Compounds*, or such as consist of a quadruple and a single binary compound, as Analcime and Eudyalite.

VII. *Sextuple Binary Compounds*, or such as contain a quadruple and a binary compound, as Tourmaline and Helvine.

An Appendix, which contains the Crystallized Minerals that have either not been analysed, or only examined imperfectly, presents an *eighth* group.

The Isomorphous bodies, which replace one another, have not been looked upon as different substances, so that Argentiferous Gold has been put into the first class, and Cobaltic Arsenical Pyrites into the third class.

The ORDERS are constituted in reference to the Electro-negative elements of the minerals; but, for the sake of simplicity, several of the electro-negative elements have been grouped together, so as to make in all only Four Orders, which are:—

1) Amalgams, Osmiumides, Antimoniurets, Arseniurets, and Tellurets,
2) Sulphurets and Seleniurets.
3) Chlorides and Fluorides.
4) Oxidised Compounds.

These Orders naturally fail in the first Class, and, indeed, appear in full in no Class but the second.

Minerals which contain compounds of different Orders, are placed in the Order to which their electro-negative compounds belongs. Thus, Arsenical Pyrites is placed among the Sulphurets, and Apatite among the Oxidised Compounds.

The GENUS depends upon Isomorphism, the SPECIES upon chemical composition. Hence all minerals which are held to be isomorphous, are grouped into one genus, while differences in the chemical composition of the minerals belonging to the same Genus, mark the Species.

The Genera are distinguished in the Table by thick figures, the Species by slight figures. Where no word is attached to the thick figure, the blank indicates that no convenient name has been found for the Genus. The arrangement of the Genera is according to the number of the atoms of their electro-negative elements; those Genera which contain fewest electro-negative atoms being placed highest in the Table.

The Genera of Minerals with hemihedral forms have not been separated from those which contain only homohedral forms. They are, however, distinguished by stars, which are set against the thick figures. The minerals whose crystals present hemihedral forms with parallel faces, are distinguished by one star *, and those which present hemihedral forms with inclined faces, by two stars **.

The note of interrogation affixed to some minerals, intimates that their crystallisation is not sufficiently understood."—*Elemente der Krystallographie*, Berlin, 1838, pages 155 to 158, *abridged.*

The reader will consult the first part of this work for an explanation of what is meant by six systems of Crystallisation.

FIRST CLASS.—SIMPLE BODIES.

Octahedral System.	Pyramidal System.	Rhombohedral System.
1. 1. Copper 2. Silver 3. Gold 4. Argentiferous Gold 5. Platinum 6. Platin-Iridium 2. Bismuth 3.** Diamond Iron Lead Titanium		1.* 1. Antimony 2. Arsenic 3. Tellurium 2. Graphite

Prismatic System.	Oblique Prismatic System.	Doubly Obliq. Prism. Syst.
1. Sulphur		

SECOND CLASS.

	Octahedral System.	Pyramidal System.	Rhombohedral System.
Arseniurets, &c.	**4.** Amalgam **5.** 1. Tin White Cobalt 2. Arsenical Nickel **6.** Tesseral Pyrites		**3.** Osmium-Iridium **4.** Antimonial Nickel **5.** Copper Nickel (?) **6.** Telluric Silver
Sulphurets and Seleniurets.	**7.** Sulphuret of Manganese **8.**** Zinc Blende **9.** 1. Galena 2. Seleniuret of Lead 3. Do. of Lead and Cobalt 4. Do. of Lead and Mercury 5. Do. of Lead and Silver **10.** Sulphuret of Silver **11.** Sulphuret of Cobalt **12.*** Iron Pyrites		**7.** Sulphuret of Nickel **8.*** Cinnabar **9.** Sulphuret of Molybdenum
Chlorides and Fluorides.	**13.** Muriate of Ammonia **14.** Chloride of Sodium **15.** Chloride of Silver **16.** 1. Fluorspar 2. Yttrocerite	**1.** Chloride of Mercury	**10.** Fluorcerium
Oxides.	**17.** Red Oxide of Copper **18.** Oxide of Arsenic	**2.** Braunite **3.** 1. Oxide of Tin 2. Rutile **4.** Anatase	**11.** Ice **12.*** 1. Corundum 2. Specular Iron 3. Titanitic Iron Ore **13.** Sulphato-tricarbonate of Lead **14.** Quartz

BINARY COMPOUNDS.

Prismatic System.	Oblique Prismatic System.	Doubly Ob. Pris. Sys.
2. Antimonial Silver 3. Arsenical Iron		
4. 1. Vitreous Copper 2. Sulphuret of Silver and Copper 5. Sulphuret of Bismuth 6. 1. Grey Antimony 2. Orpiment 7. White Iron Pyrites	1. Realgar	
8. Red Oxide of Zinc. 9. White Antimony 10. Pyrolusite		

THIRD CLASS.

	Octahedral System.	Pyramidal System.	Rhombohedral System.
Non-Oxidised Minerals.	**19.*** 1. Bright White Cobalt 2. Nickel Glance 3. Sulpho-antimonite of Nickel **20.** Purple Copper	**5.**** Copper Pyrites **6.** Cryolite	**15.*** Tetradymite **16.*** Polybasite **17.*** Red Silver 1. Light Red Silv. 2. Dark Red Silv. **18.** Magnetic Iron Pyrites
Oxides.	**21.** 1. Spinel 2. Pleonaste 3. Automalite 4. Magnetic Iron Ore 5. Franklinite 6. Chromate of Iron **22.**** 1. Boracite 2. Rhodizite	**7.** Hausmannite **8.** Phosphate of Yttria **9.*** Fergusonite **10.*** 1. Tungstate of Lime 2. Tungstate of Lead 3. Molybdate of Lead **11.** Zircon	**19.*** 1. Calcareous Spar 2. Dolomite 3. Brown Spar 4. Bitter Spar 5. Mesitin Spar 6. Carbonate of Iron 7. Carbonate of Manganese 8. Galmei **20.*** Nitrate of Soda **21.** Talc **22.*** Phenakite **23.*** Willelmine

DOUBLE BINARY COMPOUNDS.

Prismatic System.	Oblique Prismatic System.	Doubly Ob. Pris. Sys.
11. Arsenical Pyrites	**2.** Plagionite	
1. Common ArsenicalPyr.	**3.** Myargyrite	
2. Cobaltic Arsenical Pyr.		
12. Brittle Sulphuret of Silver		
13. Berthierite (?)		
14. Jamesonite		
15. Zinkenite		
16. Antimonial Copper		
17. Sternbergite		
18. Mendipite (?)	**4.** Red Antimony	**1.** Boracic Acid
		2. Diaspore
19.	**5.** Tungstate of Iron	**3.** Cyanite
1. Manganite	**6.** Chromate of Lead	
2. Prismatic Iron Ore	**7.** Gadolinite	
20. Tantalite	**8.** Tabular Spar	
21. Columbite	**9.** Augite	
22. Aeschynite	1. Diopside	
23.	2. Sahlite	
1. Witherite	3. Hedenbergite	
2. Strontianite	4. Rhodonite	
3. Arragonite	5. Basaltic Augite	
4. Junkerite	6. Hypersthene	
5. White Lead Ore	7. Diallage	
24. Phosph. of Manganese		
25. Nitre		
26. Staurolite		
27. Andalusite		
28. Olivine		
29. Sulphate of Potash		
30. Thenardite		
31.		
1. Sulphate of Barytes		
2. Sulphate of Strontian		
3. Sulphate of Lead		
32. Anhydrite		

FOURTH CLASS.

	Octahedral System.	Pyramidal System.	Rhombohedral System.
Oxides.		12. Murio-Carbonate of Lead. 13. Mellite.	24. 1. Apatite from Ehrenfriedersdorf. 2. Apatite from Snarum 3. Phosphate of Lead from Poullaouen 4. Arseniate of Lead from Johann Georgenstadt 25.* Copper Mica. 26.* Dioptase. 27. Coquimbite.

TREBLE BINARY COMPOUNDS.

Prismatic System.	Oblique Prismatic System.	Doub. Obliq. Pris. Sys.
33. Muriate of Copper.	10. Wagnerite	4. Blue Vitriol
	11. Lithia Mica (?)	
34. Wavellite.		
35.	12. Malachite	
1. Olivenite	13. Carbonate of Soda	
2. Libethenite.	14. Trona	
36. Euchroite.	15. Phosphate of Copper	
37. Haidingerite.	from Rheinbreitenbach	
38. Siliceous Oxide of Zinc.	16. Oblique Prismatic Ar-	
39. Picrosmine.	seniate of Copper	
40. Mascagnine.	17.	
41. Brochantite.	1. Vivianite	
42.	2. Cobalt Bloom	
1. Sulphate of Magnesia.	18. Huraulite	
2. Sulphate of Zinc	19. Heterosiderite	
	20. Pharmacolite	
	21. Strahlerz	
	22. Tincal	
	23. Glauber's Salt	
	24. Gypsum	
	25. Sulphate of Iron	

FIFTH CLASS.

	Octahedral System.	Pyramidal System.	Rhombohedral System.
Sulphurets.	**23.**** Fahlerz, or Grey Copper 1. Arsenical Grey C. 2. Mixed Grey C. 3. Antimonial Grey C. **24.** Sulphuret of Tin		
Oxides.	**25.** Garnet: 1. Almandine 2. Cinnamon Stone 3. Grossular 4. Common Garnet 5. Melanite 6. Manganesian Garnet 7. Rothoffite **26.** Leucite	**14.** Idocrase **15.** Gehlenite **16.** Wernerite **17.*** Humboldtilite	**28.** Vanadiate of Lead **29.** One-axed Mica **30.** Nepheline **31.** Beryll

QUADRUPLE BINARY COMPOUNDS.

Prismatic System.	Oblique Prismatic System.	Doub. Obliq. Pris. Sys.
43. Needle Ore 44. Bournonite		
45. Topas	26. Baryto-Calcite 27. Azure Copper Ore 28. Triphyline 29. Vauquelinite 30. Titanite 31. Epidote 1. Zoisite 2. Pistacite 3. Mangan-Epidote 4. Bucklandite 32. Couzeranite 33. Euclase 34. Two-axed Mica 35. Acmite 36. Hornblende 1. Tremolite 2. Actynolite 3. Arfvedsonite 4. Basaltic Hornblende 5. Anthophyllite 37. 1. Felspar 2. Rhyacolite 38. Glauberite 39. Azure Lead Ore 40. Leadhillite 41. Lanarkite	5. Latrobite 6. 1. Anorthite 2. Labradorite 3. Oligoclase 4. Albite 7. Petalite
46. Amblygonite (?) 47. Chiastolite (?) 48. Chrysoberyll 49. Lievrite 50. Allanite 51. Dichroite 52. Spodumen (?)		

SIXTH CLASS.

	Octahedral System.	Pyramidal System.	Rhombohedral Syst.
Oxides.	**27.** Sodalite	**18.** Uranite	**32.** Pyrosmalite
	28.** Bismuth Blende	1. Copper Uranite	**33.** Eudialyte
	29. Analcime	2. Lime Uranite	**34.** Chabasite
	30. Cube Ore	**19.** Apophyllite	**35.** Levyne
	31. Alum		**36.** Alunite
	1. Potash Alum		
	2. Ammonia Alum		

SEVENTH CLASS.

	Octahedral System.	Pyramidal System.	Rhombohedral Syst.
Oxides.	**32.**** Helvine		**37.** Tourmaline
	33.		
	1. Lapis Lazuli		
	2. Hauyne		
	3. Nosian		

APPENDIX.

Octahedral System.	Pyramidal System.	Rhombohedral Syst.
34. Pyrochlore	**20.** Black Tellurium	**38.** Palladium from Tilkerode
35. Pyrope	**21.** Mellilite	**39.*** Crichtonite
36. Cancrinite	**22.** Oerstedtite	**40.** Chlorite
37. Uwarowite	**23.** Somervillite	**41.** Cronstedtite
	24.** Edingtonite	**42.*** Sideroschisolite
		43. ·Pinite
		44.* Dreelite

QUINTUPLE BINARY COMPOUNDS.

Prismatic System.	Oblique Prismatic System.	Doub. Obliq. Pris. Sys.
53. Scorodite	42. Gay Lussite	
54. Prehnite	43. Laumonite	
55. Pyrophyllite	44. Mesotype	
56. Harmotome	1. Natrolite	
1. Potash Harmotome	2. Mesolite	
2. Barytes Harmotome	3. Scolezite	
57. Thomsonite	45. Stilbite	
58. Desmine	46. Brewsterite	
59. Epistilbite	47. Datolite	
60. Polyhallite	48. Red Iron Vitriol	
	49. Johannite	

SEXTUPLE BINARY COMPOUNDS.

Prismatic System.	Oblique Prismatic System.	Doub. Obliq. Pris. Sys.
61. Caledonite		8. Axinite

NON-ANALYSED MINERALS.

Prismatic System.	Oblique Prismatic System.	Doub. Obliq. Pris. Sys.
62. White Tellurium	50. Graphic Tellurium	9. Babingtonite
63. Schilfglaserz	51. Flexible Sulphuret of Silver	
64. Fluellite	52. Humite	
65. Polymignite	53. Monazite	
66. Brookite	54. Turnerite	
67. Lenticular Copper Ore		
68. Lazulite		
69. Childrenite		
70. Forsterite		
71. Sillimanite		
72. Mengite		
73. Koenigite		
74. Monticellite		
75. Herderite		
76. Hopeite		

Section II.

A CATALOGUE of CRYSTALLIZED MINERALS, SHOWING THE COMBINATIONS THAT OCCUR IN NATURE.

I have arranged the Minerals in the following Catalogue in the same order as in the preceding Table, and I have marked them with the same numbers which distinguish them there. This affords a ready opportunity of comparing my system of Crystallography with the German system.

The Minerals belonging to each of the Six Systems of Crystallisation are collected into a distinct group, but no attention is paid to the chemical differences of the Genera or Species, the present object being simply the exhibition of their Crystallographical characters.

The two figures prefixed to each symbol show the Class and Order to which the combination belongs, according to my method of classification. The thick figure indicates the Class, and the slight figure the Order. In Sect. III. the Minerals will be re-arranged according to this classification.

In many cases the localities are given where particular combinations have been found in the greatest perfection.

The Authorities affixed to each combination, refer to the descriptions or figures upon which the symbol is founded. The references are in abridged signs, which are explained in the following table:—

A = *Allan*, *R.* Manual of Mineralogy, Edinburgh, 1834. The number refers to a figure in the plates.

B = *Brooke*, *H. J.* Familiar Introduction to Crystallography, London, 1823. The large figures refer to pages, the small figures to subjects.

D = *Dana*, *J. D.* System of Mineralogy, including an extended Treatise on Crystallography, New Haven, U.S. 1837. The large figures refer to pages, the small figures to diagrams. The small figures without large figures refer to subjects on the plates.

G = *Geiger*, *P. L.* Handbuch der Pharmaceutische Mineralogie, neu bearbeitet von Dr. Clamor Marquart, Heidelberg, 1838. The references are to pages.

H = *Haüy*, *L'Abbé*. Traité de Minéralogie, seconde edition, Paris, 1822. The references are in general to the Atlas, the large figure referring to the plate, the small one to the subject. When the reference is to a page, the volume is noted in numerals.

J = *Jameson*, *Robert*. System of Mineralogy, Edinburgh, 1820. The numerals refer to volumes, the large figures to pages, the small figures to subjects. Small figures alone refer to diagrams on the plates.

L = *Leonhard*, *C. C. Von.* Handbuch der Oryktognosie, Zweite Auflage, Heidelberg, 1826. The large figures refer to a page, the small figures to the number of the description of the crystal on that page.

M = *Mohs*, *F.* Treatise on Mineralogy, Translated by W. Haidinger, Edinburgh, 1825. The numerals (i.) (ii.) refer to the volume. The large figure to the page or plate. The small figure to the subject.

Md = *Model.*—Sim. Md. signifies similar to Model — in kind, but not equal in degree; similar in its general figure, but not in its angles; or similar in its planes and angles, but not equal in the comparative sizes of the forms which compose it. References are made thus, for the sake of giving a certain amount of comparative information, in cases where exact models of the combinations in question are not contained in the collection. The accompanying references to exact figures or descriptions of the combinations will prevent misconception.

Mr = *Miller*, *W. H.* Treatise on Crystallography, Cambridge, 1839. The references are to diagrams on the plates.

P = *Phillips. W.* Introduction to Mineralogy, fourth edition, by R. Allan, London. 1837. The large figures refer to pages, the small figures to subjects.

R = *Rose, Gustav.* Elemente der Krystallographie, Zweite Auflage, Berlin, 1838. The small figures refer to subjects on the plates. The large figures to pages.

S = *Shepard, C. U.* Treatise on Mineralogy, New Haven, U. S. 1832—35. The large figures refer to pages, the small figures to diagrams.

T = *Thomson, Thomas.* Outlines of Mineralogy and Geology, London, 1836. The numerals refer to volumes, the figures to pages.

CLASS I.—MINERALS BELONGING TO THE OCTAHEDRAL SYSTEM OF CRYSTALLISATION.

The AXES of all Combinations belonging to this Class are = $p^a\ m^a\ t^a$.

The constituent FORMS of the Combinations of this Class are all unequiaxed, excepting P,M,T, MT. PM, PT, and PMT. The forms that occur are as follows:—

a.) Homohedral Forms.

P,M,T.
MT.PM,PT.
$MT_{\frac{4}{3}}.PM_{\frac{4}{3}},P_{\frac{4}{3}}T.$
$MT_{\frac{3}{2}}.PM_{\frac{3}{2}},P_{\frac{3}{2}}T.$
$MT_2.PM_2,P_2T.$
$MT_4.PM_4,P_4T.$
$MT_{\frac{3}{2}},M_{\frac{3}{2}}T.PM_{\frac{3}{2}},P_{\frac{3}{2}}M,PT_{\frac{3}{2}},P_{\frac{3}{2}}T.$
$MT_2,M_2T.PM_2,P_2M,PT_2,P_2T.$
$MT_{\frac{5}{2}},M_{\frac{5}{2}}T.PM_{\frac{5}{2}},P_{\frac{5}{2}}M,PT_{\frac{5}{2}},P_{\frac{5}{2}}T.$
$MT_3,M_3T.PM_3,P_3M,PT_3,P_3T.$
$MT_5,M_5T.PM_5,P_5M,PT_5,P_5T.$
$MT_{10},M_{10}T.PM_{10},P_{10}M,PT_{10},P_{10}T.$
PMT.
$3\ P_{\frac{3}{4}}MT.$
$3\ P_{\frac{1}{2}}MT.$
$3\ P_{\frac{1}{3}}MT.$
$3\ P_{\frac{3}{2}}MT.$
$3\ P_2MT.$
$3\ P_3MT.$
$3\ P_{\frac{1}{3}}M_{\frac{1}{2}}T.$
$3\ P_{\frac{1}{3}}MT_{\frac{1}{2}}.$
$3\ P_{\frac{1}{4}}M_{\frac{1}{2}}T.$
$3\ P_{\frac{1}{5}}M_{\frac{1}{5}}T.$
$6\ P_{\frac{1}{3}}M_{\frac{1}{2}}T.$
$6\ P_{\frac{1}{4}}M_{\frac{1}{3}}T.$
$6\ P_{\frac{1}{4}}M_{\frac{1}{2}}T.$
$6\ P_{\frac{1}{11}}M_{\frac{1}{5}}T_{\frac{1}{3}}.$
$6\ P_{\frac{1}{7}}M_{\frac{1}{3}}T.$

b.) Hemihedral Forms.

$\frac{1}{2}$ PMT Znw.
$\frac{1}{2}$ pmt Zne.
$\frac{1}{2}\ (3\ P_{\frac{1}{2}}MT).$
$\frac{1}{2}\ (3\ P_{\frac{1}{3}}MT).$
$\frac{1}{2}\ (3\ P_{\frac{3}{2}}MT).$
$\frac{1}{2}\ (3\ P_2MT).$
$\frac{1}{2}\ (3\ P_3MT).$
$\frac{1}{2}\ (6\ P_{\frac{1}{3}}M_{\frac{1}{2}}T).$
$\frac{1}{2}\ (6\ P_{\frac{1}{3}}M_{\frac{1}{3}}T).$

c.) Twin Crystals of common occurrence.

PMT $\times$ 2.
$\frac{1}{2}$ PMT $\times$ 2.
($\frac{1}{2}$ PMT, $\frac{1}{2}$ pmt) $\times$ 2.
(MT. PM, PT) $\times$ 2.
$3\ P_{\frac{1}{3}}MT \times 2.$
$3\ P_2MT \times 2.$
$(MT_2.PM_2,\ P_2T) \times 2.$

d.) Forms which produce unequiaxed Combinations, occurring as irregular Crystals of the Minerals of this Class.

P_{+}.	T.	PM.	$M\frac{7}{10}T$.
M.	MT.	PT.	$P\frac{7}{10}T$.

Every natural Combination belonging to this Class, has *one of the foregoing Forms or Combinations* PREDOMINANT. When this predominant form is put into position, all the subordinate forms fall also into position, and can be found in their proper polaric situations.

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1. *An Isomorphous Group*, 1, 2, 3, 4, 5, 6 :—

1. NATIVE COPPER. Kupfer. Cuivre natif.
Cleavage = 0.

1. 1. P,M,T..................Siberia...Model 1. P307[1]. H86[1]. L711[1]. J[163].
2. 1. PMT ......New Haven...Model 15. D[4]. H86[2]. P307[4]. L711[3]. J[167].
3. 1. P,M,T.pmt.........New Haven....Cornwall.......P307[2] .L711[2]. D[2].
3. 1. P,M,T,mt.pm,pt....Siberia...Model 27. H86[6]. L711[4]. D[5]. J iii 90[2].
3. 1. P,M,T.mt.pm,pt,pmt.......Model 31. S[320]. P307[7]. H86[8]. L711[6]. J[165].
3. 1. P,M,T.PMT,.......................................Model 29. H86[3]. J[164].
3. 1. p,m,t.MT.PM,PT. ....Model 28. D[6].
3. 1. p,m,t.PMT. ....Model 30. P307[3]. D[3].
3. 4. $P,M,T,mt_2,m_2t.pm_2,p_2m,pt_2,p_2t$............Sim. Model 45. L711[7]. D[10].
3. 4. P,M,T,MT,mt$\frac{3}{2}$,m$\frac{3}{2}$t.PM,pm$\frac{3}{2}$,p$\frac{3}{2}$m,PT,pt$\frac{3}{2}$,p$\frac{3}{2}$t,PMT.......Siberia...R[22a]
4. 1. MT.PM,PT....Cornwall...Model 63. P307[6]. L711[5]. D[7]. H86[4]. J[166].
4. 1. MT.PM,PT,pmt ....Model 65. D[8].
4. 1. mt.pm,pt,PMT.....................................Model 64. P307[5]. D[9].
4. 3. $MT_2,M_2T.PM_2,P_2M,PT_2,P_2T$. ...Naalsoe in Faroe...Md. 68. D[11]. S[321]. L711[8]. R[22].

2. NATIVE SILVER. Silber. Argent natif.
Cleavage = 0.

1. 1. P,M,T.......Cornwall....Mexico....Md. 1. J[163]. L700[1]. P293. H86[1].
2. 1. PMT.........Kongsberg.......Md. 15. J[167]. D[4]. L700[3]. P293. H86[2].
2. 1. PMT × 2.......................................Model 16. D[129]. L700[6].
2. 3. 3 P$\frac{1}{3}$MT........Kongsberg......Similar Model 22. J iii 70[6]. R[7]. D[16].
2. 3. pmt, 3 P$\frac{1}{3}$MT ....Kongsberg.......R[8].
3. 1. P,M,T.mt.pm,pt ....Model 27. H86[6].
3. 1. P,M,T.pmt..D[2]. J iii 70[3].
3. 1. P,M,T,mt.pm,pt,pmt..................................Model 31. L700[4].
3. 1. p,m,t.PMT............Kongsberg......Model 30. D[3]. L700[2]. P294.
4. 1. MT.PM,PT...Model 63. J iii 70[5]. J[166].
4. 1. MT.PM,PT,pmt. ....Model 65. L700[5].
5. 3. $M\frac{7}{10}T$. $P\frac{7}{10}T$. Axes : $p\frac{1}{+}$ $m\frac{2}{-}$ t^{2} * ....H iii 250[m]. L700[8].
6. 1. $\frac{1}{2}$PMT...Model 117. J iii 70[4].

* An irregular combination. The axes are $p\frac{1}{+}$ $m\frac{2}{-}$ t^{2}, and the combination is a rhombic prism, but the interfacial angles are the same as those of the regular octahedron.

3. NATIVE GOLD. Gold. Or natif.

Cleavage = 0.

1. 1. P,M,T,......Veroespatak...Model 1. D[1]. H86[1]. L708[1]. J[163]. P336[1].

2. 1. PMT, ...Veroespatak, Schlangenberg, Mexico, La Gardette,... Model 15. D[4]. H86[2]. L708[3]. P336[3]. J[167].

2. 3. 3 P$\frac{1}{3}$MT, ...Veroespatak in Transylvania ...Similar Model 22. R[7]. D[16]. J iii 57[4]. L708[5].

2. 3. P$\frac{1}{3}$MT × 2, ...S[324]. L708[9].

2. 3. pmt, 3 P$\frac{1}{3}$MT....................................Veroespatak......R[8].

2. 4. PMT, 3 p$\frac{1}{3}$mt. ....R[10a].

3. 1. P,M,T,mt.pm,pt. ....Siberia......Model 27. L708[6]. D[5].

3. 1. P,M,T,mt.pm,pt,pmt. ....Tijuco......Model 31. L708[7]. J[165].

3. 1. P,M,T.PMT. ....Brazil......Model 29. H86[3]. J[164].

3. 1. P,M,T.$\frac{1}{2}$pmt..................................Model 118. J iii 56[3b].

3. 1. P,M,T. 3 p$\frac{1}{3}$mt.......Veroespatak. Tijuco...Model 39. L708[4]. D[14].

3. 1. p,m,t.PMT ....Brazil......Model 30. J iii 56[1b]. D[3]. P336[2].

3. 4. p,m,t. 3 P$\frac{1}{3}$MT, ...D[15].

3. 4. P,M,T,MT, mt_2,m_2t.PM,pm_2, p_2m, PT,pt_2,p_2t, PMT, 3 p$\frac{1}{3}$mt,......P336[7].

4. 1. MT.PM,PT...Katherinenburg...Model 63. L708[8]. D[7]. P336[5].J[166].

4. 1. mt.pm,pt,PMT.................................Model 64. D[9]. P336[4].

4. 3. MT_2,M_2T.PM_2,P_2M,PT_2,P_2T. ....Similar Model 68. R[29]. S[323].

6. 1. $\frac{1}{2}$PMT..Model 117. J iii 57[5].

4. ARGENTIFEROUS GOLD. Electrum. Auriferous Silver.

1. 1. P,M,T.................Schlangenberg. Kongsberg......P337. L710.

5. NATIVE PLATINUM. Platin.

Cleavage = 0.

1. 1. P,M,T ....P.338. H iii 226. G58.

6. PLATIN-IRIDIUM.

1. 1. P,M,T? ....Rose, page 160.

2. NATIVE BISMUTH. Wismuth. Bismuth natif.

Cleavage = PMT

1. 1. P,M,T, ...Model 1. J iii 108[3].

2. 1. PMT. ...Johann-Georgenstadt...Md. 15. P276. L693[1]. H iv 203[1]. J iii 108[1].

4. 1. MT.PM,PT ....Wittichen. Bieber......Model 63. L693[2].

4. 1. MT.PM,PT,pmt, ...Model 65. G55.

4. 1. mt.pm,pt,PMT,...Model 64. G55.

5. 1. mt.pm,pt,$\frac{1}{2}$PMT, $\frac{1}{2}$pmt,................................M ii 430[2].

6. 1. $\frac{1}{2}$PMT, $\frac{1}{2}$pmt, ...Model 118. A129. M ii 430[1]. J iii 108[2].

8.** DIAMOND. Diamant. Demant.

Cleavage = PMT.

1. 1. P,M,T, ...Model 1. H120[312]. P384[3]. L670[1].

2. 1. PMT, ...Model 15. J[1]. H120[341]. P384[1]. L670[5].
2. 1. 3 P_3MT, ...Similar Model 17. D[30]. H120[343].
2. 3. PMT, 6p$\frac{1}{3}$m$\frac{1}{2}$t, ...J i 3[7].
2. 3. 6 P$\frac{1}{3}$M$\frac{1}{2}$T, ...Similar Model 23. H120[345]. L670[6]. R[12]. P384[6]. J[3].
2. 3. $\frac{1}{2}$ (6 P$\frac{1}{3}$M$\frac{1}{2}$T) ....Model 24. D[41]. A119.
3. 1. P,M,T,mt.pm,pt, ...Model 27. H120[344]. D[5]. J i 5[15].
3. 1. p,m,t.PMT, ...Model 30. P384[2]. L670[2]. D[3].
3. 1. P,M,T.PMT, 6 p$\frac{1}{3}$m$\frac{1}{2}$t, ...P384[7].
4. 1. (MT.PM,PT) × 2,..D331[2].
4. 1. MT.PM,PT,..................Model 63. D[7]. H120[346]. P384[5]. L670.
4. 1. mt.pm,pt,PMT, ...Model 64. D[9]. P384[4]. L670[4].

NATIVE IRON. Eisen. Fer natif.
Cleavage = pmt.

2. 1. PMT, ...Model 15. P208. G53.

NATIVE LEAD. Blei. Plomb natif.

2. 1. PMT, ...Model 15. D393.
3. 1. P,M,T.pmt, ...Similar Model 29. G54.

TITANIUM. Titan.
Cleavage = P,M,T, mt.pm,pt.

1. 1. P,M,T, ...Model 1.

4. AMALGAM. Amalgam. Mercure argental.
Cleavage = MT.PM,PT.

2. 1. PMT. ....Model 15. H88[27]. L698[3]. D[4].
2. 3. 3P$\frac{1}{2}$MT. ....Model 22. D[16].
3. 1. p,m,t,MT.PM,PT..........................Model 28. S[315]. D[6]. H88[28].
3. 1. p,m,t,MT.PM,PT,3p$\frac{1}{2}$mt. ....Landsberg......H89[29]. L698[4].
3. 1. p,m,t,MT.PM,PT,pmt,3p$\frac{1}{2}$mt.................................L698[5].
3. 4. p,m,t,MT,MT_3,M_3T.PM,PM_3,P_3M,PT,PT_3,P_3T,pmt,3P$\frac{1}{2}$MT,6p$\frac{1}{3}$m$\frac{1}{2}$t. D392. P378. L698[6]. S[318].
3. 4. P,M,T,MT,mt_3,m_3t.PM,pm_3,p_3m,PT,pt_3,p_3t,pmt,3P$\frac{1}{2}$MT,6P$\frac{1}{3}$M$\frac{1}{2}$T. L698[6]. H89[30].
4. 1. MT.PM,PT..................Model 63. H88[25]. P378. L698[1]. D392.
4. 1. MT.PM,PT,pmt. ....Model 65. L698[2]. D[8]. S[316].
4. 1. mt.pm,pt,PMT....................................Model 64. H88[26]. D[9].
4. 4. MT.PM,PT,3p$\frac{1}{2}$mt...................................Model 69. J iii 87[b].

5. *An Isomorphous Group*, 1, 2:—

1. TIN WHITE COBALT. Grey Cobalt. Speiskobalt. Cobalt arsenical.
Cleavage = p,m,t,mt.pm,pt,PMT.

1. 1. P,M,T. ......Riechelsdorf......Model 1. P286[1]. L681[3]. H iv 222[1].
2. 1. PMT..............Saalfeld......Model 15. P286[4]. L680[1]. H iv 222[2].
3. 1. P,M,T.pmt....Riechelsdorf. Joachimstal. P286[2]. L681[2]. H iv 222[3].
3. 1. P,M,T,mt.pm,pt,PMT. ...Riechelsdorf....Sm. Md. 31. R[18]. L681[4].

3. 1. p,m,t.PMT ....Riechelsdorf......Model 30. P286[3]. L681[2].
3. 1. p,m,t,mt.pm,pt,PMT, 3p½mt. .... Scheerer.
5. 3. $M\frac{7}{10}T$. $P\frac{7}{10}T$. Axes p:m:t. ....L681[1].

2. ARSENICAL NICKEL. Arsenik-Nickel. Binarseniet of Nickel.
3. 1. P,M,T,mt.pm,pt. ....Model 27. D[6].
3. 1. P,M,T.pmt............................Similar Model 29. D[5]. R162[5].

6. TESSERAL PYRITES. Tesseralkies.
1. 1. P,M,T. ....Model 1. R.162[6]. S229.
3. 1. p,m,t,mt.pt,PMT, 3p½mt, ...Scheerer.

7. SULPHURET OF MANGANESE. Manganglanz. Manganèse sulfuré. Manganblende.
Cleavage = mt.pm,pt,PMT.
1. 1. P,M,T....................................Model 1. D429. L656. P245.
2. 1. PMT, ...Model 15. D429. S24.
3. 1. P,M,T.pmt................................Similar to Model 29. L656.

8.** ZINC BLENDE. Sulphuret of Zinc. Zinkblende. Zinc sulfuré.
Cleavage = MT.PM,PT.
1. 1. P,M,T, ...Cornwall......Model 1. P372[6]. L621[4]. D[1]. S70[6].
2. 1. PMT, ...Siberia......Model 15. S70[5]. H113[275]. P372[4]. L620[3].
2. 1. PMT × 2.....................Schemnitz......Model 16. D[129]. L621[9].
3. 1. P,M,T.PMT, ...Model 29. H114[277].
3. 1. p,m,t.PMT,..Model 30. P372[7].
3. 1. p,m,t,MT.PM,PT, ... Model 28. D[3]. L621[4].
3. 1. p,m,t,mt.pm,pt,PMT...Kapnik... Model 33. H114[279]. L621[5]. S70[7].
3. 1. p,m,t,MT.PM,PT,PMT, ½ (3p½mt) ....P372[10]. S[67].
3. 1. p,m,t.½PMT,½pmt, ...Model 118, with edges replaced. H113[276].
4. 1. MT.PM,PT,......Schemnitz. Cornwall. Derbyshire. Kapnik...... Model 63. P372[1]. H113[272]. L620[1].
4. 1. MT.PM,PT,pmt, Kapnik. Schemnitz. Md.65. S[65].D[8].P372[2].L620[2].
4. 1. mt.pm,pt,PMT, ...Model 64. L620[2]. S70[4]. P372[3]. D[9]. H114[278].
5. 4. MT.PM,PT, ½pmt[Znw], ½ (3p½mt)[Zne]. ...Kapnik...Model 95. P372[9]. H114[283]. S[66].
5. 4. MT.PM,PT, ½ (3p½mt) ....H114[280]. D429.
6. 1. ½PMT,......Alston Moor. Kapnik...Md. 117. R[25]. P372[6]. H113[273]. L621[6]. D[30].
6. 1. ½PMT, ½pmt,.................Model 118. D[32]. R[31]. H113[274]. P372[5].

9. *An Isomorphous Group*, 1, 2, 3, 4, 5 :—

1. GALENA. Sulphuret of Lead. Bleiglanz. Plomb sulfuré.
Cleavage = P,M,T.
1. 1. P,M,T, ...Saxony. Derbyshire...Md. 1. H90[37]. P350[1]. L625[1]. J[220].
2. 1. PMT, ...Heidelberg, Bleiberg, Derbyshire...Model 15. J[224]. L625[4]. P350[2]. H90[38].

2. 1. 3 P_2MT. ....Model 17. S[286].
2. 1. 3 P_2MT × 2,..S[289].
2. 1. PMT, 3 $p_2$mt, ...Similar Model 15, with bevelled edges. P350[5].
2. 4. PMT, 3$p\frac{1}{2}$mt...H90[41]. L625[6].
3. 1. P,M,T.pmt, ...Hartz, Derbyshire......R[14]. L625[3]. J[231]. H90[39].
3. 1. P,M,T.PMT, ...Model 29. R[15]. L625[3].
3. 1. P,M,T,mt.pm,pt,PMT, ...Neudorf, Freiberg, Schemnitz......
Similar Model 31. J[232]. H90[45]. L625[3].
3. 1. P,M,T,mt.pm,pt,pmt,...............Schemnitz......Model 31. L625[3].
3. 1. P,M,T.pmt, 3$p\frac{1}{2}$mt, ...L625[6].
3. 1. P,M,T.PMT, 3$p_2$mt, 3$p_3$mt,.........Andreasberg, Wittichen......R[14a].
3. 1. p,m,t.PMT......Hartz, Derbyshire......Md. 30. L625[3]. P350[3]. R[16].
3. 1. P,M,T.PMT, 3$p_2$mt,...............Pfaffenberg......H90[44]. J[226]. L625[7].
3. 1. P,M,T,mt.pm,pt,PMT, 3$p_2$mt,.......................H90[46]. J[229]. L625[8].
3. 1. P,M,T,mt.pm,pt,PMT, 3$p\frac{1}{3}$mt, 3$p_2$mt, ...P350[6].
3. 1. p_+,M,T.PMT. Axes: $p^2_+ m^2 t^2$, ...J[230], H90[40].L625 note.
3. 4. P,M,T,mt_+,m_+t.pm_+,p_+m,pt_+,p_+t, ...Similar Model 45. J[232].
3. 4. P,M,T.PMT,3$p\frac{1}{2}$mt, ... Schemnitz......R[16a]. J[227].
3. 4. P,M,T.PMT, 3$p\frac{1}{3}$mt, ...H90[43]. J[227].
4. 1. mt.pm,pt,PMT, ...Model 64. J[226]. P350[4]. H90[42]. L625[5].
5. 3. $M\frac{7}{10}$T. $P\frac{7}{10}$T. Axes: $p^2_+ m^2_- t^2$,...................................J[225].

2. Seleniuret oe Lead. Selenblei. Plomb seleniuré.
1. 1. P,M,T? ....Model 1. R162.

3. Seleniuret of Lead and Cobalt. Selenkobaltblei.
1. 1. P,M,T? ....Model 1. R162.

4. Seleniuret of Lead and Mercury. Selenquecksilberblei.
1. 1. P,M,T? ....Model 1. R162.

5. Seleniuret of Lead and Silver. Selensilberblei.
1. 1. P,M,T? ....Model 1. R162.

10. Sulphuret of Silver. Silberglanz. Argent sulfuré.
Cleavage = p,m,t,mt.pm,pt.
1. 1. P,M,T,......Freiberg, Mexico...Model 1. P296. L636[1]. H86[1]. D[1].
2. 1. PMT, ...Schemnitz, Freiberg...Model 15. P296. L636[2]. H86[2].
2. 3. 3 $P\frac{1}{2}$MT,............Freiberg......Model 22. D[16]. L636[6], H86[5]. S[466].
2. 4. PMT, 3$p\frac{1}{2}$mt, ...H86[7].
3. 1. P,M,T,mt.pm,pt, ...Model 27, L636[7]. H86[6]. S260[3].
3. 1. P,M,T,mt.pm,pt,pmt, ...Model 31. L636[4]. H86[8]. J iii 340[1].
3. 1. P,M,T.PMT, ...Schemnitz......Model 29. L636[3]. H86[3]. D[3].
3. 1. P,M,T. 3$p\frac{1}{2}$mt, ... Freiberg......Model 39. L636[5]. S[465]. D[14].
3. 1. p,m,t,MT.PM,PT,pmt, ... Model 34. P296[1].
3. 4. p,m,t. 3$P\frac{1}{2}$MT, ...D[15].

4. 1. MT.PM,PT,...Schemnitz, Freiberg...Md. 63. P296. H86[4]. L636[8].
4. 1. MT.pmt. Axes: $p\frac{a}{+}m^a t^a$,....................................J iii 340[2].

11. SULPHURET OF COBALT. Kobaltkies. Schwefel Kobalt.
Cleavage = p,m,t. pmt.
2. 1. PMT, ...Siegen......Model 15. L653[1].
3. 1. P,M,T.pmt,...............................Similar Model 29. D[2].
3. 1. p,m,t.PMT, ...Siegen......Model 30. L652[2]. D[2].
5. 3. $M\frac{7}{10}T$. $P\frac{7}{10}T$. Axes: $p\frac{a}{+}m\frac{a}{-}t^a$ ....Siegen......L653[1].

12.* IRON PYRITES. Eisenkies. Fer sulfuré.
Cleavage = $P,M,T,mt_2.pm_2,p_2t$,PMT.
1. 1. P,M,T,.........Cornwall......Model 1. J[199]. L657[3]. H106[194]. P210[4].
1. 1. P_-,M,T. Axes: $p\frac{a}{-}m^a t^a$.......................................Model 2.
1. 1. P_+,M,T. Axes: $p\frac{a}{+}m^a t^a$. ....Model 3.
1. 2. P_+,M_-,T. Axes: $p\frac{a}{+}m\frac{a}{-}t^a$.......................Similar to Model 5.
2. 1. PMT, ...Persberg...Model 15. J[203]. L657[5]. P210[1]. H106[195]. D[4].
2. 1. 3 P_2MT × 2,..............Schoharie, New York.........S[270]. D406[2].
2. 3. 3 $P\frac{1}{2}$MT, ...Corsica...Model 22. S[265]. J295[12]. H106[197]. D[16]. L658[15].
2. 3. 3 $P\frac{1}{3}M\frac{1}{2}$T, ...Model 35. R[45].
2. 4. PMT, 3 $p\frac{1}{3}m\frac{1}{2}t$, ...J[204]. D[18].
3. 1. P,M,T.pmt, ... Similar Model 29. L657[7]. J[202]. D[2]. P210[3]. H106[200].
3. 1. P,M,T. 3 $p\frac{1}{2}$mt,...Md. 39. R[19]. D[14]. S[266]. J iii 293[11]. H107[205]. L657[11].
3. 1. P,M,T.pmt, 3$p\frac{1}{3}m\frac{1}{2}t$,...Monroe, Connecticut. Facebay...R[53a]. Mr[88].
H107[210]. L658[18]. S[264].
3. 1. p,m,t.PMT, ...Model 30. D[3]. P210[2].
3. 4. $P,M,T,mt_2.pm_2,p_2t$,.......................................Elba......R[53]. D[42].
3. 4. $P,M,T,MT_2.PM_2,P_2T$, ...Cornwall, Elba......Sim. Md. 47. J[200].
H106[201]. L657[2]. P210[8]. S[262]. D[43].
3. 4. $P,M,T,MT_2.PM_2,P_2T$,PMT, ...Sim. Md. 48. L657[6]. H107[209]. S[261].
3. 4. $P,M,T,mt_2.pm_2,p_2t$, 3 $p\frac{1}{3}m\frac{1}{2}t$, ...Rossie, New York......D407.
3. 4. $P,M,T,mt_2,m\frac{3}{2}t.pm_2,p\frac{3}{2}m,p_2t,pt\frac{3}{2}$,pmt, 3$p\frac{1}{2}$mt,3$p_2$mt,3$p\frac{1}{3}m\frac{1}{2}t$,3$p\frac{1}{4}m\frac{1}{2}t$,
Peru......H108[216].*
3. 4. p,m,t.PMT, 3$p\frac{1}{3}m\frac{1}{2}t$,...J[205].
3. 4. p,m,t.PMT, 3$p\frac{1}{2}$mt, 3P_2MT, 6$p\frac{1}{3}m\frac{1}{2}t$, ...H108[215].
3. 4. p,m,t. 3$P\frac{1}{2}$MT, ...D[15].
3. 4. p,m,t.3$P\frac{1}{3}M\frac{1}{2}$T, Brosso Thal, Md. 46. R[47]. H106[202]. J[209]. S[267]. L657[9].
3. 4. $p,m,t,MT_2.PM_2,P_2T$, 3$P\frac{1}{3}M\frac{1}{2}$T, 3$p\frac{1}{4}m\frac{1}{2}t$,............Piedmont......R[47a].
4. 1. mt.pm,pt,PMT, ...Model 64. H106[204].
4. 3. $MT_2.PM_2,P_2T$, 3$P\frac{1}{3}$MT, 3$p\frac{1}{2}$mt, 3$p_2$mt, ...H108[214].
4. 4. $mt_2,m_2t.pm_2,p_2m,pt_2,p_2t$,PMT, 3$p\frac{1}{2}$mt,..........................H107[211].
5. 4. $MT_2.PM_2,P_2T$, ...Elba...Md. 91. R[49]. D[44]. H106[198]. S[263]. P210[7].
J[201]. L657[1].
5. 4. ($MT_2.PM_2,P_2T$) × 2, ...L658[17].
5. 4. $MT_2.PM_2,P_2T$, pmt, ...Similar Model 92. D[47].
5. 4. $MT_2.PM_2,P_2T$,PMT,......Elba...Md. 92. R[52]. J[207]. H107[206]. L657[4].
R210[6]. D[46].

5. 4. $MT_2.PM_2,P_2T$, $3p_{\frac{1}{3}}m_{\frac{1}{2}}t$, ...Elba......R[51]. H107[208]. S[269]. L657[8].
5. 4. $MT_2.PM_2,P_2T$,pmt, $3p_{\frac{1}{3}}m_{\frac{1}{2}}t$, ...Elba......R[51a]. H107[212].
5. 4. mt,MT_2.pm,PM_2,pt,P_2T,PMT, $3p_{\frac{1}{3}}m_{\frac{1}{2}}t$,..............L658[14]. H107[213].
5. 4. $mt_2.pm_2,p_2t$,PMT, ...Model 93. J[206]. D[45]. P210[5]. H107[207]. S[209].

* "This variety of iron pyrites—the *Parallélique*—offers at present the *maximum* relative to the number of faces observed upon different bodies produced by crystallisation. The number of faces upon this crystal is 134.—The symbol for it is:—

$$\text{“}\,MP\overset{1}{\underset{2}{A}}B\overset{2}{C}G^{22}GB\overset{\frac{3}{2}}{\underset{\frac{3}{2}}{C}}G^{\frac{3}{2}\frac{3}{2}}G(A^{\frac{1}{2}}B^2G^1)(\overset{\frac{1}{2}}{A}B^1C^2)({}^{\frac{1}{2}}AG^2C^1)$$

M P d c″ c′ e y″ y′ y o″ o′ o

$$(A^{\frac{3}{2}}B^2G^1)(\overset{\frac{3}{2}}{A}B^1C^2)({}^{\frac{3}{2}}AG^2C^1)(A^2B^2G^1)(\overset{2}{A}B^2C^1)({}^2AG^2C^1)$$

f″ f′ f s″ s′ s

$$(A^{\frac{5}{3}}G^1B^3)(\overset{\frac{5}{3}}{A}B^1C^3)({}^{\frac{5}{3}}AC^1G^3)\,\text{”}$$

n″ n′ n

—HAÜY, *Traité de Minéralogie. Paris*, 1822, *t. iv. p.* 57.

In W. PHILLIPS's *Mineralogy*, London, 1837, page 210, there is a figure nearly resembling Haüy's parallélique crystal, and said to be "given on the authority of Haüy," which contains 194 planes, or 60 above Haüy's maximum quantity. The combination exhibited by Phillips, contains the following planes:

P,M,T, mt,mt_2,$m\frac{3}{2}t$. pm,pm_2,$p\frac{3}{2}m$, pt,p_2t,$pt\frac{3}{2}$, pmt, $3p_{\frac{1}{3}}mt$, $3p_{\frac{1}{2}}mt$, $3p_2mt$, $3p_{\frac{1}{4}}m_{\frac{1}{2}}t$, $6p_{\frac{1}{3}}m_{\frac{1}{2}}t$.

Now, the Forms of Iron Pyrites are collectively as follows:

P,M,T, MT,MT_2,M_2T,$M\frac{3}{2}T$. PM,PM_2,P_2M,$P\frac{3}{2}M$, PT,PT_2,P_2T,$PT\frac{3}{2}$. PMT, $3P_{\frac{1}{3}}MT$,$3P_{\frac{1}{2}}MT$, $3P_2MT$, $3P_{\frac{1}{3}}M_{\frac{1}{2}}T$, $3P_{\frac{1}{3}}MT_{\frac{1}{2}}$, $3P_{\frac{1}{4}}M_{\frac{1}{2}}T$.

These forms contain 206 planes in all, but Haüy's maximum crystal is without the forms MT.PM,PT; M_2T,P_2M,P_2T; 3 $P_{\frac{1}{3}}MT$; and 3 $P_{\frac{1}{3}}MT_{\frac{1}{2}}$; which contain together 72 planes. Consequently, Phillips's crystal is a diagram intended merely to represent all the forms of Iron Pyrites, and is not a drawing of a real crystal.

The Eidogens whose intersections produce the forms of Iron Pyrites, have the following dimensions:

90° 00′ = P,M,T,MT,PM,PT.
109° 28′ = PMT,$P_{\frac{1}{2}}MT$,$PM_{\frac{1}{2}}T$,$PMT_{\frac{1}{2}}$.
129° 31′ = $P_{\frac{1}{3}}MT$,$PM_{\frac{1}{3}}T$,$PMT_{\frac{1}{3}}$.
112° 37′ = $M\frac{3}{2}T$,$P\frac{3}{2}M$,$PT\frac{3}{2}$.
126° 52′ = MT_2,M_2T.PM_2,P_2M,PT_2,P_2T.
141° 3′ = P_2MT,PM_2T,PMT_2.
149° 00′ = $P_{\frac{1}{3}}M_{\frac{1}{2}}T$,$PM_{\frac{1}{3}}T_{\frac{1}{2}}$,$P_{\frac{1}{2}}MT_{\frac{1}{3}}$.
154° 47′ = $P_{\frac{1}{4}}M_{\frac{1}{2}}T$,$PM_{\frac{1}{4}}T_{\frac{1}{2}}$,$P_{\frac{1}{2}}MT_{\frac{1}{4}}$.

Here are no less than *eight Eidogens or primary forms* of crystallised Iron Pyrites, that is to say, evidence of the operation of the power of crystallisation, within the limits of eight forms of different dimensions. Yet crystallographers have chosen to fix the term *primary form* exclusively upon *one* combination produced by the action of *one* Eidogen, differing, however, among themselves as to which one of the many they should elevate to the post of honour,—thus, Haüy chose the Cube = P,M,T; Leonhard, the Pentagonal Dodecahedron = MT_2,PM_2,P_2T; and Phillips, the Octahedron = PMT. But it does not appear to me, that any one of these forms or combinations, can be said to have exercised any more control over, or to have been in any more peculiar manner affected by the power of crystallisation, than any one or all of the other forms; so that there is no evidence of the existence of any single primary form adapted to originate all the other forms, and there is no benefit derived from the assumption of such a primary form.

13. MURIATE OF AMMONIA, Salmiak. Ammoniac muriatée.
Cleavage = PMT.
1. 1. P,M,T,.........St. Etienne by Lyons......Model 1. L588[2]. D[1]. S[169].
1. 1. P_+,M,T, ...L588[2].
2. 1. PMT,...Model 15. J iii 12[1]. H ii 223[2]. L588[1]. P201. H55[167]. D175.
2. 3. 3P½MT, Glan in Bavaria. Sim. Md. 22. J iii 12[5]. L588[3]. H55[168]. D[16].
3. 1. P,M,T,mt.pm,pt, ...Model 27. J iii 12[3].
4. 1. MT.PM,PT,................................Model 63. L588[4]. J iii 12[4].
4. 1. M,T.pm,pt, ...J iii 12[2].

14. CHLORIDE OF SODIUM. Muriate of Soda. Steinsalz. Rock Salt. Soude muriatée.
Cleavage = P,M,T, mt.pm,pt.
1. 1. P,M,T, ...Tyrol......Model 1. J iii 3. H53[171]. R[13]. L584[1].
1. 1. P_-,M,T,...................................Model 2. J iii 3. L584[1].
1. 1. P_+,M,T,...................................Model 3. J iii 3. L584[1].
3. 1. P,M,T,mt.pm,pt,...............................Model 27. L584[3]. D[5].
3. 1. P,M,T. pmt, ...Salzberg...Sim. Md. 29. P200. L584[2]. H53[173]. S[145].
3. 4. P,M,T,mt_2,m_2t.pm_2,p_2m,pt_2,p_2t, ... Similar Model 45. S[146]. D[10].
4. 1. MT.PM,PT,..............Berchtesgaden......Model 63. L584[4]. D[7].

15. CHLORIDE OF SILVER. Hornerz. Argent muriatée.
Cleavage = 0.
1. 1. P,M,T,...Peru, Cornwall...Md. 1. H iii 294[1]. D[1]. L581[1]. J ii 351[1a].
1. 1. P_+,M,T, ...Poland......Model 3. H iii 294[1]. L581[1].
2. 1. PMT,............Siberia......Model 15. D[4]. S268[2]. L581[5]. J ii 351[2].
3. 1. P,M,T,mt.pm,pt, ...Joh. Georgenstadt...Md. 27. D[5]. L581[2]. J ii 351[1b]. S[253].
3. 1. P,M,T.pmt, ...Cornwall...Sim. Md. 29. S[259]. L581[4]. D[2]. J ii 351[1c].
3. 1. p,m,t,MT.PM,PT, ...Model 28. D[6].
3. 1. p,m,t.PMT, ...Cornwall......Model 30. D[3]. L581 .
4. 1. MT.PM,PT,......Siberia......Md. 63. D[7]. L581[3]. J ii 351[3]. S268[4].

16. *An Isomorphous Group,* 1 *and* 2:—
1. FLUORSPAR. Flusspath. Chaux fluatée.
Cleavage = p,m,t,mt.pm,pt,PMT.
1. 1. P,M,T, Kongsberg.Derbyshire. Md.1. L574[5]. P173[3]. R[13]. H27[2].J[155].
2. 1. PMT, Derbyshire. Puy-de-Dôme. Md.15. P173[1].H27[1]. L574[1]. J[159].
2. 1. 3 P_3MT, ...Similar Model 17. L575[12].
2. 1. PMT, 3$p_3$mt, ...Kongsberg......R[28]. Mr[26].
3. 1. P,M,T,mt.pm,pt,......Derbyshire, Drammen...Md. 27. H28[14]. R[17]. L575[9]. J[156].
3. 1. P,M,T,mt.pm,pt,pmt,...............................Model 31. P173[4].
3. 1. P,M,T,mt.pm,pt, 3p⅓mt,...P173[10]. R[19a].
3. 1. P,M,T.PMT,...Derbyshire, Zinnwald...Md. 29. H28[10]. L574[4]. J[158].
3. 1. P,M,T. 3p½mt,......Saxony......Model 39. P173[9]. H28[12]. J ii 591[7].

3. 1. P,M,T.6 $p\frac{1}{4}m\frac{1}{2}t$,...Münstherthal Baden, Derbyshire...Sim. Md. 40. P173[11]. H28[15]. R[30]. Mr[27]. J ii 591[8]. L575[11].
3. 1. P,M,T,MT.PM,PT, $3p\frac{1}{2}mt$, $6p\frac{1}{4}m\frac{1}{2}t$, $6p\frac{1}{11}m\frac{1}{3}t\frac{1}{5}$.........England...R[198].
3. 1. p,m,t.PMT,..Model 30. P173[2].
3. 1. P,M,T,mt.pm,pt,PMT, Cornwall, Zinnwald. Md. 33. H28[16]. L575[8].
3. 1. p,m,t,MT.PM,PT,PMT, $3p\frac{1}{2}mt$, ...D185[1].
3. 4. P,M,T,mt_3,m_3t.pm_3,p_3m,pt_3,p_3t,...Alston Moor. Zinnwald. Durham. Md. 45. P173[7]. R[21]. H28[13]. L575[7]. J[161].
3. 4. P,M,T,mt,mt_3,m_3t.pm,pm_3,p_3m,pt,pt_3,p_3t, ...H29[19].
3. 4. P,M,T,mt_3,m_3t.pm_3,p_3m,pt_3,p_3t, $6p\frac{1}{4}m\frac{1}{2}t$, ...Münsterthal...L575[10].
3. 4. P,M,T,mt,mt_3,m_3t.pm,pm_3,p_3m,pt,pt_3,p_3t, $3p\frac{1}{2}mt$,...............H29[20].
3. 4. P,M,T,MT,$mt\frac{3}{2}$,$m\frac{3}{2}t$.PM,$pm\frac{3}{2}$,$p\frac{3}{2}m$,PT,$pt\frac{3}{2}$,$p\frac{3}{2}t$,PMT,...England...R[214].
3. 4. p,m,t. $6P\frac{1}{4}M\frac{1}{2}T$, ...P173[12].
3. 4. P,M,T,mt,$mt\frac{3}{2}$,mt_{10},$m\frac{3}{2}t$,$m_{10}t$.pm,$pm\frac{3}{2}$, pm_{10},$p\frac{3}{2}m$,$p_{10}m$,pt,$pt\frac{3}{2}$,pt_{10},$p\frac{3}{2}t$, $p_{10}t$,pmt, $3p\frac{1}{2}mt$, 5 $(6\,p_xm,t_z)$.*
3. 4. p,m,t,MT_3,M_3T.PM_3,P_3M,PT_3,P_3T,...Zinnwald...Sim. Md. 45. R[23].
4. 1. MT.PM,PT, ...Châlons...Model 63. P173[6]. H28[9]. L574[3]. J[157].
4. 1. mt.pm,pt,PMT, ...Saxony...Model 64. P173[5]. H28[11]. L574[2]. J[160].
4. 3. MT_3,M_3T.PM_3,P_3M,PT_3,P_3T,......Derbyshire......Sim. Model 68. P173[8]. L574[6]. H27[4]. J[162].
4. 4. mt.pm,pt,PMT,$3p\frac{1}{2}mt$,....................................H28[18]. L575[14].
4. 4. mt,mt_3,m_3t.pm,pm_3,p_3m,pt,pt_3,p_3t,PMT, ...H28[17]. L575[13].

* A crystal from Devonshire, in the possession of W. Phillips, exhibited all these faces, in number 338. The figure shown in his *Mineralogy*, page 174, is an imaginary crystal, containing 434 planes, as follow:—

P,M,T,MT, $3mt_+$, $3m_+t$. PM, $3pm_+$, $3p_+m$, PT, $3pt_+$, $3p_+t$, PMT, 4 $(3p\_mt)$, 5 $(6p_xm,t_z)$.

The measurements given are not sufficient to supply a more significant symbol for either of these complex combinations.

2. Yttrocerite. Yttrocerit.

2. 1. PMT? ....Model 15. R162[16].

17. Red Oxide of Copper. Rothkupfererz. Cuivre oxidulé.

Cleavage = p,m,t. PMT.

1. 1. P,M,T,...........Moldava......Model 1. J[176]. P317[4]. H99[124]. L567[4].
2. 1. PMT,.........Siberia, Cornwall, Chessy......Model 15. J[177]. P317[1]. H99[123]. L566[1].
2. 1. PMT, $3p_3mt$,..P317[9]. J[181]. L567[10].
2. 1. 3 P_3MT, ...Gumeschewskoi......Sim. Model 17. J[182]. L567[11].
2. 4. PMT, $3p\frac{1}{2}mt$, ...P317[10]. J[83]. L567[5].
2. 4. PMT, $6p_xm_yt_z$, ...J_{184}. L567[7].
3. 1. p,m,t,MT.PM,PT, ...Model 28. H100[128].
3. 1. P,M,T,mt.pm,pt,pmt,..................................Model 31. P317[3].
3. 1. p,m,t,mt.pm,pt,PMT, ...Cornwall...Model 33. H100[130]. L567[12].
3. 1. p,m,t,MT.PM,PT,PMT, $3p\frac{1}{2}mt$, ...Gumeschewskoi......R[180].
3. 1. p,m,t.PMT, ...Cornwall...Model 30. J[175]. P317[2]. H99[126]. L567[3].

3. 4. P,M,T.PMT, 3p½mt, ...Gumeschewskoi......R[16a].
3. 4. p,m,t,mt,mt₂,m₂t.pm,pm₂,p₂m,pt,pt₂,p₂t,PMT, ...J[180]. L567[13].
3. 4. P,M,T,MT,mt₅,m₅t.PM,pm₅,p₅m,PT,pt₅,p₅t,PMT, ...R[32].
3. 4. p,m,t,MT,mt₂,m₂t.PM,pm₂,p₂m,PT,pt₂,p₂t, PMT, 3p½mt,3p₃mt,6pₓmᵧtᵤ.
P317[11]. L567[14].
4. 1. MT.PM,PT, ...Chessy......Model 63. J[178]. P317[8]. H99[125]. L567[9].
4. 1. MT.PM,PT, pmt, ...Model 65. P317[7].
4. 1. mt.pm,pt,PMT,...Chessy...Model 64. J[177]. P317[6]. H100[127]. L567[8].
4. 4. MT.PM,PT,PMT, 3p₃mt, ...Gumeschweskoi ......R[3a].
4. 4. mt₂,m₂t.pm₂,p₂m,pt₂,p₂t,PMT, ...J[179]. L567[6].
5. 3. $M\frac{7}{10}T.P\frac{7}{10}T$. Axes: p‡ m± t°,...........................J[174]. L566[1].
5. 3. $M\frac{7}{10}T.\frac{1}{2}P\frac{7}{10}T$. Axes: p‡ m± t°,................P317[2]. J[173]. L566[2].
6. 2. ¼PMT Znw,Nse, ¾pmt,...J[172].

18. Oxide of Arsenic. Arsenikblüthe. Arsenic oxidé.
Cleavage = PMT.
2. 1. PMT, ...Model 15. P281.
5. 3. $M\frac{7}{10}T.P\frac{7}{10}T$. Axes: m‡ t± t°, ...L334.

19.* *An Isomorphous Group*, 1, 2. 3 :—

1. Bright White Cobalt. Kobaltglanz. Cobalt gris. Cobaltine. Silver White Cobalt.
Cleavage = P,M,T,mt₂.pm₂,p₂t.
1. 1. P,M,T, ...Hokanbo, Tunaberg...Md. 1. P285[1]. L654[3]. H iv 228[1].
2. 1. PMT,.........Tunaberg...Model 15. L654[5]. J[260]. P285[4]. H iv 228[2].
3. 1. P,M,T.pmt, ...Tunaberg...Sim. Mod. 29. P285[2]. L655[7]. M ii 456.
3. 1. P,M,T.PMT,........................Tunaberg......Model 29. L655[7].
3. 1. p,m,t.PMT,.........Tunaberg...Model 30. P285[3]. L655[7]. M ii 456.
3. 4. P,M,T,mt₂.pm₂,p₂t,.........Tunaberg......Sim. Md. 47. R[53]. L654[2].
3. 4. P,M,T,mt₂.pm₂,p₂t,pmt,...Tunaberg, Sim. Md. 48. L655[6]. R[54]. Mr[31].
3. 4. P,M,T,MT₂.PM₂,P₂T, ...Tunaberg...Model 47. P285[5]. L654[2].
3. 4. P,M,T,MT₂,mt₄.PM₂,pm₄,P₂T,p₄t,PMT,3p⅔m½t.........Mr[32]. P285[9].
5. 4. MT₂.PM₂,P₂T, ...Hokanbo, Tunaberg...Model 91. P285[6]. L654[1].
5. 4. MT₂.PM₂,P₂T,PMT,.........Tunaberg...Model 92. L654[4]. P285[7].
5. 4. mt₂.pm₂, p₂t, PMT, ...Tunaberg......Model 93. P285[8]. R[48].

2. Nickel Glance. Nickelglanz. Sulpho-Arsenide of Nickel.
Cleavage = P,M,T.
1. 1. P,M,T,..............................Sweden, Hartz......Model 1. D[400].
3. 1. P,M,T.pmt, ...Hartz......Similar Model 29. S81.

3. Sulpho-Antimonite of Nickel. Nickeliferous Grey Antimony. Nickelantimonglanz.
Cleavage = P,M,T.
1. 1. P,M,T, ...Hartzgerode......Model 1. T ii 531.
2. 1. PMT, ...Model 15. T ii 531.
3. 1. P,M,T.pmt,..............................Similar Model 29. T ii 531.

20. PURPLE COPPER. Buntkupfererz. Variegated Copper.
Cleavage = pmt.

1. 1. P,M,T, ... Model 1. D[1].
2. 1. PMT, ...Cornwall...... Model 15. S254[2].
2. 1. PMT × 2,.................................Model 16. L643[2]. D[129].
3. 1. P,M,T.pmt, ... Cornwall...... Similar Model 29. P310. D[2].
3. 1. p,m,t.PMT, ...Model 30. L643[1]. D[3]. S254[3].

21. *An Isomorphous Group*, 1, 2, 3, 4, 5, 6:—

1. SPINEL. Spinell. Alumine magnésiée.
Cleavage = pmt.

2. 1. PMT, ...Ceylon......Model 15. J[12]. L542[1]. H51[151]. P81[1]. R[1].
2. 1. PMT × 2, ...Model 16. L542[6]. J[16]. P81[2]. H51[154].
2. 1. PMT, 3p,mt, ...S202[2]. D[21].
4. 1. MT.PM,PT,...................Model 63. L542[3]. H52[156]. P81[5]. J[19].
4. 1. mt.pm,pt,PMT, ...Ceylon... Md. 64. L542[2]. H52[157]. P81[3]. R[2]. J[17].
4. 1. MT.PM,PT,PMT, 3p$\frac{1}{3}$mt, ...P82[1]. L542[4].
4. 1. MT.pm,pt. Axes: p$\frac{+}{}$m[a]t[a],...J[18].
4. 4. MT.PM,PT,PMT, 3p$\frac{1}{3}$mt,...H52[155].
5. 3. M$\frac{7}{10}$T. P$\frac{7}{10}$T. Axes: p$\frac{a}{+}$m$\frac{a}{-}$t[a], ... J[20]. H ii 168[a]. L542[1].
6. 1. $\frac{1}{2}$PMT, ... Model 117. J[15].
6. 1. $\frac{1}{2}$PMT, $\frac{1}{2}$pmt,...Model 118. J[14].
6. 1. ($\frac{1}{2}$PMT, $\frac{1}{2}$pmt) × 2,* ....J i 44[7].
6. 2. $\frac{1}{4}$PMT Znw, Nse, $\frac{3}{4}$ pmt,* ....J i 45[12].

* Many of the crystals of spinel are segments of the combinations PMT, $\frac{1}{2}$PMT, and $\frac{1}{2}$PMT,$\frac{1}{2}$pmt, the sections of these forms being always parallel to a plane, and the segments occurring either singly or combined, two, three, or more together, in variously inverted positions. See P81$_4$. J i p. 44, and H ii p. 168. It would be useless to give symbols for such irregular combinations. Their plane angles show the character of the crystallisation, and their interfacial and re-entering angles distinguish the octahedral from the tetrahedral forms.

2. PLEONASTE. Zeilanit. Black Spinel. Spinelle noir. Ceylanite.
Cleavage = PMT.

2. 1. PMT, ...Amity, N.Y. Ceylon...Md. 15. J[9]. H ii 170[1]. P83[1]. L543[1].
2. 4. PMT,3$\frac{1}{3}$mt,..J[11].
3. 1. p,m,t,MT.PM,PT,PMT,3p$\frac{1}{3}$mt.........Hamburgh, New Jersey...S[416].
4. 1. MT.PM,PT,.........................Model 63. J[12]. H ii 170[2]. P83[3].
4. 1. MT.PM,PT,pmt, ...Model 65. P83[2].
4. 1. mt.pm,pt,PMT, ... Model 64. S202[1]. J[10]. L543[2]. H ii 170[5].
4. 4. MT.PM,PT,PMT, 3p$\frac{1}{3}$mt, ...H ii 170[4]. pl. 52[158].
4. 4. mt.pm,pt,PMT, 3p$\frac{1}{3}$mt, ...Vesuvius......R[10]. L543[4].
4. 4. MT.PM,PT,PMT, 3p$\frac{1}{3}$mt, 3p,mt, ...P83[4]. L543[5].

3. AUTOMALITE. Gahnite. Zinciferous Spinel.
Cleavage = PMT.

2. 1. PMT, ...Model 15. L544. H ii 171. J89[1].
2. 1. PMT × 2, ... Model 16. H ii 171. D[129].

6. 1. ½PMT, ½pmt, ...Model 118. J i 39[2].
6. 1. ½PMT × 2, ...J i 39[5].

4. MAGNETIC IRON ORE. Oxydulated Iron. Magneteisenerz. Fer oxidulé.
Cleavage = pmt.
1. 1. P,M,T, ...Arendal. Steyermark......Model 1. S20[4]. L553[6].
2. 1. PMT,...Fahlun. Piedmont...Md.15.L553[1]. H103[165]. P215[1]. R[1]. J[193].
2. 1. PMT × 2, ...Model 16. J iii 190[7]. L553[12]. H iii 562[c].
2. 1. 3 P⅔MT, ...Zillerthal ...... Similar Model 17. S282. L553[6].
2. 1. PMT, 3p⅔mt, ...J[196]. L553[5].
2. 4. PMT, 3p⅓mt, ...Traversella...... L553[9]. R[10a]. H iii 562[4].
3. 1. P,M,T.pmt, ...Similar to Model 29. J[198].
3. 1. p,m,t,MT.PM,PT, ...Model 28. M ii 400. L553[4].
3. 1. p,m,t,MT. PM,PT,pmt, 3p⅓mt, ... L553[11].
3. 1. p,m,t,MT.PM,PT,PMT, 3p⅓mt, ...L553[11].
3. 1. p,m,t.PMT, ...Zillerthal......Model 30. J iii 189[c]. L553[7].
4. 1. MT.PM,PT, ...Piedmont...Model 63. H103[167]. L553[3]. P215[3]. R[4].
4. 1. MT.PM,PT,pmt, ...Normarken......Model 65. R[8].
4. 1. mt.pm,pt,PMT,... Traversella...Md.64. H103[166]. L553[2]. P215[2]. J[191].
4. 1. MT.pm,pt. Axes: p$_{+}^{4}$m^{2}t^{2}, ...J[197].
4. 4. MT.PM,PT, 3p⅓mt, ...Zillerthal......Model 69. L553[10].
4. 4. MT.PM,PT,PMT, 3p⅓mt, ...Piedmont......R[9]. Mr[29].
4. 4. mt.pm,pt,PMT, 3p⅓mt, ... Piedmont...... L553[10].
5. 3. M$\frac{7}{10}$T.P$\frac{7}{10}$T. Axes: p$_{+}^{4}$ m$_{-}^{2}$ t^{2}, ... L553[1].
5. 3. M$\frac{7}{10}$T. ½P$\frac{7}{10}$T, Zw, Ne. Axes: p$_{+}^{4}$ m$_{-}^{2}$ t^{2}... Bournon...L553 note. J iii 189[1].
6. 1. ½PMT, ½pmt, ...Model 118. J iii 190[5].

5. FRANKLINITE. Franklinit. Zinc oxidé ferrifère.
Cleavage = pmt.
3. 1. p,m,t,mt.pm,pt,PMT, ...Model 33. L551[2].
4. 1. mt.pm,pt,PMT, ...Model 64. M ii 403[1]. L551[1]. P219.
4. 1. mt.pm,pt,PMT,3p$_{+}$mt, ...M ii 403[2].

6. CHROMATE OF IRON. Chromeisenerz. Fer chromaté.
Cleavage = pmt.
2. 1. PMT, ...Baltimore......Model 15. L558. P275. M ii 396.
4. 1. mt.pm,pt,PMT, ... Hoboken, New Jersey...Model 16. S[131]. D[9].

22.** *An Isomorphous Group*, 1, 2:—
1. BORACITE. Borazit. Borate of Magnesia. Magnésie boratée.
Cleavage = pmt.
1. 1. P,M,T, ...Segeberg, Holstein......Model 1. H46[101]. M ii 348.
3. 1. P,M,T,MT.PM,PT,½PMT,½pmt, ...Luneberg......Model 35. R[39].
3. 1. P,M,T,mt.pm,pt,½pmt, ...Luneberg... Md. 36. R[40]. H46[103]. D347[1].
3. 1. P,M,T,MT.PM,PT,pmt, ½ (3p⅓mt)Z^{2}nw, ... P187. H46[104].

3. 1. P,M,T,MT.PM,PT,$\frac{1}{2}$PMT Znw, $\frac{1}{2}$pmt Zne, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2ne, $\frac{1}{2}$(6p$\frac{1}{3}$m$\frac{1}{3}$t) Z^3n^2w,.........Luneberg......R[39 a,b].

3. 1. p,m,t,MT.PM,PT,$\frac{1}{2}$pmt, ... Luneberg ......R[42]. D347[1].

3. 1. P,M,T,MT.PM,PT, $\frac{1}{2}$PMT Znw, $\frac{1}{2}$PMT Zne, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2nw, ... Luneberg......R[41].

3. 1. p,m,t,MT.PM,PT, $\frac{1}{2}$pmt Znw, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2ne, ... H46[105].

3. 1. P,M,T,MT.PM,PT,$\frac{1}{2}$PMT Znw, $\frac{1}{2}$ (6p$\frac{1}{4}$m$\frac{1}{3}$t) Zne,............... H46[106].

3. 1. p,m,t.$\frac{1}{2}$PMT, ...Luneberg......R[37].

3. 1. p,m,t,mt.pm,pt, $\frac{1}{2}$PMT, ... Luneberg...Model 37. R[38]. L288[1]. Mr[38].

3. 1. p,m,t,mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$pmt, ... L288[2].

4. 1. MT.PM,PT, $\frac{1}{2}$pmt, ... H46[102].

2. Rhodizite. Rhodizit. G. Rose, Pogg. Ann. xxxiii. 253.

4. 1. MT.PM,PT, ...Siberia......Model 63. D[7]. R164.

4. 1. MT.PM,PT,pmt, ...Siberia......Model 65. D[8]. R164.

4. 1. MT.PM,PT, $\frac{1}{2}$pmt, ...Siberia...... G. R. in Pogg.

4. 1. mt.pm,pt,PMT, ...Siberia......Model 64. D[9]. R164.

23.** Fahlerz:—*An Isomorphous Group*, 1, 2, 3:—

1. Arsenical Grey Copper. Arsenikfahlerz. Tennantite. Cleavage = mt.pm,pt.

3. 1. P,M,T,mt.pm,pt,pmt, ...Model 31. P313[2]. S[431].

3. 1. p,m,t,MT.PM,PT,pmt, ...Model 34. P313[3]. S[432].

3. 1. P,M,T,MT.PM,PT,PMT, 3p$\frac{1}{2}$mt, 3p$\frac{3}{4}$mt, ... P313[6].

3. 1. p,m,t.PMT,.......................................Model 30. P313[1]. L604[1].

3. 1. p,m,t,mt.pm,pt,PMT, ...Model 33. L604[2].

4. 1. MT.PM,PT, ... Model 63. P313[4]. L604[5].

4. 1. mt.pm,pt,PMT, ... Model 64. L604[3].

4. 4. MT.PM,PT, 3P$\frac{1}{2}$MT,...............................Model 69. P313[5].

2. Mixed Grey Copper. Vermischtes Fahlerz. Cuivre gris. Panabase. Cleavage = $\frac{1}{2}$ pmt.

3. 1. p,m,t. $\frac{1}{2}$ PMT, ... H97[103]. L648[3].

3. 1. p,m,t,mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$pmt, ...H98[109]. L648[11].

3. 4. p,m,t. $\frac{1}{2}$PMT, $\frac{1}{2}$(3p$\frac{1}{2}$mt)..............................Anhalt......L648[10].

3. 4. p,m,t,mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$(3p$\frac{1}{2}$mt)...................................L648[6].

3. 4. p,m,t,mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$ pmt, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2nw, ...H98[112]. L648[13].

3. 4. p,m,t,mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$ (3P$\frac{1}{2}$MT) Z^2nw, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2ne,...L648[15].

3. 4. p,m,t,mt.pm,pt, $\frac{1}{2}$ (3P$\frac{1}{2}$MT) Z^2nw, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2ne, $\frac{1}{2}$(3P$\frac{1}{2}$MT)Z^2nw, L648[16].

3. 4. P,M,T, mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$pmt, $\frac{1}{2}$ (3P$\frac{1}{2}$MT) Z^2nw, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2ne, ... P312. L648[17].

4. 1. MT.PM,PT, ...Model 63. J[219].

5. 1. mt.pm,pt, $\frac{1}{2}$PMT,...Kapnik. Dillenberg. Md.78. R[32]. H97[104]. L684[4].

5. 4. mt.pm,pt,$\frac{1}{2}$PMT, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2nw,............Felsobanya, Clausthal, Schemnitz......Model 94. R[33]. H98[107]. L648[5].

5. 4. mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$PMT, $\frac{1}{2}$(3P$\frac{1}{2}$MT) Z^2nw, $\frac{1}{2}$(3p$\frac{1}{2}$mt) Z^2ne,...H98[111].

5\. 4. mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$pmt, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2nw, ... H98[105]. L648[12].
5\. 4. MT.PM,PT, $\frac{1}{2}$PMT, $\frac{1}{2}$ (3P$\frac{1}{2}$MT) Z^2nw, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2ne,...Dillenberg... R[33a]. H98[110]. L648[14].
5\. 4. mt.pm,pt, $\frac{1}{2}$PMT, $\frac{1}{2}$ (3P$\frac{1}{2}$MT) Z^2nw, $\frac{1}{2}$ (6p$\frac{1}{2}$m$\frac{1}{3}$t) Z^3n^2w,...Ilanz...R[33b].
5\. 4. MT.PM,PT, $\frac{1}{2}$ (3P$\frac{1}{2}$MT) Z^2nw, $\frac{1}{2}$ (3p$\frac{3}{2}$mt) Z^2n^2w, ... Dillenberg...R[34].
6\. 1. $\frac{1}{2}$PMT,.........Heidelberg......Model 117. R[25]. H97[100]. J[213]. L648[1].
6\. 1. $\frac{1}{2}$PMT,$\frac{1}{2}$pmt,......Kapnik......Model 118. H97[102]. R[31]. J[214]. L648[2].
6\. 1. ($\frac{1}{2}$PMT, $\frac{1}{2}$pmt) × 2, ... L648[18].
6\. 1. $\frac{1}{2}$PMT Znw, $\frac{1}{2}$ (3p$\frac{1}{2}$mt) Z^2ne, ... H97[105]. L648[7].
6\. 3. $\frac{1}{2}$(3P$\frac{1}{2}$MT)......Clausthal...Sim. Md. 119. H97[101]. R[29]. J[217]. L648[9].
6\. 4. $\frac{1}{2}$PMT, $\frac{1}{2}$ (3p$\frac{1}{2}$mt), $\frac{1}{2}$ (3p$\frac{1}{3}$mt) ....Clausthal......R45.
6\. 4. $\frac{1}{2}$PMT Znw, $\frac{1}{2}$(3P$\frac{1}{2}$MT) Z^2nw, ... R[28]. J[216]. H97[106]. L648[8].

3\. ANTIMONIAL GREY COPPER. Antimonfahlerz. Schwartzerz. Black Copper. Cuivre gris arsenifère.
Cleavage = 0.
3\. 1. p,m,t,mt.pm,pt,$\frac{1}{2}$PMT, ... Model 37. J iii 321[b].
3\. 1. p,m,t,mt.pm,pt,$\frac{1}{2}$PMT,$\frac{1}{2}$pmt,..................................J iii 321[c].
4\. 1. MT.PM,PT, ... Model 63. J iii 321[2].
4\. 1. MT.PM,PT,$\frac{1}{2}$pmt, ...P313[7].
6\. 1. $\frac{1}{2}$PMT,...Model 117. J iii 321[a].

24\. SULPHURET OF TIN. Zinnkies. Etain sulfuré. Tin Pyrites.
Cleavage = p,m,t,mt.pm,pt.
1\. 1. P,M,T ? ....Cornwall......Model 1. M iii 163. L624.

25\. GARNET. Granat. Grenat: comprehending the following species:—
1. Almandine. Precious Garnet. Edler Granat.
2. Cinnamon Stone. Kaneelstein.
3. Grossular. Green Garnet. Aplome.
4. Common Garnet. Gemeiner Granat.
5. Melanite. Black Garnet. Pyreneite.
6. Manganesian Garnet. Mangangranat.
7. Rothoffite. Colophonite. Brown Garnet.

Cleavage = MT.PM,PT.
2\. 3. 3 P$\frac{1}{2}$MT,......Arendal......Model 22. H61[39]. L488[4]. J[57]. P14[4]. R[6].
2\. 3. 6 P$\frac{1}{7}$M$\frac{1}{3}$T,.........Mussa in Piedmont.......Similar Model 23. P18.
3\. 1. p,m,t,MT.PM,PT, ... Czilklowa, Siberia...Model 28. L488[2]. P16.
4\. 1. MT.PM,PT,......Arendal......Model 63. H60[36]. P14[1]. R[4]. L488[1].
4\. 1. MT.pm,pt. Axes: p$\frac{1}{4}$ m[a] t[a],..................................L488[1] note.
4\. 4. MT.PM,PT, 3p$\frac{1}{2}$mt,......Melanite from Frascati......Sim. Md. 69. J[58]. H61[40]. P14[2]. R[5]. L488[3].
4\. 4. MT.PM,PT, $\frac{1}{2}$(3p$\frac{1}{2}$mt).......................................L488[3] note.
4\. 4. MT.PM,PT, 3P$\frac{1}{2}$MT, 3p$\frac{3}{2}$mt,......................Brossothal......R[5a].
4\. 4. MT,mt$\frac{3}{2}$,m$\frac{3}{2}$t.PM,pm$\frac{3}{2}$,p$\frac{3}{2}$m,PT,pt$\frac{3}{2}$,p$\frac{3}{2}$t, 3p$\frac{1}{2}$mt,...Friedberg...R31.
4\. 4. mt.pm,pt, 3P$\frac{1}{2}$MT,...Grossular from Siberia...Md. 69. P14[3]. R24.
4\. 4. MT.PM,PT, 3P$\frac{1}{2}$MT, 6p$\frac{1}{3}$m$\frac{1}{2}$t, ...Arendal...R[11]. L488[5]. Mr[30].

4. 4. MT.PM,PT, 3P$\frac{1}{2}$MT, 6p$\frac{1}{4}$m$\frac{1}{3}$t, ... Ciklowa......R35.
4. 4. MT,mt$_2$,m$_2$t. PM,pm$_2$,p$_2$m, PT,pt$_2$,p$_2$t, 3P$\frac{1}{2}$MT, ...Dognatzka......
L488[6]. R[5b]. J[80]. H61[43].

26. LEUCITE. Leucit. Amphigène.
Cleavage = p,m,t,mt.pm,pt.

2. 1. PMT, ...Model 15. L435[2].
2. 3. 3 P$\frac{1}{2}$MT, ...Model 22. R[6]. L435[3]. P105. H78[318].
3. 1. P,M,T.pmt,..................................Similar Model 29. L435[1].

27. SODALITE. Sodalit.
Cleavage $\stackrel{.}{=}$ MT.PM,PT.

3. 1. p,m,t,MT.PM,PT, 3p$\frac{1}{2}$mt,........................Vesuvius...... L461[2].
4. 1. MT.PM,PT,......Vesuvius. Greenland...Model 63. L461[1]. P134.

28.** BISMUTH BLENDE. Wismuthkieselerz. Arsenical Bismuth.
Cleavage = mt.pm.pt.

6. 1. $\frac{1}{2}$PMT, ...Model 117. D[33].
6. 3. $\frac{1}{2}$ (3P$\frac{1}{2}$MT) .... Similar Model 119. P279. D[34].
6. 4. $\frac{1}{2}$PMT Znw, $\frac{1}{2}$ (3P$\frac{1}{2}$MT) Z'nw, ...D[35].

29. ANALCIME. Cubicite. Analcim. Kubizit. Sarcolite.
Cleavage = p,m,t.

1. 1. P,M,T, ... Model 1. L202[1]. P138[1]. J[78]. H85[387].
2. 3. 3P$\frac{1}{2}$MT, ...Kilpatrick Hills...Md. 22. H85[388]. L202[3]. D[16]. J[80].
R[6]. P138[3].
2. 4. 3 P$\frac{1}{2}$MT, 3P$\frac{1}{3}$MT,...........................Catania ...:.. Levy 45[2].
3. 1. P,M,T. 3p$\frac{1}{2}$mt, ...Model 39. H85[389]. L202[2]. D[14]. J[79]. P138[2]. R[19].
3. 1. P,M,T. 3P$\frac{1}{2}$MT, ...Similar Model 39. L202[2]. P139[1].
3. 1. P,M,T,mt.pm,pt,PMT, (Sarcolite) .... Model 31. P139[2].
3. 4. p,m,t. 3 P$\frac{1}{2}$MT, ...D[15].

30. ARSENIATE OF IRON. Wurfelerz. Cube Ore. Fer Arseniaté.
Cleavage = p,m,t.

1. 1. P,M,T, ...Model 1. J ii 341[1]. L165[1]. P235[1].
3. 1. P,M,T,mt.pm,pt,........................Model 27. J ii 342[3]. L165[2].
3. 1. P,M,T,mt pm,pt,pmt, ...Model 31. J ii 342[4]. L165[5]. P235[4].
3. 1. P,M,T.pmt, ... Similar Model 29. L165[3].
3. 1. P,M,T. $\frac{1}{2}$pmt,.........Cornwall......Model 38. R[37]. P235[2]. J ii 342[2].
3. 1. P,M,T,mt.pm,pt, $\frac{1}{2}$pmt,........................Cornwall......Levy 70[4].
3. 1. P,M,T.pmt, 3p$_2$mt,..L165[4].
3. 1. P,M,T. $\frac{1}{2}$pmt, $\frac{1}{2}$ (3p$_2$mt)..P235[3].
6. 3. $\frac{1}{2}$ (3P$\frac{1}{46}$MT), ... Cornwall ...... Levy 70[2]. P235[5].

Rose has not marked this mineral with **, as having hemihedral forms with inclined faces, although he has given an example of such a form.

31. ALUM. Alaun. Alun. Alumine sulfatée.
Comprehends the following varieties:—
1. Potash Alum.
2. Ammonia Alum.
3. Soda Alum.

Cleavage = pmt.

1. 1. P,M,T, ...Model 1. H98[124].
2. 1. PMT, ...Model 15. H48[125].
2. 1. PMT × 2, ...Model 16. H ii 117[5].
3. 1. P,M,T.pmt, ...Similar Model 29. H48[126].
3. 1. P,M,T, mt. pm,pt,PMT, ...Model 33. H48[126].

The forms of factitious crystals of alum are often very irregular, presenting any number and combination of the planes of the cube and rhombic dodecahedron, subordinate to the planes of the octahedron. The following symbols represent some of these combinations:—

$\frac{1}{2}$ mt nw,ne. PMT.
P. $\frac{1}{4}$ pm Zn, PMT.
p, $\frac{1}{2}$ m n, $\frac{1}{2}$ t e, $\frac{1}{2}$mt ne, se. PMT.
p,m,t. $\frac{1}{4}$ PM Zn, $\frac{1}{4}$ pm Nn, $\frac{1}{4}$ PT Nw, $\frac{1}{4}$ pt Zw, PMT.

These combinations prove that, although there is no substantive crystal, which can be called a hemi-rhombic-dodecahedron, yet the planes of the three zones of that combination, namely, the forms MT, PM, and PT, are all susceptible of producing either hemihedral or tetartohedral forms.

In the same manner, the crystals of commercial alum present all kinds of odd numbers of the planes of the cube.

32.** HELVINE. Helvin.

Cleavage = $\frac{1}{2}$pmt, or 0.

5. 1. mt.pm,pt,$\frac{1}{2}$PMT, ...Model 78. L463[3].
6. 1. $\frac{1}{2}$PMT, ... Model 117. R[25]. L463[1].
6. 1. $\frac{1}{2}$PMT, $\frac{1}{2}$pmt, ... Schwartzenberg......Model 118. L463[2]. P243.

33. *An Isomorphous Group, comprehending:*

1. LAPIS LAZULI. Lazurstein. Azurestone.
2. HAÜYNE. Haüyn.
3. NOSIAN. Nosin. Spinellane.

Cleavage = mt.pm,pt.

2. 1. PMT, ...Model 15. L457[4].
3. 1. p,m,t,MT.PM,PT, ...Model 28. L457[5].
3. 1. p,m,t,MT.PM,PT, 3p$\frac{1}{2}$mt, ... L457[6]
4. 1. MT.PM,PT, ...Model 63. P131. H iii 56[1]. R[4]. P111. L457[1].
4. 1. MT.PM,PT,pmt, ... Model 65. L457[3].
4. 1. MT.PM,PT, 3p$\frac{1}{2}$mt, ...Model 64. H iii 56[2]. L457[2].

34. PYROCHLORE. Octahedral Titanium Ore.

Cleavage = 0.

2. 1. PMT, ...Norway......Model 15. P260.

35. PYROPE. Red Garnet.
Cleavage = MT.PM,PT.
4. 1. MT.PM,PT, ... Model 63. R174.

36. CANCRINITE, a variety of Sodalite.
Cleavage = mt.pm,pt.
4. 1. MT.PM PT,....................Siberia......Model 63. P134. R174.

37. UWAROWITE.
4. 1. MT.PM,PT,.....................................Model 63. P408. R174.

The reader is requested to make the following addition to the list of authorities given at page 14. The work in question came into my hands only a few days before the printing of this page, and, therefore, is not quoted in the previous section:—

Ly = *Léry, A.* Description d'une Collection de Minéraux, formée par M. Henri Heuland, et appartenant a M. Ch. Hampden Turner, de Rooksnest dans le Comté de Surrey. London, 1837, 3 vols. 8vo, with a folio Atlas.—The large figures refer to the plates, and the small figures to the subjects. The letter v = *variety*, and refers to descriptions given in the text.—This work contains about twelve hundred figures of crystals, apparently drawn with great accuracy, but it is published without a single measurement of their angles, which greatly detracts from its utility. There is a Table of Angles spoken of in the preface, but none printed in the book.

CLASS II.—MINERALS BELONGING TO THE PYRAMIDAL SYSTEM OF CRYSTALLISATION.

The AXES of all Combinations belonging to this Class are = $p^{a}_{x}\ m^{a}\ t^{a}$.
Different Combinations of the same Mineral have Axes which are sometimes = $p^{a}_{+}\ m^{a}\ t^{a}$, and sometimes = $p^{a}_{-}\ m^{a}\ t^{a}$, but never = $p^{a}\ m^{a}\ t^{a}$.
The FORMS are either equiaxed or unequiaxed. The following are those of most frequent occurrence :—

a.) Homohedral Forms.

	Forms of the North Zone.	Forms of the East Zone.	Forms of the Octahedral Zones.	Forms of the Prismatic Zones.
	P	P.	P.	M.
Obtuse Octahedrons.	$P\frac{1}{4}M$.	$P\frac{1}{4}T$.	$P\frac{1}{4}MT$.	T.
	$P\frac{2}{7}M$.	$P\frac{2}{7}T$.	$P\frac{1}{3}MT$.	MT.
	$P\frac{1}{3}M$.	$P\frac{1}{3}T$.	$P\frac{3}{8}MT$.	$M_{-}T, M_{+}T$.
	$P\frac{1}{2}M$.	$P\frac{1}{2}T$.	$P\frac{2}{5}MT$.	$M\frac{1}{3}T, M_{3}T$.
	$P\frac{5}{9}M$.	$P\frac{5}{9}T$.	$P\frac{5}{12}MT$.	$M\frac{1}{2}T, M_{2}T$.
	$P\frac{3}{5}M$.	$P\frac{3}{5}T$.	$P\frac{1}{2}MT$.	$M\frac{2}{3}T, M\frac{3}{2}T$.
	$P\frac{5}{8}M$.	$P\frac{5}{8}T$.	$P\frac{4}{7}MT$.	
	$P\frac{2}{3}M$.	$P\frac{2}{3}T$.	$P\frac{3}{5}MT$.	
	$P\frac{3}{4}M$.	$P\frac{3}{4}T$.	$P\frac{2}{3}MT$.	
	$P\frac{4}{5}M$.	$P\frac{4}{5}T$.	$P\frac{3}{4}MT$.	
	$P\frac{5}{6}M$.	$P\frac{5}{6}T$.	$P\frac{4}{5}MT$.	
			$P\frac{5}{6}MT$.	

	Forms of the North Zone.	Forms of the East Zone.	Forms of the Octahedral Zones.	Dioctahedrons which always occur subordinately.
	PM.	PT.	PMT.	
Acute Octahedrons.	$P\frac{10}{9}M$.	$P\frac{10}{9}T$.		$p\frac{1}{3}m\frac{1}{2}t$, $p\frac{1}{3}mt\frac{1}{2}$.
	$P\frac{3}{2}M$.	$P\frac{3}{2}T$.	$P\frac{5}{4}MT$.	$p\frac{1}{2}m\frac{1}{3}t$, $p\frac{1}{2}mt\frac{1}{3}$.
	$P\frac{5}{3}M$.	$P\frac{5}{3}T$.	$P\frac{4}{3}MT$.	$pm_{-}t_{+}$, $pm_{+}t_{-}$.
	P_2M.	P_2T.	$P\frac{3}{2}MT$.	p_2mt_3, p_2m_3t.
	$P\frac{9}{4}M$.	$P\frac{9}{4}T$.	$P\frac{5}{3}MT$.	p_3mt_2, p_3m_2t.
	$P\frac{5}{2}M$.	$P\frac{5}{2}T$.	P_2MT.	p_4mt_2, p_4m_2t.
	$P\frac{8}{3}M$.	$P\frac{8}{3}T$.	$P\frac{5}{2}MT$.	p_4mt_3, p_4m_3t.
	P_3M.	P_3T.	P_3MT.	p_5mt_3, p_5m_3t.
	P_4M.	P_4T.		$p_{+}mt_{-}$, $p_{+}m_{-}t$.
	M.	T.	MT.	$p_xm_yt_z$, $p_xm_zt_y$.

2.) *Hemihedral Forms.*

$\frac{1}{2}(M_{-}T, M_{+}T)$.	$\frac{1}{2}P\frac{1}{4}MT$.
$\frac{1}{4}MT$.	$\frac{1}{2}P\frac{1}{3}MT$.
$\frac{1}{2}P\frac{2}{3}M$.	$\frac{1}{2}P\frac{2}{3}MT$.
$\frac{1}{2}PMT$.	$\frac{1}{2}(p_xm_yt_z, p_xm_zt_y.)$
$\frac{1}{2}(3P\ MT)$.	$\frac{1}{2}(p\frac{1}{3}m\frac{1}{2}t, p\frac{1}{3}mt\frac{1}{2}.)$

The Table shows the order in which these Forms are arranged, when several of them occur together upon one Combination. The angle of P upon PM, or of PM upon M, is 135°, and all the forms quoted betwixt P and PM make with P a more obtuse angle than 135°, and appear upon a combination exactly in the order in which they are placed in the Table. On the other hand, the planes betwixt PM and M make with P a more acute angle than 135°, and correspondingly a more obtuse angle with M. In short, the series from P to M, from P to T, or from P to MT, represents the gradual passage from a horizontal to a vertical plane, the forms PM, PT, and PMT, which are equiaxed, constituting the middle points of each oblique series.

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1. CHLORIDE OF MERCURY. Quecksilberhornerz. Mercure muriaté.

Axes: $p_x^a\ m^a\ t^a$. Cleavage = m,t.

$P\frac{5}{2}M$ Zn on Nn : 136° Brooke. Cot. 68° = 0.404 = $P\frac{1.000}{0.404}M$ = $P\frac{5}{2}M$.

$P\frac{5}{6}M$ Zn on Mn : 129° 32′ = $P\frac{5}{6}M$ on m^a : 39° 32′ Cot. 1.21166 = $P\frac{10}{12}M$ = $P\frac{5}{6}M$.

$P\frac{2}{3}MT$ on MT = 119° 30′. 119° 30′ — 90° = 29° 30′ = cot. 1.7675.

1.7675 × sec 45° = 2.4996 = $P\frac{10}{25}MT$ = $P\frac{2}{3}MT$.

2. 1. $P\frac{5}{2}M$, $P\frac{5}{2}T$, ... J ii 357[3]. L580[2].

3. 1. P_{+},MT.$p\frac{5}{2}m$,$p\frac{5}{2}t$,$p\frac{5}{2}mt$. Factitious. Brooke, Annals Phil. Oct. 1823.

4. 1. M,T. $P\frac{5}{2}M$, $P\frac{5}{2}T$, ... J ii 356[1]. L580[3].

4. 1. MT. $P\frac{5}{2}M$, $P\frac{5}{2}T$, ... J ii 357[2]. L580[1]. A[65]. M ii 156[97].

4. 1. M,T. $p\frac{5}{2}m$, $p\frac{5}{2}t$, $p\frac{2}{3}mt$, ... L580[4].

4. 1. M,T,mt. $p\frac{5}{2}m$, $p\frac{5}{2}t$, $p\frac{2}{3}mt$, ... L580[5]. M ii 157[10?].
4. 1. M,T,mt. $p\frac{5}{2}m$, $p\frac{5}{6}m$, $p\frac{5}{2}t$, $p\frac{5}{6}t$, $p\frac{2}{3}mt$, ... D249. P380.

2. BRAUNITE. Brachytypous Manganese Ore, HAIDINGER. Edinburgh Journal of Science, January, 1826.

Axes: $p^a_x\, m^a\, t^a$. Cleavage = PMT.

PMT Znw on Nnw = 108° 39′. Haidinger. Cot. 54° 19½′ = 0.7179.

$0.7179 \times 1.4142 = 1.015 = P\frac{1.000}{1.015}MT = PMT.$

P_2MT Znw on Nnw = 140° 30′. Cot. 70° 15′ = 0.359.

$0.359 \times 1.4142 = 0.5077 = P\frac{1.0000}{0.5077}MT = P\frac{10}{5}MT = P_2MT.$

2. 1. PMT,P_2MT, ... Haidinger 2[18]. L759[2].
2. 3. PMT,P_2M_3T,P_2MT_3, ... Haid. 2[19] L759[4].
5. 1. p. PMT, ... Haid. 2[17]. L759[1].
5. 1. p. pmt, P_2MT, ... Haid. 2[20]. L759[3].

3. *An Isomorphous Group*, 1, 2:—

1. OXIDE OF TIN. Zinnstein. Etain Oxidé. Zinnerz.

Axes: $p^a_x\, m^a\, t^a$. Cleavage = m,t,mt. $P\frac{2}{3}M$,$P\frac{2}{3}T$.

$P\frac{2}{3}M$ Zn on Nn : 67° 52′ P. Cot. 33° 56′ : $1.4863 = P\frac{1.00}{1.49}M = P\frac{10}{15}M.$

$P\frac{2}{3}MT$ Znw on Nnw = 86° 58′ Haüy. Cot. 43° 29′ = 1.0544.

$1.0544 \times 1.4142 = 1.493 = P\frac{1000}{1493}MT = P\frac{10}{15}MT = P\frac{2}{3}MT.$

2. 1. $P\frac{2}{3}M$, $P\frac{2}{3}T$, (Axes: $p^a_2\, m^a_3\, t^a_3$) ....L355[1]. H112[253]. P250[1]. J[186].
3. 1. P_+,M,T, MT. $p\frac{2}{3}m$,$p\frac{2}{3}t$,$p\frac{2}{3}mt$, ...H112[262].
4. 1. MT. $P\frac{2}{3}M$,$P\frac{2}{3}T$, ...L355[2]. H112[254]. P250[2]. J[190].
4. 1. MT. $P\frac{2}{3}MT$, ... L355[6]. H112[255]. P250[4]. J[187].
4. 1. (MT. $P\frac{2}{3}MT$) × 2, ... Similar Model 62. H113[563]. J[193]. S[438].
4. 1. (M,T. $P\frac{2}{3}M$, $P\frac{2}{3}T$) × 2,................................Model 62. S[438].
4. 1. MT. $P\frac{2}{3}M$,$P\frac{2}{3}T$,p_3m_2t, p_3mt_2, ... J[191]. H112[257].
4. 1. MT. $P\frac{2}{3}MT$,p_3m_2t,p_3mt_2,....................................P250[5]. J[193].
4. 1. M,T,mt. $P\frac{2}{3}MT$,...........................Cornwall......D363[1]. S[435].
4. 1. M,T,mt. $p\frac{2}{3}m$,$p\frac{2}{3}t$,$P\frac{2}{3}MT$,...........Goshen, Massachusets,......S[436].
4. 1. m,t,MT.$P\frac{2}{3}MT$,...L355[7]. H112[256].
4. 1. MT.$P\frac{2}{3}M$,$P\frac{2}{3}T$, $p\frac{2}{3}mt$, ... P250[3].
4. 1. m,t,MT.$p\frac{2}{3}m$,$p\frac{2}{3}t$, $p\frac{2}{3}mt$, ... H112[260]. R[63]. J[189].
4. 1. MT. $p\frac{2}{3}mt$,P_3M_2T,P_3MT_2, ... J[192]. P250[6]. H112[258].
4. 1. MT. $p\frac{2}{3}m$,$p\frac{2}{3}t$,$p\frac{2}{3}mt$, P_3M_2T, P_3MT_2, ... H112[259].
4. 4. m,t,MT,mt_3,m_3t.$p\frac{2}{3}m$,$p\frac{2}{3}t$,$P\frac{2}{3}MT$, ... H112[261]. J[189].
4. 4. MT,$m\frac{3}{2}t$,$m\frac{2}{3}t$.$p\frac{2}{3}m$,$p\frac{2}{3}t$,$p\frac{2}{3}mt$,p_3m_2t,p_3mt_2, ...D363[2]. S[437].

2. RUTILE. Rutil. Titane oxidé.

Axes: $p^a_x\, m^a\, t^a$. Cleavage = m,t,MT.

$P\frac{2}{3}M$ on M : 122° 51′. 122° 51′ — 90° = 32° 51′. Cot. 1.5487 =

$P\frac{100}{155}M = P\frac{10}{15}M = P\frac{2}{3}M.$

$P\frac{2}{3}MT$ on MT : 132° 20′. 132° 20′ — 90° = 42° 20′. Cot. 1.0977.

$1.0977 \times 1.4142 = 1.5521 = P\frac{100}{155}MT = P\frac{10}{15}MT = P\frac{2}{3}MT.$

1. 1. $P\frac{5}{11}$,M,T. (Haüy's primitive form) Md. 2. L360[1]. P255[1]. H iv 333.
1. 1. $P\frac{1}{3}$,MT,...(Ditto, position reversed) ......Md. 2. H iv 234. L360[1].

2. 1. P$\frac{2}{3}$MT, ... H iv 235[1]. H117[310r].
3. 1. P$\frac{1}{3}$,m,t, MT. .... L360[3].
3. 1. P$\frac{1}{3}$,m,t,MT.p$\frac{2}{3}$mt, ... Aschaffenberg...... L360[4].
3. 1. P$\frac{1}{3}$,MT. p$\frac{2}{3}$mt, ... L360[7].
3. 4. P$\frac{1}{3}$, m$\frac{1}{2}$t,m$_2$t,MT, ... L360[5].
4. 1. MT.P$\frac{2}{3}$M,P$\frac{2}{3}$T, ... Aschaffenberg...... L360[9].
4. 1. MT. P$\frac{2}{3}$MT, ... L360[8].
4. 1. m,t,MT.P$\frac{2}{3}$MT, ... Aschaffenberg...... L360([4]). M ii 377[2].
4. 3. MT$_2$,M$_2$T.P$\frac{1}{3}$MT, ... H117[309]. L360[6].
4. 4. M,T,mt,mt$_3$,m$_3$t.p$\frac{2}{3}$m,p$\frac{2}{3}$t,P$\frac{2}{3}$MT, ... H117[310].
4. 4. m,t,MT,mt$_3$,m$_3$t.p$\frac{2}{3}$m,p$\frac{2}{3}$t,P$\frac{2}{3}$MT, ... S[385]. P255[2]. D359[2].
5. 1. (m,t,MT. $\frac{1}{2}$P$\frac{2}{3}$M) $\times$ 2, ... H117[311].
5. 3. (MT$_2$. $\frac{1}{2}$P$\frac{2}{3}$M) $\times$ 2, ... H117[312]. J[170].
5. 4. (MT,MT$_2$. $\frac{1}{2}$P$\frac{2}{3}$M) $\times$ 2, ... H117[313].
5. 4. (M,T,MT,mt ,m$_3$t. $\frac{1}{2}$P$\frac{2}{3}$M) $\times$ 2, ... P255[3]. S[386].

4. ANATASE. Anatas. Titane Anatase. Ootahedrite.

Axes: p_1^2 m^2 t^2. Cleavage = P. P$\frac{5}{2}$M, P$\frac{5}{2}$T.

P$\frac{5}{2}$M Zn on Nn : 137° 10′. H. tan 68° 35′ : 2.5495 = P$\frac{2.55}{1.00}$M = P$\frac{2.5}{1.0}$M = P$\frac{5}{2}$M.

P$\frac{5}{2}$M Zn on Nn : 136° 22′ M. tan. 68° 11′ = 2.498 = P$\frac{2.498}{1.000}$M = P$\frac{25}{10}$M = P$\frac{5}{2}$M.

P$\frac{1}{2}$M Zn on Nn : 53° 6′ M. cot. 26° 33′ = 2.0013 = P$\frac{1}{2}$M = PM$_2$.

2. 1. P$\frac{5}{2}$M,P$\frac{5}{2}$T, ... Model 13. L358[1]. H117[314].
2. 1. (P$\frac{5}{2}$M,P$\frac{5}{2}$T) $\times$ 2, ... L358[20].
2. 1. P$\frac{5}{2}$M,p$\frac{1}{2}$m,P$\frac{5}{2}$T,p$\frac{1}{2}$t, ... Model 14. M ii 379[f. 105]. L358[3]. H117[316].
2. 1. P$\frac{5}{2}$M,p$\frac{1}{3}$m,p$\frac{1}{2}$m,P$\frac{5}{2}$T,p$\frac{1}{3}$t,p$\frac{1}{2}$t ? .... L358[6].
2. 1. P$\frac{5}{2}$M,P$\frac{5}{2}$T,p$\frac{1}{3}$mt,p$\frac{1}{2}$mt ? .... L358[5].
2. 1. P$\frac{5}{2}$M,p$\frac{1}{2}$m,P$\frac{5}{2}$T,p$\frac{1}{2}$t,p$\frac{5}{2}$mt, ... R[57].
2. 1. P$\frac{5}{2}$M,p$\frac{1}{2}$m,P$\frac{5}{2}$T,p$\frac{1}{2}$t,p$\frac{5}{2}$mt, ... L358[14].
2. 1. P$\frac{5}{2}$M,P$\frac{5}{2}$T,p$\frac{1}{3}$m$\frac{1}{2}$t,p$\frac{1}{3}$mt$\frac{1}{2}$, ... H117[317].
3. 1. p,mt.P$\frac{5}{2}$M,P$\frac{5}{2}$T, ... L358[8].
3. 1. p,mt. P$\frac{5}{2}$M,P$\frac{5}{2}$T, p$\frac{5}{2}$mt, ... L368[9].
3. 1. p,m,t. P$\frac{5}{2}$M,P$\frac{5}{2}$T,p$\frac{5}{2}$mt, ... L358[15].
3. 1. p,m,t, mt. P$\frac{5}{2}$M,P$\frac{5}{2}$T, p$\frac{5}{2}$mt, ... L358[16].
4. 1. M,T,mt.P$\frac{5}{2}$M,P$\frac{5}{2}$T, ... L358[13].
5. 1. p.P$\frac{5}{2}$M,P$\frac{5}{2}$T, ... M ii 379[1]. L358[2]. H117[315].
5. 1. (p.P$\frac{5}{2}$M,P$\frac{5}{2}$T) $\times$ 2, ... Dauphiny...... L358[20].
5. 1. p. P$\frac{5}{2}$M, p$\frac{1}{2}$m,P$\frac{5}{2}$T,p$\frac{1}{2}$t, ... L358[4].
5. 1. p. P$\frac{5}{2}$M,P$\frac{5}{2}$T,p$\frac{1}{3}$mt,p$\frac{1}{2}$mt ? .... L358[7].
5. 1. p. P$\frac{5}{2}$M,P$\frac{5}{2}$T,p$\frac{5}{2}$mt,p$\frac{5}{4}$mt, ... M ii 379[3].
5. 1. p. P$\frac{5}{2}$M,P$\frac{5}{2}$T,p$\frac{5}{2}$mt, ... L358[12]. R[58].
5. 1. p. P$\frac{5}{2}$M,P$\frac{5}{2}$T, p$_3$mt, ... L358[10].
5. 1. p. P$\frac{5}{2}$M,P$\frac{5}{2}$T,p$\frac{5}{2}$mt,p$_3$mt, ... L358[11].
5. 1. P$\frac{5}{2}$M,P$\frac{5}{2}$T,p$\frac{1}{3}$mt,p$\frac{1}{2}$mt,p$_3$mt, ... L358[17].
5. 1. p+. P$\frac{5}{2}$M,p$\frac{1}{2}$m, P$\frac{5}{2}$T,p$\frac{1}{2}$t,p$\frac{5}{2}$mt,p$\frac{1}{3}$mt$\frac{1}{2}$,p$\frac{1}{3}$m$\frac{1}{2}$t, ... M ii 379[fig. 100].

5.** COPPER PYRITES. Kupferkies. Cuivre Pyriteux. Cu Fe S.
Axes: $p_2^1 m^a t^a$ (sometimes $p^a m^a t^a$?) Cleavage = p. P_2M, P_2T.

The following measurements and computations tend to prove the existence of equiaxed pyramidal forms upon combinations of the two-and-one-axed Class:

Forms of the Octahedral Zones.

PMT = P = 109° 53′. 108° 40′. Mohs.

108° 40′ ÷ 2 = 54° 20′.
log cot 54° 20′ = 9.8559376
log sec 45° = 10.1505150

log cot 44° 34½′ = 10.0064526
cot 44° 34½′ = 1.0149465 = PMT.

$P\frac{1}{4}$MT = P—4 (d) = 38° 15′ Mohs.
38° 25′ ÷ 2 = 19° 12½′.
log cot 19° 12½′ = 10.4583286
log sec 45° = 10.1505150

log cot 13° 49½′ = 10.6088436
cot 13° 49½′ = 4.0636171 = $P\frac{1}{4}$MT.

$P\frac{1}{3}MT = \frac{2\sqrt{2}}{3}$ P—3(e) = 49° 50′ Mohs.
49° 50′ ÷ 2 = 24° 55′.
log cot 24° 55′ = 10.3329786
log sec 45° = 10.1505150

log cot 18° 11′ = 10.4834936
cot 18° 11′ = 3.0445018 = $P\frac{1}{3}$MT.

$P\frac{1}{2}$MT = P—2 = 69° 44′. Mohs.
69° 44′ ÷ 2 = 34° 52′.
log cot 34° 52′ = 10.1569261
log sec 45° = 10.1505150

log cot 26° 14′ = 10.3074411
cot 26° 14′ = 2.0292873 = $P\frac{1}{2}$MT.

Forms of the Octahedral Zones.

P_2MT = P + 2 = 140° 31′. Mohs.
140° 31′ ÷ 2 = 70° 15½′.
log cot 70° 15½′ = 9.5553374
log sec 45° = 10.1505150

log cot 63° 4′ = 9.7058524
cot 63° 4′ = 0.5080607 = P_2MT.

Forms of the N. and E. Zones.

PM, PT = P — 1 (b) = 120° 30′, 89° 9′. Mohs.
89° 9′ ÷ 2 = 44° 34½′. Cot 1.0149465
= $P\frac{10000}{10149}$M = $P\frac{1}{1}$M = PM.

$P\frac{2}{3}M, P\frac{2}{3}T = \frac{2\sqrt{2}}{3}$ P — 2 (g) = 66° 36′. Mohs.
66° 36′ ÷ 2 = 33° 18′. cot 1.5223545.
= $P\frac{1000}{1522}$M = $P\frac{10}{15}$M = $P\frac{2}{3}$M.

$P\frac{3}{2}M, P\frac{3}{2}T, = \frac{3}{2\sqrt{2}}$ P (h) = 111° 56′. Mohs.
111° 56′ ÷ 2 = 55° 58′. cot. 0.6753553
= $P\frac{1000}{0675}$M = $P\frac{3}{2}$M.

P_2M, P_2T = P + 1 (c) = 126° 11′. Mohs.
126° 11′ ÷ 2 = 63° 5½′. cot 0.5075119
= $P\frac{1000}{0507}$M = $P\frac{10}{5}$M = P_2M.

The axes m^a t^a of these forms are all, according to the results of the above calculations, invariably 1½ per cent. greater than they should be, to agree with the short symbols that I have given to express their rela-

tion to the axis p^a. To make the symbols agree with the reckoning, I should say:

$P\frac{1.0000000}{1.0149465}M$, $P\frac{1.0000000}{1.0149465}T$, $P\frac{1.0000000}{1.0149465}MT$, instead of PM,PT,PMT;
also, $P\frac{1.0000000}{0.3073119}M$, $P\frac{1.0000000}{0.3073119}T$, instead of P_2M,P_2T;
and $P\frac{1.0000000}{2.0298273}MT$ instead of $P\frac{1}{2}MT$;

and I admit that these proportions may possibly represent the combinations which occur in nature, although I may at the same time be permitted to inquire whether or not the natural combinations have in this case been correctly measured.

It will be observed that, according to Mohs, the form PMT measures across the equator 108°. 40′, which measurement gives for the axes p^a and m^a the relation of 1 to 1.0149465. But Phillips (*Introduction to M.* p. 315°) states this angle to be 180° — 71° 10′ = 108° 50′, which gives for the axes p^a m^a the relation of 1 to 1.0118215. But, again, he states that a plane of the form PMT makes with the horizontal plane P Z an angle of 126°. Now, 126° — 90° = 36° = PMT on p^a. This gives 54° as the value of the ½ angle at the equator, and the relation of the axes p^a m^a comes thence to be that of 1 to 1.0274253. Finally, we find Haüy stating the relation of the axes to be that of 1 to 1. Hence we have:

	p^a	to	m^a or t^a.
Haüy,	1	:	1.0000000.
Phillips,	1	:	1.0118213.
Mohs,	1	:	1.0149465.
Phillips,	1	:	1.0274253.

Both of Phillips's measurements cannot be correct, because they are not consistent; and we have no means of calculating whether either of them is correct. Mohs has taken the *mean* of these as the *true* measure, from which it may be concluded, that Mohs's angles are those of arithmetic and not of nature; and that, consequently, all the above calculations give only approximate values for the lengths of the axes, and are insufficient to prove the impropriety of denoting the forms by symbols with short fractions, such as those which I have given in the Table.

2. 1. PMT, ...(Mohs's assumed primitive)......L645[1]. J iii 311[1a].
2. 1. P_2M,P_2T, ...(Phillips's assumed primitive)......P315[1].
2. 1. P_2M,P_2T,pmt,..................Freiberg......L645[5]. P315[2]. M ii 470[2].
3. 1. p,m,t.PMT, ...J iii 311[1c].
3. 1. p,m,t. ½PMT, ... H97[108].
5. 1. p. PMT,..L645[2].
5. 1. P‿ ½PMT,½PMT, ... Cornwall ......M ii 470[1].
5. 1. p.pm,pt,PMT,...Model 77. L645[3].
5. 1. p.p_2m,p_2t,½PMT,½PMT,..............................D408[1]. A[165]. L645[4].
5. 1. p.pm,p_2m,pt,p_2t,½PMT,½PMT, ... Freiberg ......D408[2]. M ii 470[3].

5. 1. p.pm,p$\frac{4}{3}$m,p$\frac{3}{2}$m,p$_2$m,pt,p$\frac{4}{3}$t,p$\frac{3}{2}$t,p$_2$t, $\frac{1}{2}$PMT,$\frac{1}{2}$PMT,$\frac{1}{2}$p$\frac{1}{3}$mt Z^{2}ne, $\frac{1}{2}$p$\frac{1}{4}$mt Z^{2}ne, $\frac{1}{2}$(p$\frac{1}{3}$m$\frac{1}{2}$t,p$\frac{1}{3}$mt$\frac{1}{2}$) Z^{2}ne,............M ii 470$^{4\ \text{fig. 173}}$

6. 1. $\frac{1}{2}$PMT,......... (Haüy's assumed primitive)......H97^{100}. J iii 311^{2b}.

6. 1. $\frac{1}{2}$PMT, $\frac{1}{2}$pmt, ... P315^3. J iii 311^{2a}. H97^{102}.

6. 1. p$_2$m,p$_2$t,$\frac{1}{2}$PMT,$\frac{1}{2}$PMT,..D408^2.

6. 3. $\frac{1}{2}$(3P$\frac{1}{2}$MT),... H97^{101}.

6. CRYOLITE. Kryolith. Alumine fluatée alkaline.

Axes: p^2_x m^2 t^2. Cleavage = p,m,t.

1. 1. P$_\infty$M,T? .... P204. L570. R164.

7. HAUSMANNITE. Black Manganese. Manganese oxidé hydrate.

Axes: p^2_xm^2 t^2. Cleavage = p$\frac{4}{3}$m,p$\frac{4}{3}$t.

P$\frac{4}{3}$M Zn on Nn : 117° 54′ Haid. cot. 58° 57′ = 0.602 = P$\frac{10}{6}$M = P$\frac{4}{3}$M.

2. 1. P$\frac{4}{3}$M,P$\frac{4}{3}$T, ... L760^1. Haidinger 2^{11}.*.

2. 1. (P$\frac{4}{3}$M,P$\frac{4}{3}$T) × 2,.....................M ii $^{106,\ 107}$. L760^4. Haidinger 2^{13}.

2. 1. P$\frac{4}{3}$M,p$\frac{2}{3}$m,P$\frac{4}{3}$T,p$\frac{2}{3}$t, ...M ii ^{105}. L760^2. Haidinger 2^{12}.

2. 1. P$\frac{4}{3}$M,p$\frac{2}{3}$m,P$\frac{4}{3}$M,p$\frac{2}{3}$t,p$\frac{2}{6}$mt, ... L760^3. Haid. p. 46.$^{\text{com.}\ 2}$.

* Edinburgh Journal of Science, Jan. 1826. Mohs ii 416.

8. PHOSPHATE OF YTTRIA. Phosphorsaure Yttererde.

Axes: p^2_x m^2 t^2. Cleavage = M,T.

4. 1. m,t.PM,PT..................Lindenaes in Norway......P194. L276.

9.* FERGUSONITE. Fergusonit.

Axes: p$^2_+$m^{2}t^2. Cleavage = p$_2$m,p$_2$t.

1. 3. P, $\frac{1}{2}$MT$_+$, $\frac{1}{2}$M$_+$T, ...M iii 98^1.

3. 3. p, $\frac{1}{2}$MT$_+$,$\frac{1}{2}$M$_+$T. P$_2$M,P$_2$T, ... M iii 98^3. $^{\text{fig. 110}}$. P274. A^{139}.

3. 3. p, $\frac{1}{2}$mt$_+$,$\frac{1}{2}$m$_+$t. p$_2$m,p$_2$t, $\frac{1}{2}$(p$_x$m,t$_x$, p$_x$m t$_y$) .... M iii 98^4. $^{\text{fig. 193}}$.

10.* *An Isomorphous Group*, 1, 2, 3:—

1. TUNGSTATE OF LIME. Tungstein. Schéelin Calcaire.

Axes: p$^2_+$m^2 t^2. Cleavage = P$\frac{3}{2}$M,P$\frac{3}{2}$T,P$\frac{3}{2}$MT.

P$\frac{3}{2}$M Zn on Nn : 113° 36′. Zn on Ze : 107° 27′ = P$\frac{0.65444}{1.00000}$M = P$\frac{100}{66}$M = P$\frac{3}{2}$M.

P$\frac{3}{2}$MT Znw on Zne 100° 8′, Znw on Nnw : 130° 20′ = P$\frac{1.0000}{0.6544}$M = P$\frac{100}{66}$MT = P$\frac{3}{2}$MT.

2. 1. P$\frac{3}{2}$M,P$\frac{3}{2}$T, ...Ly. v.^{1} H119^{329}. P182^3. L347^3.

2. 1. P$\frac{3}{2}$MT, ... H119^{328}. P182^1. L347^1.

2. 1. P$\frac{3}{2}$M,P$\frac{3}{2}$T,p$\frac{3}{2}$mt,........................Ly v.^{2}. H119^{330}. P182^2. L347^2.

2. 1. P$\frac{3}{2}$M,p$\frac{2}{7}$m,P$\frac{3}{2}$T,p$\frac{2}{7}$t, ...M ii 114^2.

2. 1. P$\frac{3}{2}$M,p$\frac{10}{9}$m,P$\frac{3}{2}$T,p$\frac{10}{9}$t, ...M ii 114^3.

2. 4. P$\frac{3}{2}$M,P$\frac{3}{2}$T, p$\frac{3}{2}$mt, $\frac{1}{2}$pm$_-$t$_+$, $\frac{1}{2}$pm$_+$t$_-$, ...M ii 114^4.

2. 4. P$\frac{3}{2}$M,P$\frac{3}{2}$T,p$\frac{3}{2}$mt, $\frac{1}{2}$p$_+$mt$_-$, $\frac{1}{2}$p$_+$m$_-$t, ...M ii 114^5. Ly 79^1.

2. 4. P$\frac{3}{2}$M,P$\frac{3}{2}$T,p$\frac{3}{2}$mt,$\frac{1}{2}$pm$_-$t$_+$,$\frac{1}{2}$pm$_+$t$_-$,$\frac{1}{2}$p$_+$mt$_-$,$\frac{1}{2}$p$_+$m$_-$t,...M ii 114^6. $^{\text{fig. 108}}$.

5. 1. p. P$\frac{3}{2}$M,p$\frac{2}{7}$m,P$\frac{3}{2}$T,p$\frac{2}{7}$t,P$\frac{3}{2}$MT, ...Ly79^2.

5. 1. $P.\,P\frac{2}{7}M,\,P\frac{2}{7}T$, ... Mii 114[1].
5. 4. $p.\,P\frac{3}{2}M,\,p\frac{10}{9}m,\,p_4m,\,P\frac{3}{2}T,\,p\frac{10}{9}t,\,p_4t,\,p\frac{2}{3}mt,\,P\frac{3}{2}MT,\,p\frac{2}{3}mt,p_4mt_2,p_4m_2t$, P183.

2. Tungstate of Lead. Scheelbleierz. Plomb Tungstaté.
Axes: $p^a_{+}m^at^a$. Cleavage = $P.\,P\frac{3}{2}MT$.

1. 1. P_+,M,T,..L345[1].
1. 1. P_-,M,T,..L345([1]).
2. 1. P_3MT, ... Levy, Annals Phil. xxviii. 364. L345[4].
2. 1. $P\frac{3}{2}M,p\frac{2}{3}m,\,P\frac{3}{2}T,\,p\frac{2}{3}t,p\frac{3}{2}mt$, ...Ly 59[2].
3. 1. $P_+,M,T.p\frac{3}{2}mt$, ... L345[2].
3. 1. $P_+,M,T.p\frac{3}{2}m,p\frac{3}{2}t$, ... P370.
4. 1. $MT.p\frac{2}{3}m,p\frac{2}{3}t,P\frac{3}{2}MT$,..................Levy Annals Phil. xxviii. 364[1].
4. 1. $MT.\,p\frac{2}{3}m,p\frac{2}{3}t,P\frac{3}{2}MT,P_3MT$,......... idem.
5. 1. $P_+.\,P\frac{2}{3}M,P\frac{2}{3}T$, ...L345[5].
5. 1. $P.\,P\frac{3}{2}MT$, ...L345[3].

3. Molybdate of Lead. Gelbbleierz. Plomb molybdaté.
Axes: $p^a_{-}m^at^a$. Cleavage = $p\frac{4}{3}m,p\frac{4}{3}t,p\frac{4}{3}mt$.
$P\frac{4}{3}M$ Zn on Zw 128°. Zn on Nn : 76° 40′. H = $P\frac{1.0000}{1.2647}M = P\frac{4}{3}M$.
$P\frac{4}{3}MT$ Znw on Zne : 116° 22′. Znw on Nnw : 96° 22′ H. = $P\frac{1.0000}{1.2649}MT$
$= P\frac{4}{3}MT$.

1. 1. P_-,M,T,..P367 . H95[82]. L341[9].
1. 1. P_-,m,t,mt..P367[5]. H95[86]. L341[10].
1. 4. $P_-,mt,m\frac{2}{3}t,m\frac{3}{2}t$,..H95[87]. L341[11].
2. 1. $P\frac{4}{3}M,P\frac{4}{3}T$, ... Bleiberg......Ly v.[1]. P367[1]. H94[77]. L341[1].
2. 1. $p\frac{4}{3}m,p\frac{4}{3}t,P\frac{4}{3}MT$,..H95[81]. L341[4].
3. 1. $P_-,M,T.p\frac{4}{3}m,p\frac{4}{3}t$, ... P367[2]. H95[83]. L341[8].
3. 1. $p,mt.P\frac{4}{3}M,P\frac{4}{3}T$,..H95[84]. L341[3].
3. 1. $p,mt.P\frac{4}{3}M,P\frac{4}{3}T,p\frac{3}{2}mt$, ... P367[4]. H95[88]. L341[6].
4. 1. $MT.P\frac{3}{2}MT$, ... P367[6]. Ly58[7].
5. 1. $P_-.p\frac{4}{3}m,p\frac{4}{3}t,p\frac{3}{2}mt$,..H95[85]. L341[5].
5. 1. $P_-.P\frac{4}{3}M,P\frac{4}{3}T$,.......................Model 76. Ly58[9]. H95[80]. L341[2].
5. 1. $P.P\frac{3}{2}MT$, ...Ly58[8].

Levy quotes many other combinations of this mineral, but his descriptions are not accompanied by measurements of angles, so that they cannot be accurately expressed in symbols.

11. Zircon. Zirkon. Hyacinth.
Axes: $p^a_xm^a\,t^a$. Cleavage = $m,t.\,P\frac{2}{3}MT$.
$P\frac{2}{3}MT$ Znw on Nnw : 84° 20′. Znw on Zne : 123° 19′ = $P\frac{100}{156}MT$
$= P\frac{2}{3}MT$.
P_2MT on MT : 159° 35′ P. 159° 35′ — 90° = 69° 35′ = $P\frac{10000}{5263}MT$
$= P\frac{10}{5}MT = P_2MT$.

2. 1. $P\frac{2}{3}MT$,.........Model 12. ......$p^a_-m^at^a$,......L388[1]. R[55]. H58[19]. P95[1].
4. 1. $M,T.P\frac{2}{3}MT$,............$p^a_+m^at^a$,......S[490]. J[6]. L388[5]. R[62]. H58[20]. P95[2].
4. 1. $M,T.P\frac{2}{3}MT$, ...Ceylon ....$p^a_-m^at^a$,......H58[20a]. Mii 368[2].

4. 1. MT.P$\frac{2}{3}$MT,...p$^a_+$m^{a}t^a,...St. Gotthardt...Model 61. L388^2.M ii 368^1.
R^{61}. J^3. H59^{21}. S^{491}.

4. 1. m,t.P$\frac{2}{3}$MT,....................p$^a_-$m^{a}t^a, ... P95^3.

4. 1. mt.P$\frac{1}{3}$MT, ...p$^a_-$m^{a}t^a,.................... L388^3. H59^{22}.

4. 1. M,T,mt.P$\frac{2}{3}$MT, ...p$^a_+$m^{a}t^a,...Md. 60. H59^{24}. J^7. S^{492}. L388^6.

4. 1. m,t,MT.P$\frac{2}{3}$MT, ...p$^a_+$m^{a}t^a, ... P95^4.

4. 1. m,t,MT.p$\frac{2}{3}$m,p$\frac{2}{3}$t,P$\frac{2}{3}$MT, ... p$^a_+$m^{a}t^a,..............................R^{63}.

4. 1. MT.P$\frac{2}{3}$MT,p$_2$mt, ... p$^a_+$m^{a}t^a,...............H59^{26}. S^{493}. L388^4..

4. 1. M,T.P$\frac{2}{3}$MT,p$_2$mt$_3$,p$_2$m$_3$t, ...p$^a_+$m^{a}t^a,...............R^{64}. H59^{23}. L388^9.

4. 1. MT.P$\frac{2}{3}$MT,p$_2$mt$_3$,p$_2$m$_3$t,......p$^a_+$m^{a}t^a,...J^4. S^{495}. H59^{25}. P95^5. L388^7.

4. 1. MT.P$\frac{2}{3}$MT,p$_2$mt,p$_2$mt$_3$,p$_2$m$_3$t, p$^a_+$m^{a}t^a,.........H59^{28}. P95^6. J^5. L388^8.

4. 1. M,T,mt.P$\frac{2}{3}$MT,p$_2$mt$_3$,p$_2$m$_3$t, p$^a_+$m^{a}t^a, ...M ii 368^3. H59^{27}. J^6. L388^{10}.

4. 1. m,t,MT.p$\frac{2}{3}$m,p$\frac{2}{3}$t,P$\frac{2}{3}$MT,p$_2$mt$_3$,p$_2$m$_3$t, p$^a_+$m^{a}t^a,...H60^{30}. S^{496}. L388^{11}.

4. 1. m,t,MT.P$\frac{2}{3}$MT,p$_2$mt, ... p$^a_+$m^{a}t^a, ...S^{494}.

4. 1. m,t,MT.p$_2$mt,P$\frac{2}{3}$MT,p$_2$mt$_3$,p$_2$m$_3$t,.........Norway......M ii 368 .$^{fig. 98}$.

4. 1. m,t,MT.p$\frac{2}{3}$m,p$\frac{2}{3}$t,P$\frac{2}{3}$MT,p$_2$mt$_3$,p$_2$m$_3$t,p$_4$mt$_3$,p$_4$m$_3$t,......Carinthia......
M ii 369^5.

4. 1. m,t,MT.P$\frac{2}{3}$M,P$\frac{2}{3}$T,p$\frac{2}{3}$mt,P$_2$MT$_3$,P$_2$M$_3$T,p$_4$mt$_3$,p$_4$m$_3$t,p$_5$mt$_3$,p$_5$m$_3$t ?.........
Sanalpe......M ii 369^6.$^{fig. 99}$.

12. MURIO-CARBONATE OF LEAD. Hornbleierz.

Axes: p$^a_+$m^{a}t^a. Cleavage = p,M,T.

P$\frac{3}{2}$M on P : 123° 6′. 123° 6′ — 90° = 33° 6° = P$\frac{1\cdot000}{0\cdot658}$M = P$\frac{3}{2}$M.

1. 1. P$_+$,M,T,...L295^1. P362.

1. 1. P$_+$,M,T,mt,..L295^2. P362.

3. 1. P$_+$,M,T.p$\frac{3}{2}$m,p$\frac{3}{2}$t,.....................................Ly 56^2. L295^4.

3. 1. P$_+$,M,T.pmt,...L295^3.

3. 1. P$_+$,M,T,mt.p$\frac{3}{2}$m,p$\frac{3}{2}$t, ...L295^5. P362. D226.

3. 1. P$_+$,M,T,mt.pmt,.......................................L295^8. D226.

3. 1. P$_+$,M,T.p$\frac{3}{2}$m,p$\frac{3}{2}$t, ...L295^4.

3. 4. P$_+$,M,T,m$\frac{1}{3}$t,m$_3$t.p$\frac{3}{2}$m,p$\frac{3}{2}$t,..L295^6.

3. 4. P$_+$,M,T, mt,m$\frac{1}{3}$t,m$_3$t.p$\frac{3}{2}$m,p$\frac{3}{2}$t, ...L295^7.

3. 4. P$_+$,M,T,MT,m$\frac{1}{3}$t,m$_3$t.p$\frac{3}{2}$m,p$\frac{3}{2}$t,p$_3$mt$_2$,p$_3$m$_2$t, ... Brooke, Phil. Mag.
Series III. xi. 175.

4. 1. MT.PMT, ... L295^{10}.

4. 1. M,T,mt.PMT, ... L295^9.

13. MELLITE. Honigstein. Mellate of Alumina.

Axes: p$^a_-$m^{a}t^a. Cleavage = p$\frac{3}{4}$mt.

P$\frac{3}{4}$M Zn on Nn : 73° 44′ M. = P$\frac{1\cdot0000}{1\cdot3333}$M = P$\frac{3}{4}$M.

P$\frac{3}{4}$MT Znw on Nnw : 93° 22′ H. = P$\frac{1\cdot0000}{1\cdot3334}$MT = P$\frac{3}{4}$MT.

2. 1. P$\frac{3}{4}$MT, ... Thuringia......L790^1. H120^{347}.

3. 1. p,m,t.P$\frac{3}{4}$MT,..............................L790^2. H120^{349}. P395. R^{56}.

3. 1. P,M,T.p$\frac{3}{4}$m,p$\frac{3}{4}$t,P$\frac{3}{4}$MT, ...M iii 56^4.$^{fig. 104}$.

4. 1. M,T. P$\frac{3}{4}$MT, ... L790^3. H120^{348}. M iii 56^2.

4. 4. m$_-$t,m$_+$t. P$\frac{3}{4}$MT, ... L790^4.

5. 1. P$_-$. p$\frac{3}{4}$mt, ... M iii 56^1.$^{dim. fig. 98}$.

14. IDOCRASE. Vesuvian. Egeran. Pyramidal Garnet.

Axes: $p_{+}m^{a}t^{a}$. Cleavage = p, M,T, mt.

$P\frac{3}{4}M$ Zn on Nn = 74° 14′ Mohs. = $P\frac{1\cdot0000}{1\cdot3214}M = P\frac{3}{4}M$.
$P\frac{1}{4}M$ Zn on Nn = 28° 19′ M. = $P\frac{1\cdot0000}{3\cdot9636}M = P\frac{1}{4}M$.
$P\frac{3}{8}MT$ Znw on Nnw = 56° 8′ M. = $P\frac{1\cdot00}{2\cdot63}MT = P\frac{3}{8}MT$.

1. 1. P_{+},MT, ... H72[156]. P20[1]. L483[1].
1. 1. P_{+},M,T,MT, ...(Egeran)......Model 4. H72[157]. P21[2]. L483[2].
1. 4. P_{+},M,T,MT,$m\frac{1}{3}t,m_{3}t$, ... L483[4].
3. 1. P_{+}, MT. $p\frac{3}{8}mt$, ... L483[3].
3. 1. p_{+},M,T,MT.$P\frac{3}{4}M,P\frac{3}{4}T$, ... Siberia...Md. 42. M ii 354[1]. H72[158]. J[53]. P20[2]. L483[5].
3. 1. p_{+},M,T,MT.$p\frac{3}{8}mt$, ... P80[3]. L483[6].
3. 1. p_{+},M,T, MT. $P\frac{3}{4}M,P\frac{3}{4}T$, $p\frac{1}{2}m\frac{1}{3}t$, $p\frac{1}{2}mt\frac{1}{3}$, ... H73[159]. L483[8].
3. 4. p_{+},m,t,MT,$m\frac{1}{3}t,m_{3}t$.$P\frac{3}{4}M,P\frac{3}{4}T$,........M ii 354[2]. H73[160]. J[54]. L483[7].
3. 4. p_{+},M,T, MT,$m\frac{1}{3}t,m_{3}t$.$P\frac{3}{4}M,P\frac{3}{4}T$. $p\frac{1}{2}m\frac{1}{3}t,p\frac{1}{2}mt\frac{1}{3}$,......H73[161]. L483[10].
3. 4. P_{+}, M,T, MT, $m\frac{1}{3}t,m_{3}t.p\frac{3}{4}m,p\frac{3}{4}t,p\frac{1}{2}m\frac{1}{3}t,p\frac{1}{2}mt\frac{1}{3}$, ... R[65].
3. 4. p_{+},m,t,MT,$m\frac{1}{3}t,m_{3}t$.$P\frac{3}{4}M,P\frac{3}{4}T,p\frac{3}{8}mt$, J[55]. H73[162]. M ii 354[3]. L483[9].
3. 4. p_{+}, M,T, MT, $m\frac{1}{3}t$, $m_{3}t$. $p\frac{1}{4}m$, $P\frac{3}{4}M$, $p_{3}m$, $p\frac{1}{4}t$, $P\frac{3}{4}T$, $p_{3}t$, $p\frac{3}{8}mt$, $p\frac{1}{2}m\frac{1}{3}t$, $p\frac{1}{2}mt\frac{1}{3}$,........H73[163]. L483[11].
3. 4. P_{+},M,T,MT,$m\frac{1}{3}t,m_{3}t$.$p\frac{3}{4}m$,$P_{3}M$,$p\frac{3}{4}t$,$P_{3}T$,$p\frac{3}{8}mt$, 3 ($p_{x}m_{y}t_{z}$, $p_{x}m_{z}t_{y}$) ...... H73[164]. L483[12].
3. 4. P_{+},M,T, MT, $m\frac{1}{3}t,m_{3}t.p\frac{1}{4}m$, $P\frac{3}{4}M$, $p\frac{3}{2}m,p\frac{9}{4}m,p_{3}m$, $p\frac{1}{4}t$, $P\frac{3}{4}T$, $p\frac{3}{2}t,p\frac{9}{4}t,p_{3}t$, $p\frac{3}{8}mt$, 4($p_{x}m_{y}t_{z}$, $p_{x}m_{z}t_{y}$)........P21. fig.
3. 4. P_{+}, M,T, MT, $m\frac{1}{2}t,m_{2}t$, $m\frac{1}{3}t,m_{3}t$. $P\frac{3}{4}M$, $P\frac{3}{2}M,p_{3}m$, $P\frac{3}{4}T,p\frac{3}{2}t,p_{3}t$, $p\frac{3}{8}mt$, 5($p_{x}m_{y}t_{z}$, $p_{x}m_{z}t_{y}$)........Vesuvius......M ii 355[4]. fig. ii [96].

15. GEHLENITE. Axes: $p_{x}^{a}m^{a}t^{a}$. Cleavage = P,m,t.
1. 1. P_{x},M,T,........Fassa in Tyrol......P22. L212. M iii 103.

16. WERNERITE. Pyramidal Felspar. Paranthine. Scapolite. Meionite.

Axes: $p_{x}^{a}m^{a}t^{a}$. Cleavage = p,m,t,mt.

$P\frac{3}{5}M$ Zn on Nn = 62° 56′. tan 31° 28′ = 6120 = $P\frac{612}{1000}M = P\frac{3}{5}M$.
$P\frac{2}{5}MT$ Znw on Nnw = 60°. tan 30° × sec 45° = 2.449 = $P\frac{10}{25}MT = P\frac{2}{5}MT$.

1. 1. P_{+},M,T, mt,........(Paranthine)......L473[1]. H75[184].
1. 1. P_{+},M,T,$\frac{1}{4}$mt nw, ... J. J. G.
3. 4. P,M,T,MT,$M\frac{1}{3}T,M_{3}T$.$P\frac{3}{5}M,P\frac{3}{5}T,p\frac{2}{5}mt$, $p_{x}m_{y}t_{z}$, $p_{x}m_{z}t_{y}$,...(Meionite)... Vesuvius......P150.
4. 1. M,T,MT.$P\frac{3}{5}M,P\frac{3}{5}T$, ... (Wernerite)...Model 59. L473[2]. H75[183].
4. 1. M,T,MT.$p\frac{2}{5}mt$, ...(Paranthine)......L473[3]. H75[185].
4. 1. m,t,MT.$P\frac{3}{5}M,P\frac{3}{5}T$,......Akudlek, Greenland...M ii 265[1]. H ii 585.
4. 1. M,T,MT.$P\frac{3}{5}M,P\frac{3}{5}T,p\frac{2}{5}mt$, ... (Scapolite)......M ii 265[2]. P143.
4. 1. M,T,mt.$P\frac{3}{5}M,P\frac{3}{5}T,p\frac{2}{5}mt$, $p_{x}m_{y}t_{z}$, $p_{x}m_{z}t_{y}$, ... Vesuvius...M ii 265[3].
4. 4. M,T,mt,$m\frac{1}{3}t,m_{3}t$.$P\frac{3}{5}M,P\frac{3}{5}T$, $p_{x}m_{y}t_{z}$, $p_{x}m_{z}t_{y}$, ...Vesuvius...M ii 265[4].

17.* HUMBOLDTILITE. Axes: $p^a_+ m^a t^a$. Cleavage = P.

1\. 1. P_+,M,T,mt, ... Vesuvius...... D447.
1\. 1. P_+,M,T, m_-t, m_+t, ... Vesuvius...... D447.

18. URANITE. Uranglimmer. Two varieties:—
1\. Copper Uranite. Kupferuranit. Chalkolite.
2\. Lime Uranite. Uran Mica. Kalkuranit.
Axes: $p^a_- m^a t^a$. Cleavage = P,m,t.

1\. 1. P_-,MT, ... G230[1]. L141[1]. H116[305]. P269[1].
1\. 1. $P_=$, mt, ... P269[2].
1\. 1. P,m,t,MT, ... L141[6].
1\. 4. $P_=$, m, mt, ... P269[8].
2\. 1. P_-MT, ... L141[4]. H iv 321[1].
3\. 1. P_-,m,t,mt, ... G230[2].
3\. 1. P,m,t. P_-MT, ... G230[5].
3\. 1. P,MT.p_-mt, ... L141[5].
3\. 1. P_-,MT.p_-m,p_-t, ... P269[5].
3\. 1. P_-,MT.p_-mt, ... G230[3]. L141[2]. P269[4].
3\. 1. P_-,m,t,MT. p_-m,p_-t,p_-mt, ... L141[7]. P269[3].
3\. 4. P,m,t,MT. m_-t,m_+t, ... L141[9].
3\. 4. P_-,m,t,MT,m_-t,m_+t. pm,p_-m,pt,p_-t,$p\frac{1}{2}$mt,$p\frac{4}{7}$mt,$p\frac{2}{3}$mt,$p\frac{4}{3}$mt, ... P269[7]. L141[10].
5\. 1. P. P_-MT, ... G230[4]. L141[3].

19. APOPHYLLITE. Fischaugenstein. Fish-eye-stone.
Axes: $p^a m^a_4 t^a_4$. Cleavage = P,m,t.
$P\frac{5}{4}$MT Znw on Zne : 104° 2′. Znw on Nnw 121° = $P\frac{10000}{8002}$MT
= $P\frac{5}{4}$MT.

1\. 1. $P\frac{5}{4}$,M,T, ... Model 3. L214[1]. H85[293]. P109[1].
2\. 1. $P\frac{5}{4}$MT, ... L214[7].
3\. 1. $P\frac{5}{4}$,M,T.$p\frac{5}{4}$mt, ... Model 41. L214[2]. H85[295].
3\. 1. $P\frac{5}{4}$,M,T,mt.$p\frac{5}{4}$mt, ... L214[9].
3\. 1. p_+,M,T.$P\frac{5}{4}$MT, ... P109[3]. L214[3].
3\. 1. P_-,m,t.$p\frac{5}{4}$mt, ... M ii [11]. P109[2]. L214[4].
3\. 1. $P\frac{5}{4}$,M,T.pm,pt,$p\frac{5}{4}$mt, ... L214[8].
3\. 4. p_+,M,T,$m\frac{1}{2}t$,m_2t.$P\frac{5}{4}$MT, ... H85[297]. L214[10].
3\. 4. P,M,T, $m\frac{1}{2}t$,m_2t. $p\frac{1}{4}$m,$p\frac{5}{8}$m,$p\frac{1}{4}$t,$p\frac{5}{8}$t,$P\frac{5}{4}$MT,$p\frac{1}{4}$mt,$p\frac{5}{12}$mt, ... M ii [fig. 98].
4\. 1. M,T.$P\frac{5}{4}$MT, $p^a_+ m^a t^a$, ... M ii [97]. H85[294]. L214[5].
4\. 4. M,T,$m\frac{1}{2}t$,m_2t.$P\frac{5}{4}$MT, ... R[66]. H85[296]. L214[11]. M ii 244[3].
5\. 1. p.$P\frac{5}{4}$MT, ... L214[6].

20. BLACK TELLURIUM. Blättererz. Nagyagererz.
Axes: $p^a_- m^a t^a$. Cleavage = P.

1\. 1. P_-,M,T, ... L689[1]. P343[1].
1\. 1. P_-,M,T,mt, ... L689[2].
3\. 1. P_-,M,T.$p\frac{8}{3}$m,$p\frac{8}{3}$t, ... L689[3].
3\. 1. P_-,M,T.$p\frac{4}{3}$mt, ... L689[4].

3. 1. P_{-},M,T.p$\frac{8}{3}$m,p$\frac{8}{3}$t,p$\frac{4}{3}$mt, ... L689[6].
3. 1. P_{-},M,T,mt.p$\frac{8}{3}$m,p$\frac{8}{3}$t, ... L689[7].
3. 1. P_{-},M,T,mt.p$\frac{8}{3}$m,p$\frac{8}{3}$t,p$\frac{4}{3}$mt, ... L689[9].
5. 1. P_{-}.P$\frac{8}{3}$M,P$\frac{8}{3}$T, ... L689[4].
5. 1. P_{-}.p$\frac{8}{3}$m,p$\frac{8}{3}$t,p$\frac{4}{3}$mt, ... A^{160}. L689[8]. Ly80[2] P343[2]. D424.

21. Mellilite. Mellilith. Axes: $p^{a}_{+}m^{a}t^{a}$. Cleavage = ?
1. 1. P_{+},M,T,mt...P48. Levy 46[2].

22. Oerstedtite. Axes: $p^{a}_{+}m^{a}t^{a}$. Cleavage = ?
3. 1. P,M,T,mt.pm,pt,p$\frac{2}{4}$mt, ... Dana 368.
4. 1. M,T,mt.P$\frac{3}{4}$MT,p$\frac{4}{3}$mt,p‡mt,$p_{x}m_{y}t_{z}$,$p_{x}m_{z}t_{y}$? ... Forchhammer, Pogg. Ann. xxxv. 630.

23. Somervillite. Axes: $p^{a}_{-}m^{a}t^{a}$. Cleavage = P.
3. 4. P,ᴍ,ᴛ,MT,m$\frac{1}{2}$t,m_{2}t.p$\frac{2}{3}$m,p$\frac{2}{3}$t, ...P406.

24.** Edingtonite. Axes: $p^{a}_{-}m^{a}t^{a}$. Cleavage = M,T.
5. 1. MT. $\frac{1}{2}$P$\frac{1}{3}$MT Znw, Zse, Nne, Nsw, $\frac{1}{2}$P$\frac{2}{3}$MT Zne, Zsw, Nnw, Nse, P150.

Class III.—Minerals belonging to the Rhombohedral System of Crystallisation.

The Axes of most Combinations belonging to this Class are = $p^{a}_{x}m^{a}_{15}t^{a}_{13}$; but in Combinations which include the twelve-sided prism, = m,T,m_{2}t$\frac{15}{13}$, M$\frac{15}{13}$$T_{2}$, the length of m^{a} fluctuates between $\frac{13}{13}$ and $\frac{15}{13}$, without reaching either of those limits; while the Axes of the Combinations of Tourmaline are out of all rule.

The following are the chief Forms and Combinations of the Rhombohedral System:—

1.) The horizontal planes = P.
2.) The six-sided Prism = T,M$\frac{15}{13}$$T_{2}$.
The twelve-sided Prism = m,T,m_{2}t$\frac{15}{13}$,M$\frac{15}{13}$$T_{2}$.
The twenty-four-sided Prism = m,T,m_{2}t$\frac{15}{13}$,M$\frac{15}{13}$$T_{2}$, 3$m_{x}$t.

The positions of these prisms, and the number of their planes, are immutable. Therefore, m, m_{2} t$\frac{15}{13}$ cannot occur without T, M$\frac{15}{13}$ T_{2}, nor can 3m_{x}t occur without both the other combinations.

3.) A series of Rhombohedrons, whose general symbol is this:—

$$\tfrac{1}{2}P_{x}T,\ \tfrac{1}{2}P_{x}M\tfrac{15}{13}T_{2}$$

and whose characteristics (*i. e.* the equivalents of $_{x}$ in this symbol, or, in other words, the length of p^{a} when t^{a} is considered unity,) are these:—

$\frac{39}{1}$, $\frac{16}{1}$, $\frac{14}{1}$, $\frac{13}{1}$, $\frac{8}{1}$, $\frac{6}{1}$, $\frac{11}{2}$, $\frac{16}{3}$, $\frac{5}{1}$, $\frac{4}{1}$, $\frac{3}{1}$, $\frac{8}{3}$, $\frac{5}{2}$, $\frac{2}{1}$, $\frac{20}{11}$, $\frac{9}{5}$, $\frac{7}{4}$, $\frac{5}{3}$, $\frac{3}{2}$, $\frac{10}{7}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{15}{13}$, $\frac{8}{7}$, $\frac{1}{1}$, $\frac{10}{11}$, $\frac{8}{9}$, $\frac{13}{15}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{10}{13}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{6}{11}$, $\frac{1}{2}$, $\frac{5}{11}$, $\frac{4}{9}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{1}{3}$, $\frac{2}{9}$, $\frac{1}{220}$, $\frac{1}{300}$.

Every rhombohedron may take one of four different positions on a combination, and may therefore be represented by any one of the following symbols:—

$\frac{1}{2}P_xT$ Zw, $\frac{1}{2}P_xM^{1\,5}_{1\,3}T_r$.	$\frac{1}{2}P_xM$ Zn, $\frac{1}{2}P_xM_2T^{1\,5}_{1\,3}$.
$\frac{1}{2}P_xT$ Ze, $\frac{1}{2}P_xM^{1\,5}_{1\,3}T_r$.	$\frac{1}{2}P\,M$ Zs, $\frac{1}{2}P_xM_2T^{1\,5}_{1\,3}$.

4.) A series of scalenohedrons, or six-sided pyramids with scalene triangular faces and twelve-sided equators.

5.) A series of six-sided pyramids, with isosceles triangular faces and six-sided equators. These are produced by the simultaneous occurrence of two equal and similar rhombohedrons in inverse positions. Thus, P_xT, $P_x\,M^{1\,5}_{1\,3}\,T_2$ contains $\frac{1}{2}\,P_xT$ Zw Ne, $\frac{1}{2}\,P_xM^{1\,5}_{1\,3}T_2$ Zne Zse Nnw Nsw $+$ $\frac{1}{2}P_xT$ Ze Nw, $\frac{1}{2}P_xM^{1\,5}_{1\,3}T_2$ Znw Zsw Nne Nse.

6.) The crystals of Tourmaline present many irregular forms that do not occur among the crystals of any other mineral of this system. The principal of these irregular forms are:—

The three-sided Prism $=$ M n, $\frac{1}{2}M_2T^{1\,5}_{1\,3}$ se sw.

The two nine-sided Prisms $= \begin{cases} \text{M n,t, } \frac{1}{2}M_2T^{1\,5}_{1\,3} \text{ se sw, } m^{1\,5}_{1\,3}t_r. \\ \text{m n,T, } \frac{1}{2}m_2t^{1\,5}_{1\,3} \text{ se sw, } M^{1\,5}_{1\,3}T_r. \end{cases}$

A variety of hemi-rhombohedrons (consequently Tetarto-Forms.)

A variety of hemi-scalenohedrons (idem.)

The scalenohedrons referred to in the following Table have not been calculated, and are indicated merely by a provisional symbol. Except among the crystals of calcareous spar, they occur only subordinately. The symbol that serves to denote the variety which is commonly called *dog's tooth spar*, or by Haüy, *Metastatique*, (Model 116,) is:

$$\tfrac{1}{2}PM^{1\,5}_{1\,3}T\tfrac{2}{3},\ \tfrac{1}{2}PM^{1\,5}_{2\,6}T\tfrac{1}{2},\ \tfrac{1}{2}PM^{\,5}_{1\,3}T_r.$$

As this is the principal scalenohedron of the varieties peculiar to calcareous spar, I shall distinguish it in the following catalogue by the symbol $\frac{1}{2}(3P_1M^{\pm}T^{-}_{+})$. More acute scalenohedrons will be marked $\frac{1}{2}(3P_{+}M^{\pm}T^{-}_{+})$, and more obtuse varieties, $\frac{1}{2}(3P_{-}M^{\pm}T^{-}_{+})$. In all these symbols, the sign placed immediately after the letter P, and referring to p^a, is the only one that is varied.

The Combinations of Calcareous Spar, Ruby Silver, and other minerals of this system being often very complex, in consequence of their comprehending many varieties of hemihedral and unequiaxed Forms, the symbols which serve to denote them become necessarily cumbrous. Yet it is not possible to shorten them by any other process than that of *abridgement;* in other words, it is not allowable to *omit* any symbols, but only to *change* them for shorter symbols, giving definitions of these shorter symbols. I am averse to this method, because I think it is worse to burthen the memory with definitions of abridged symbols, than to

burthen the printed paper with cumbrous symbols unabridged. I shall, however, explain a method of abridging the symbols, in so far as respects the chief forms of this Class, and leave it to the reader to employ as he pleases, the full symbols or the abridgments.

Examples of a Method of denoting Combinations of Rhombohedrons and Scalenohedrons by means of Abridged Symbols.

Let R_x denote a rhombohedron, the $_x$ signifying the variable quantity indicated by $_x$ in the full symbol $\frac{1}{2}P_xT$, $\frac{1}{2}P_xM\frac{1}{1}\frac{5}{3}T_2$.

Let S denote the scalenohedron $\frac{1}{2}(3P_1M^{+}_{-}T^{-}_{+})$.

S_-, the obtuse scalenohedron $\frac{1}{2}(3P_-M^{+}_{-}T^{-}_{+})$.

S_+, the acute scalenohedron $\frac{1}{2}(3P_+M^{+}_{-}T^{-}_{+})$.

Let R_x be changed to $\textsc{r}_x$ and r_x, and let S_x be changed to $\textsc{s}_x$ and s_x, to indicate middle and small sized planes.

It is scarcely worth while to abridge the symbols which indicate the regular six, twelve, and twenty-four sided prisms, but it is extremely easy to do so when brevity of expression is considered indispensable. For example, let $T,M\frac{1}{1}\frac{5}{3}T_2$ be represented by V, (the initial of the word *vertical*,) and $m,m_2t\frac{1}{1}\frac{5}{3}$ by *v*. Then will

$$P,V, \quad = P,T,M\frac{1}{1}\frac{5}{3}T_2 \quad = \text{the 6 sided prism.}$$
$$P,V,v, \quad = P,m,T,m_2t\frac{1}{1}\frac{5}{3},M\frac{1}{1}\frac{5}{3}T_2 \quad = \text{the 12 sided prism.}$$
$$P,V,v,3m_xt = P,m,T,m_2t\frac{1}{1}\frac{5}{3},M\frac{1}{1}\frac{5}{3}T_2,3m_xt \quad = \text{the 24 sided prism.}$$

The six-sided pyramid $= PT,PM\frac{1}{1}\frac{5}{3}T_2$, which, as shown in 5) above, contains two equal and similar Rhombohedrons in opposite positions, may be expressed in abridged signs as follows :—

$$R_1\,Zw + R_1\,Ze, \text{ or } 2R_1\,Zw\,Ze.$$
$$\text{Also, } R_1\,Zn + R_1\,Zs, \text{ or } 2R_1\,Zn\,Zs.$$

The abridged signs, representative of the principal combinations, are given in the following catalogue within brackets, after the full symbol corresponding to each of them. About fifty of the most complex combinations of calcareous spar, placed at the end of the list relating to that mineral, are particularised in abridged signs alone. The expansion of the abridged signs into full symbols can be readily effected by substituting the equivalent symbols in the place of each R, S, and V. Thus, fig. 153, plate 21, Haüy =

$$V.\,\textsc{r}_1Zw,\ R_4Zw,\ r\tfrac{1}{2}Ze,\ r\tfrac{4}{5}Ze,\ r_2Ze,\ S_1,$$

Becomes, $T,M\frac{1}{1}\frac{5}{3}T_2$. $\frac{1}{2}\textsc{pt}Zw$, $\frac{1}{2}P_4TZw$, $\frac{1}{2}p\frac{1}{2}tZe$, $\frac{1}{2}p\frac{4}{5}tZe$, $\frac{1}{2}p_2tZe$, $\frac{1}{2}\textsc{pm}\frac{1}{1}\frac{5}{3}\textsc{t}_2$, $\frac{1}{2}P_4M\frac{1}{1}\frac{5}{3}T_2$, $\frac{1}{2}p\frac{1}{2}m\frac{1}{1}\frac{5}{3}t_2$, $\frac{1}{2}p\frac{4}{5}m\frac{1}{1}\frac{5}{3}t_2$, $\frac{1}{2}p_2m\frac{1}{1}\frac{5}{3}t_2$, $\frac{1}{2}(PM\frac{1}{1}\frac{5}{3}T\frac{2}{3}$, $PM\frac{1}{2}\frac{5}{6}T\frac{1}{2}$, $PM\frac{1}{1}\frac{5}{3}T_2$.)

I. *An Isomorphous Group*, 1, 2, 3:—

1. **Native Antimony.** Antimon. Antimoine natif.

Cleavage $= P.\frac{1}{2}P\frac{3}{4}T$, $\frac{1}{2}P\frac{3}{4}M\frac{1}{1}\frac{5}{3}T_2 = (P.R\frac{3}{4})$.

5. 5. $P.\frac{1}{2}P\frac{3}{4}T\ Zw$, $\frac{1}{2}P\frac{3}{4}M\frac{1}{1}\frac{5}{3}T_2 = (P.R\frac{3}{4})$......Similar Model 114. P343.

2. NATIVE ARSENIC. Arsenik. Arsenic natif.
Cleavage = p.
5. 5. $p.\frac{1}{2}P\frac{4}{3}T$ Zw, $\frac{1}{2}P\frac{4}{3}M\frac{1}{1}\frac{5}{3}T_i = (p. R\frac{4}{3})$... Sim. Mod. 86. P280. A226.

3. NATIVE TELLURIUM. Tellur.
Cleavage = $p\frac{3}{2}t,p\frac{3}{2}m\frac{1}{1}\frac{5}{3}t_2 = (2R\frac{3}{2})$.
3. 5. $P_+,T,M\frac{1}{1}\frac{5}{3}T_2.p\frac{3}{2}t,p\frac{3}{2}m\frac{1}{1}\frac{5}{3}t_2 = (P_+,V.2r\frac{3}{2})$......Sim. Mod. 58. P340.

2. GRAPHITE. Plumbago. Black Lead.
Cleavage = P.
1. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2 = (P_-,V)$ .... Model 7. L674[1]. P385.
3. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2.\ p_xt,p_xm\frac{1}{1}\frac{5}{3}t_2 = (P\_V.\ 2r_x$ ZwZe) Sim. Md. 58. L674[2].
3. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2.\ p_xm,p_xm_2t\frac{1}{1}\frac{5}{3} = (P_-,V.\ 2r_x$ ZnZs) Sim. Md. 56. L674[3].

3. OSMIUM-IRIDIUM. Iridium osmié.
Cleavage = P.
1. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2 = (P_-,V)$ ....Model 7. Ly47[1].
3. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2.\ p\frac{1}{2}t,p\frac{8}{3}t,p\frac{1}{2}m\frac{1}{1}\frac{5}{3}t_2,p\frac{8}{3}m\frac{1}{1}\frac{5}{3}t_2 = (P_-,V.\ 2r\frac{1}{2},2r\frac{8}{3})$ P340. Ly47[2].
5. 5. $P_-.P\frac{1}{2}T,P\frac{8}{3}T,P\frac{1}{2}M\frac{1}{1}\frac{5}{3}T_2,P\frac{8}{3}M\frac{1}{1}\frac{5}{3}T_2 = (P_-.\ 2R\frac{1}{2},2R\frac{8}{3})$ ... Ly v. 2.

4. ANTIMONIAL NICKEL. Antimonnickel.
1. 5. $P_x,T,M\frac{1}{1}\frac{5}{3}T_2 = (P_x,V)$..........................Model 7. Geiger 63.

5. COPPER NICKEL. Kupfernickel. Arsenical Nickel.
1. 5. $P_x,T,M\frac{1}{1}\frac{5}{3}T_2? = (P_x,V)$..................................Rose 162.

6. TELLURIC SILVER. Tellursilber. Weiss-Tellur.
1. 5. $P_x,T,M\frac{1}{1}\frac{5}{3}T_2$? ....Rose 162.

7. SULPHURET OF NICKEL. Haarkies. Nickel sulfuré.
1. 5. $P_x,T,M\frac{1}{1}\frac{5}{3}T_2$, ... Model 7. Rose 162.

8*. CINNABAR. Sulphuret of Mercury. Zinnober.
Cleavage = $T,M\frac{1}{1}\frac{5}{3}T_2 = (V)$.
1. 5. $P_+,T,M\frac{1}{1}\frac{5}{3}T_2 = (P_+,V.)$..............................Model 7. H89[32].
2. 5. $\frac{1}{2}P\frac{8}{3}T,\frac{1}{2}P\frac{8}{3}M\frac{1}{1}\frac{5}{3}T_2 = (R\frac{8}{3})$.................Model 88. H89[31]. Ly50[1].
3. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2.\ \frac{1}{2}p\frac{8}{3}t,\ \frac{1}{2}PT,\ \frac{1}{2}p\frac{8}{3}m\frac{1}{1}\frac{5}{3}t_2,\ \frac{1}{2}PM\frac{1}{1}\frac{5}{3}T_2$, ... H89[34].
3. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2.\ \frac{1}{2}p\frac{8}{3}t,\ \frac{1}{2}P\frac{8}{9}T,\ \frac{1}{2}p\frac{8}{3}m\frac{1}{1}\frac{5}{3}t_2,\ \frac{1}{2}P\frac{8}{9}M\frac{1}{1}\frac{5}{3}T_2$, ... H89[35].
3. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2.\ \frac{1}{2}p\frac{4}{3}t,\ \frac{1}{2}P\frac{8}{9}T,\ \frac{1}{2}p\frac{4}{3}m\frac{1}{1}\frac{5}{3}t_2,\ \frac{1}{2}P\frac{8}{9}M\frac{1}{1}\frac{5}{3}T_2$, ... H89[36].
5. 5. $P_+,\ \frac{1}{2}P\frac{8}{3}T,\ \frac{1}{2}p\frac{8}{9}t,\ \frac{1}{2}p\frac{8}{3}t,\ \frac{1}{2}P\frac{8}{3}M\frac{1}{1}\frac{5}{3}T_2,\ \frac{1}{2}p\frac{8}{9}m\frac{1}{1}\frac{5}{3}t_2,\ \frac{1}{2}p\frac{8}{3}m\frac{1}{1}\frac{5}{3}t_2$
$= (P_+.R\frac{8}{3},r\frac{8}{9},r\frac{8}{3})$......H89[33]. Ly50[6].

9. SULPHURET OF MOLYBDENUM. Molybdänglanz.
Cleavage = P.
1. 5. $P_-,T,M\frac{1}{1}\frac{5}{3}T_2$, ...: Model 7. J[231]. H116[306]. P249. L667[1].
4. 5. $T,M\frac{1}{1}\frac{5}{3}T_2.P_xT,P_xM\frac{1}{1}\frac{5}{3}T_2 = (V.\ 2R_x)$......Sim. Md. 47. J[232]. P667[2]. H116[307].

10. FLUORIDE OF CERIUM. Fluorcerium. Cérium fluaté.
Cleavage = P.

1. 5. $P_{-},T,M\frac{1}{1}\frac{5}{3}T_2$. = (P_{-},V) .... Fahlun ......Model 7. P267.

1. 5. $P_{-},m,T,m_2t\frac{1}{1}\frac{5}{3},M\frac{1}{1}\frac{5}{3}T_2$ = (P_{-},V,v) ... Model 10. L571. H iv 399.

11. ICE. Eis.

1. 5. $P\ ,T,M\frac{1}{1}\frac{5}{3}T_2$? = (P_x,V) .... Model 7. Rose 162.

12.* *An Isomorphous Group*, 1, 2, 3:—

1. CORUNDUM. Korund. Corindon. Sapphire. Oriental Ruby.
Cleavage = $P\frac{3}{2}T,P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2$ = $(2R\frac{3}{2})$.

1. 5. $P_{+},T,M\frac{1}{1}\frac{5}{3}T_2$, ...Bengal...Sim. Md. 7. J[23]. P66[3]. L536[4]. H47[112].

2. 5. $\frac{1}{2}P\frac{3}{2}T,\frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2$ = $(R\frac{3}{2})$... Bengal ... Sim. Md. 89. P66[1]. L536[1]. H47[107].

2. 5. $P\frac{9}{5}T,P\frac{9}{5}M\frac{1}{1}\frac{5}{3}T_2$ = $(2R\frac{9}{5}$ Zw Ze)...Sim. Md. 26, but more acute... P66[5]. L536[7]. H47[108].

2. 5. $P\frac{8}{3}T,P\frac{8}{3}M\frac{1}{1}\frac{5}{3}T_2$ = $(2R\frac{8}{3}$ Zw Ze) ...Sim. Md. 26, but more acute... P66[4]. L536[7]. J[25]. H47[109].

2. 5. $P\frac{16}{3}T, P\frac{9}{5}T, P\frac{16}{3}M\frac{1}{1}\frac{5}{3}T_2, P\frac{9}{5}M\frac{1}{1}\frac{5}{3}T_2$ = $(2R\frac{16}{3}$ ZwZe, $2R\frac{9}{5}$ ZwZe) H47[114].

3. 5. $P_{+},T,M\frac{1}{1}\frac{5}{3}T_2.P\frac{9}{5}T,P\frac{9}{5}M\frac{1}{1}\frac{5}{3}T_2$, ... Sim. Md. 58. J[27]. L536[6]. H47[116].

3. 5. $P,T.M\frac{1}{1}\frac{5}{3}T_2.\frac{1}{2}p\frac{3}{2}m$ Zn, $\frac{1}{2}p\frac{3}{2}m_2t\frac{1}{1}\frac{5}{3}$, ...Model 57. L536[3]. H48[118].

3. 5. $P_{+}T,M\frac{1}{1}\frac{5}{3}T_2.\frac{1}{2}p\frac{3}{2}m,p\frac{9}{5}t,\frac{1}{2}p\frac{3}{2}m_2t\frac{1}{1}\frac{5}{3},p\frac{9}{5}m\frac{1}{1}\frac{5}{3}t_2$, ..P65[4]. L536[5]. H48[119].

3. 5. $P_{+},t,m\frac{1}{1}\frac{5}{3}t_2.P\frac{8}{3}T,P\frac{8}{3}M\frac{1}{1}\frac{5}{3}T_2$ = $(P_{+},v.\ 2R\frac{8}{3}$ ZwZe) ....H48[120].

3. 5. $P,T,M\frac{1}{1}\frac{5}{3}T_2.\ p\frac{3}{2}m,p\frac{3}{2}m_2t\frac{1}{1}\frac{5}{3}$ = $(P,V.\ 2r\frac{3}{2}$Zn Zs) Sim. Md, 56. P65[1].

4. 5. $T,M\frac{1}{1}\frac{5}{3}T_2.P\frac{3}{2}M,P\frac{3}{2}M_2T\frac{1}{1}\frac{5}{3}$ = $(V.\ 2R\frac{3}{2}$Zn Zs)....................P65[2].

4. 5. $T,M\frac{1}{1}\frac{5}{3}T_2.\ \frac{1}{2}P\frac{3}{2}M,\frac{1}{2}P\frac{3}{2}M_2T\frac{1}{1}\frac{5}{3}$ = $(V.\ R\frac{3}{2}$ Zn)....................P66[2].

5. 5. $P.P\frac{9}{5}T,P\frac{9}{5}M\frac{1}{1}\frac{5}{3}T_2$ = $(P.2R\frac{9}{5})$ Sim. Md. 96. J[26]. P65[3]. L536[8]. H47[112].

5. 5. $P\ddagger.P\frac{16}{3}T,P\frac{16}{3}M\frac{1}{1}\frac{5}{3}T_2$ = $(P\ddagger.\ 2R\frac{16}{3})$ ....H47[115].

5. 5. $P_{+}.\frac{1}{2}p\frac{3}{2}m,P\frac{8}{3}T,\frac{1}{2}p\frac{3}{2}m_2t\frac{1}{1}\frac{5}{3},P\frac{8}{3}M\frac{1}{1}\frac{5}{3}T_2$ = $(P_{+}.\ R\frac{3}{2}$Zn,$2R\frac{8}{3}$Zw) H48[117].

5. 5. $P_{-}.\frac{1}{2}P\frac{3}{2}T,\frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2$ = $(P_{-}.R\frac{3}{2})$...Sim. Md. 114. Ly v. 1. H47[111]. L536[2].

2. SPECULAR IRON. Eisenglanz. Fer oligiste.
Cleavage = $\frac{1}{2}p\frac{3}{2}t,\frac{1}{2}p\frac{3}{2}m\frac{1}{1}\frac{5}{3}t_2$ = $(r\frac{3}{2})$.

2. 5. $\frac{1}{2}P\frac{3}{2}T,\frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2$ = $(R\frac{3}{2})$...Sim. Md. 83. L545[1]. P217[1]. H104[170].

2. 5. $\frac{1}{2}P\frac{3}{8}T,\frac{1}{2}P\frac{3}{8}M\frac{1}{1}\frac{5}{3}T_2$ = $(R\frac{3}{8})$...........Sim. Md. 85. L546[4]. H104[171].

2. 5. $\frac{1}{2}P\frac{3}{2}T,\ \frac{1}{2}p\frac{3}{8}t,\ \frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2,\ \frac{1}{2}p\frac{3}{8}m\frac{1}{1}\frac{5}{3}t_2$ = $(R\frac{3}{2},\ r\frac{3}{8})$......L545[3]. H104[173].

2. 5. $\frac{1}{2}P\frac{3}{2}T,\ \frac{1}{2}p\frac{3}{8}t,\ \frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2,\ \frac{1}{2}p\frac{3}{8}m\frac{1}{1}\frac{5}{3}t_2,\ \frac{1}{2}(3p_xm\mp t\pm)$ = $(R\frac{3}{2},\ r\frac{3}{8},\ s_x)$ ... P217[6]. H104[181].

2. 5. $\frac{1}{2}P\frac{3}{2}T$ Zw, $\frac{1}{2}P\frac{3}{8}T$ Zw, $\frac{1}{2}p\frac{3}{16}t$ Ze, $\frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2,\ \frac{1}{2}P\frac{3}{8}M\frac{1}{1}\frac{5}{3}T_2,\ \frac{1}{2}p\frac{3}{16}m\frac{1}{1}\frac{5}{3}t_2,$ $\frac{1}{2}(3P_xM\mp T\pm)$ = $(R\frac{3}{2}$Zw, $R\frac{3}{8}$Zw, $r\frac{3}{16}$Ze, $s_x)$.........H105[184].

2. 5, $\frac{1}{2}P\frac{3}{2}T,\ \frac{1}{2}p\frac{3}{8}t,\ \frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2,\ \frac{1}{2}p\frac{3}{8}m\frac{1}{1}\frac{5}{3}t_2,\ 2\,[\frac{1}{2}(3p_xm\mp t\pm)]$ = $(R\frac{3}{2},r\frac{3}{8},2s_x)$ ... H105[186].

2. 5. $P,t,\ m\frac{1}{1}\frac{5}{3}t_2.\ P\frac{7}{4}M,P\frac{7}{4}M_2T\frac{1}{1}\frac{5}{3}$ = $(P,v.\ 2\,R\frac{7}{4}$ ZnZs) .... H105[183].

5. 5. $P.\frac{1}{2}P\frac{3}{2}T,\ \frac{1}{2}P\frac{3}{2}M\frac{1}{1}\frac{5}{3}T_2$ = $(P.\ R\frac{3}{2})$......Sim. Md. 114. L545[2] P217[2], H104[172].

5. 5. $P.P\frac{7}{4}T,P\frac{7}{4}M\frac{1}{1}\frac{5}{3}T_2$ = $(P.\ 2R\frac{7}{4})$ ....Sim. Md. 96. H104[174].

5. 5. $P.P_5T,P_5M\frac{15}{13}T_2 = (P.\,2R_5)$ ....Sim. Md. 96. P217[5]. H104[175].

5. 5. $P_-.P\frac{7}{4}T,\frac{1}{2}p\frac{3}{2}t,P\frac{7}{4}M\frac{15}{13}T_2.\ \frac{1}{2}p\frac{3}{2}m\frac{15}{13}t_2 = (P_-.\,2R\frac{7}{4},r\frac{3}{2})$.........H104[176].

5. 5. $p.P\frac{3}{2}T,P\frac{3}{2}M\frac{15}{13}T_2,\ \frac{1}{2}(3p_xm^{-}_{+}t^{+}_{-}) = (p.\,2R\frac{3}{2},s_x)$...............H104[177].

5. 5. $P_-.\frac{1}{2}P\frac{3}{2}T,\frac{1}{2}p\frac{3}{2}t,\frac{1}{2}P\frac{3}{2}M\frac{15}{13}T_2,\frac{1}{2}p\frac{3}{2}m\frac{15}{13}t_2=(P_-.\,R\frac{3}{2},r\frac{3}{2})$ L546[5]. H104[179].

5. 5. $P_-.p\frac{3}{2}t,p\frac{3}{2}m\frac{15}{13}t_2 = (P_-.\,2r\frac{3}{2})$..........................P217[4]. H104[180].

5. 5. $P_-,t,m\frac{15}{13}t_2.p\frac{3}{2}m,p\frac{3}{2}m_2t\frac{15}{13} = (P_-,v.\,2r\frac{3}{2}\ ZnZs)$ ....H105[183].

3. Titanitic Iron Ore. Titaneisenerz.

Cleavage = P.

5. 5. $P_-.\frac{1}{2}P\frac{3}{2}T,\frac{1}{2}P\frac{3}{2}M\frac{15}{13}T_2,\ \frac{1}{4}(3p_xm^{-}_{+}t^{+}_{-})$ .... M ii [fig. 141, 142].

5. 5. $P_-.\frac{1}{2}P\frac{3}{2}T,\frac{1}{2}P\frac{3}{2}M\frac{15}{13}t_2,\ \frac{1}{2}(3p_xm^{-}_{+}t^{+}_{-})$ .... M ii [f. 143].

5. 5. $P_-.\frac{1}{2}P\frac{3}{2}T\ Zw,\ \frac{1}{2}p_-t\ Ze,\ \frac{1}{2}p_+t\ Ze,\ \frac{1}{2}P\frac{3}{2}M\frac{15}{13}T_2,\ \frac{1}{2}p_-m\frac{15}{13}t_2,\ \frac{1}{2}p_+m\frac{15}{13}t_2,$
$\frac{1}{4}(3P_xM^{-}_{+}T^{+}_{-}) = (P_-.\,R\frac{3}{2}\ Zw,\ r_-\ Ze,\ \dot{r}_+\ Ze,\ \frac{1}{2}S_x)$.........M ii [f. 144].

13. Sulphato-tricarbonate of Lead. Schwerbleierz.

Cleavage = P.

2. 5. $R\frac{5}{2},$.................................Brooke, Edin. Phil. Jour. iii. 119.

2. 5. $R\frac{5}{2},\ 3r_-,$............................... idem.

4. 5. $T,\ M\frac{15}{13}T_2.\ R\frac{5}{2},$ .... idem.

14. Quartz. Quarz. Rock Crystal.

Cleavage $= P\frac{5}{4}T,P\frac{5}{4}M\frac{15}{13}T_2 = (2R\frac{5}{4})$.

2. 5. $P\frac{5}{4}T,P\frac{5}{4}M\frac{15}{13}T_2 = (2R\frac{5}{4})$ ......Sim. Mod. 26...J[67]. P2[3]. R[67]. H55[2].

2. 5. $\frac{1}{2}P\frac{5}{4}T,\frac{1}{2}P\frac{5}{4}M\frac{15}{13}T_2 = (R\frac{5}{4})$.........Sim. Mod. 83......P2[1]. R[71]. H55[1].

2. 5. $\frac{1}{2}P\frac{5}{4}T\ Zw,\ \frac{1}{2}p\frac{5}{4}t\ Ze,\ \frac{1}{2}P\frac{5}{4}M\frac{15}{13}T_2,\ \frac{1}{2}p\frac{5}{4}m\frac{15}{13}t_2, = (R\frac{5}{4}Zw,\ r\frac{5}{4}Ze)$ ...... P2[2].

3. 5. $P_+,T,M\frac{15}{13}T_2.\ P\frac{5}{4}T,\ P\frac{5}{4}M\frac{15}{13}T_2,$.................................. H56[9].

4. 5. $T,M\frac{15}{13}T_2.\ P\frac{5}{4}T,P\frac{5}{4}M\frac{15}{13}T_2 = (V.\,2R\frac{5}{4}\ ZwZe)$ ...... Model 73...J[76].
P2[6]. R[68]. H55[3].

4. 5. $T,M\frac{15}{13}T_2.\ \frac{1}{2}P\frac{5}{4}T\ Zw,\ \frac{1}{2}p\frac{5}{4}t\ Ze,\ \frac{1}{2}P\frac{5}{4}M\frac{15}{13}T_2,\ \frac{1}{2}p\frac{5}{4}m\frac{15}{13}t_2$
$= (V.\,R\frac{5}{4}\ Zw,r\frac{5}{4}\ Ze)$.........H56[4].

4. 5. $t,m\frac{15}{13}t_2.\ \frac{1}{2}P\frac{5}{4}T,\ \frac{1}{2}p\frac{5}{4}t,\ \frac{1}{2}P\frac{5}{4}M\frac{15}{13}T_2,\ \frac{1}{2}p\frac{5}{4}m\frac{15}{13}t_2,$ ....P2[4]. H56[5].

4. 5. $T_-,M\frac{15}{13}T_2.\ P\frac{5}{4}T,\ P\frac{5}{4}M\frac{15}{13}T_2,$ ....H56[6].

4. 5. $T,M\frac{15}{13}T_2.\ \frac{1}{2}P\frac{5}{4}T\ Zw,\ \frac{1}{2}p\frac{5}{4}t\ Ze,\ p\frac{5}{4}m\frac{15}{13}t_2 = (V.\,R\frac{5}{4}\ Zw,\ r\frac{5}{4}\ Ze)$ H56[3].

4. 5. $T,M\frac{15}{13}T_2.\ \frac{1}{2}p_5m,\ P\frac{5}{4}T,\ \frac{1}{2}p_5m_2t\frac{15}{13},\ P\frac{5}{4}M\frac{15}{13}T_2. = (V.\,r_5\ Zn,\ 2\ R\frac{5}{4}\ ZwZe)$
J[68]. H57[10].

4. 5. $T,M\frac{15}{13}T_2.p\frac{5}{2}m,P\frac{5}{4}T,p\frac{5}{2}m_2t\frac{15}{13},P\frac{5}{4}M\frac{15}{13}T_2,$ ....H57[12].

4. 5. $T,M\frac{15}{13}T_2.\ p\frac{5}{4}t,\ P_{39}T,p\frac{5}{4}m\frac{15}{13}t_2,P_{39}M\frac{15}{13}T_2,$.......................H57[13].

4. 5. $T,M\frac{15}{13}T_2.\ p_5t,\ P\frac{5}{4}T,\ p_5m\frac{15}{13}t_2,\ P\frac{5}{4}M\frac{15}{13}T_2,$ ....P2[11]. H57[14].

4. 5. $T,M\frac{15}{13}T_2.\ P\frac{5}{4}T,\ P\frac{5}{4}M\frac{15}{13}T_2,\ \frac{1}{4}(3p_+m^{+}_{-}t^{-}_{+})Ze,\ \frac{1}{4}(3p_+m^{+}_{-}t^{-}_{+})\ Zw$...........
P2[8]. H57[15]. J[69].

4. 5. $T,M\frac{15}{13}T_2.P\frac{5}{4}T,P\frac{5}{4}M\frac{15}{13}T_2,\ 2\left[\frac{1}{2}(3p_+m^{+}_{-}t^{-}_{+})\right] = (V.\,2R\frac{5}{4}ZwZe,\ 2s_+)$...
H58[16].

4. 5. $T,M\frac{15}{13}T_2.P\frac{5}{4}T,P_4T,P\frac{5}{4}M\frac{15}{13}T\ ,P_4M\frac{15}{13}T_2,\ \frac{1}{2}(3p_+m^{+}_{-}t^{-}_{+})$..............H58[17].

4. 5. $T,M\frac{15}{13}T_2.\ P\frac{5}{4}T,\ P\frac{5}{4}M\frac{15}{13}T_2,\ 3\left[\frac{1}{2}(3p_xm^{+}_{-}t^{-}_{+})\right] = (V.\,2\,R\frac{5}{4}\ Zw\ Ze,\ 3s_+)$
H58[18].

4. 5. $t,m\frac{15}{13}t_2.\ P\frac{5}{4}T,\ P\frac{5}{4}M\frac{15}{13}T_2 = (v.\,2R\frac{5}{4}\ Zw\ Ze)$..........................P2[9].

4. 5. $t,m\frac{15}{13}t_2.p_5m,\ P\frac{5}{4}T,\ p_5m_2t\frac{15}{13},\ P\frac{5}{4}M\frac{15}{13}T_2 = (v.\,2r_5\ ZnZs,\ 2R\frac{5}{4}\ ZwZe)$ P2[7].

15.* TETRADYMITE. Cleavage = P.

2. 5. $\frac{1}{2}P\frac{6}{11}T, \frac{1}{2}P\frac{6}{11}M\frac{1\,5}{1\,3}T_2 = (R\frac{1}{2}?)$ ....S230.

16*. POLYBASITE.

1. 5. $P_-, T, M\frac{1\,5}{1\,3}T_2 = (P_-, V)$.....................Mexico......D417. P300.

3. 5. $P, T, M\frac{1\,5}{1\,3}T_2.\ 3p_x t, 3p_x m\frac{1\,5}{1\,3}t_2 = (P, V.\ 3r_x\ Ze, 3r_x\ Zw)$ .... S114.

3. 5. $P, T, M\frac{1\,5}{1\,3}T_2.\ 6p_x t, 6p_x m\frac{1\,5}{1\,3}t_2 = (P, V.\ 6r_x\ Ze, 6r_x\ Z\hat{w})$ .... S114.

17.* RED SILVER. Rothgültigerz. Ruby Silver. Argent rouge. Argent antimonié sulfuré. Two varieties:

1. LIGHT RED SILVER; Sulphuret of Silver and Arsenic.
2. DARK RED SILVER; Sulphuret of Silver and Antimony.

Cleavage = $PT, PM\frac{1\,5}{1\,3}T_2$.

(The forms $PT, P\frac{1}{2}T, P_2T$ should perhaps be $P\frac{10}{11}T, P\frac{5}{11}T, P\frac{20}{11}T$.)

1. 5. $P, T, M\frac{1\,5}{1\,3}T_2 = (P, V.)$..............................Model 7......H87[11].

2. 5. $\frac{1}{2}p\frac{1}{2}t, \frac{1}{2}p\frac{1}{2}m\frac{1\,5}{1\,3}t_2, \frac{1}{2}(3P_+M^-_+T^+_-) = (r\frac{1}{2}, S_+)$......................H87[12].

2. 5. $\frac{1}{2}(3P_+M^-_+T^+_-), \frac{1}{2}(3p_-m^-_+t^+_-) = (S_+, s_-)$..............................H87[13].

2. 5. $\frac{1}{2}p_+t, \frac{1}{2}p_+m\frac{1\,5}{1\,3}t_2, \frac{1}{2}(3P_xM^-_+T^+_-) = (r_+, S_x)$ ....H87[14].

3. 5. $p, T, M\frac{1\,5}{1\,3}T_2.\ \frac{1}{2}P\frac{1}{2}M, \frac{1}{2}P\frac{1}{2}M_2T\frac{1\,5}{1\,3} = (p, V.\ r\frac{1}{2}\ Zn)$ .... H87[16].

4. 5. $T, M\frac{1\,5}{1\,3}T_2.\frac{1}{2}PM, \frac{1}{2}PM_2T\frac{1\,5}{1\,3}. = (V.\ R_1\ Zn)$.........................H87[10].

4. 5. $T, M\frac{1\,5}{1\,3}T_2.\ \frac{1}{2}PM\ Zn, \frac{1}{2}p\frac{1}{2}m\ Zs, \frac{1}{2}PM_2T\frac{1\,5}{1\,3}, \frac{1}{2}p\frac{1}{2}m_2t\frac{1\,5}{1\,3}$ ....H87[15].

4. 5. $T, M\frac{1\,5}{1\,3}T_2.\ \frac{1}{2}PM\ Zn, \frac{1}{2}p\frac{1}{2}m\ Zs, \frac{1}{2}p_2m\ Zs, \frac{1}{2}PM_2T\frac{1\,5}{1\,3}, \frac{1}{2}p\frac{1}{2}m_2t\frac{1\,5}{1\,3},$
$\frac{1}{2}p_2m_2t\frac{1\,5}{1\,3}, \frac{1}{2}(3p_+m^+_-t^-_+) = (V.\ R_1\ Zn, r\frac{1}{2}\ Zs, r_2\ Zs, s_+)$......H88[23].

4. 5. $T, M\frac{1\,5}{1\,3}T_2.\ \frac{1}{2}(3P_-M^-_+T^+_-), \frac{1}{2}(3p_+m^-_+t^+_-) = (V.\ s_-, s_+)$.........H88[20].

4. 5. $T, M\frac{1\,5}{1\,3}T_2.\ \frac{1}{2}PM\ Zn, \frac{1}{2}p\frac{1}{2}m\ Zs, \frac{1}{2}PM_2T\frac{1\,5}{1\,3}, \frac{1}{2}p\frac{1}{2}m_2t\frac{1\,5}{1\,3}, \frac{1}{2}(3p_-m^-_+t^+_-)$
$= (V.\ R_1 Zn, r\frac{1}{2}Zs, s_-)$.........H88[21].

4. 5. $m, T, m_2t\frac{1\,5}{1\,3}, M\frac{1\,5}{1\,3}T_2.\ \frac{1}{2}P\frac{1}{2}M, \frac{1}{2}P\frac{1}{2}M_2T\frac{1\,5}{1\,3} = (V, v.\ R\frac{1}{2}\ Zn)$......H88[19].

4. 5. $m, T, m_2t\frac{1\,5}{1\,3}, M\frac{1\,5}{1\,3}T_2.\ \frac{1}{2}PM\ Zn, \frac{1}{2}p\frac{1}{2}m\ Zs, \frac{1}{2}PM_2T\frac{1\,5}{1\,3}, \frac{1}{2}p\frac{1}{2}m_2t\frac{1\,5}{1\,3}$
$= (V, v.\ R_1\ Zn, r\frac{1}{2}\ Zs)$.........H88[22].

4. 5. $m, T, m_2t\frac{1\,5}{1\,3}, M\frac{1\,5}{1\,3}T_2.\ \frac{1}{2}PM\ Zn, \frac{1}{2}p\frac{1}{2}m\ Zs, \frac{1}{2}PM_2T\frac{1\,5}{1\,3}, \frac{1}{2}p\frac{1}{2}m_2t\frac{1\,5}{1\,3},$
$\frac{1}{2}(3p_+m^+_-t^-_+) = (V, v.\ R_1\ Zn, r\frac{1}{2}\ Zs, s_+)$.........H88[24].

5. 5. $P.PT, P_2T, PM\frac{1\,5}{1\,3}T_2, P_2M\frac{1\,5}{1\,3}T_2. = (P.\ 2R_1, 2R_2)$ ....H88[17].

18. MAGNETIC IRON PYRITES. Magnetkies. Fer sulfuré magnétique.

Cleavage = $P, T, M\frac{1\,5}{1\,3}T_2 = (P, V)$.

1. 5. $P_-, T, M\frac{1\,5}{1\,3}T_2 = (P_-, V)$ .... Model 7. L665[1].

1. 5. $P, m, T, m_2t\frac{1\,5}{1\,3}, M\frac{1\,5}{1\,3}T_2 = (P, V, v)$ .... Model 10. L665[2].

3. 5. $P, T, M\frac{1\,5}{1\,3}T_2.pm, pm_2t\frac{1\,5}{1\,3} = (P, V.\ 2r_1\ ZnZs)$.....................L665[3].

3. 5. $P_-, T, M\frac{1\,5}{1\,3}T_2.\ p\frac{1}{3}t, p\frac{1}{3}m\frac{1\,5}{1\,3}t_2,$ .... D404. L665[4].

3. 5. $P, m, T, m_2t\frac{1\,5}{1\,3}, M\frac{1\,5}{1\,3}T_2.\ pm, pm_2t\frac{1\,5}{1\,3} = (P, V, v.\ 2r_1\ ZnZs)$......L665[7].

3. 5. $P, m, T, m_2t\frac{1\,5}{1\,3}, M\frac{1\,5}{1\,3}T_2.\ pm, p\frac{1}{3}t, p\frac{2}{9}t, pm_2t\frac{1\,5}{1\,3}, p\frac{1}{3}m\frac{1\,5}{1\,3}t_2, p\frac{2}{9}m\frac{1\,5}{1\,3}t_2$
$= (P, V, v.\ 2r_1\ ZnZs, 2r\frac{1}{3}\ ZwZe, 2r\frac{2}{9}\ ZwZe)$......L665[8].

3. 5. $P,m,T,m_2t\frac{1\,5}{1\,3}, M\frac{1\,5}{1\,3}T_2.\ pm,p\frac{8}{9}t,pm_2t\frac{1\,5}{1\,3}, p\frac{8}{9}m\frac{1\,5}{1\,3}t_2$
$= (P,V,v.\ 2r_1\ ZnZs, 2r\frac{8}{9}\ ZeZw)$......P213.

4. 5. $T,M\frac{1\,5}{1\,3}T_2.\ P\frac{1}{3}T, P\frac{1}{3}M\frac{1\,5}{1\,3}T_2$, ... Sim. Model 74. L665[6].

5. 5. $P.\ P\frac{1}{3}T, P\frac{1}{3}M\frac{1\,5}{1\,3}T_2 = (P.\ 2R\frac{1}{3}\ ZwZe)$...... Sim. Model 96. L665[5].

19.* *An Isomorphous Group of Carbonates*, which crystallise or cleave in nearly the same form as the cleavage rhombohedron of calcareous spar.—I have written the symbols as if the forms were all precisely alike, but the following table corrects the error of this uniformity by giving the numbers derivable from the most recent measurements of each form.

[The angle quoted is that across the obtuse edge of the Rhombohedron.]

1. Calcareous Spar:
 105° 5′ Mohs $= \frac{1}{2}P\frac{1:0000000}{1\cdot0128817}T, \frac{1}{2}P\frac{1:0000000}{1\cdot0128817}M\frac{1\,5}{1\,3}T_r$
 Do.
 104° 28$\frac{2}{3}$′ Haüy $= \frac{1}{2}P\frac{1:0000000}{1\cdot0002929}T, \frac{1}{2}P\frac{1:0000000}{1\cdot0002929}M\frac{1\,5}{1\,3}T_r$
2. Dolomite:
 106° 15′ Mohs $= \frac{1}{2}P\frac{1:0000000}{1\cdot0403643}T, \frac{1}{2}P\frac{1:0000000}{1\cdot0403643}M\frac{1\,5}{1\,3}T_r$
3. Brown Spar:
 107° 30′ Levy $= \frac{1}{2}P\frac{1:0000000}{1\cdot0698702}T, \frac{1}{2}P\frac{1:0000000}{1\cdot0698702}M\frac{1\,5}{1\,3}T_r$
4. Carbonate of Magnesia:
 107° 22′ Levy $= \frac{1}{2}P\frac{1:0000000}{1\cdot0667358}T, \frac{1}{2}P\frac{1:0000000}{1\cdot0667358}M\frac{1\,5}{1\,3}T_r$
5. Mesitin Spar: ? — Rose
6. Carbonate of Iron:
 107° 0′ Mohs $= \frac{1}{2}P\frac{1:0000000}{1\cdot0580867}T, \frac{1}{2}P\frac{1:0000000}{1\cdot0580867}M\frac{1\,5}{1\,3}T_r$
7. Carbonate of Manganese:
 106° 51′ Mohs $= \frac{1}{2}P\frac{1:0000000}{1\cdot0543942}T, \frac{1}{2}P\frac{1:0000000}{1\cdot0543942}M\frac{1\,5}{1\,3}T_r$
8. Carbonate of Zinc:
 107° 40′ Mohs $= \frac{1}{2}P\frac{1:0000000}{1\cdot0734333}T, \frac{1}{2}P\frac{1:0000000}{1\cdot0734333}M\frac{1\,5}{1\,3}T_r$

I have adopted Haüy's measurement, and written the symbol $\frac{1}{2}PT$, $\frac{1}{2}PM\frac{1\,5}{1\,3}T_2$, as equivalent to the whole of the above cited forms. It may be that this is inaccurate, but, at any rate, the differences of the forms are not such as to serve as a useful means of discriminating the minerals from one another, while, at the same time, chemistry affords us methods of discrimination which are both simple and certain.

The Cleavage of all the varieties is $= \frac{1}{2}PT, \frac{1}{2}PM\frac{1\,5}{1\,3}T_2$.

1. CALCAREOUS SPAR. Calc Spar. Carbonate of Lime. Chaux Carbonaté. Kalkspath. Kohlensaurer Kalk.

Prisms.

1. 5. $P_+, T, M\frac{1\,5}{1\,3}T_2. = (P_+,V)$ ...(Haüy's Prismatique)...Md. 7. H6[22].

1. 5. $P_+, \frac{1}{2}T, \frac{1}{2}t, \frac{1}{2}M\frac{1\,5}{1\,3}T_2, \frac{1}{2}m\frac{1\,5}{1\,3}t_2$, ... (alternante)...... H6[22] i. 315[a].

1. 5. $P_+,T,m\frac{1\,5}{1\,3}t_2$, ...Model 8...(comprimée)......H6[22] i. 315[b].

1. 5. $P_+, t, M\frac{1\,5}{1\,3}T_2$................................ (evasée)......H6[22] i. 315[c].

1. 5. $P_{-}, T, M^{1\,5}_{1\,3}T_2 = (P_{-}, v)$ .... (raccourcie)......H6[22] i. 315[d].
1. 5. $P^{-}_{-}, t, m^{1\,5}_{1\,3}t_2 = (P^{-}_{-}, v)$ ....(lamelliforme)......H6[23] i. 315[e].
1. 5. $P_{+}, M, T, M_2T^{15}_{13}, M^{15}_{13}T_2 = (P_{+}, V, v)$ Cumberland...Md. 10. H11[73].

Rhombohedrons.

2. 5. $\frac{1}{2}PT, \frac{1}{2}PM^{1\,5}_{1\,3}T_2 = (R_1)$ ... (Haüy's Primitive)...Md. 83. H4[1]. R[71].
2. 5. $\frac{1}{2}P\frac{1}{2}T, \frac{1}{2}P\frac{1}{2}M^{1\,5}_{1\,3}T_2 = (R\frac{1}{2})$ ......(Haüy's Equiaxe)...Md. 85. H4[2].
2. 5. $\frac{1}{2}P_2T, \frac{1}{2}P_2M^{1\,5}_{1\,3}T_2 = (R_2)$ ....(Haüy's Inverse)...Md. 89. H4[3].
2. 5. $\frac{1}{2}P_4T, \frac{1}{2}P_4M^{1\,5}_{1\,3}T_2 = (R_4)$ .... (Haüy's Contrastante)...H4[7].
2. 5. $\frac{1}{2}P_5T, \frac{1}{2}P_5M^{1\,5}_{1\,3}T_2 = (R_5)$ ... (Haüy's Mixte) ...Derbyshire...H4[8].
2. 5. $P_5T, P_5M^{1\,5}_{1\,3}T_2 = (2R_5$ ZwZe$)$... (Haüy's Leptomorphique)...H4[4].
2. 5. $\frac{1}{2}P\frac{3}{2}T, \frac{1}{2}P\frac{3}{2}M^{1\,5}_{1\,3}T_2 = (R\frac{3}{2})$.........(Haüy's Cuboïde)...........H4[9].
2. 5. $\frac{1}{2}PT$ Zw, $\frac{1}{2}p^1_2t$ Ze, $\frac{1}{2}PM^{1\,5}_{1\,3}T_2, \frac{1}{2}p^1_2m^{1\,5}_{1\,3}t_2 = (R_1$ Zw, r^1_2 Ze$)$
(Haüy's Semi-émarginé)...Isère......H5[11].
2. 5. $\frac{1}{2}pt$ Zw, $\frac{1}{2}P_2T$ Ze, $\frac{1}{2}pm^{1\,5}_{1\,3}t_2, \frac{1}{2}P_2M^{1\,5}_{1\,3}T_2 = (r_1$ Zw, R_2 Ze$)$......(Haüy's
Unitaire)......Lyons. Ireland......H5[12].
2. 5. $\frac{1}{2}PT, \frac{1}{2}P_4T, \frac{1}{2}PM^{1\,5}_{1\,3}T_2, \frac{1}{2}P_4M^{1\,5}_{1\,3}T_2 = (R_1, R_4)$ ......Derbyshire......H5[17].
2. 5. $\frac{1}{2}pt$ Zw, $\frac{1}{2}P_5T$ Ze, $\frac{1}{2}pm^{1\,5}_{1\,3}t_2, \frac{1}{2}P_5M^{1\,5}_{1\,3}T_2 = (r_1$ Zw, R_5 Ze$)$ ....H5[18].
2. 5. $\frac{1}{2}p^1_2t, \frac{1}{2}P_2T, \frac{1}{2}p^1_2m^{1\,5}_{1\,3}t_2, \frac{1}{2}P_2M^{1\,5}_{1\,3}T_2 = (r^1_2, R_2)$...Clermont-Farrand, H6[25].
2. 5. $\frac{1}{2}P^1_2T, \frac{1}{2}P_5T, \frac{1}{2}P^1_2M^{1\,5}_{1\,3}T_2, \frac{1}{2}P_5M^{1\,5}_{1\,3}T_2. = (r^1_2, R_5)$ ......Derbyshire...H7[32].
2. 5. $\frac{1}{2}P_{13}T$ Zw, $\frac{1}{2}P^1_2T$ Ze, $\frac{1}{2}P_{13}M^{1\,5}_{1\,3}T_2, \frac{1}{2}P^1_2M^{1\,5}_{1\,3}T_2. = (R_{13}$ Zw, r^1_2 Ze$)$ ...H7[33].
2. 5. $\frac{1}{2}P_{14}T, \frac{1}{2}P^1_2T, \frac{1}{2}P_{14}M^{1\,5}_{1\,3}T_2, \frac{1}{2}P^1_2M^{1\,5}_{1\,3}T_2. = (R_{14}, R^1_2)$ ....H7[34].
2. 5. $\frac{1}{2}P_2T$ Zw, $\frac{1}{2}p_4t$ Ze, $\frac{1}{2}P_2M^{15}_{13}T_2, \frac{1}{2}p_4m^{15}_{13}t_2 = (R_2$ Zw, r_4 Ze$)$ .... H8[42].
2. 5. $\frac{1}{2}P^5_4T$ Zw, $\frac{1}{2}p_4t$ Ze, $\frac{1}{2}P^5_4M^{15}_{13}T_2, \frac{1}{2}p_4m^{15}_{13}t_2. = (R^5_4$ Zw, r_4 Ze$)$ .... H9[53].
2. 5. $\frac{1}{2}pt$ Zw, $\frac{1}{2}P_4T$ Zw, $\frac{1}{2}P_5T$ Ze, $\frac{1}{2}pm^{15}_{13}t_2, \frac{1}{2}P_4M^{15}_{13}T_2, \frac{1}{2}P_5M^{15}_{13}T_2.$
$= (r_1$ Zw, R_4 Zw, R_5 Ze$)$......Jura......H10[65].
2. 5. $\frac{1}{2}P^1_2T, \frac{1}{2}p^4_5t, \frac{1}{2}P_{14}T, \frac{1}{2}P^1_2M^{15}_{13}T_2, \frac{1}{2}p^4_5m^{15}_{13}t_2, \frac{1}{2}P_{14}M^{15}_{13}T_2 = (R^1_2, r^4_5, R_{14})$ ...H13[91].
2. 5. r_1Zw, R_4Zw, R_5Ze, r^4_5 Ze, ...H16[111].

Scalenohedrons.

2. 5. $\frac{1}{2}(3_1PM^{\pm}_{}T^{-}_{+}) = (S_1)$...(Haüy's Métastatique)...Md. 116. H4[5]. R[79].
2. 5. $\frac{1}{2}(3P^{+}_{+}M^{\pm}T^{-}_{+}) = (S^{+}_{+})$ .... (Haüy's Axigraphe) ......H4[6].
2. 5. $\frac{1}{2}(3P_{-}M^{\pm}T^{-}_{+}), \frac{1}{2}(3P_{+}M^{\pm}T^{-}_{+}) = (S_{-}, S_{+})$ ....H7[36].
2. 5. $\frac{1}{2}(3p_1m^{\pm}t^{-}_{+}), \frac{1}{2}(3P_{-}M^{\pm}T^{-}_{+}) = (s_1, S_{-})$...........Simplon......H7[37].

Rhombohedrons with Scalenohedrons.

2. 5. $\frac{1}{2}pt, \frac{1}{2}pm^{1\,5}_{1\,3}t_2, \frac{1}{2}(3P_1M^{\pm}T^{-}_{+}) = (r_1, S_1)$......Derbyshire......R[81]. H5[14].
2. 5. $\frac{1}{2}pt, \frac{1}{2}pm^{1\,5}_{1\,3}t_2, \frac{1}{2}(3P_{+}M^{\pm}T^{-}_{+}) = (r_1, S_{+})$ ....Oisans......R[81]. H5[15].
2. 5. $\frac{1}{2}P^1_2T, \frac{1}{2}P^1_2M^{1\,5}_{1\,3}T_2, \frac{1}{2}(3p_{+}m^{-}_{+}t^{\pm}) = (R^1_2, s_{+})$ ....H6[26].
2. 5. $\frac{1}{2}P^1_2T, \frac{1}{2}P^1_2M^{1\,5}_{1\,3}T_2, \frac{1}{2}(3p^{+}_{+}m^{-}_{+}t^{\pm}) = (R^1_2, s^{+}_{+})$ ....H6[28].
2. 5. $\frac{1}{2}P^1_2T, \frac{1}{2}P^1_2M^{1\,5}_{1\,3}T_2, \frac{1}{2}(3P^{+}_{+}M^{\pm}T^{-}_{+}) = (r^1_2, S^{+}_{+})$ ....H6[29].
2. 5. $\frac{1}{2}P_2T, \frac{1}{2}P_2M^{15}_{13}T_2, \frac{1}{2}(3P_{+}M^{\pm}T^{-}_{+}) = (R_2, S_{+})$........................H7[30].

2. 5. $\frac{1}{2}p_2t,\ \frac{1}{2}p_2m_{13}^{15}t_2,\ \frac{1}{2}(3P\ddagger M\pm T\mp) = (r_2,\ S\ddagger)$ ....H8[44].

2. 5. $\frac{1}{2}P_4T,\ \frac{1}{2}P_4M_{13}^{15}T_2.\ \frac{1}{2}(3p_1m\pm t\mp) = (R_4\ s_1)$ ...... Derbyshire......H8[45].

2. 5. $\frac{1}{2}P_5^4T,\ \frac{1}{2}P_5^4M_{13}^{15}T_2,\ \frac{1}{2}(3P_1M\pm T\mp) = (R_5^4,\ S_1)$ ... Bex, Switzerland...H8[46].

2. 5. $\frac{1}{2}P_4^5T,\ \frac{1}{2}P_4^5M_{13}^{15}T_2,\ \frac{1}{2}(3P_1M\pm T\mp) = (R_4^5,\ S_1)$ ....H8[47].

2. 5. $\frac{1}{2}p_4t,\ \frac{1}{2}p_4m_{13}^{15}t_2,\ \frac{1}{2}(3P\ddagger M\pm T\mp) = (r_4,\ S\ddagger)$..............Hartz......H8[48].

2. 5. $\frac{1}{2}p_2^7t,\ \frac{1}{2}p_2^7m_{13}^{15}t_2,\ \frac{1}{2}(3P\ddagger M\pm T\mp) = (r_2^7,\ S\ddagger)$........................ H9[49].

2. 5. $\frac{1}{2}pt$ Zw, $\frac{1}{2}p_2t$ Ze, $\frac{1}{2}pm_{13}^{15}t_2,\ \frac{1}{2}p_2m_{13}^{15}t_2,\ \frac{1}{2}(3P\ddagger M\pm T\mp) = (r_1$ Zw, r_2 Ze, $S\ddagger)$
H9[57].

2. 5. $\frac{1}{2}PT$ Zw, $\frac{1}{2}p_5^4t$ Ze, $\frac{1}{2}PM_{13}^{15}T_2,\ \frac{1}{2}p_5^4m_{13}^{15}t_2,\ \frac{1}{2}(3P_1M\pm T\mp) = (R_1$ Zw, r_5^4 Ze, $S^1)$
Mexico............H10[62].

2. 5. $\frac{1}{2}PT,\ \frac{1}{2}PM_{13}^{15}T_2,\ \frac{1}{2}(3P_+M\pm T\mp),\ \frac{1}{2}(3P_-M\pm T\mp) = (R_1, S_+, S_-)$ H10[63].

2. 5. $\frac{1}{2}p_2t,\ \frac{1}{2}p_2m_{13}^{15}t_2.\ \frac{1}{2}(3P_+M\pm T\mp),\ \frac{1}{2}(3p_1m\pm t\mp) = (r_2, S_+, s_1)$.........H14[94].

2. 5. $\frac{1}{2}P_2T$ Zw, $\frac{1}{2}p_{14}t$ Ze, $\frac{1}{2}P_2M_{13}^{15}T_2,\ \frac{1}{2}p_{14}m_{13}^{15}t_2,\ \frac{1}{2}(3p_+m\pm t\mp)$
$= (R_2$ Zw, r_{14}Ze, $s_+)$......Lyons...H14[95].

2. 5. $\frac{1}{2}P_4T$ Zw, $\frac{1}{2}p_2t$ Ze, $\frac{1}{2}P_4M_{13}^{15}T_2,\ \frac{1}{2}p_2m_{13}^{15}t_2,\ \frac{1}{2}(3p_1m\pm t\mp) = (R_4$ Zw, r_2Ze, $s_1)$
H14[100].

2. 5. $R_4,\ S_1,\ S\ddagger,$ ....H15[103].

2. 5. r_1Zw, r_2Ze, $s_1,\ S_+,$ ....H15[106].

2. 5. r_1Zw, R_4Zw, r_2^3 Ze, $S_1,$ ....H15[107].

2. 5. R_1Zw, r_4Zw, $S_1,\ s_-,$ ....H15[108].

2. 5. R_1Zw, R_4Zw, R_{14}Ze, $s_1,$ ....H16[109].

2. 5. $R_1,\ r_4,\ s\ddagger,\ s_-,$ ....H16[110].

2. 5. R_4Zw, r_2^1Ze, r_2Ze, $s_1,$ ....H17[121].

2. 5. r_4Zw, r_8Ze, $S_1,\ S\ddagger,$ ....H18[131].

2. 5. $r_2^1,\ r_2,\ s_1,\ S_+,$ ....H18[132].

2. 5. r_1Zw, r_2^1Ze, r_4^5 Zn, r_4^5Zs, $S_+,\ s\ddagger,$ ....H19[135].

2. 5. r_1Zw, R_5^4Ze, r_4^5Ze, R_2Ze, $S_1,$ ....H19[137].

2. 5. R_4Zw, R_{14}Ze, r_2^1Ze, r_5^4Ze, r_4^5Zn, r_4^5Zs, $S_1,$ ....H20[140].

Complete Prisms with Incomplete Pyramids.

3. 5. $p_+, T, M_{13}^{15}T_2.\ \frac{1}{2}P_2^1T,\ \frac{1}{2}P_2^1M_{13}^{15}T_2,$ ...... Kongsberg. Andreasberg ... H10[66].

3. 5. $P_+, T, M_{13}^{15}T_2.\ \frac{1}{2}P_2T,\ \frac{1}{2}P_2M_{13}^{15}T_2,$ ....Derbyshire......H11[69].

3. 5. $P_+, T, M_{13}^{15}T_2.\ P_5M,\ P_5M_2T_{13}^{15} = (P_+, T, M_{13}^{15}T_2.\ 2R_5$ ZnZs)......H11[73].

3. 5. $P, t, m_{13}^{15}t_2.\ \frac{1}{2}(3P\ddagger M\pm T\mp) = (P.t, m_{13}^{15}t_2,\ S\ddagger) = (P, v.\ S\ddagger)$......H11[74].

3. 5. $P_+, T,\ M_{13}^{15}T_2.\ \frac{1}{2}p_2^3t,\ \frac{1}{2}p_2^3m_{13}^{15}t_2 = (P_+, V,\ r_2^3)$......Derbyshire......H11[75].

3. 5. $P_+, T, M_{13}^{15}T_2.\ \frac{1}{2}p_4^5t,\ \frac{1}{2}p_4^5m_{13}^{15}t_2 = (P_+, V,\ r_5^4)$........................H12[76].

3. 5. $p_+, t, m_{13}^{15}t_2.\ \frac{1}{2}P_4M,\ \frac{1}{2}P_4M_2T_{13}^{15} = (p_+, v.\ R_4$ Zn) ....H12[77].

3. 5. p_+, V. R_1Zw, r_2^1Ze, ...H15[105].

3. 5. p_+, V. R_2^1Ze, $s_-,$ ....H16[112].

3. 5. P, V. r_2^1Ze, r_2Ze, ...H16[113].

3. 5. P_+, V. r_2Ze, $S_1,$ ....H16[114].

3. 5. P_+, V, *v*. r_4Zw, ...H16[116].

3. 5. p_+, V. R_5^3Ze, r_2Ze, ...H16[115].

3. 5. P_+, V. r_4Zw, R_5Ze, ...H16[117].

3. 5. p_{+},v. R_2Zn, r_2^3Zn, s_{+}, ... H19[138].
3. 5, P_{+}, v, $3m_x$t. R_2, $s_{\ddagger}$, ... H19[139].
3. 5. P_{+}, V, *v*. R_4Zn, r_2Zs, ... H19[140].
3. 5. P_{+}, V, *v*. S_{+}, s_{+}, ... H19[141].
3. 5. p,v, *v*. r^1_2, R_2, $s_{\ddagger}$, ... H20[148].
3. 5. P_{+}, v. r_4Zw, r_2Ze, $R_{14}Ze$, $s_{\ddagger}$, ... H20[149].

Incomplete Prisms with Complete Pyramids.

a.) Without Scalenohedrons.

4. 5. T,$M^{1\,5}_{1\,3}T_2$. P^5_4M, $P^5_4M_2T^{1\,5}_{1\,3}$, ... Derbyshire. Cumberland... H7[35].
4. 5. T,$M^{1\,5}_{1\,3}T_2$. P^2_3M, $P^2_3M_2T^{1\,5}_{1\,3}$, ... H7[38].
4. 5. T, $M^{15}_{13}T_2$. PT, $PM^{15}_{13}T_2$. = (V. $2R_1$) ...... Sim. Md. 73. H10[64].
4. 5. T,$M^{1\,5}_{1\,3}T_2$. $\frac{1}{2}$PM Zn, $\frac{1}{2}PM_2T^{1\,5}_{1\,3}$ = (V. R_1 Zn) ...... Md. 71. H5[13]. R[78].
4. 5. T,$M^{1\,5}_{1\,3}T_2$. $\frac{1}{2}$ PT Zw, $\frac{1}{2}PM^{1\,5}_{1\,3}T_2$ = (V. R_1 Zw) ...Md. 72. H5[16]. R[77].
4. 5. T,$M^{1\,5}_{1\,3}T_2$. $\frac{1}{2}P^1_2T$, $\frac{1}{2}P^1_2M^{1\,5}_{1\,3}T_2$, ... Sim. Model 72. H7[30].
4. 5. T,$M^{1\,5}_{1\,3}T_2$. $\frac{1}{2}p^1_2m$, $\frac{1}{2}p^1_2m_2t^{1\,5}_{1\,3}$, ... Cumberland...Sim. Model 71. H6[27].
4. 5. T,$M^{15}_{13}T_2$, $\frac{1}{2}P_2T$, $\frac{1}{2}P_2M^{15}_{13}T_2$. = (V. R_2) ...... Lyons...... H8[41].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}P^4_5T$, $\frac{1}{2}P^4_5M^{15}_{13}T_2$, ... Sim. Model 72. H9[50].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}P^5_4T$, $\frac{1}{2}P^6_4M^{15}_{13}T_2$, ... Sim. Model 72. H9[51].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}P^3_2T$, $\frac{1}{2}P^3_2M^{15}_{13}T_2$, ...Castelnaudary...Sim. Model 72. H9[52].
4. 5. T, $M^{15}_{13}T_2$. $\frac{1}{2}$pm Zn, $\frac{1}{2}P_2M$ Zs, $\frac{1}{2}pm_2t^{15}_{13}$, $\frac{1}{2}P_2M_2T^{15}_{13}$, ... Lyons...... H9[58].
4. 5. T, $M^{15}_{13}T_2$. $\frac{1}{2}$PT Zw, $\frac{1}{2}p_2t$ Ze, $\frac{1}{2}PM^{15}_{13}T_2$, $\frac{1}{2}p_2m^{15}_{13}t_2$, ...Derbyshire... H10[59].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}p^1_2t$ Zw, $\frac{1}{2}P_2T$ Zw, $\frac{1}{2}p^1_2m^{15}_{13}t_2$, $\frac{1}{2}P_2M^{15}_{13}T_2$, ...Derbyshire. H12[79].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}P^1_2T$, $\frac{1}{2}p^4_5t$, $\frac{1}{2}P^1_2M^{15}_{13}T_2$, $\frac{1}{2}p^4_5m^{15}_{13}t_2$ = (V. R^1_2, r^4_5) ...... H13[88].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}p^1_2t$, $\frac{1}{2}P^3_2T$, $\frac{1}{2}p^1_2m^{13}_{13}t_2$, $\frac{1}{2}P^3_2M^{15}_{13}T_2$, ... Hartz ...... H13[89].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}P^1_2T$, $\frac{1}{2}P_{14}T$, $\frac{1}{2}P^1_2M^{15}_{13}T_2$, $\frac{1}{2}P_{14}M^{15}_{13}T_2$, ... Freyberg ...... H13[90].
4. 5. t,$m^{1\,5}_{1\,3}t_2$. $\frac{1}{2}P^1_2T$, $\frac{1}{2}P^1_2M^{1\,5}_{1\,3}T_2$, ... Derbyshire. Norway...... H7[31].
4. 5. t, $m^{15}_{13}t_2$. $\frac{1}{2}$PT Zw, $\frac{1}{2}p^1_2t$ Ze, $\frac{1}{2}PM^{15}_{13}T_2$, $\frac{1}{2}p^1_2m^{15}_{13}t_2$, ... Isère...... H9[56].
4. 5. m,T,$m_2t^{15}_{13}$, $M^{15}_{13}T_2$, $\frac{1}{2}P^1_2T$, $\frac{1}{2}P^1_2M^{15}_{13}T_2$. = (V, *v*. R^1_2) ...... H12[80].

b.) With Scalenohedrons.

4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}(3P_1M^{\pm}T^{\mp})$ = (V. S_1) ...... Derbyshire...... H8[41].
4. 5. T, $M^{15}_{13}T_2$. $\frac{1}{2}$PM, $\frac{1}{2}PM_2T^{15}_{13}$, $\frac{1}{2}(3p_{\ddagger}m^{\pm}t^{\mp})$ = (V. R_1 Zn, $s_{\ddagger}$) ... H10[60].
4. 5. T, $M^{15}_{13}T_2$. $\frac{1}{2}$pt, $\frac{1}{2}pm^{15}_{13}t_2$, $\frac{1}{2}(3p_1m^{\pm}t^{\mp})$ = (V. r_1, s_1) ...... H10[61].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}P^1_2T$, $\frac{1}{2}P^1_2M^{15}_{13}T_2$, $\frac{1}{2}(3P_1M^{\pm}T^{\mp})$ = (V. R^1_2, S_1) ...... H12[81].
4. 5. T,$M^{15}_{13}T_2$. $\frac{1}{2}p^1_2t$, $\frac{1}{2}p^1_2m^{15}_{13}t_2$, $\frac{1}{2}(3P_1M^{\pm}T^{\mp})$ ...... H12[82].
4. 5. [T,$M^{15}_{13}T_2$. $\frac{1}{2}p^1_2t$, $\frac{1}{2}p^1_2m^{15}_{13}t_2$, $\frac{1}{2}(3P_1M^{\pm}T^{\mp})$] × 2, ... H12[83].
4. 5. T,$M^{15}_{13}T_2$, $\frac{1}{2}(3p_1m^{\pm}t^{\mp})$, $\frac{1}{2}(3p_{-}m^{\pm}t^{\mp})$ = (V. s_1, s_{-}) ...... H14[93].
4. 5. V. r^1_2 Ze, r^5_4Zn, r^5_4Zs, S_1, ... H17[118].
4. 5. V. r^1_2 Ze, s_1, s_{-}, ... H17[119].
4. 5. V. R^1_2 Ze, r_2Ze, s_1, ... H17[120].
4. 5. V. R^1_2Ze, s_1, s_{+}, ... H17[122].
4. 5. V. $R_{13}Zw$, R^1_2Ze, s_{+}, ... H18[126].

4\. 5. V. $R_{13}Zw, S_+, s_-$, ... $H18^{127}$.
4\. 5. V. $R_{13}Zw, R^1_2Ze, s_1$, ... $H18^{129}$.
4\. 5. V. R_2Ze, s_1, s_-, ... $H18^{130}$.
4\. 5. V. $r_1Zw, R^1_2Ze, r_2Ze, s_1$, ... $H19^{134}$.
4\. 5. V. r_1Zw, r^4_5Ze, S_1, s_-, ... $H19^{136}$.
4\. 5. V. $r^1_2Ze, r_2Ze, r^5_4Zn, r^5_4Zs, S_1$, ... $H19^{142}$.
4\. 5. V. $R^1_2Ze, r^7_2Ze, 2s_+$, ... $H20^{144}$.
4\. 5. V. $R^1_2Ze, r_5Ze, R_4Zw, s_1$, ... $H20^{145}$.
4\. 5. V. $r_1Ze, r^5_4Zn, r^5_4Zs, s_1, s_-$, ... $H20^{146}$.
4\. 5. v, *v*. $r_4Zw, r^5_4Zn, r^5_4Zs, s_{\ddagger}$, ... $H20^{147}$.
4\. 5. V. $R_1Zw, R_4Zw, r^1_2Ze, r^4_5Ze, r_2Ze, S_1$, ... $H21^{148}$.
4\. 5. T, $M^{15}_{13}T_2.\ \frac{1}{2}(3P_1M\pm T\mp) = (v.\ S_1)$ .... Derbyshire...... $H8^{43}$.
4\. 5. T, $M^{15}_{13}T_2.\ p^5_4m,\ p^5_4m_2t^{15}_{13}.\ \frac{1}{2}(3P_1M\pm T\mp) = (v.\ 2r^5_4\ ZnZs, S_1)$ $H13^{92}$. R^{87}.
4\. 5. T, $M^{15}_{13}T_2.\ \frac{1}{2}p_2t\ Ze,\ \frac{1}{2}p_2m^{15}_{13}t_2,\ \frac{1}{2}(3P_1M\pm T\mp)$ .... $H14^{99}$.
4\. 5. v. S_1, s_-, ... $H15^{101}$.
4\. 5. v. $S_{\ddagger}, s_+$, ... $H15^{102}$.
4\. 5. v. R_4, s_-, ... $H15^{104}$.
4\. 5. v. R_4Zw, r^1_2Ze, s_1, ... $H17^{122}$.
4\. 5. v. R_4Zw, r^1_2Ze, s_-, ... $H17^{125}$.
4\. 5. $t, m^{15}_{13}t_2.\ \frac{1}{2}P^1_2T,\ \frac{1}{2}P^1_2M^{15}_{13}T_2,\ \frac{1}{2}(3p_+m\pm t\mp) = (v.\ R^1_2, s_+)$ .... $H13^{86}$.
4\. 5. $t, m^{15}_{13}t_2.\ \frac{1}{2}p^1_2t,\ \frac{1}{2}p^1_2m^{15}_{13}t_2,\ \frac{1}{2}(3P_{\ddagger}M\pm T\mp) = (v.\ r^1_2, S_{\ddagger})$...Norway. $H13^{87}$.
4\. 5. $t, m^{15}_{13}t_2.\ \frac{1}{2}P_2T,\ \frac{1}{2}P_2M^{15}_{13}T_2,\ \frac{1}{2}(3p_1m\pm t\mp) = (v.\ R_2, s_1)$......India...$H14^{95}$.
4\. 5. $t, m^{15}_{13}t_2.\ \frac{1}{2}P_2M,\ \frac{1}{2}P_2M_2T^{15}_{13},\ \frac{1}{2}(3p_{\ddagger}m\pm t\mp) = (v.\ R_2Zn, s_{\ddagger})$......... $H14^{97}$.
4\. 5. $t, m^{15}_{13}t_2.\ \frac{1}{2}P_2M,\ \frac{1}{2}P_2M_2T^{15}_{13},\ \frac{1}{2}(3p_{\ddagger}m\pm t\mp)$ .... $H14^{98}$.
4\. 5. v. r^1_2Ze, R_5Ze, s_1, ... $H17^{121}$.
4\. 5. v. $r^5_4Zn, r^5_4Zs, S_1.\ 2s_{\ddagger}$, ... $H18^{123}$.
4\. 5. v. R_4Zw, S_1, S_+, ... $H19^{138}$.
4\. 5. v. $R_4Zn, r^4_5Zs, r_2Zs, s_1, s_-$, ... $H21^{151}$.
4\. 5. v. $r_4Zw, r_5Ze, S_1, 2s_{\ddagger}$, ... $H21^{152}$.
4\. 5. V, *v*. r^1_2Ze, s_+, s_+, ... $H20^{143}$.
4\. 5. V, *v*. $R_1Zw, r_{13}Zw, s_-, s_+, s_{\ddagger}$, ... $H21^{154}$.
4\. 5. V, *v*. $R_{13}Zw, R^1_2Ze, R_2Ze, s_-, s_{\ddagger}$, ... $H21^{155}$.

Pyramids with the apex truncated.

5\. 5. $P_-.\ \frac{1}{2}PT,\ \frac{1}{2}PM^{15}_{13}T_2 = (P_-.R_1)$...(Haüy's Basé) Md. 86. R^{73}. $H5^{10}$.
5\. 5. $p_-.\ \frac{1}{2}P^1_2T,\ \frac{1}{2}P^1_2M^{15}_{13}T_2. = (p_-.\ R^1_2)$...Mexico...Sim. Model 86. $H5^{19}$.
5\. 5. $p.\ \frac{1}{2}P_2T,\ \frac{1}{2}P_2M^{15}_{13}T_2. = (p.\ R_2)$ .... Offenbanya...... $H6^{20}$.
5\. 5. $p_+.\ \frac{1}{2}P_4T,\ \frac{1}{2}P_4M^{15}_{13}T_2 = (p_+.R_4)$ .... Derbyshire ...... $H6^{23}$.
5\. 5. $p.\ \frac{1}{2}P^3_2T,\ \frac{1}{2}P^3_2M^{15}_{13}T_2. = (p.\ R^3_2)$ .... Hartz...... $H6^{24}$.
5\. 5. $P.\ \frac{1}{2}pt\ Zw,\ \frac{1}{2}P_2T\ Ze,\ \frac{1}{2}pm^{15}_{13}t_2,\ \frac{1}{2}P_2M^{15}_{13}T_2 = (P.\ r_1\ Zw, R_2\ Ze)$...... $H9^{54}$.
5\. 5. $P.\ \frac{1}{2}pt,\ \frac{1}{2}P_4T,\ \frac{1}{2}pm^{15}_{13}t_2,\ \frac{1}{2}P_4M^{15}_{13}T_2 = (P.\ r_1, R_4)$ .... $H9^{55}$.
5\. 5. $p_+.\ \frac{1}{2}p^1_2t,\ \frac{1}{2}P_5T,\ \frac{1}{2}p^1_2m^{15}_{13}t_2,\ \frac{1}{2}P_5M^{15}_{13}T_2. = (p_+.\ r^1_2, R_5)$ ...... Hartz...$H11^{67}$.
5\. 5. $p_+.\ \frac{1}{2}P_2T\ Zw,\ \frac{1}{2}p_{13}t\ Ze,\ \frac{1}{2}P_2M^{15}_{13}T_2,\ \frac{1}{2}p_{13}m^{15}_{13}t_2 = (p_+.R_2Zw, r_{13}Ze)$ $H11^{70}$.
5\. 5. $p_+.\ \frac{1}{2}p_2t,\ \frac{1}{2}P_{14}T,\ \frac{1}{2}p_2m^{15}_{13}t_2,\ \frac{1}{2}P_{14}M^{15}_{13}T_2. = (p_+.\ r_2, R_{14})$ ... Hartz...$H11^{71}$.
5\. 5. $p.\ \frac{1}{2}P^3_2T\ Zw,\ \frac{1}{2}p_5t\ Ze,\ \frac{1}{2}P^3_2M^{15}_{13}T_2,\ \frac{1}{2}p_5m^{15}_{13}t_2 = (p.\ R^3_2Zw, r_5\ Ze)$ ... $H12^{78}$.

5. 5. $P.\frac{1}{2}p_2t$ Ze, $\frac{1}{2}p_2m^{15}_{13}t_2$, $\frac{1}{2}(3P_1M\pm T\mp) = (P.r_2$ Ze, $S_1)$...Mexico...H11[69].
5. 5. $P_+.\frac{1}{2}(3P\ddagger M\pm T\mp) = (P_+.S\ddagger)$........................Hartz......H6[21].

2. Dolomite. Bitter Spar. Carbonate of Lime and Magnesia.
Rhomb Spar. Rautenspath.
2. 5. $\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (R_1)$ .... P168. Ly12[1].
2. 5. $r_1, R_2,$ ....Ly v. 5.
2. 5. $R_4, R_1,$..Ly12[3].
3. 5. p_+, v. r_1Zn, R_4Zn, r_2Zs, $s\ddagger$, ...Ly12[6].
4. 5. v. R_1Zn,r_4Zn,..Ly12[4].
4. 5. v. r_1Zn, R_4Zn, r_2Zs, ...Ly12[5].
5. 5. $P.\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (P.R_1)$ ....Ly12[2].
5. 5. $p_+.R_4,$ ....Ly v. 4.
5. 5. $P.r_1, R_2,$ ....Ly v. 6.

3. Brown Spar. Braunspath. Carbonate of Magnesia and Iron.
Breunnerite. Hallite. Pearl Spar.
2. 5. $\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (R_1)$ ....Ly v. 1. P168.

4. Carbonate of Magnesia. Talkspath.
2. 5. $\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (R_1)$..Ly v. 1.

5. Mesitin Spar. Mesitinspath.
2. 5. $\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2$?

6. Carbonate of Iron. Eisenspath. Spathose Iron.
1. 5. $P,t,m^{15}_{13}t_2 = (P, v)$ ....Ly69[2].
2. 5. $\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (R_1)$..Ly v. 1.
2. 5. $\frac{1}{2}P^1_2T, \frac{1}{2}P^1_2M^{15}_{13}T_2 = (R^1_2)$..Ly v. 2.
2. 5. $\frac{1}{2}P_5T, \frac{1}{2}P_5M^{15}_{13}T_2 = (R_5)$ ....Ly v. 3.
2. 5. $\frac{1}{2}PT, \frac{1}{2}p_5t, \frac{1}{2}PM^{15}_{13}T_2, \frac{1}{2}p_5m^{15}_{13}t_2 = (R_1, r_5)$..........................Ly v. 4.
4. 5. V. R_1Zn, S_1, ... Ly70[5].
4. 5. $P_-,T,M^{15}_{13}T_2.\frac{1}{2}pt$ Zw, $\frac{1}{2}p^1_2t$Ze, $\frac{1}{2}pm^{15}_{13}t_2, \frac{1}{2}p^1_2m^{15}_{13}t_2 = (P_-,V.r_1$Zw, r^1_2Ze)
Ly70[6].
5. 5. $P.\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (P.R_1)$..Ly v. 5.
5. 5. $P_+.\frac{1}{2}P_5T, \frac{1}{2}P_5M^{15}_{13}T_2 = (P_+.R_5)$ ....Ly70[3].
5. 5. $P_-.\frac{1}{2}PT$ Zw, $\frac{1}{2}P^1_2T$ Ze, $\frac{1}{2}PM^{15}_{13}T_2, \frac{1}{2}P^1_2M^{15}_{13}T_2 = (P_-.R_1$Zw, R^1_2Ze) Ly70[4].

7. Carbonate of Manganese. Manganspath. Red Manganese.
2. 5. $\frac{1}{2}P^1_2T, \frac{1}{2}P^1_2M^{15}_{13}T_2 = (R^1_2)$..Ly v.1.
2. 5. $S_1,$ ....Ly v.2.
5. 5. $P.\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (P.R_1)$..P246.

8. Galmei. Carbonate of Zinc. Calamine. Zinc spath.
1. 5. $P_x,T,M^{15}_{13}T_2 = (P_x, V)$ ....H iv 184.
2. 5. $\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (R_1)$ .... Ly73[1]. P375. H iv 181.
2. 5. $\frac{1}{2}P_2T, \frac{1}{2}P_2M^{15}_{13}T_2 = (R_2)$ ....P375. H iv 184.

2. 5. $^1_2P^1_2T, ^1_2P^1_2M^{15}_{13}T_2 = (R^1_2)$ .... Ly v. 2. P375.
3. 5. $^1_2P_2T, ^1_2p^1_2t, ^1_2P_2M^{15}_{13}T_2, ^1_2p^1_2m^{15}_{13}t_2 = (R_2, r^1_2)$ .... Ly73[1].

9? PLUMBO-CALCITE. Carbonate of Lime and Lead.
2. 5. R_1, ... Johnstone.

20*. NITRATE OF SODA. Saltpetersaures Natron. Cubic Nitre.
Cleavage = $\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (R_1)$.
2. 5. $\frac{1}{2}PT, \frac{1}{2}PM^{15}_{13}T_2 = (R_1)$ .... Model 83. P198. H ii 214.

21. TALC. Talk. Hexagonal Talc.
Cleavage = P.
1. 5. $P\_,T,M^{15}_{13}T_2 = (P\_,V)$...Model 7. T i 357. Ly v. 1. H71[130]. P120.

22.* PHENAKITE. Cleavage = $T,M^{15}_{13}T_2 = (V)$.
4. 5. $t,m^{15}_{13}t_2. ^1_2P^4_5M, ^1_2P^4_5M_2T^{15}_{13}$,............Beirich, Pogg. Ann. Bd 34 fig [11].
4. 5. $T,M^{15}_{13}T_2. P^4_9T, P^4_9M^{15}_{13}T_2$, ... Sim. Model 73. idem fig [13].
4. 5. $T,M^{15}_{13}T_2. ^1_2p^4_5m, P^4_9T, ^1_2p^4_5m_2t^{15}_{13}, P^4_9M^{15}_{13}T_2 = (V. r^4_5Zn, 2R^4_9Zw\ Ze)$
idem fig. [14].
4. 5. $(T,M^{15}_{13}T_2. ^1_2P^4_5M, ^1_2P^4_5M_2T^{15}_{13}) \times 2$, ...idem

23.* WILLELMINE. Willemit. Cleavage = P.
4. 5. $T,M^{15}_{13}T_2. ^1_2PT, ^1_2PM^{15}_{13}T_2 = (V. R_1)$.................Model 72. Ly82[2].

24. *An Isomorphous Group of Phosphates,* 1, 2, 3, 4 :—
1. APATITE from Ehrenfriedersdorf. Phosphate of Lime. Chaux phosphatée.
2. APATITE from Snarum.
Cleavage = $p,T,M^{15}_{13}T_2 = (p,V)$.
1. 5. $P\_, T,M^{15}_{13}T_2 = (P\_,V)$ ....Model 7. J[148]. P171[1]. H26[1].
1. 5. $P,m,T,m_2t^{15}_{13}, M^{15}_{13}T_2 = (P,V,v)$ .... Model 10. J[149]. H26[5].
3. 5. $P,T,M^{15}_{13}T_2. p^4_5t, p^4_5m^{15}_{13}t_2$,..................Model 58. J[151]. P171[2]. H26[3].
3. 5. $P,T,M^{15}_{13}T_2. p^2_5t, p^2_5m^{15}_{13}t_2$,...................................Model 58. H26[4].
3. 5. $P,T,M^{15}_{13}T_2. p^{10}_7m, p^4_5t, p^{10}_7m_2t^{15}_{13}, p^4_5m^{15}_{13}t_2 = (P,V. 2r^{10}_7\ ZnZs, 2r^4_5\ Ze\ Zw)$
H26[7]. J[153].
3. 5. $P,T,M^{15}_{13}T_2. p^{10}_7m, p^2_5t, p^{10}_7m_2t^{15}_{13}, p^2_5m^{15}_{13}t_2$, ...H26[8].
3. 5. $P,m,T, m_2t^{15}_{13}, M^{15}_{13}T_2. P^2_5T, P^2_5M^{15}_{13}T_2 = (P,V,v. 2R^2_5ZwZe)$ J[150]. H26[9].
3. 5. $P,m,T, m_2t^{15}_{13}, M^{15}_{13}T_2. p^{10}_7t, p^{10}_7m^{15}_{13}t_2$, 2 s,............................H26[10].
3. 5. $P,m,T, m_2t^{15}_{13}, M^{15}_{13}T_2. 2r^{10}_7Zn\ Zs, 2r^2_5ZeZw$, ...Sim. Md. 52. H26[11].
3. 5. $P,m,T, m_2t^{15}_{13}, M^{15}_{13}T_2. 2r^{10}_7\ ZnZs, 2r^4_5\ ZeZw, 2r^2_5\ ZeZw$,............H27[12].
3. 5. $P,m,T, m_2t^{15}_{13}, M^{15}_{13}T_2. 2r^{10}_7\ ZnZs, 2r^4_5\ ZeZw, 2r^2_5\ ZeZw$,............H27[13].
3. 5. $P,m,T, m_2t^{15}_{13}, M^{15}_{13}T_2. 2r^{10}_7\ ZnZs, 2r^2_5\ ZeZw, 2r^4_5\ ZeZw, 2r^2_5\ ZeZw$, 2s,
H27[14].
4. 5. $T,M^{15}_{13}T_2. P^4_5T, P^4_5M^{15}_{13}T_2 = (V. 2R^4_5Zw\ Ze)$....................J[188]. H26[2].
4. 5. $m,T,m_2t^{15}_{13}, M^{15}_{13}T_2. P^4_5T, P^4_5M^{15}_{13}T_2$,......................P171[3]. H26[6]. J[154].

3. PHOSPHATE OF LEAD. Braunbleierz von Paoullaouen. Plomb phosphaté.

Cleavage $= p^{13}_{15}t,\ p^{13}_{15}m^{15}_{13}t_2 = (2r^{13}_{15})$.

1. 5. $P_{+}, T, M^{15}_{13}T_2 = (P_{+}, V)$... Hofsgrund. Clausthal. Beresof...Md. 7. J^{131}. $L273^{5}$. $H93^{70}$.

1. 5. $P_{+}, M, T, M_2T^{15}_{13}, M^{15}_{13}T_2 = (P_{+}, V, v)$.........Huelgoet......Model 10. J^{132}. $L273^{6}$. $H93^{71}$.

2. 5. $P^{13}_{15}T, P^{13}_{15}M^{15}_{13}T_2$, (*Primitive*) $= (2R^{13}_{15}\ Zw\ Ze)$...Md. 26. Leonhard.

3. 5. $P_{+}, T, M^{15}_{13}T_2 . p^{13}_{15}t, p^{13}_{15}m^{15}_{13}t_2$......Joh. Georgenstadt......Sim. Md. 58. $L273^{3}$. J^{131}. $H94^{75}$.

3. 5. $P_{+}, m, T, m_2t^{15}_{13}, M^{15}_{13}T_2 . p^{13}_{15}t, p^{13}_{15}m^{15}_{13}t_2$...............$L273^{4}$. P363. $H94^{76}$.

4. 5. $T, M^{15}_{13}T_2 . P^{13}_{15}T, P^{13}_{15}M^{15}_{13}T_2$...... Beresof. Cornwall...Sim. Model 74, but the prism longer, like Model 73......J^{133}. $L273^{2}$. M^{117}. $H93^{72}$.

4. 5. $T, M^{15}_{13}T_2 . P^{26}_{15}T, P^{26}_{15}M^{15}_{13}T_2$,...............Sim. Model 74. $L273^{2}$. $H94^{73}$.

5. 5. $p_{-} . P^{13}_{15}T, P^{13}_{15}M^{15}_{13}T_2$ ... Joh. Georgenstadt...Md. 96. $L273^{1}$. $H94^{74}$.

4. ARSENIATE OF LEAD. Grünbleierz von Johann-Georgenstadt. Plomb phosphato-arséniaté.

Cleavage $= t,\ m^{15}_{13}t_2$.

1. 5. $P, T, M^{15}_{13}T_2$, ... Model 7. $Ly\ 52^{1}$. P364.

3. 5. $P, T, M^{15}_{13}T_2 . p^{5}_{6}t, p^{5}_{6}m^{15}_{13}t_2$......Sim. Model 58. $Ly\ 52^{4}$. T i 574. P364.

3. 5. $P_{-}, T, M^{15}_{13}T_2 . p^{5}_{3}t, p^{5}_{3}m^{15}_{13}t_2$ ....Sim. Model 58. $Ly\ 52^{5}$.

3. 5. $P, T, M^{15}_{13}T_2 . p^{5}_{6}t, p^{5}_{3}t, p^{5}_{6}m^{15}_{13}t_2, p^{5}_{3}m^{15}_{13}t_2$ ....$Ly\ 52^{7}$.

4. 5. $T, M^{15}_{13}T_2 . P^{5}_{6}T, P^{5}_{6}M^{15}_{13}T_2$,.......................Sim. Model 74. $Ly\ 52^{2}$.

4. 5. $T, M^{15}_{13}T_2 . P^{5}_{6}T, p^{5}_{3}t, P^{5}_{6}M^{15}_{13}T_2, p^{5}_{3}m^{15}_{13}t_2$.............................$Ly\ 52^{6}$.

5. 5. $P_{-} . p^{5}_{6}t, p^{5}_{6}m^{15}_{13}t_2$.....................Sim. Model 76, but flatter. $Ly\ 52^{3}$.

25.* COPPER MICA. Kupferglimmer. Rhomboidal Arseniate of Copper.

Cleavage $= P . \frac{1}{2}p_3t, \frac{1}{2}p_3m^{15}_{13}t_2 = (P . r_3)$.

2. 5. $\frac{1}{2}p_3t, \frac{1}{2}P_{\frac{1}{2}\frac{1}{0}}T, \frac{1}{2}p_3m^{15}_{13}t_2, \frac{1}{2}P_{\frac{1}{2}\frac{1}{0}}M^{15}_{13}T_2, = (r_3, R_{\frac{1}{2}\frac{1}{0}})$ .... $P330^{3}$.

5. 5. $P^{-}_{-} . \frac{1}{2}p_3t$ Zw, $\frac{1}{2}p^{3}_{2}t$ Ze, $\frac{1}{2}p_3m^{15}_{13}t_2, \frac{1}{2}p^{3}_{2}m^{15}_{13}t_2$ .... $Ly\ 65^{3}$. $P330^{2}$.

5. 5. $P^{-}_{-} . \frac{1}{2}p_3t, p^{3}_{2}t, \frac{1}{2}p_3m^{15}_{13}t_2, p^{3}_{2}m^{15}_{13}t_2$...$Ly\ 65^{4}$.

5. 5. $P^{-}_{-} . \frac{1}{2}p_3t, \frac{1}{2}p_3m^{15}_{13}t_2$, Tingtang, Cornwall. $H102^{149}$. $Ly\ 65^{2}$. P330. $M\,ii^{119}$.

26.* DIOPTASE. Emerald Copper. Kupfer-Smaragd.

Cleavage $= \frac{1}{2}P^{2}_{3}T, \frac{1}{2}P^{2}_{3}M^{15}_{13}T_2 = (R^{2}_{3})$.

4. 5. $T, M^{15}_{13}T_2 . \frac{1}{2}P^{4}_{5}M, \frac{1}{2}P^{4}_{5}M_2T^{15}_{13}$...Sim. Md. 71. M^{118}. $H100^{135}$. $P323^{2}$. R^{73}.

27. COQUIMBITE. Persulphate of Iron from Chili.

Cleavage $= t,\ m^{15}_{13}t_2 . p^{9}_{5}t, p^{9}_{5}m^{15}_{13}t_2 = (v . 2r^{9}_{5})$.

3. 5. $p_{+}, T, M^{15}_{13}T_2 . P^{9}_{5}T, P^{9}_{5}M^{15}_{13}T_2 = (p_{+} V . 2R^{9}_{5}\,ZeZw)$...D178. T i 450.

28. VANADIATE OF LEAD. Vanadinbleierz.

1. 5. $P, T, M^{15}_{13}T_2 = (P, V)$, ...Model 7. T i 573.

29. ONE-AXED MICA. Einaxiger Glimmer. Mica with one axis of double refraction.

Cleavage = P.

1. 5. $P_{-}, T, M^{15}_{13} T_2$, ... Siberia. Vesuvius. Model 7. J[105]. H82[209]. P102.
1. 5. $P^{-}_{-}, t, m^{15}_{13} t_2.\ p_6 t,\ p_6 m^{15}_{13} t_2$ .... J[107]. H82[210].

30. NEPHELINE. Sommite.

Cleavage = $p, t, m^{15}_{13} t_2$ = (p,v).

1. 5. $P, T, M^{15}_{13} T_2$ = (P,V)...Vesuvius. Md.7. L469[1]. J[101]. Mii 250[1]. H62[49].
1. 5. $P, m, T,\ m_2 t^{15}_{13},\ M^{15}_{13} T_2$ .... Sim. Model 10. Ly28[2].
3. 5. $P, T, M^{15}_{13} T_2.\ p^1_2 t, p^1_2 m^{15}_{13} t_2$, Sim. Md. 58. L469[2]. J[102]. M[112]. H62[50]. Ly28[3].
3. 5. $P, T, M^{15}_{13} T_2.\ p^1_2 t,\ pt,\ p^1_2 m^{15}_{13} t_2,\ pm^{15}_{13} t_2$, ... L469[3]. P132.
3. 5. $P, m, T,\ m_2 t^{15}_{13},\ M^{15}_{13} T_2.\ p^1_2 t,\ p^1_2 m^{15}_{13} t_2$, ... Ly28[4].
3. 5. $P, m, T,\ m_2 t^{15}_{13},\ M^{15}_{13} T_2.\ p^1_2 t,\ pt,\ p^1_2 m^{15}_{13} t_2,\ pm^{15}_{13} t_2$, ... Ly28[5].

31. BERYL. Emerald. Smaragd. Emeraude. Aquamarine.

Cleavage = $P, T, M^{15}_{13} T_2$.

1. 5. $P, T, M^{15}_{13} T_2$, ... Peru......Model 7. Ly33[1]. L392[1]. H71[140].
1. 5. $P, m, T,\ m_2 t^{15}_{13},\ M^{15}_{13} T_2$, ... Sim. Model 10. Ly v. 2. L392[2]. J[35]. H71[141].
3. 5. $P, T, M^{15}_{13} T_2.\ pm,\ pm_2 t^{15}_{13}$, ... Model 56. J[36]. Ly v. 3. L392[3]. H71[142].
3. 5. $P, T, M^{15}_{13} T_2.\ p^{15}_{26} t,\ p^{15}_{26} m^{15}_{13} t_2$, ...Sim. Model 58. J[38]. L392[4]. H71[143].
3. 5. $P, T, M^{15}_{13} T_2.\ pm,\ p^{15}_{13} t,\ pm_2 t^{15}_{13},\ p^{15}_{13} m^{15}_{13} t_2$,......J[37]. Ly v. 6. L392[7]. H71[145].
3. 5. $P, T, M^{15}_{13} T_2.\ pm,\ p^{15}_{26} t,\ pm_2 t^{15}_{13},\ p^{15}_{26} m^{15}_{13} t_2$, Md. 52. Ly v. 5. L392[5]. H71[146].
3. 5. $P, T, M^{15}_{13} T_2.\ pm,\ p^{15}_{13} t,\ p^{15}_{26} t,\ pm_2 t^{15}_{13},\ p^{15}_{13} m^{15}_{13} t_2,\ p^{15}_{26} m^{15}_{13} t_2$,......Ly v. 7. L392[6]. H71[147].
3. 5. $P, T, M^{15}_{13} T_2,\ 3m_x t.\ pm,\ p^{15}_{26} t,\ pm_2 t^{15}_{13},\ p^{15}_{26} m^{15}_{13} t_2$, ... Ly33[4].
3. 5. $P, m, T,\ m_2 t^{15}_{13},\ M^{15}_{13} T_2.\ pm,\ p^{15}_{26} t,\ pm_2 t^{15}_{13},\ p^{15}_{26} m^{15}_{13} t_2$, ...Model 52. Ly v. 8. L392[8]. H72[148].
3. 5. $P, m, T,\ m_2 t^{15}_{13},\ M^{15}_{13} T_2.\ pm,\ p_{\frac{1}{300}} t,\ p^{15}_{26} t,\ p^{15}_{13} t,\ pm_2 t^{15}_{13},\ p_{\frac{1}{300}} m^{15}_{13} t_2,\ p^{15}_{26} m^{15}_{13} t_2,\ p^{15}_{13} m^{15}_{13} t_2$, ...P99.
3. 5. $P, m, T,\ m_2 t^{15}_{13},\ M^{15}_{13} T_2.\ pm,\ p^{15}_{13} t,\ p^{15}_{26} t,\ pm_2 t^{15}_{13},\ p^{15}_{13} m^{15}_{13} t_2,\ p^{15}_{26} m^{15}_{13} t_2$ = (P,V, *v*. $2r_1$ZnZs, $2r^{15}_{13}$ ZwZe, $2r^{15}_{26}$ ZwZe).........Ly33[3].
4. 5. $T, M^{15}_{13} T_2.\ pm,\ P^{15}_{13} T,\ pm_2 t^{15}_{13},\ P^{15}_{13} M^{15}_{13} T_2$, ... Ly33[2].

32. PYROSMALITE. Fer muriaté. Cleavage = P.

1. 5. $P, T, M^{15}_{13} T_2$, ...Model 7. L772[1]. J iii [561]. P227. M iii 143. Ly v. 1.
3. 5. $P, T, M^{15}_{10} T_2.\ p^5_8 t,\ p^5_4 t,\ p^5_8 m^{15}_{13} t_2,\ p^4_5 m^{15}_{13} t_2$ = (P, V. r^5_8, r^5_4) ......P227. L772[2].

33. EUDIALYTE. Cleavage = P. $\frac{1}{2} p^5_8 m,\ \frac{1}{2} p^5_8 m_2 t^{15}_{13}$ = (P. r^5_8).

3. 5. $P, T, M^{15}_{15} T_2.\ \frac{1}{2} P^5_2 M,\ \frac{1}{2} p^5_8 m,\ \frac{1}{2} P^5_2 M_2 T^{15}_{13},\ \frac{1}{2} p^5_8 m_2 t^{15}_{13}$ = (P,v. R^5_2Zn, r^5_8Zn) M[127]. P153.
3. 5. P,v, *v*. R^5_2 Zn, r^5_8 Zn, r^5_4Zs, ... Ly27[2]. L391[2].

34. CHABASITE. Chabasie.

Cleavage= R^5_4 = Rhombohedron of 94° 46′.

2. 5. R^5_4, ... J[81]. Ly44[1]. L199[1]. H84[264].
2. 5. $R^5_4 \times 2$, ...Ly44[2]. L199[2].

2. 5. $R^5_4 Zw, r^5_8 Ze,$ .... L199[2].
2. 5. $R^5_4 Zw, r^5_2 Ze,$ .... L199[3].
2. 5. $R^5_4 Zw, R^5_8 Ze, r^5_2 Ze,$ .... Model 102. L199[4]. J[82]. M[150]. H84[285].
2. 5. $r^5_4 Zw, r^5_2 Ze, S_{-},$ .... L199[6]. H84[286].
2. 5. $r^5_4 Zw, r^5_8 Ze, r^5_2 Ze, S_{-},$ .... P145[2].
2. 5. $(R^5_4 Zw, r^5_2 Ze) \times 2,$ .... L44[3]. L199[7].
2. 5. $(R^5_4 Zw, R^5_8 Ze, r^5_2 Ze) \times 2,$ .... L199[7].
2. 5. $(R^5_2, S_{-}) \times 2,$ .... Ly44[4].
2. 5. $(R^5_8, R^5_4, S_{-}) \times 2,$ .... Ly44[5].
2. 5. $(R^5_4, R^5_8, R^5_2, S_{-}) \times 2,$ .... Ly44[6].
4. 5. v. $R^5_4 Zn, R^5_8 Zs, r^5_2 Zs,$ .... L199[5].
4. 5. (v. $R^5_4 Zn, R^5_8 Zs, r^5_2 Zs) \times 2,$ .... M[173]. L199[7].
4. 5. (V. $R^5_4, R^5_8, R^5_2, S_{-}) \times 2,$ .... Ly44[7].

35. LEVYNE. Cleavage = $R_2 = (\frac{1}{2}P_2T, \frac{1}{2}P_2M^{15}_{13}T_2)$.
5. 5. (P. pm, P_2T, $pm_2t^{15}_{13}$, $P_2M^{15}_{13}T_2) \times 2,$ .... P146. Ly45[2].

36. ALUNITE. Alum-stone. Alaunstein. Cleavage = P.
2. 5. R^4_3, ... Ti307. L131[1]. Ly19[1].
5. 5. P. R^4_3, ... Sim. Model 86. Ti 307. M[111]. L131[2]. Ly19[2].
5. 5. p. $R^4_3, r_{-}, r^{-}_{-},$ .... L131[3]. P203.

37. TOURMALINE. Turmalin.

Axes varying from $p\frac{a}{+}m^a_{15}t^a_{13}$ to $p\frac{a}{+}m^a_{13}t^a_{15}$, in which respect this mineral differs from all others belonging to the rhombohedral system.

The prism $T, M\frac{1}{1}\frac{5}{3}T_2$ never occurs in a complete state, but is always irregularly modified by hemihedral varieties of the prism $M, M_2T\{\frac{5}{3}$.

The obtuse rhombohedral terminations of the prisms are extremely irregular, dissimilar combinations of rhombohedrons occurring at the two ends of almost every crystal.

When the crystals are heated over a spirit lamp, and then allowed to cool gradually, one end exhibits *positive* electricity and the other end *negative* electricity, during the time of cooling. It is the *zenith, or upper end* of the combinations described below, which exhibit NEGATIVE *electricity.*

When the symbol of a rhombohedron is embraced between the signs $\frac{1}{2}$ and N, (for example, $\frac{1}{2}r_1Nn$,) it signifies that only three planes of the rhombohedron are present on the combination, and that these are all situated on the nadir end of the crystal. But if Z instead of N is used in the symbol, it signifies that the three rhombohedral planes are all on the zenith end of the crystal.

The references are to figures which accompany an article on the connection between the form and the electrical properties of Tourmaline, written by G. Rose, and printed in Poggendorff's Annalen for Oct. 1836.

Black Tourmalines :—

5. 5. PZ, Mn, t, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $m^{15}_{13}t_{..}$ R^1_2Zn, $\frac{1}{2}r^1_1 Zs$, $\frac{1}{2}r_1Nn$, Bavaria, Rose 11.

4. 5. Mn, $\frac{1}{2}M_2T^{15}_{13}$ se sw. R^1_2 Zn, ...Ceylon......Rose 1.
4. 5. Mn, $\frac{1}{2}M_2T^{15}_{13}$ se sw. R^1_2 Zn, $\frac{1}{2}r_1$ Nn,..................Arendal......Rose 2.
4. 5. Mn, t, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $m^{15}_{13}t_2$. R^1_2 Zn, $\frac{1}{2}r_1$Nn,......Siberia. St. Gotthardt.
Zillerthal. Schneeberg......Rose 3
4. 5. Mn, t, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $m^{15}_{13}t_2$. R^1_2 Zn, $\frac{1}{2}r^1_4$Zs, $\frac{1}{2}r_1$Nn,......Sweden...Rose 4.
4. 5. Mn, t, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $m^{15}_{13}t_2$. $\frac{1}{2}R^1_2$ Zn, $\frac{1}{2}r^1_2$ Ns, $\frac{1}{2}r_1$Nn, $\frac{1}{2}s_2$N,...Arendal. R.5.
4. 5. mn, T, $\frac{1}{2}m_2t^{15}_{13}$ se sw, $M^{15}_{13}T_2$. R^1_2 Zn, ... Greenland......Rose 6.
4. 5. mn, T, $\frac{1}{2}m_2t^{15}_{13}$ se sw, $M^{15}_{13}T_2$. $\frac{1}{2}r^1_2$ Zn, $\frac{1}{2}r_2$Zn, $\frac{1}{2}R_1$Zs, $\frac{1}{2}R^1_2$Ns, Andreasberg,
Rose 7.
4. 5. Mn, ms, T, $\frac{1}{2}m_2t^{15}_{13}$ ne nw, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $M^{15}_{13}T_2$. R^1_2Zn, $\frac{1}{2}r^1_4$Zs, $\frac{1}{2}r_1$Nn,
Silesia......Rose 9.
4. 5. Mn, ms, T, $\frac{1}{2}m_2t^{15}_{13}$ ne nw, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $M^{15}_{13}T_2$, $\frac{1}{2}m^4_5$t n²e n²w. R^1_2 Zn,
r_1Zs, $\frac{1}{2}r^1_4$ Zs,...............Rose 10.

Green Tourmalines:

3. 5. PZ, mn, T, $\frac{1}{2}m_2t^{15}_{13}$ se sw, $M^{15}_{13}T_2$. $\frac{1}{2}r^1_2$Zn, $\frac{1}{2}r^1_4$ Zs, $\frac{1}{2}r^1_2$ Ns, $\frac{1}{2}r_1$ Nn,
St. Gotthardt......Rose 14.
3. 5. PZ, p N, M n, t, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $m^{15}_{13}t_2$. $\frac{1}{2}r^1_2$Zn, $\frac{1}{2}r^1_4$Zs, $\frac{1}{2}R^1_2$Ns, $\frac{1}{2}s_2$Nn, $\frac{1}{2}s_3$Ns,
Saxony......Rose 15.
4. 5. Mn, t, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $m^{15}_{13}t_2$. $\frac{1}{2}R^1_2$Zn, $\frac{1}{2}R^1_2$Ns, $\frac{1}{2}r_1$Nn, $\frac{1}{2}s_2$Nn, Brasils, R.12.

Brown Tourmalines:

4. 5. Mn, t, $\frac{1}{2}M_2T^{15}_{13}$ se sw, $m^{15}_{13}t_2$. R^1_2Zn Ns, $\frac{1}{2}r_1$Nn,...St. Gotthardt,...R.16.

Red Tourmalines:

3. 5. PZ, mn, T, $\frac{1}{2}m_2t^{15}_{13}$ se sw, $M^{15}_{13}T_2$. $\frac{1}{2}r^1_2$Ns, $\frac{1}{2}s_2$Ns,.........Katharinenburg,
Rose 17.
3. 5. P, Mn,t,$\frac{1}{2}M_2T^{15}_{13}$ se sw, $m^{15}_{13}t_2$. $\frac{1}{2}r^1_4$ Zs, $\frac{1}{2}r^1_2$Ns, ...Elba......Rose 18.
3. 5. pZ, mn, T, $\frac{1}{2}m_2t^{15}_{13}$ se sw, $M^{15}_{13}T_2$. $\frac{1}{2}R^1_4$Zs, $\frac{1}{2}R^1_2$Ns, ... Saxony...Rose 19.
4. 5. mn, T, $\frac{1}{2}m_2t^{15}_{13}$ se sw, $M^{15}_{13}T_2$. R^1_2Zs Nn, $\frac{1}{2}r^1_4$Zn, ... Saxony...Rose 20.

The contrast betwixt the prisms of Tourmaline and the regular hexagonal prisms may be thus shown:—The figure e o w s, page 17, Part I., is a rhombus having angles of 120° at o and s, and of 60° at e and w. When the acute angles are replaced by the lines *ez* and *wx*, the resulting figure has six equal sides and six angles of 120° each. This is the prism = $T,M^{1}_{1}{}^{5}_{3}T_2$ of the rhombohedral system. But when the rhombus e o w s, is divided by the single line o s, the two products, e o s and s o w, are three-sided forms, having three angles of 60° each, and, therefore, are equilateral triangles. This is the form of the equator of the Tourmaline crystal, and in the example marked "Rose 1," it occurs without modification.

38. PALLADIUM from Tilkerode. Seleniet of Palladium.
1. 5. P_-,T,$M^{15}_{13}T_2$,...Cleavage = P,..............Model 7. D388. Ti656.

39.* CRICHTONITE. Fer oxidulé titané. Cleavage = P.
2. 5. R_6, ...Ly69¹. H iv 99¹.

2. 5. R_6 Zw, r_{16} Ze, ...Ly69[3]. H iv 99[3].
5. 5. P. R_{∞}..Ly69[2]. T i 467. P257. H iv 99[2].
5. 5. P. R^4_{∞} ....H iv 99[4].

40. CHLORITE. Talc?
1. 5. P,T,$M^{15}_{13}T_{\infty}$..Model 7. R174.

41. CRONSTEDTITE. Hydrous Silicate of Iron.
Cleavage = P,t,$m^{15}_{13}t_{\infty}$.
1. 5. P,T,$M^{15}_{13}T_{\infty}$ ....Model 7. T461. P223. L211[1].
1. 5. P,m,T, $m_{\infty}t^{15}_{13}$, $M^{15}_{13}T_{\infty}$ .... Model 10. L211[2].

42.* SIDEROSCHISOLITE.
1. 5. P,T,$M^{15}_{13}T_{\infty}$,...Cleavage = P. .... Model 7. P225.

43. PINITE. Cleavage = t, $m^{15}_{13}t_{\infty}$.
1. 4. P,M,T, $m_{\infty}t^{15}_{13}$, $m^{15}_{13}t_{\infty}$.........Axes: $p^a_x m^a t^a$,..H62[54].
1. 5. P,T,$M^{15}_{13}T_{\infty}$,...Greenland. Salzbourg. Md. 7. Ly v. 1 L464[1]. H62[51].
1. 5. P,m,T, $m_{\infty}t^{15}_{13}$, $M^{15}_{13}T_{\infty}$, Puy-de-Dôme, Md. 10. Ly v 2. L464[2]. H62[52].
3. 5. P,m,T, $m_{\infty}t^{15}_{13}$, $M^{15}_{13}T_{\infty}$. p^8_7t, $p^8_7m^{15}_{13}t_{\infty}$..Ly29[2]. L464[3]. H62[53].
3. 5. P,m,T, $m_{\infty}t^{15}_{13}$, $M^{15}_{13}T_{\infty}$. p^7_4m, p^4_7t, p^8_7t, $p^7_4m_{\infty}t^{15}_{13}$, $p^4_7m^{15}_{13}t_{\infty}$, $p^8_7m^{15}_{13}t_{\infty}$,
L464[4]. P114.

44.* DREELITE. Dréelith.
2. 5. R^4_{∞},.........(Cleavage = ?)..D203.

CLASS IV.—MINERALS BELONGING TO THE PRISMATIC SYSTEM OF CRYSTALLISATION.

The AXES of all Combinations belonging to this Class are = $p^a_x m^a_y t^a_z$. In many cases the Axes are = $p^a_+ m^a_- t^a$, as in the examples P_+,M_-,T and $P^{19}_{10}M^{8}_{10}T$.

The constituent FORMS of the Combinations of this Class are as follow:—

Zones.	Forms.	Number of varieties of the unequiaxed Forms.
Prismatic,............	M, M_-T, M_+T, T,...............	About 70.
North, ...	P, P_-M, P_+M, M,...............	— 20.
East, ...	P, P_-T, P_+T, T, ...	— 50.
Octahedral, ...	$P_xM_yT_z$..........................	— 60.

No other Forms than these occur upon crystals of this Class. There are two or three hemihedral octahedrons, and many twin crystals. There are no Hemihedral Forms in the Prismatic, North, or East Zones.

1. SULPHUR. Schwefel. Soufre.
Cleavage = $m^{8}_{10}t$. $p^{19}_{10}m^{8}_{10}t$.
2. 3. $P^{19}_{10}M^{8}_{10}T$,.........Model 21. L596[1]. Ly2. J[213]. R[85]. P383[1]. H331.

2. 3. P$\frac{19}{10}$M$\frac{8}{10}$T, p$\frac{4}{10}$m$\frac{8}{10}$t,........................L596[3]. J[248]. P383[6]. H337.
2. 5. p‡t,P$\frac{19}{10}$M$\frac{8}{10}$T,..L596[14].
2. 5. p$\frac{19}{10}$t, P$\frac{19}{10}$M$\frac{8}{10}$T, p$\frac{4}{10}$m$\frac{8}{10}$t, ...L596[12]. J[251]. H339.
3. 5. p$_+$,m$\frac{8}{10}$t. p$\frac{19}{10}$t, P$\frac{19}{10}$m$\frac{8}{10}$t,..Ly6.
3. 5. p$_+$,m$\frac{8}{10}$t. p$\frac{19}{10}$t, P$\frac{19}{10}$M$\frac{8}{10}$T, p$\frac{4}{10}$m$\frac{8}{10}$t, ...M[18]. L596[8]. Ly8.
4. 5. t,m$\frac{8}{10}$t. P$\frac{19}{10}$M$\frac{8}{10}$T, p$\frac{4}{10}$m$\frac{8}{10}$t, ...L596[13].
4. 5. m$_-$. P$\frac{19}{10}$M$\frac{8}{10}$T, ...L596[7]...Model 70. J[246]. P383[4]. H334.
4. 3. M$\frac{8}{10}$T. P$\frac{19}{10}$M$\frac{8}{10}$T,.............L596[9]...Model 66. J[247]. P383[5]. H335.
5. 3. p$_+$. P$\frac{19}{10}$M$\frac{8}{10}$T,.........L596[2]. Ly3...Model 80. J[245]. P383[2]. H333.
5. 3. p$_+$. P$\frac{19}{10}$M$\frac{8}{10}$T, p$\frac{4}{10}$m$\frac{8}{10}$t, ...L596[4]. Ly5. J[249]. H338.
5. 5. p$_+$. p$\frac{19}{10}$t, P$\frac{19}{10}$M$\frac{8}{10}$T, p$\frac{4}{10}$m$\frac{8}{10}$t, ...L596[5]. Ly[7]. R[88]. H340.
5. 5. p$_+$,m$_-$. P$\frac{19}{10}$M$\frac{8}{10}$T,...L596[6].
5. 5. p$_+$. p$\frac{19}{10}$t, P$\frac{19}{10}$M$\frac{8}{10}$T,..L596[11].
6. 5. $\frac{1}{2}$P$\frac{19}{10}$M$\frac{8}{10}$T, $\frac{1}{2}$p$\frac{19}{10}$m$\frac{8}{10}$t,..............................J[244]. P383[1]. H332.
6. 5. p$\frac{19}{10}$t, P$\frac{19}{10}$M$\frac{8}{10}$T,................ L596[10]. Ly4...Model 120. J[250]. H336.

2. ANTIMONIAL SILVER. Antimonsilber. Argent antimonial.
Cleavage = p,t,m$\frac{7}{12}$t.p$\frac{7}{8}$t.

1. 2. P$_2$,M$_-$,T, ...L685[6].
1. 5. P,T,M$\frac{7}{12}$T,.............................Sim. Model 7. Ly v 1. L685[1].
2. 3. P$\frac{7}{8}$M$\frac{7}{12}$T, ...L685[2].
2. 5. P$\frac{7}{8}$T, P$\frac{7}{8}$M$\frac{7}{12}$T,...L685[8].
3. 5. P,T,M$\frac{7}{12}$T. P$\frac{7}{8}$M$\frac{7}{12}$T,...Ly47[2].
3. 5. P,T,M$\frac{7}{12}$T. P$\frac{7}{8}$T, P$\frac{7}{8}$M$\frac{7}{12}$T,...............................Ly47[3]. L685[2].
3. 5. P,m,t, M$\frac{7}{12}$T, m$\frac{12}{7}$t. p$\frac{7}{8}$t, p$\frac{7}{8}$m$\frac{7}{12}$t,...............................L685[4].
4. 5. T,M$\frac{7}{12}$T. P$\frac{7}{8}$T, P$\frac{7}{8}$M$\frac{7}{12}$T,...L685[3].
4. 5. M$\frac{7}{12}$T. P$\frac{7}{8}$M$\frac{7}{12}$T, ...L685[5].
5. 5. P. P$\frac{7}{8}$T, P$\frac{7}{8}$M$\frac{7}{12}$T,...L685[7].

3. ARSENICAL IRON. Arsenikeisen.
Cleavage = P,m$\frac{8}{9}$t.

5. 3. M$\frac{8}{9}$T. P$_2$M, ... Stiria......Sim. Model 82. Mohs ii [1].

4. *An Isomorphous Group*, 1, 2:—

1. VITREOUS COPPER. Kupferglanz. Cuivre sulfuré. Sulphuret of Copper. Copper Glance.
2. SULPHURET OF SILVER AND COPPER. Silberkupferglanz.

Cleavage = p$_2$t, p$_2$m$\frac{15}{13}$t$_2$.

1. 5. P,T,M$\frac{15}{13}$T$_2$...Cornwall...Sim. Md. 7. L640[1]. P308[1]. Ly60[1]. H98[114].
1. 5. P,m,T, m$_2$t$\frac{15}{13}$, M$\frac{15}{13}$T$_2$, ... Mexico ......Model 10. Ly60[3].
2. 5. P$_2$T, P$_2$M$\frac{15}{13}$T$_2$, ...Sim. Md. 26, but more acute. P308[5]. H98[115]
2. 5. P$\frac{1}{2}$T, P$\frac{1}{2}$M$\frac{15}{13}$T$_2$, ... Similar to Model 26. P308[3].
2. 5. P$_2$T, P$\frac{1}{2}$T, P$_2$M$\frac{15}{13}$T$_2$, P$\frac{1}{2}$M$\frac{15}{13}$T$_2$ = (2R$_2$, 2R$\frac{1}{2}$)
3. 5. P,T,M$\frac{15}{13}$T$_2$. p$_2$t, p$_2$m$\frac{15}{13}$t$_2$, ...Sim. Md. 58. P308[4]. Ly61[6]. H99[118].
3. 5. P,T,M$\frac{15}{13}$T$_2$. p$\frac{1}{2}$t, p$\frac{1}{2}$m$\frac{15}{13}$t$_2$,......Sim. Md. 58. P308 . Ly61[4]. H99[119].
3. 5. P,T,M$\frac{15}{13}$T$_2$. p$\frac{1}{2}$t, p$_2$t, p$\frac{1}{2}$m$\frac{15}{13}$t$_2$, p$_2$m$\frac{15}{13}$t$_2$, ... Ly61[8]. H99[120].
3. 5. P,m,T, m$_2$t$\frac{15}{13}$, M$\frac{15}{13}$T$_2$. p$\frac{1}{2}$t, p$\frac{1}{2}$m$\frac{15}{13}$t$_2$ = (P, V, *v*. 2r$\frac{1}{2}$) ....H99[121].

3. 5. P,m,T, $m_2t\frac{1}{1}\frac{5}{3}$, $M\frac{1}{1}\frac{5}{3}T_2$. $p\frac{1}{2}t$, pt, p_2t, $p\frac{1}{2}m\frac{1}{1}\frac{5}{3}t_2$, $pm\frac{1}{1}\frac{5}{3}t_2$, $p_2m\frac{1}{1}\frac{5}{3}t_2$ = (P,V, *v*. $2r\frac{1}{2}$, $2r_{II}$, $2r_2$)………H99[122].

3. 5. ($P,T,M\frac{1}{1}\frac{5}{3}T_2$. $p\frac{1}{2}t$, $p\frac{1}{2}m\frac{1}{1}\frac{5}{3}t_2$) × 2,……………………………Ly61[5].

4. 5. $m,T,m_2t\frac{1}{1}\frac{5}{3}$, $M\frac{1}{1}\frac{5}{3}T_2$. $P\frac{1}{2}T$, $P\frac{1}{2}M\frac{1}{1}\frac{5}{3}T_2$ = (V, *v*. $2R\frac{1}{2}$) ………Ly61[7].

5. 5. P_. P_2T, $P_2M\frac{1}{1}\frac{5}{3}T_2$,………………Sim. Md. 96, but flatter. H99[116].

5. 5. P_. pt, $pm\frac{1}{1}\frac{5}{3}t_2$,……………………Sim. Md. 96, but flat. H99[117].

5. SULPHURET OF BISMUTH. Wismuthglanz. Bismuth sulfuré.

Cleavage = $p,t,m\frac{7}{15}t$.

1. 3. $P,M\frac{7}{15}T$, ……………………………… Sim. Model 6. L616[1].

1. 5. $P,t,M\frac{7}{15}T$,………………………………………L616[2].

3. 5. $P,t,M\frac{7}{15}T$. p_xm,p_xt,p_yt, ……………………………P277. L616[3].

6. *An Isomorphous Group*, 1, 2:—

1. SULPHURET OF ANTIMONY. Antimonglanz. Antimoine sulfuré. Grey Antimony. Grau Spiesglanzerz.

Cleavage = P,m,t. $p\frac{98}{100}m\frac{96}{100}t$.

1. 5. $P,T,M\frac{96}{100}T$, ………………………Sim. Model 8. L605[7]. H116[301].

1. 5. P,M,T, $m\frac{96}{100}t$, ……………………………………………… H116[304].

4. 3. $M\frac{96}{100}T$. $P\frac{98}{100}M\frac{96}{100}T$,………………Sim. Md. 67. L605[1]. H116[299].

4. 3. $M\frac{96}{100}T$. $P\frac{98}{100}M\frac{96}{100}T$, $p\_m\frac{96}{100}t$, ……………………………… L605[2].

4. 3. $M\frac{96}{100}T$. $P\_M\frac{96}{100}T$, ……………………………………L605[3].

4. 5. t, $M\frac{96}{100}T$. $P\frac{98}{100}M\frac{96}{100}T$, ……………………M ii [6]. L605[6]. H116[300].

4. 5. m,t, $M\frac{96}{100}T$. $P\frac{98}{100}M\frac{96}{100}T$,……………………L605[5]. H116[302].

4. 5. m,t, $M\frac{96}{100}T$. $P\_M\frac{96}{100}T$, ………………………………H iv 291[5].

2. ORPIMENT. Auripigment. Arsenic sulfuré jaune.

Cleavage = m.

5. 5. M,T, $M\frac{3}{4}T$, $m\frac{3}{4}t$, $m\frac{3}{2}t$. $P\frac{9}{8}T$, $p_xm_yt_z$, ………………… D434. P283.

5. 5. M,T, $M\frac{3}{2}T$, $m\frac{3}{4}t$. $P\frac{9}{8}T$,…………………………………Levy 74[2].

7. WHITE IRON PYRITES. Speerkies. Fer sulfuré blanc.

Cleavage = $p,M\frac{3}{4}T$.

1. 3. $P,M\frac{3}{4}T$,…Cornwall. Derbyshire…Sim. Model 6. L661[1]. H109[223].

3. 3. $P,M\frac{3}{4}T$. $p\frac{1}{3}t$, ……………Sim. Model 44. L661[2]. Ly v. 2. H109[226].

3. 3. $p,m\frac{3}{4}t$. $p\frac{6}{4}m$, $p\frac{6}{3}t$, $P\frac{6}{3}M\frac{3}{4}T$, ………………………L661[7]. H109[229].

3. 5. $p,m\frac{3}{4}t$. $P\frac{6}{4}M$, $P\frac{6}{3}T$, ……………… Joachimstal……L661[5]. H109[228].

4. 5. $m\frac{3}{4}t$. $P\frac{6}{4}M$, $P\frac{6}{4}T$,………………… Freiberg……L661[6]. H109[227].

5. 3. $M\frac{3}{4}T$. $P\frac{1}{3}T$,………………Derbyshire……L661[3]. Ly v. 1. H109[224].

5. 5. $m\frac{3}{4}t$. $P\frac{1}{3}T$, ……………………… Similar to Model 82. H109[225].

8. RED OXIDE OF ZINC. Zinkoxyd.

1. 3. $P,M\frac{1}{2}T$,……(Cleavage = $m\frac{1}{2}t$)…………Sim. Md. 6. L563. P373.

9. WHITE ANTIMONY. Weissantimonerz. Antimoine oxidé.

5. 5. $T\_,M\frac{3}{2}T$. $P\frac{10}{7}T$, $p_xm_yt_z$,…(Cleavage = T)………P348. Mohs ii [14].

10. PYROLUSITE. Grey Ore of Manganese.

Cleavage = m,t, $m\frac{94}{100}t$.

1. 3. $P,M\frac{94}{100}T$, m_t, ...Elgersburg......Levy 75[3].
1. 5. P,m, $M\frac{94}{100}T$, ...Thuringia......Levy 75[2].
3. 5. P,M,t, $M\frac{94}{100}T$. p_t, ...P238. D376.

11. ARSENICAL PYRITES. Arsenikkies. Mispickel. Fer arsenical.

1. Common Arsenical Pyrites.
2. Cobaltic Arsenical Pyrites.

Cleavage = $p,M\frac{8}{3}T$.

1. 3. $P,M\frac{8}{3}T$,...Freiberg...Sim. Md. 6. L663[1]. Ly v. 1. P214[1]. H105[198].
3. 3. $P,M\frac{8}{3}T.\ p\frac{9}{3}m$,.................Sim. Model 44. T497[2]. L663[7]. P214[3].
3. 5. $P,M\frac{8}{3}T.\ p\frac{9}{3}M$,..L663[8]. P214[4].
3. 3. $p, M\frac{8}{3}T.\ p\frac{5}{10}T$, ...Sim. Model 104. P214[2].
5. 5. $m\frac{8}{3}t.\ P\frac{9}{3}M$, ...Sim. Model 82. T497[3]. P214[5].
5. 3. $M\frac{8}{3}T.\ p\frac{4}{7}m,\ P\frac{5}{10}T$,...(Cobaltic) .... Scheerer.
5. 3. $M\frac{8}{3}T.\ P\frac{6}{3}T$,...Cornwall. Tunaberg. Freiberg. L663[2]. Ly v.4. H105[199].
5. 3. $M\frac{8}{3}T.\ P\frac{5}{10}T$, ... Freiberg. Bohemia...M[8]. L663[3]. Ly v. 2. H105[199].
5. 3. $M\frac{8}{3}T.\ P\frac{6}{10}T,\ p\frac{4}{3}t$, ...Tunaberg......L663[4]. Ly v. 5. H105[191].
5. 3. $M\frac{8}{3}T.\ P\frac{3}{10}T,\ p\frac{6}{3}t$, ... R[88]. L663[5]. H105[192].
5. 5. $T,M\frac{8}{3}T.\ P\frac{6}{3}T,\ P\frac{6}{3}M\frac{8}{3}T$, ...L663[6]. H105[193].
5. 3. $(M\frac{8}{3}T.\ P\frac{5}{10}T) \times 2$, ...Freiberg......L663[9]. Ly67[2].

12. BRITTLE SULPHURET OF SILVER. Sprödglaserz.

Cleavage = $m\frac{5}{8}t.\ p_x m_y t_z$.

1. 5. $P\_,T,M\frac{5}{8}T$,...................................Schemnitz......Levy 50[2].
3. 5. $P\_,T,M\frac{5}{8}T.\ p\frac{4}{3}t,\ p_x m_y t_z$, ... Mexico......P298. Levy 50[3].
3. 5. $P\_,T,M\frac{5}{8}T.\ 2p_x t,\ 2p_x m_y t_z$, ...Freiberg......Levy 50[4].
3. 5. $P\_,T,M\frac{5}{8}T.\ 3p_x t,\ p_x M_y T_z$, ...Freiberg......Levy 50[5].
3. 5 $P\_,T,M\frac{5}{8}T.\ 3p_x t,\ 2p_x m_y t_z$, ...Freiberg......Levy 50[6].

13. BERTHIERITE.

1. 3. P,M_xT?......(Cleavage = $m_x t$?)..............Auvergne......P344.

14. JAMESONITE. Cleavage = P.

1. 3. $P,M\frac{5}{6}T$,...Cornwall. Siberia. Hungary...Sim. Md. 6. P346. D420.

15. ZINKENITE. Cleavage = 0.

4. 5. $T,M\frac{15}{13}T_2.\ P\frac{5}{4}T,\ P\frac{5}{4}M\frac{15}{13}T_2 = (V.2R\frac{5}{4})$ ...... Sim. Md. 74. P347.

16. ANTIMONIAL COPPER. Kupferantimonglanz. $Cu_2S + Sb_2S_3$.

Cleavage = p,T.

1. 5. $P,T\_,\ m\frac{5}{6}t,\ m\frac{5}{12}t$, ... G. Rose, Pogg. Ann. xxxv. 360.

17. STERNBERGITE. Cleavage = P.

5. 5. $P\_,T.\ 2p_x m\ t_z$,..P297.

18. MENDIPITE. Muriate of Lead. Berzélite.

1. 3. $P, M\frac{1}{3}T$,......(Cleavage = $M\frac{1}{3}T$) ....P361.

19. *An Isomorphous Group*, 1, 2:—

1. MANGANITE. Grey Oxide of Manganese. Hydrated Deutoxide of Manganese.

Cleavage = $p, M_2T, M\frac{5}{6}T, m\frac{5}{12}t$.

4. 3. $M\frac{5}{6}T, M_2T, m\frac{5}{12}t.\ P\frac{1}{13}M\frac{1}{6}T, p\frac{3}{2}m\frac{5}{6}t, pm\frac{5}{3}t, \frac{1}{2}p\frac{2}{3}m_2t$ Znw Zse, Haidinger, Edin. Jour. of Science, Jan. 1826, fig. 2, 3, 4.

5. 3. $M\frac{5}{6}T, M_2T, M\frac{4}{3}T, m\frac{5}{12}t.\ p\frac{2}{3}m, p\frac{5}{9}t, P\frac{1}{4}M\frac{2}{3}T, p\frac{5}{9}m\frac{5}{6}t, p\frac{3}{2}m\frac{5}{6}t$, idem fig. 5.

2. PRISMATIC IRON ORE. Nadeleisenerz. Brown Iron Ore. Fer hydro-oxidé.

Cleavage = T.

4. 5. $T_{-}, M\frac{92}{100}T.\ P_xM\frac{92}{100}T$, ... Levy 69[2].

4. 5. $T_{-}, M\frac{92}{100}T, m\frac{46}{100}t.\ P_xm\frac{92}{100}T$, ... Levy 69[3].

4. 5. $M, T_{-}, M\frac{92}{100}T, M\frac{46}{100}T.\ p_{+}m, p_{-}m_xt, 2p_{+}m_xt$, ... P220[2].

5. 5. $T_{-}, M\frac{92}{100}T, m\frac{46}{100}t.\ P_{-}T$, ...Levy 69[4].

20. TANTALITE. }
21. COLUMBITE. } Cleavage = M,T.

3. 5. $P_{+}, M, T, m\frac{6}{5}t, m\frac{2}{5}t, m\frac{5}{2}t.\ p\frac{7}{4}m, P_{-}M_xT, p_{+}m_xt$,...D371[3]. P272. T485.

3. 5. $P_{+}, M, T, M\frac{6}{5}T, m\frac{2}{5}t, m\frac{5}{2}t.\ p\frac{7}{4}m, P\frac{1}{3}T, P_{-}M_xT, 2p_{+}m_xt$, ... D371[8].

3. 5. $M, T, m\frac{6}{5}t, m\frac{5}{2}t, m\frac{2}{5}t.\ P_{-}M_xT$, ...D371[1].

22. AESCHYNITE. Cleavage = p.

4. 3. $M\frac{1}{2}T.\ P_xM_yT_z$..............................P261. Brooke.

23. *An Isomorphous Group of Carbonates*, 1, 2, 3, 4, 5:—

1. WITHERITE. Carbonate of Barytes. Baryte Carbonatée.

Cleavage = $t.\ p\frac{4}{3}t, p\frac{4}{5}m\frac{3}{5}t$.

2. 5. $P\frac{4}{3}T, P\frac{4}{3}M\frac{3}{5}T$, ...Levy 15[2].

2. 5. $P\frac{1}{3}T, p\frac{4}{3}t, P\frac{1}{3}M\frac{3}{5}T, p\frac{4}{3}m\frac{3}{5}t$,..............................H43[77].

3. 5. $P_{+}, T, M\frac{3}{5}T.\ p\frac{4}{3}t, p\frac{4}{3}m\frac{3}{5}t$,..............................L330[1]. H43[76].

3. 5. $p_{+}, T, M\frac{3}{5}T.\ P\frac{4}{3}T, p\frac{2}{3}t, p\frac{1}{3}t, P\frac{4}{3}M\frac{3}{5}T, p\frac{2}{3}m\frac{3}{5}t, p\frac{1}{3}m\frac{3}{5}t$,...H43[78]. Levy 15[3].

4. 5. $T, M\frac{3}{5}T.\ P\frac{4}{3}T, P\frac{4}{3}M\frac{3}{5}T$,..............................L330[2]. H42[75].

5. 5. $T, M\frac{3}{5}T.\ p\frac{4}{3}t, p\frac{2}{3}t, p\frac{1}{3}t$, ...L330[3]. M ii[22].

5. 5. $T, M\frac{3}{5}T.\ P\frac{4}{3}T$,..............................M ii[9].

2. STRONTIANITE. Strontian carbonatée. Strontites.

Cleavage = $M\frac{3}{5}T.\ p\frac{6}{5}t$.

1. 5. $P_{-}, T, M\frac{3}{5}T$, ...Sim. Model 7. Levy 18[1]. H45[93].

3. 5. $P, T, M\frac{3}{5}T.\ p\frac{6}{5}t, p\frac{6}{5}m\frac{3}{5}t$, ... Sim. Model 58. L328[1]. Ly18[2]. H45[94].

3. 5. $P, T, M\frac{3}{5}T.\ p\frac{6}{5}t, p\frac{2}{5}t, p\frac{6}{5}m\frac{3}{5}t, p\frac{22}{5}m\frac{3}{5}t$,..............................L328[3]. H45[95].

3. 5. $P, T, M\frac{3}{5}T.\ P\frac{6}{5}T, p\frac{6}{5}m\frac{3}{5}t, p\frac{22}{5}m\frac{3}{5}t$, ... D200. M ii[38]. Levy 18[3].

4. 5. $T, M\frac{3}{5}T.\ P\frac{6}{5}T, P\frac{6}{5}M\frac{3}{5}T$,..............................L328[2].

3. ARRAGONITE. Carbonate of Lime (and Strontian ?).

Cleavage = t,M$\frac{5}{8}$T. p$\frac{7}{10}$t.

1. 3. P,M$\frac{5}{8}$T, ... Spain......Sim. Model 6. H23[3].

3. 3. p,M$\frac{5}{8}$T. P$\frac{10}{7}$T, ... Spain......H23[5].

4. 5. T$_-$,M$\frac{5}{8}$T. P$\frac{7}{10}$T, p$_5$t,.. Ly11[8].

5. 3. M$\frac{5}{8}$T. P$\frac{7}{10}$T, ... Spain......Sim. Model 20. H23[1].

5. 3. M$\frac{5}{8}$T. P$\frac{10}{7}$T, ... Spain......Sim. Model 19. H23[2].

5. 5. M,T, M$\frac{5}{8}$T. P$\frac{7}{10}$T, ... Piedmont ......Sim. Model 97. H23[6].

5. 5. M,T,M$\frac{5}{8}$T. P$\frac{7}{10}$T,....................................Model 97. H23[5].

5. 5. T,M$\frac{5}{8}$T. P$\frac{7}{10}$T, ...Piedmont......Sim. Model 111. Ly11[7]. H23[4].

5. 5. T,M$\frac{5}{8}$T. p$\frac{5}{14}$t, P$\frac{7}{10}$T, p$\frac{10}{7}$t. 3P$_x$M$_y$T$_z$,..............................Mr6[64].

[Many twin crystals. See Levy and Haüy.]

4. JUNKERITE. Cleavage = t,M$\frac{5}{7}$T.

5. 3. M$\frac{5}{7}$T. P$_x$T, ...T i 449.

5. WHITE LEAD ORE. Weissbleierz. Plomb carbonaté. Carbonate of Lead. Kohlensaures Blei.

Cleavage = p,m$\frac{6}{10}$t.

2. 5. P$\frac{10}{7}$T, P$\frac{7}{10}$M$\frac{6}{10}$T, ...L290[5]. J[138]. H91[54].

3. 3. P,M$\frac{6}{10}$T. P$\frac{7}{10}$T, ... L290[1].

3. 5. P,T,M$\frac{6}{10}$T. p$\frac{10}{7}$t, p$\frac{7}{10}$m$\frac{6}{10}$t, ... J[136]. H92[59].

3. 5. P,T,M$\frac{6}{10}$T. p$\frac{14}{3}$t, p$_2$t, p$\frac{10}{7}$t, p$\frac{7}{10}$m$\frac{6}{10}$t, ... L290[9]. H92[63].

3. 5. P$_-$,M,T, m$\frac{6}{10}$t, M$\frac{18}{10}$T. p$\frac{4}{7}$m, p$\frac{5}{14}$t, p$\frac{7}{10}$t,..................L290[10]. H92[64].

4. 5. T, M$\frac{6}{10}$T. P$\frac{10}{7}$T, P$\frac{7}{10}$M$\frac{6}{10}$T,..................L290[6]. J[137]. R[96]. H92[56].

5. 3. M$\frac{6}{10}$T. P$\frac{7}{10}$T. Axes: p^{a}m$^a_6$t$^a_{10}$, ...Model 82[a]. H91[51].

The letters stamped on Model 82[a] in agreement with the system of Haüy, who considers this combination to be a rectangular octahedron, give rise to the symbol P$\frac{10}{6}$M, P$\frac{10}{7}$T. To make the Model agree with the symbol M$\frac{6}{10}$T. P$\frac{7}{10}$T, it must be turned on the axis m^a from east to west, 90°, so as to make p^a and t^a change places.

5. 5. T$_-$, M$\frac{6}{10}$T. p$\frac{7}{4}$m, P$\frac{10}{7}$T, ... Model 110. R[90]. Sim. H92[57].

5. 5. T$_-$, M$\frac{6}{10}$T. p$\frac{4}{7}$m, P$\frac{5}{14}$T, ... Sim. Model 110. J[139]. H92[57]. L290[7].

5. 5. M$\frac{6}{10}$T. P$_2$T,...................................M ii [3]. L290[2]. J[135]. H91[53].

5. 5. T,M$\frac{6}{10}$T. P$\frac{10}{7}$T,..M ii [9]. L290[3]. H91[55].

5. 5. T, m$\frac{6}{10}$t. p$\frac{4}{7}$m, P$\frac{7}{10}$T, P$\frac{7}{10}$M$\frac{6}{10}$T, ... H92[58].

5. 5. T,M$\frac{6}{10}$T. p$\frac{4}{7}$m, p$\frac{10}{7}$t, p$\frac{5}{14}$t...................................L290[8]. H92[60].

5. 5. M,T, m$\frac{6}{10}$t, m$\frac{18}{10}$t. P$\frac{7}{10}$T, P$\frac{7}{10}$M$\frac{6}{10}$T,..................................H92[61].

5. 5. T,M$\frac{6}{10}$T. p$\frac{4}{7}$m, P$\frac{5}{14}$T, p$\frac{7}{10}$t, p$\frac{10}{7}$t, P$\frac{7}{10}$M$\frac{6}{10}$T, ...H92[62].

24. PHOSPHATE OF MANGANESE. Triplit. Manganèse phosphaté.

1. 1. P$_x$,M$_-$,T,...(Cleavage = P,m,t) ...... Limoges...P248. Ly iii 304.

25. NITRE. Salpeter. Potasse nitraté.

Cleavage = T,m$\frac{11}{19}$t.

[The following descriptions apply to manufactured Crystals.]

2. 5. P$\frac{26}{19}$T, P$\frac{26}{38}$M$\frac{22}{38}$T,.........Axes: p$^a_{26}$m$^a_{22}$t$^{a'}_{19}$...Sim. Model 26. H52[161].

3. 5. p$_+$,m, T$_-$, m$\frac{11}{19}$t. p$\frac{13}{19}$t, ... H53[164].

4. 5. $T, M\frac{11}{19}T.\ p\frac{26}{19}t,\ p\frac{26}{38}m\frac{22}{38}t$, ... Axes: $p_{+}^{a}m^{a}t_{-}^{a}$...Sim. Model 73. Aikin. H52[163].

4. 5. $T, M\frac{11}{19}T.\ p\frac{26}{19}t,\ p\frac{13}{19}t,\ p\frac{13}{38}t,\ p\frac{26}{38}m\frac{22}{38}t,\ p\frac{13}{38}m\frac{22}{38}t,\ p\frac{13}{76}m\frac{44}{76}t$, ...H53[166].

5. 3. $M\frac{11}{19}T.\ P\frac{13}{19}T$,...Axes: $p_{13}^{a}m_{11}^{a}t_{19}^{a}$...Similar to Md. 20. Aikin. H52[159].

5. 5. TW, te, $M\frac{11}{19}T.\ P\frac{13}{19}T$, ...J. J. G.

5. 5. $T_{-}, M\frac{11}{19}T.\ P\frac{13}{19}T$,............Axes: $p_{+}^{a}m^{a}t_{-}^{a}$......M ii 9. Levy. H52[162].

5. 5. $T_{-}, m\frac{11}{19}t.\ p\frac{26}{19}t$, ...Aikin, Chemical Dictionary.

5. 5. $T_{-}, M\frac{11}{19}T.\ p\frac{26}{19}t,\ p\frac{13}{19}t,\ p\frac{13}{38}t$, ...M ii [23]. H53[165].

26. STAUROLITE. Staurolith. Staurotide.

Cleavage = $T, m\frac{8}{17}t$.

1. 3. $P_{-}, M\frac{8}{17}T$,...Morbihan. Sim. Md. 6. M ii 366[1]. L409[1]. P75[1]. H61[44].

1. 5. $P_{+}, T, M\frac{8}{17}T$, ... Cayenne. St. Gotthardt...J[61]. Ly v.[1]. L409[2]. M ii 366[1]. P75[2]. H61[45].

1. 5. $(P_{+}, T, M\frac{8}{17}T) \times 2$, ... Two individuals crossing at a right angle. Model 9. J[63]. Ly v.[3]. L409[4]. P75[4]. H62[47].

1. 5. $(P_{+}, T, M\frac{8}{17}T) \times 2$, ... Two individuals crossing at alternate angles of 60° and 120°,......J[64]. Ly v.[4]. L409[4]. P75[5]. H62[48].

3. 5. $P_{+}, T, M\frac{8}{17}T.\ P\frac{17}{12}M$, ... Aschaffenberg...Model 55. P75[3]. H61[46]. M[12]. J[62]. Ly v. 2. L409[3].

27. ANDALUSITE. Feldspath apyre. Andalousite.

Cleavage = $m\frac{98}{100}t$.

1. 3. $P, M\frac{98}{100}T$, ... Lisenz, Tyrol......Ly42[1]. M v.[1]. L405[1].

3. 3. $P, M\frac{98}{100}T.\ p\frac{71}{100}t$,............Lisenz, Tyrol......Ly42[2]. M ii[3]. L405[2].

3. 3. $P, M\frac{98}{100}T.\ p\frac{71}{98}m$, ...L405[3].

3. 3. $P, M\frac{98}{100}T.\ p\frac{71}{98}m,\ p\frac{71}{100}t$, ... Ly42[3]. L405[4].

3. 3. $P, M\frac{98}{100}T,\ M\frac{52}{100}T.\ p\frac{84}{100}t,\ p_{x}m_{y}t_{z}$, ... P107.

3. 3. $P, M\frac{98}{100}T.\ p\frac{71}{100}t,\ P_{x}M_{y}T_{z}$, ...Ly42[4].

3. 3. $P, M\frac{98}{100}T,\ m\frac{52}{100}t.\ p\frac{71}{98}m,\ p\frac{71}{100}t,\ p_{x}m_{y}t_{z}$, ... Ly42[6].

3. 5. $P, m, M\frac{98}{100}T.\ p\frac{71}{100}t$, ...L405[5].

3. 5. $P, m, M\frac{98}{100}T.\ p\frac{71}{98}m,\ p\frac{71}{100}t$, ...L405[6].

3. 5. $P, m, M\frac{98}{100}T.\ p\frac{71}{100}t,\ P_{x}M_{y}T_{z}$, ...Ly42[5].

5. 5. $M\frac{98}{100}T.\ P\frac{71}{100}T$,..M v[2]. L405[7].

5. 5. $m, M\frac{98}{100}T.\ p\frac{71}{98}m,\ P\frac{71}{100}T$,...L405[8].

28. OLIVINE. Chrysolite. Péridot. Krysolith.

Cleavage = p, m, T.

1. 2. $P\frac{5}{9}, M\frac{4}{9}, T$, ... Eisenach......L531[1].

3. 2. $p_{+}, M_{-}, T.\ p\frac{5}{4}m,\ P\frac{10}{9}T,\ p\frac{5}{9}t,\ p\frac{10}{9}m\frac{8}{9}t,\ p\frac{5}{9}m\frac{4}{9}t$, ... Ly32[5].

3. 5. $p_{+}, M_{-}, T,\ M\frac{4}{9}T.\ P\frac{5}{4}M,\ P\frac{5}{9}M\frac{4}{9}T$,..........................L531[5]. H70[130].

3. 5. $p_{+}, M_{-}, T,\ M\frac{4}{9}T.\ P\frac{5}{4}M,\ P\frac{10}{9}T,\ p\frac{5}{9}m\frac{4}{9}t$,...Model 51. L531[3]. R[98]. Ly32[3]. H70[129].

3. 5. $p_{+}, M_{-}, T,\ M\frac{4}{9}T.\ P\frac{5}{4}M,\ P\frac{10}{9}T,\ p\frac{5}{9}t,\ p\frac{5}{9}m\frac{4}{9}t$, ... L531[4]. Ly32[4]. H70[133].

3. 5. $p_{+}, M_{-}, T,\ m\frac{4}{9}t,\ m\frac{8}{9}t,\ m\frac{16}{9}t.\ P\frac{5}{4}M,\ P\frac{10}{9}T,\ p\frac{5}{9}t,\ p\frac{5}{9}m\frac{4}{9}t,\ p\frac{10}{9}m\frac{8}{9}t$, found in meteoric Iron,...........L531[11]. P85.

3. 5. $p_{+}, M_{-}, T,\ m\frac{4}{9}t,\ m\frac{8}{9}t.\ P\frac{5}{4}M,\ p\frac{10}{9}t,\ p\frac{5}{9}t,\ p\frac{5}{9}m\frac{4}{9}t$, ...Ly32[7].

3. 5. p_+, $M_\llcorner$, T, $M\frac{4}{9}T$, $m\frac{8}{9}t$. $P\frac{4}{4}M$, $p\frac{10}{9}t$, $p\frac{8}{9}t$, $p\frac{8}{9}m\frac{4}{9}t$, $p\frac{10}{9}m\frac{8}{9}t$, ...Ly32[9].
3. 5. p_+, $M_\llcorner$, t, $M\frac{4}{9}T$, $m\frac{8}{9}t$. $P\frac{4}{4}M$, $P\frac{10}{9}T$, $p\frac{8}{9}t$, $p\frac{5}{9}m\frac{4}{9}t$, ...L531[6]. H70[135].
3. 5. p_+, $M_\llcorner$, T, $M\frac{4}{9}T$, $m\frac{8}{9}t$, $m\frac{16}{9}t$. $P\frac{4}{4}M$, $P\frac{10}{9}T$, $P\frac{8}{9}M\frac{4}{9}T$, $p\frac{10}{9}m\frac{8}{9}t$, ...Ly32[6].
3. 5. p_+, m, T, $m\frac{8}{9}t$, $M\frac{16}{9}T$. $p\frac{4}{4}m$, $P\frac{10}{9}T$, $P\frac{8}{9}M\frac{4}{9}T$, ...L531[8]. H70[134].
3. 5. P_+, T, $M\frac{8}{9}T$, $m\frac{16}{9}t$. $P\frac{10}{9}T$, ... Ly32[2].
3. 5. p_+,t,$M\frac{4}{9}T$, $m\frac{8}{9}t$. $P\frac{4}{4}M$, $P\frac{10}{9}T$, ... L531[7]. H70[131].
5. 2. $M_\llcorner$,T. $P\frac{10}{9}T$, ...Baden......L531[2].
5. 5. $M_\llcorner$, T, $M\frac{4}{9}T$, $m\frac{8}{9}t$. $P\frac{4}{4}M$, $P\frac{10}{9}T$, $p\frac{8}{9}m\frac{4}{9}t$, $p\frac{10}{9}m\frac{8}{9}t$, ...Ly32[4].

29. Sulphate of Potash. Schwefelsaures Kali. Potasse sulfatée.
Cleavage = t,$m\frac{4}{7}t$.

1. 5. P_x,T,$M\frac{4}{7}T$. Axes: $p_x^a m_+^1 t^a$,..H53[100].
2. 5. $P\frac{2}{3}T$,$P\frac{2}{7}M\frac{4}{7}T$,...................................P196. R123. H53[103].

30. Thenardite. Sulphate of Soda. Cleavage = P,$m\frac{1}{2}t$.

2. 3. $P_+M\frac{1}{2}T$, ... Aranjuez......Sim. Model 21. D[76]. P407.
3. 3. P_+,$M\frac{1}{2}T$. $p_+m\frac{1}{2}t$, ...Aranjuez......D75.

31. *An Isomorphous Group of Sulphates*, 1, 2, 3:—

1. Sulphate of Barytes. Schwerspath. Baryte sulfatée. Heavy Spar.
Cleavage = P,m,t,$M\frac{4}{3}T$.

1. 3. P,$M\frac{4}{3}T$. Axes: $p_x^a m_4^a t_5^a$, ...Model 6. R[92]. Ly15[1]. L256[1]. H33[1].
1. 5. $P_{\overline{\llcorner}}$, m, $m\frac{4}{3}t$,......$p_-^a m^a t_+^a$,..................................L256[2]. H33[9].
1. 5. $P_{\overline{\llcorner}}$, t, $m\frac{4}{3}t$, ... $p_-^a m_+^a t^a$,.....................J[142]. Ly v[3]. L256[5]. H33[8].
1. 5. $P_{\overline{\llcorner}}$, m, $M\frac{4}{3}T$, $m\frac{4}{13}t$,......$p_-^a m^a t_+^a$,..........................Ly15[3]. H34[16].
3. 3. P_-,$M\frac{4}{3}T$. $p\frac{4}{3}m$, ... $p_-^a m_4^a t_5^a$,......Model 44. Ly v[4]. L256[7]. H33[5]. S[229].
3. 3. P_-, $M\frac{4}{3}T$. $p\frac{4}{3}m\frac{4}{3}t$,......$p_-^a m_4^a t_5^a$,..................................H33[7].
3. 3. P_-, $M\frac{4}{3}T$, $m\frac{8}{13}t$. $p\frac{4}{3}t$, $p\frac{4}{3}m\frac{4}{3}t$,......$p_-^a m^a t_+^a$,........................H36[29].
3. 3. p_+, $M\frac{3}{4}T$. $P\frac{4}{3}M$, $p\frac{2}{4}t$, $p\frac{3}{3}m\frac{3}{4}t$, ... $p_+^a m_-^a t^a$,........................H36[31].
3. 3. p_+, $M\frac{3}{4}T$. $P\frac{2}{4}T$, $p\frac{2}{4}m\frac{3}{4}t$, $p\frac{3}{3}m\frac{3}{4}t$,......$p_+^a m_3^a t_4^a$, ...H37[33].
3. 3. P_+, $M\frac{4}{3}T$. $p\frac{3}{4}m$, $P\frac{2}{4}T$, $p\frac{4}{3}m\frac{4}{3}t$, $P\frac{4}{3}M\frac{4}{3}T$,......$p_+^a m_4^a t_5^a$, ...H38[40].
3. 5. P_-,m,t_+, $M\frac{4}{3}T$. $p\frac{4}{3}m$, $p\frac{4}{3}t$, $p_-^a m^a t_+^a$,...Md. 50. Ly v[28]. L256[20]. H38[45].
3. 5. p_+, M, t, $M\frac{4}{3}T$. $P\frac{2}{4}T$, $p\frac{2}{4}m\frac{4}{3}t$,......$p_+^a m_-^a t^a$, ...H38[46].
3. 5. p_+, M, T, $M\frac{3}{4}T$. $p\frac{4}{3}m$, $p\frac{2}{2}t$, $p\frac{2}{4}t$, $p\frac{2}{8}t$, $p\frac{10}{4}m\frac{3}{4}t$,......$p_+^a m^a t_-^a$......H41[64].
3. 5. p_+,M,T,$M\frac{3}{4}T$. $P\frac{4}{3}M$,$p\frac{8}{13}m$,$P\frac{2}{4}T$,$p\frac{2}{4}m\frac{3}{4}t$,$p\frac{3}{3}m\frac{3}{4}t$, $p_+^a m^a t_-^a$, L257[30].H42[70].
3. 5. p_+,$M_\llcorner$,T, $M\frac{4}{3}T$, $m\frac{4}{3}t$. $P\frac{2}{4}M$, $P\frac{2}{4}T$, $p\frac{5}{4}m\frac{4}{3}t$,...$p_+^a m_-^a t^a$, ...S[237]. H40[61].
3. 5. p_+,$M_\llcorner$,T, $M\frac{4}{3}T$,$m\frac{4}{3}t$. $P\frac{2}{4}M$, $P\frac{2}{4}T$, $p\frac{2}{4}m\frac{4}{3}t$, $p\frac{4}{3}m\frac{3}{4}t$, $p_+^a m_-^a t^a$, S[236]. H41[67].
3. 5. p_+, m, T, $M\frac{3}{4}T$. $p\frac{4}{3}m$, $P\frac{2}{4}T$, $p\frac{3}{3}m\frac{3}{4}t$, $p_+^a m^a t_-^a$,...Ly v[48]. L257[29]. H40[57].
3. 5. p_+, m, T, $M\frac{3}{4}T$. $P\frac{4}{3}M$, $p\frac{2}{2}t$, $p\frac{2}{4}t$, $p\frac{2}{8}t$,.....$p_+^a m^a t_-^a$, ...H40[58].
3. 5. P_-, m, $M\frac{4}{3}T$. $p\frac{4}{3}t$,......$p_-^a m^a t_+^a$, ...H34[15].
3. 5. $P_{\overline{\llcorner}}$, m_-, $M\frac{4}{3}T$, $m\frac{8}{13}t$. $p\frac{4}{3}m$,......$p_-^a m^a t_+^a$,........................H36[28].
3. 5. p_+, M, $M\frac{4}{3}T$. $P\frac{2}{4}M$, $p\frac{2}{4}t$, $p\frac{2}{4}m\frac{4}{3}t$,......$p_+^a m_-^a t^a$,............Ly v[29]. H38[42].
3. 5. P_+, M_-, $M\frac{4}{3}T$, $M\frac{1}{3}T$. $p\frac{2}{4}t$,......$p_+^a m_-^a t^a$,................. L256[24]. H35[25].

SULPHATE OF BARYTES *Continued.*

3. 5. P_{-}, t, M$\frac{4}{3}$T. p$\frac{4}{3}$m, p$\frac{4}{3}$t,......$p^{a}_{-}m^{a}t^{a}_{+}$, ... S[285]. H36[29].
3. 5. P_{-}, t_{+}, M$\frac{4}{3}$T. P$\frac{5}{4}$M, p$\frac{2}{3}$m, p$\frac{4}{3}$t,...... $p^{a}_{-}m^{a}t^{a}_{+}$, ...Ly v[33]. H38[41].
3. 5. p_{+}, t, M$\frac{3}{4}$T. P$\frac{4}{3}$M, P$\frac{4}{3}$T, p$\frac{3}{3}$m$\frac{3}{4}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$, ...Ly v[30]. H38[42].
3. 5. P_{-}, M$\frac{4}{3}$T. p$\frac{4}{3}$t,......$p^{a}_{-}m^{a}t^{a}_{+}$,............Ly v[6]. R[94]. S[230]. L256[10]. H33[6].
3. 5. P_{-}, M$\frac{4}{3}$T, m$\frac{8}{15}$t. p$\frac{4}{3}$m, p$\frac{4}{3}$t, ... $p^{a}_{-}m^{a}t^{a}_{+}$,........................H36[28].
4. 5. $T_{=}$, M$\frac{3}{4}$T. P$\frac{4}{3}$M, p_{3}t, p$\frac{3}{3}$m$\frac{3}{4}$t, ... $p^{a}_{+}m^{a}t^{a}_{-}$,........................H36[31].
4. 5. T_{-}, M$\frac{3}{4}$T. P$\frac{4}{3}$M, P$\frac{5}{4}$T, p$\frac{5}{2}$t, p$\frac{3}{3}$m$\frac{5}{4}$t,......$p^{a}_{+}m^{a}t^{a}_{-}$,.........Ly v[37]. H37[40].
4. 3. M$\frac{4}{3}$T. p$\frac{3}{4}$m, P$\frac{5}{4}$T,......$p^{a}_{+}m^{a}_{4}t^{a}_{5}$, ...L256[12]. H34[10a].
4. 3. M$\frac{3}{4}$T. p$\frac{4}{3}$m, p$\frac{5}{4}$t,......$p^{a}_{+}m^{a}_{3}t^{a}_{4}$,........................J[146]. L256[13]. H34[10b].
5. 3. M$\frac{4}{3}$T. P$\frac{5}{4}$T, ...$p^{a}_{+}m^{a}_{4}t^{a}_{5}$, ...Model 82. J[141]. S[229]. L56[8]. H233[2].
5. 3. M$\frac{3}{4}$T. P$\frac{5}{4}$T,......$p^{a}_{+}m^{a}_{3}t^{a}_{4}$,..............................Ly v[2]. H33[4].
5. 3. M$\frac{3}{4}$T. P$\frac{5}{4}$T, p$\frac{5}{4}$m$\frac{3}{4}$t,......$p^{a}_{+}m^{a}_{3}t^{a}_{4}$,..............................H34[12].
5. 3. M$\frac{3}{4}$T. P$\frac{4}{3}$M, P$\frac{5}{4}$T, p$\frac{4}{3}$m$\frac{3}{4}$t,......$p^{a}_{+}m^{a}_{3}t^{a}_{4}$,..........................H34[17].
5. 3. M$\frac{3}{4}$T, m$\frac{5}{6}$t. P$\frac{5}{4}$T, p$\frac{5}{8}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$,..............................H35[22].
5. 3. M$\frac{3}{4}$T, m$\frac{5}{6}$t. p$\frac{5}{4}$t, p$\frac{5}{8}$t, p$\frac{3}{3}$m$\frac{5}{4}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$, ...H37[35].
5. 3. M$\frac{3}{4}$T, m$\frac{3}{8}$t. P$\frac{4}{3}$M, P$\frac{5}{4}$T, p$\frac{5}{4}$m$\frac{3}{4}$t, p$\frac{4}{3}$m$\frac{3}{4}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$,...............H38[47].
5. 3. M$\frac{3}{4}$T. p$\frac{4}{3}$m,......$p^{a}_{+}m^{a}_{3}t^{a}_{4}$,..............................L256[11]. H33[3].
5. 5. M_{-}, M$\frac{4}{3}$T. P$\frac{4}{3}$M,......$p^{a}m^{a}_{-}t^{a}_{+}$, ...Model 100. R[91a]. J[140]. L256[16].
5. 5. $M_{=}$, M$\frac{4}{3}$T. P$\frac{3}{4}$M, $p^{a}_{+}m^{a}_{-}t^{a}$, Sim. Md. 100. R[91a]. Lyv7. J[140]. L256[18]. H34[11].
5. 5. M_{-}, M$\frac{1}{3}$T, m$\frac{4}{3}$t. P$\frac{5}{4}$T,......$p^{a}_{+}m^{a}_{-}t^{a}$, ... Ly v[15]. L256[22]. H34[13].
5. 5. $M_{=}$, M$\frac{4}{3}$T. P$\frac{3}{4}$M, p$\frac{5}{4}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$,......... Ly v[11]. J[141]. L256[17]. H35[14].
5. 5. M_{-}, m$\frac{4}{3}$t, m$\frac{8}{3}$t. P$\frac{3}{4}$M, ...$p^{a}m^{a}_{-}t^{a}_{+}$,......................Ly v[14]. H35[18].
5 5. M_{-}, m$\frac{4}{3}$t, M$\frac{8}{3}$T. P$\frac{3}{4}$M......$p^{a}_{+}m^{a}_{-}t^{a}$, ...H35[19].
5. 5. M_{-}, m$\frac{4}{3}$t, m$\frac{8}{3}$t. P$\frac{3}{4}$M, P$\frac{5}{4}$T,.........$p^{a}_{+}m^{a}_{-}t^{a}$,...Ly v[18]. S[233]. D204. H35[23].
5. 5. M_{-}, m$\frac{4}{3}$t, M$\frac{8}{3}$T. P$\frac{3}{4}$M, p$\frac{5}{4}$t, ... $p^{a}_{+}m^{a}_{-}t^{a}$,............Ly v[19]. S[233]. H35[24].
5. 5. M_{-}, M$\frac{4}{3}$T, m$\frac{3}{3}$t, m$\frac{8}{3}$t. P$\frac{3}{4}$M, P$\frac{5}{4}$T, p$\frac{5}{4}$m$\frac{8}{3}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$, ...H37[39].
5. 5. $M_{=}$, t, m$\frac{4}{3}$t. P$\frac{3}{4}$M, p$\frac{5}{4}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$, ... Ly v[17]. L256[27]. H36[27].
5. 5. M_{-}, t, M$\frac{4}{3}$T. P$\frac{3}{4}$M, p$\frac{5}{4}$t, p$\frac{15}{8}$t, ...$p^{a}_{+}m^{a}_{-}t^{a}$,...............L257[28]. H38[44].
5. 5. M_{-}, t, M$\frac{4}{3}$T, M$\frac{8}{3}$T. P$\frac{3}{4}$M, P$\frac{5}{4}$T, p$\frac{5}{4}$m$\frac{3}{4}$t, ...$p^{a}_{+}m^{a}_{-}t^{a}$,...............H40[55].
5. 5. M_{-}, t, M$\frac{4}{3}$T, m$\frac{8}{3}$t. P$\frac{3}{4}$M, P$\frac{5}{4}$T, p$\frac{15}{8}$t. p$\frac{5}{4}$m$\frac{4}{3}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$, ...H40[50].
5. 5. M_{-}, t, M$\frac{4}{3}$T. P$\frac{3}{4}$M, P$\frac{5}{4}$T, ...$p^{a}_{+}m^{a}_{-}t^{a}$,..............................A[52].
5. 5. m, T, M$\frac{3}{4}$T. p$\frac{4}{3}$m, P$\frac{5}{4}$T, p$\frac{5}{8}$t, p$\frac{3}{3}$m$\frac{3}{4}$t, ...$p^{a}_{+}m^{a}t^{a}_{-}$, ...H39[53].
5. 5. m, T_{-}, M$\frac{3}{4}$T. p$\frac{4}{3}$m, P$\frac{5}{4}$T, p$\frac{5}{2}$t, p$\frac{3}{3}$m$\frac{3}{4}$t,...$p^{a}_{+}m^{a}t^{a}_{-}$,...Ly v[47]. S[236]. H39[54].
5. 5. m, T_{-}, M$\frac{3}{4}$T. P$\frac{4}{3}$M, p$\frac{5}{2}$t, p$\frac{5}{4}$t, p$\frac{5}{8}$t, ...$p^{a}_{+}m^{a}t^{a}_{-}$, ... Ly v[46]. H39[52].
5. 5. m, T_{-}, M$\frac{3}{4}$T. p$\frac{4}{3}$m, P$\frac{5}{4}$T, p$\frac{5}{4}$m$\frac{3}{4}$t, p$\frac{3}{3}$m$\frac{3}{4}$t,......$p^{a}_{+}m^{a}t^{a}_{-}$, ...H40[56].
5. 5. t, M$\frac{3}{4}$T. P$\frac{4}{3}$M, ...$p^{a}_{+}m^{a}_{-}t^{a}$,..............................J[145].
5. 5. t, M$\frac{3}{4}$T. p$\frac{4}{3}$m, p$\frac{5}{4}$t, ...$p^{a}_{+}m^{a}_{-}t^{a}$,..............................J[147].
5. 5. t, M$\frac{3}{4}$T. p$\frac{5}{4}$t, p$\frac{5}{8}$t, p$\frac{5}{4}$m$\frac{3}{4}$t, $\frac{1}{2}P_{+}M_{-}T$ ZneZnwNseNsw, $p^{a}_{+}m^{a}_{-}t^{a}$, H39[49].
5. 5. T, M$\frac{3}{4}$T, m$\frac{6}{3}$t. P$\frac{4}{3}$M, p$\frac{5}{2}$t, p$\frac{5}{4}$t, p$\frac{5}{8}$t,......$p^{a}_{+}m^{a}t^{a}_{-}$, ...H39[50].
5. 5. T, M$\frac{3}{4}$T. p$\frac{4}{3}$m, p$\frac{5}{2}$t, p$\frac{5}{4}$t, p$\frac{5}{8}$t, p$\frac{3}{3}$m$\frac{3}{4}$t,......$p^{a}_{+}m^{a}t^{a}_{-}$,......Ly v[48]. H39[51].

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5. 5. t, M$\frac{4}{3}$T. P$\frac{5}{4}$T,......$p^{a}_{+}m^{a}_{-}t^{a}$, ...R[91]. S[231]. J[144].
5. 5. t_{+}, M$\frac{3}{4}$T, m$\frac{5}{6}$t. P$\frac{5}{4}$T,......$p^{a}_{+}m^{a}_{-}t^{a}$,..............................H35[20].
5. 5. t_{+}, M$\frac{3}{4}$T. P$\frac{5}{4}$T, p$\frac{5}{4}$m$\frac{3}{4}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$,..............................H35[21].
5. 5. t, M$\frac{3}{4}$T. P$\frac{4}{3}$M, P$\frac{5}{4}$T, p$\frac{3}{3}$m$\frac{3}{4}$t, ... $p^{a}_{+}m^{a}_{-}t^{a}$,...S[234]. L256[16]. Ly v[23]. H36[23].
5. 5. T_{-}, M$\frac{3}{4}$T. p$\frac{5}{2}$t, p$\frac{5}{4}$t, p$\frac{5}{8}$t, ... $p^{a}_{+}m^{a}t^{a}_{-}$,..................Ly v[26]. H37[24].
5. 5. T_{-}, M$\frac{3}{4}$T. P$\frac{4}{3}$M, p$\frac{5}{2}$t, p$\frac{5}{4}$t, p$\frac{5}{8}$t, ...$p^{a}_{+}m^{a}t^{a}_{-}$,............Ly v[25]. H37[27].
5. 5. T_{-}, M$\frac{3}{4}$T. P$\frac{4}{3}$M, P$\frac{5}{4}$T, p$\frac{5}{8}$t, p$\frac{3}{3}$m$\frac{3}{4}$t,......$p^{a}_{+}m^{a}t^{a}_{-}$,.........Ly v[28]. H37[28].

1. SULPHATE OF STRONTIAN. Strontspath. Strontiane sulfatée. Celestine.

Cleavage = p, m, T, P$\frac{10}{13}$M.

1. 5. P_{-}, t, m$\frac{4}{3}$t, ...$p^{a}_{-}m_{+}t^{a}$,.................... L263[3]. H43[63].
3. 3. P_{+}, M$\frac{4}{3}$T. P$\frac{9}{3}$M$\frac{4}{3}$T, ... $p^{a}_{+}m^{a}_{4}t^{a}_{5}$, ...Meudon, Paris ... Ly18[2].
3. 3. p_{+}, T, M$\frac{4}{3}$T. p$\frac{10}{13}$m, p$\frac{5}{2}$t,......$p^{a}_{+}m^{a}t^{a}_{-}$, La Catholica, Sicily...Ly18[3].
3. 5. P_{+}, t, M$\frac{4}{3}$T. P$\frac{5}{4}$T, p$\frac{3}{3}$m$\frac{4}{3}$t,......$p^{a}_{+}m^{a}_{-}t^{a}$, ... L263[8]. H54[80].
3. 5. p_{+}, m, T, M$\frac{4}{3}$T, m_{10}t. p$\frac{10}{13}$m, p$\frac{5}{2}$t, p$\frac{5}{4}$t, p$\frac{4}{3}$t, p$\frac{3}{3}$m$\frac{4}{3}$t....$p^{a}_{+}m^{a}_{-}t^{a}$, ...P193[2].
3. 5. p_{+}, T, M$\frac{4}{3}$T. P$\frac{10}{13}$M, P$\frac{5}{4}$T, p$\frac{9}{3}$m$\frac{4}{3}$t, p$\frac{3}{3}$m$\frac{4}{3}$t,...$p^{a}_{+}m^{a}_{-}t^{a}$, ... Sicily...Ly18[5].
4. 3. M$\frac{4}{3}$T. P$\frac{9}{3}$M$\frac{4}{3}$T, ...$p^{a}_{+}m^{a}_{4}t^{a}_{5}$,.................................H43[61].
5. 3. M$\frac{4}{3}$T. P$\frac{10}{13}$M, ...$p^{a}_{+}m^{a}_{4}t^{a}_{5}$,...............S[124]. L263[2]. H43[60].
5. 3. M$\frac{4}{3}$T. P$\frac{10}{13}$M, P$\frac{5}{4}$T,...............$p^{a}_{+}m^{a}_{4}t^{a}_{5}$, Ly v[1]. A[63]. S[125]. L263[4]. H44[94].
5. 3. M$\frac{4}{3}$T. p$\frac{5}{4}$t, P$\frac{9}{3}$M$\frac{4}{3}$T,............$p^{a}_{+}m^{a}_{4}t^{a}_{5}$,..............................H44[95].
5. 3. M$\frac{4}{3}$T. p$\frac{10}{13}$m, p$\frac{5}{4}$t, P$\frac{9}{3}$M$\frac{4}{3}$T,......$p^{a}_{+}m^{a}_{4}t^{a}_{5}$, ... Fassa, Tyrol......Ly18[4].
5. 5. M, M$\frac{4}{3}$T, m$\frac{2}{3}$t. P$\frac{10}{13}$T,.........$p^{a}_{+}m^{a}_{-}t^{a}$, ...Lake Erie......D201[2].
5. 5. T_{-}, M$\frac{4}{3}$T. p$\frac{10}{13}$m, p$\frac{5}{4}$t, P$\frac{5}{2}$T,......$p^{a}m_{+}t^{a}_{-}$,.........Bristol...Ly v[5]. H44[87].
5. 5. T_{-}, M$\frac{4}{3}$T. P$\frac{10}{13}$M, p$\frac{4}{3}$t, P$\frac{5}{2}$T, p$\frac{3}{3}$m$\frac{4}{3}$t,...$p^{a}m_{+}t^{a}_{-}$,.........Verona...Ly18[6].
5. 5. t, M$\frac{4}{3}$T. P$\frac{10}{13}$M, ...$p^{a}_{+}m^{a}_{-}t^{a}$, ...;.......S[123]. L263[1]. H43[83].
5. 5. t, M$\frac{4}{3}$T. P$\frac{10}{13}$M, p$\frac{5}{4}$t,...Etna...$p^{a}_{+}m^{a}_{-}t^{a}$,...D201[1]. L263[5]. Ly v[2]. H44[86].
5. 5. t, M$\frac{4}{3}$T. P$\frac{10}{13}$M, p$\frac{5}{4}$t, p$\frac{3}{3}$m$\frac{4}{3}$t,...$p^{a}_{+}m^{a}_{-}t^{a}$,...............S[126]. L263[6]. H44[88].

3. SULPHATE OF LEAD. Bleivitriol. Plomb sulfaté. Anglesite.

Cleavage = p$\frac{5}{4}$t.

3. 5. P_{+}, M, M$\frac{4}{3}$T. P$\frac{5}{4}$M, P$\frac{5}{4}$T, P$\frac{5}{4}$M$\frac{4}{3}$T,..........................L249[7]. H97[98].
3. 5. p_{+}, M, M$\frac{4}{3}$T. P$\frac{5}{4}$M, p$\frac{4}{3}$m, P$\frac{5}{4}$T, P$\frac{5}{4}$M$\frac{4}{3}$T, ... L249[8]. H97[99].
5. 3. M$\frac{4}{3}$T. P$\frac{5}{4}$T,......Anglesea...Model 82[b], with p^{a} and t^{a} reversed, ... L249[1]. H96[90].
5. 3. M$\frac{4}{3}$T. P$\frac{5}{4}$T, ... J[124]. S[21]. H96[91].
5. 5. M, M$\frac{4}{3}$T. P$\frac{5}{4}$T,..............Model 104. A[58]. J[125]. S[22]. L249[2]. H96[92].
5. 5. M, M$\frac{4}{3}$T. P$\frac{5}{4}$M, P$\frac{5}{4}$T,.................Anglesea......S[23]. L249[3]. H96[94].
5. 5. M, M$\frac{4}{3}$T. P$\frac{5}{4}$T, P$\frac{5}{4}$M$\frac{4}{3}$T, ... L249[4]. H96[93].
5. 5. M, M$\frac{4}{3}$T. P$\frac{5}{4}$M, P$\frac{5}{4}$T, P$\frac{5}{4}$M$\frac{4}{3}$T, ... S[24]. H96[96].
5. 5. T, M$\frac{4}{3}$T. P$\frac{5}{4}$T, P$\frac{5}{4}$M$\frac{1}{3}$T, p$\frac{6}{4}$m$\frac{4}{3}$t,.................................H96[97].

32. ANHYDRITE. Anhydrous Gypsum. Chaux sulfatée anhydre. Cube spar.

Cleavage = P_{12}, M_9, T_{10}, $p\frac{6}{5}t$.

1. 2. $P\frac{18}{10}$,$M\frac{9}{10}$,T,...Mont Blanc...Md. 5. M ii 63[1]. Ly14[1]. L267[1]. H32[22].

3. 2. $P\frac{18}{10}$,$M\frac{9}{10}$,T. $p\frac{6}{5}T$,...Bex...S[26]. M ii 63[2]. L267[2]. Ly v [1]. P176. H32[23].

3. 2. $P\frac{18}{10}$,$M\frac{9}{10}$,T. $p_x M_y T_z$, ...Ly14[2]. L267[3].

3. 2. $P\frac{18}{10}$,$M\frac{9}{10}$,T. $p_x M_y T_z$, $p_+m_yt_z$,..................................Ly14[3].

3. 2. $P\frac{18}{10}$,$M\frac{9}{10}$,T. $3p_xm_yt_z$, ... Steyermark...M ii [26]. S[27]. L267[4]. H32[24].

5. 5. $M\frac{9}{10}$,T, $m_x t$. $P\frac{6}{5}T$,..L267[5].

33. MURIATE OF COPPER. Atakamit. Salzsaures Kupfer.

Cleavage = T,$p\frac{7}{8}m$.

5. 3. $M\frac{8}{3}T$. $P\frac{3}{4}T$,...............Chili......J ii 344[1]. T i 621. L242[2]. Ly62[2].

5. 5. T,$M\frac{8}{3}T$. $P\frac{3}{4}T$, ...Chili ......J ii 344[2]. L242[1]. Ly62[3].

5. 5. T,$M\frac{8}{3}T$, $m\frac{4}{3}t$, $m\frac{8}{3}t$. $P\frac{3}{4}T$, $p'_xm_yt_z$, ... Chili ......Ly62[4].

5. 5. m,T,$M\frac{8}{3}T$, $m\frac{4}{3}t$. $P\frac{3}{4}T$, $p\_m_yt_z$, $p_xM_yT_z$, $p_+m_yt_z$, ... P326.

34. WAVELLITE. Alumine hydro-phosphatée. Devonite.

Cleavage = T, $M\frac{5}{9}T$.

5. 3. $M\frac{5}{9}T$.$P\frac{3}{4}M$,..J i 389[1].

5. 5. T,$M\frac{5}{9}T$. $P\frac{3}{4}M$,....................................J i 389[2]. L134[1]. D188.

5. 5. t,$M\frac{5}{9}T$, $m\frac{2}{5}t$, $m\frac{5}{12}t$. $P\frac{3}{4}M$, ...L134[2]. P157[2].

5. 5. t, $M\frac{5}{9}T$, $m\frac{2}{5}t$. $P\frac{3}{4}M$, ... Ly24[2].

35. *An Isomorphous Group of Copper Ores*, 1, 2:—

1. OLIVENITE. Right Prismatic Arseniate of Copper. Olivenerz. Cuivre arseniaté octaèdre aigu.

Cleavage = $P\frac{8}{3}T$.

5. 3. $M\frac{9}{10}T$.$P\frac{8}{3}T$, ...$p\_m^a t\frac{1}{+}$,.........Sim. Model 82[a]. H102[150]. iii 510.

5. 3. $M\frac{9}{10}T$. $p\frac{8}{3}T$, ...$p\frac{1}{+}m\_t^a$, ...Sim. Md. 82. Ly65[2]. H102[151]. iii 510.

5. 5. $M\_$,t, $M\frac{9}{10}T$. p_xm, $P\frac{8}{3}T$,.........................Cornwall......P332[2].

2. LIBETHENITE. Cuivre phosphaté. Oktaedrisches phosphorsaures Kupfer.

Cleavage = $P\frac{8}{3}M$.

3. 3. p,$M\frac{9}{10}T$. $P\frac{8}{3}T$, ...L143[4].

5. 3. $M\frac{9}{10}T$. $P\frac{8}{3}T$, ... Ly62[2]. M ii [2]. L143[1].

5. 3. $M\frac{9}{10}T$. $P\frac{8}{3}T$, $p_xm_yt_z$,..................................M ii[5]. D24[2]. L143[2].

5. 5. t, $M\frac{9}{10}T$. $P\frac{8}{3}T$, $p_xm_yt_z$, ...Libethen. Cornwall......Ly62[3]. P327[2].

5. 5. m, $M\frac{9}{10}T$. $P\frac{8}{3}T$, $p_xm_yt_z$,..L143[3].

36. EUCHROITE. Cleavage = $M\frac{3}{5}T$.

3. 3. P,$M\frac{3}{5}T$, $m\frac{6}{5}t$. $P\frac{3}{2}\frac{3}{4}T$, ... Libethen......L174[2]. M iii 95[1].

3. 5. P,t,$M\frac{3}{5}T$, $m\frac{9}{10}t$, $M\frac{6}{5}T$.$P\frac{3}{2}\frac{3}{4}T$, ...Libethen...S[174]. L174[1]. M ii[192]. P333.

37. HAIDINGERITE. Cleavage = T.

5. 5. M,T,$M\frac{5}{6}T$. P_+M, $P\frac{1}{2}T$, ...D190. P181.

38. SILICEOUS OXIDE OF ZINC. Kieselzinkerz. Electric Calamine. Zinc oxidé silicifère. Galmei.

Cleavage = $M\frac{4}{3}T$.

3. 5. p, T_{-}, $M\frac{4}{3}T$. $P_x M$, $p_x m$, $3p_x t$, ...Aix-la-Chapelle......Ly73[3].

5. 5. $T_{-}, M\frac{4}{3}T$. $P\frac{4}{7}M$, ... Siberia......Ly73[1]. H113[m].

5. 5. $T_{-}, M\frac{4}{3}T$. $P\frac{1}{2}T$, ... J ii 438[8]. H113[n].

5. 5. $T_{-}, M\frac{4}{3}T$. $P\frac{4}{7}M$, $P\frac{1}{2}T$, ...S[109]. P374.

5. 5. $T_{-}, M\frac{4}{3}T$. $P_x M$, $2p_x t$,.....................Bleiberg, Carinthia......Ly73[2].

5. 5. $T_{-}, M\frac{4}{3}T$. $2p_x m$, $p_x t$, ...Carinthia......Ly73[4].

5. 5. $T_{-}, M\frac{4}{3}T$. $P_x M$, $p_x m$, $P_x T$, $p_x t$,...........Matlock. Bleiberg......Ly73[5].

39. PICROSMINE. Pikrosmin. Silicate of Magnesia.

Cleavage = $M, T, m\frac{1}{2}t$. $p\frac{5}{3}m$.

5. 5. $M, T, M\frac{1}{2}T$. $P\frac{5}{3}M$, ...T i 172. P94.

40. MASCAGNINE. Ammoniaque sulfatée.

5. 5. $T_{-}, M_x T$. $p_x m$, $p_x t$?

41. BROCHANTITE. Cleavage = $p\frac{5}{8}t$.

5. 5. $M, t, M\frac{5}{3}T$. $P\frac{4}{15}M$, $P\frac{5}{8}T$,..P324.

5. 5. $M, M\frac{5}{3}T$. $P\frac{4}{15}M$, $p\frac{5}{8}t$, ...L724. D245.

42. *An Isomorphous Group of Sulphates*, 1, 2:—

1. SULPHATE OF MAGNESIA. Bittersalz. Magnesie sulfatée.

Cleavage = $m\frac{99}{100}t$.

[The following symbols relate to artificial crystals.]

4. 3. $M\frac{99}{100}T$. $P\frac{50}{100}M\frac{99}{100}T$, ...H45[97]

4. 5. $M, T, M\frac{99}{100}T$. $P\frac{50}{100}M\frac{99}{100}T$, ...H45[98].

4. 5. $M, T, m\frac{99}{100}t$. $m\frac{1}{2}t$, $m\frac{2}{1}t$. $P\frac{50}{100}M\frac{99}{100}T$, ...H45[99].

4. 5. $m, t, M\frac{99}{100}T$. $p\frac{1}{2}m$, $p\frac{1}{2}t$, $P\frac{50}{100}M\frac{99}{100}T$,.........................H45[100].

2. SULPHATE OF ZINC. Zincvitriol. Zinc sulfaté. White vitriol.

Cleavage = T, $m\frac{99}{100}t$.

4. 5. $T, M\frac{99}{100}T$. $P\frac{50}{100}M\frac{99}{100}T$,........................D179. L110[2]. P376.

4. 3. $M\frac{99}{100}T$. $P\frac{50}{100}M\frac{99}{100}T$, ...Ti547. J iii 21[1]. L110[1].

43. NEEDLE ORE. Nadelerz. Acicular Bismuth-Glance.

5. 3. $M\frac{1}{2}T$. $p_x m$, $p_x t$?

5. 5. $t, M\frac{1}{2}T$. $p_x m$, $p_x t$?

44. BOURNONITE. Triple Sulphuret. Endellione.

Cleavage = $M, T, m\frac{15}{16}t$.

1. 5. P_{-}, $M\frac{15}{16}$, T, $m\frac{15}{16}t$,..L613[2]. P353[1].

1. 5. $(P_x, M\frac{15}{16}, T, m\frac{15}{16}t) \times 2$, ...P353[2].

3. 2. $P_{-}, M\frac{15}{16}, T$. $p\frac{25}{24}m$, $p\frac{24}{25}t$,......................Kapnik......L613[4]. Ly51[2].

3. 2. P_{-}, $M\frac{15}{16}$, T. $p\frac{25}{24}m$, $p\frac{24}{25}t$, ... Ly51[1].

3. 2. P, $M\frac{15}{16}$, T. $p\frac{25}{24}m$, ...L613[3]. P353[3].

3. 2. P$_-$, M$\frac{15}{16}$, T. p$\frac{25}{24}$m, p$\frac{24}{23}$t, p$_x$m$\frac{15}{16}$t,.........Kapnik......L613[6]. Ly51[7].

3. 2. (P , M$\frac{15}{16}$,T. p$\frac{25}{24}$m, p$\frac{24}{23}$t, p$_x$m$\frac{15}{16}$t) × 2,........................Ly51[8].

3. 5. P$_-$, M$\frac{15}{16}$,T, M$\frac{15}{16}$T. p$\frac{25}{24}$m,p$\frac{24}{23}$t,............................L613[5]. Ly51[6].

3. 5. P$_-$, M$\frac{15}{16}$,T, M$\frac{15}{16}$T. p$\frac{25}{24}$m, p$\frac{24}{23}$t, p$_x$m$\frac{15}{16}$t, p$_+$m$\frac{15}{16}$t, ... L613[9]. Ly51[9].

3. 5. P$_-$, M$\frac{15}{16}$,T, m$\frac{15}{16}$t, m$_-$t. p$\frac{25}{24}$m, p$\frac{24}{23}$t, p m$\frac{15}{16}$t, p$_+$m$\frac{15}{16}$t,.........L51[10].

3. 5. P$_-$,M$\frac{15}{16}$,T, m$\frac{15}{16}$t, m$_-$t, M$\overline{-}$T. P$\frac{25}{24}$M, p$\frac{24}{25}$t,p$_x$m$\frac{15}{16}$t, p$_+$m$\frac{15}{16}$t,...Ly51[11].

3. 5. P$_-$, M$\frac{15}{16}$,T, m$\frac{15}{16}$t, m$_-$t, M$\overline{-}$T. p$\frac{25}{24}$m, p$_-$m, p$\frac{24}{23}$t, p$_x$m$\frac{15}{16}$t, p$_+$m$\frac{15}{16}$t,
Ly51[12].

3. 5. P$_-$, M$\frac{15}{16}$, T, M$\frac{15}{16}$T, m$_-$t, m$\overline{-}$t, m$_+$t. p$_+$m, P$\frac{25}{24}$M, 3p$_-$m, P$\frac{24}{23}$T, p$_-$t,
8 p$_x$m$_y$t$_z$,......(*Imaginary Combination*)......P353[5].

5. 2. P$_-$. P$\frac{25}{24}$M, P$\frac{24}{23}$T,..............................Kapnik..............Ly51[4].

5. 5. P$_-$. P$\frac{25}{24}$M, P$\frac{24}{25}$T, P$_x$M$\frac{15}{16}$T, ...Hartz ...... L613[8]. Ly51[5].

45. Topaz. Topas. Topaze.

Cleavage = P,m$\frac{19}{36}$t, m$\frac{19}{18}$t. P$\frac{29}{28}$M, P$\frac{28}{29}$T.

3. 3 P$_+$, M$\frac{19}{36}$T, M$\frac{19}{18}$T. p$\frac{28}{29}$t, p$\frac{33}{36}$m$\frac{19}{36}$t, p$\frac{22}{36}$m$\frac{19}{36}$t, ... S[237]. H50[140].

3. 3. P$_+$, M$\frac{19}{36}$T, M$\frac{19}{18}$T. P$\frac{28}{29}$T, p$\frac{33}{36}$m$\frac{19}{36}$t,..........................J[51]. H50[138].

3. 3. P$_+$, M$\frac{19}{36}$T, m$\frac{19}{18}$t, m$\frac{19}{12}$t. P$\frac{28}{29}$T, p$\frac{33}{36}$m$\frac{19}{36}$t, ... P77[3]. H50[142].

3. 3. P$_+$, M$\frac{19}{36}$T, M$\frac{19}{18}$T. P$\frac{28}{29}$T, p$\frac{56}{29}$t, p$\frac{33}{36}$m$\frac{19}{36}$t, p$\frac{22}{36}$m$\frac{19}{36}$t, ...H50[145].

3. 3. P$_+$,M$\frac{19}{36}$T,M$\frac{19}{18}$T.P$\frac{28}{29}$T,p$\frac{33}{36}$m$\frac{19}{36}$t,p$\frac{22}{36}$m$\frac{19}{36}$t,p$\frac{11}{18}$m$\frac{19}{18}$t, L399[11].H50[146].

3. 3. P$_+$,M$\frac{19}{36}$T,M$\frac{19}{18}$T.P$\frac{28}{29}$T,p$\frac{56}{29}$t, p$\frac{33}{36}$m$\frac{19}{36}$t, p$\frac{22}{36}$m$\frac{19}{36}$t, p$\frac{11}{18}$m$\frac{19}{18}$t, H50[149].

3. 3. P$_+$, M$\frac{19}{24}$T, M$\frac{19}{12}$T. P$\frac{42}{29}$T, p$\frac{33}{36}$m$\frac{19}{36}$t, p$\frac{11}{18}$m$\frac{19}{18}$t,.................H50[144].

3. 3. P$_+$, M$\frac{19}{36}$T, M$\frac{19}{18}$T, m$\frac{19}{12}$t. P$\frac{28}{29}$T, P$\frac{56}{29}$T, p$\frac{33}{36}$m$\frac{19}{36}$t, p$\frac{22}{36}$m$\frac{19}{36}$t, p$\frac{11}{18}$m$\frac{19}{18}$t,
S[441]. H50[150]. Mr6[87].

3. 5. P$_+$,T, M$\frac{19}{36}$T. P$\frac{29}{28}$M, p$\frac{66}{36}$m$\frac{19}{36}$t,.........................L398[7]. H50[137].

3. 5. P$_+$,T, M$\frac{19}{36}$T, M$\frac{19}{18}$T. P$\frac{28}{29}$T, p$\frac{66}{36}$m$\frac{19}{36}$t, ... H50[139].

3. 5. P$_+$,M,T, m$\frac{19}{36}$t, m$\frac{19}{18}$t. P$\frac{28}{29}$T,....................................H50[141].

4. 3. M$\frac{19}{36}$T, M$\frac{19}{18}$T. P$\frac{28}{29}$T, P$\frac{33}{36}$M$\frac{19}{36}$T, p$\frac{11}{18}$m$\frac{19}{18}$t,........................H49[134].

4. 3. M$\frac{19}{36}$T, M$\frac{19}{18}$T. P$\frac{33}{36}$M$\frac{19}{36}$T,.........................J[50]. L698[3]. H49[135].

4. 3. M$\frac{19}{36}$T, M$\frac{19}{18}$T. P$\frac{33}{36}$M$\frac{19}{36}$T, ...A[130]. R[87].

4. 3. M$\frac{19}{36}$T, M$\frac{19}{18}$T. P$\frac{28}{29}$T, p$\frac{33}{36}$m$\frac{19}{36}$t, p$\frac{22}{36}$m$\frac{19}{36}$t, p$\frac{11}{18}$m$\frac{19}{18}$t, ...H50[143].

4. 3. M$\frac{19}{36}$T,M$\frac{19}{18}$T.p$\frac{29}{28}$m, P$\frac{28}{29}$T, p$\frac{22}{36}$m$\frac{19}{36}$t,p$\frac{33}{36}$m$\frac{19}{36}$t,p$\frac{66}{36}$m$\frac{19}{36}$t, J[52]. H51[147].

4. 3. M$\frac{19}{36}$T, M$\frac{19}{18}$T. p$\frac{28}{29}$t, P$\frac{33}{36}$M$\frac{19}{36}$T, p$\frac{11}{18}$m$\frac{19}{18}$t,........................M ii [36].

5. 3. M$\frac{19}{36}$T, M$\frac{19}{18}$T. P$\frac{28}{29}$T, p$\frac{33}{36}$m$\frac{19}{36}$t, ... Md. 90. J[53]. S[439]. L398[9]. H49[136].

5. 3. M$\frac{19}{36}$T, M$\frac{19}{18}$T. p$\frac{29}{28}$m, P$\frac{28}{29}$T, p$\frac{33}{36}$m$\frac{19}{36}$t,.........................H49[133].

5. 5. M$_-$, M$\frac{19}{12}$T. p$\frac{14}{29}$t, P$\frac{28}{29}$T, ... H49[132].

5. 5. T, M$\frac{19}{36}$T, M$\frac{19}{18}$T. p$\frac{28}{29}$m, P$\frac{28}{29}$T, p$\frac{33}{36}$m$\frac{19}{36}$t, p$\frac{66}{36}$m$\frac{19}{36}$t,...........H51[148].

The Atlas to Levy's Catalogue contains 82 figures of Topaz Crystals, all different from the above, but most of them apparently consisting of combinations of the same forms.

46. Amblygonite. Cleavage = M$\frac{3}{4}$T.

1. 3. P,M$\frac{3}{4}$T, ... P158. L283.

47. CHIASTOLITE. Chiastolith. Macle. Hohlspath.

Cleavage = p,m,t.

1. 3. $P,M\frac{97}{100}T$, ... J[17]. H64[6].

48. CHRYSOBERYL. Cymophane. Prismatic Corundum.

Cleavage = M,T.

4. 5. $M,T,M\frac{7}{10}T.\ P_xM_yT_z$, ... L540[7]. H60[32].
4. 5. $M,T_-,m_-t.\ P_xM_yT_z$, ... Ly27[3].
5. 2. $M,T.\ P\frac{4}{7}M$, ... Sim. Model 8. Haddam...L540[1]. J[28]. S[132]. H60[31].
5. 2. $M,T_-.\ P_xM,\ P_xM_yT_z,\ p_+m_yt_z$, ... Ly28[5].
5. 5. $M,T,M\frac{7}{10}T.\ P\frac{4}{7}M,\ P_xM_yT_z$, ... L540[3]. J[29]. S[133]. H60[33].
5. 5. $M,T,M\frac{7}{10}T,M\frac{17}{16}T.\ P\frac{4}{7}M,\ P_xM_yT_z$, ... H60[34].
5. 5. $M,T,M\frac{7}{10}T.\ p\frac{4}{7}m,\ P_xM_yT_z,\ p_xm_yt_z$, ...L540[5]. H60[35].
5. 5. $M,T,M\frac{7}{10}T.\ P\frac{4}{7}M$, ... L540[2].
5. 5. $M,T,m\frac{8}{17}t,m\frac{16}{17}t,M\frac{24}{17}T,m\frac{36}{17}t.\ p\frac{4}{7}t,\ 2P_xM_yT_z$, ... P80[2].
5. 5. $M,T_-,m_+t.\ P_xM,\ P_xM_yT_z,\ p_+m_yt_z$, ... Ly28[6].
5. 5. $M,T_-,2m_xt.\ p_xm,\ P_xT,\ p_xm_yt_z$, ... Ly28[9].
5. 5. $M,T_-,m_-t.\ P_xT$, ... Ly27[2].
5. 5. $m,T_-,m_-t.\ p_xm,\ P_xM_yT_z$, ... Ly27[4].
5. 5. $M,T\ ,m_-t.\ P_xM,\ P_xM_yT_z,\ p_+m_yt_z$, ... Ly28[7,8].

49. LIEVRITE. Ilvaite. Fer calcaréo-siliceux.

Cleavage = $t,m\frac{8}{9}t.\ p\frac{8}{9}m$.

3. 3. $P_+,M\frac{8}{9}T,m\frac{8}{9}t.\ p\frac{8}{9}m,\ p\frac{6}{9}t,\ p\frac{4}{6}m\frac{8}{6}t$, ...$p_+^am_-^at^a$,......L529[6]. H110[220].
3. 3. $p,M\frac{8}{9}T,M\frac{4}{9}T,M\frac{8}{8}T.\ P\frac{6}{9}M,\ p\frac{6}{9}t,\ P\frac{4}{9}M\frac{6}{9}T$, ... Ly69[7].
4. 3. $M\frac{8}{9}T.\ P\frac{4}{9}M\frac{6}{9}T$, ... $p_+^am_6^at_9^a$, ...J iii 539[1]. M ii 415[1]. L528[2]. H110[233].
5. 3. $M\frac{8}{9}T.\ P\frac{6}{9}M$, ...$p_4^am_6^at_9^a$, ...Elba......H110[234].
5. 3. $M\frac{6}{9}T.\ P\frac{8}{9}M$, ... $p_+^am_6^at_9^a$, ... M ii 415[2]. L528[1]. H110[237].
5. 3. $M\frac{8}{9}T.\ P\frac{6}{9}T$, ... $p_+^am_8^at_9^a$, ... L528[3]. H110[239].
5. 3. $M\frac{8}{9}T.\ P\frac{6}{9}T,\ p\frac{4}{6}m\frac{8}{6}t$, $p_+^am_8^at_9^a$, ... L528[4]. H110[240].
5. 3. $M\frac{8}{9}T,m\frac{8}{6}t.\ p\frac{8}{9}m,\ P\frac{6}{9}T,\ p\frac{4}{6}m\frac{8}{6}t$,......$p_+^am_-^at^a$,............L529[5]. H110[241].
5. 3. $M\frac{8}{9}T.\ p\frac{6}{9}t,\ P\frac{4}{6}M\frac{8}{6}T$, ...$p_+^am_8^at_9^a$,......S[489]. D379. M ii [4]. Ly69[3]. Rose [88].
5. 3. $M\frac{8}{9}T,m\frac{8}{8}t.\ P\frac{4}{9}M\frac{6}{9}T$,...$p_+^am^at_-^a$, ... Ly69[2].
5. 3. $M\frac{6}{9}T,m\frac{8}{8}t.\ p\frac{6}{9}m,\ p\frac{4}{9}m\frac{6}{9}t$, ... Ly69[4].
5. 3. $M\frac{6}{9}T,M\frac{4}{9}T,M\frac{8}{8}T.\ P\frac{6}{9}M,P\frac{4}{9}M\frac{6}{9}T$,.. Ly69[5].
5. 3. $M\frac{6}{9}T,M\frac{4}{9}T,M\frac{8}{8}T.\ P\frac{6}{9}M,\ p\frac{6}{9}t,\ P\frac{4}{9}M\frac{6}{9}T$,.. Ly69[6].

50. ALLANITE. Prismatic Cerium Ore.

Cleavage = $m,\frac{1}{2}m_zt$.

5. 5. $M,\frac{1}{2}M_zT,\frac{1}{2}m\frac{5}{4}t.\ \frac{1}{2}p_-m,\ 4(\frac{1}{4}p_xm_yt_z)$.........P264. D365. A[128]. M ii [165].

51. DICHROITE. Iolite. Cordiérite. Peliom.

Cleavage = $m,T,m_zt\frac{15}{13},M\frac{15}{13}T_z$.

1. 5. $P_-,T,M\frac{15}{13}T_z$, ... Model 7. L467[1]. H76[188].
1. 5. $P_-,m,T,m_zt\frac{15}{13},M\frac{15}{13}T_z$,........................Model 10. L467[2]. H76[189].
3. 5. $P_-,m,T,m_zt\frac{15}{13},M\frac{15}{13}T_z.\ p\frac{11}{10}t,p\frac{1}{10}m\frac{15}{13}t_z$, L467[3]. P42. H76[194]. Ly38[2].
3. 5. $P_-,m,T,m_zt\frac{15}{13},M\frac{15}{13}T_z.\ 3(p_xt,\ p_xm\frac{15}{13}t_z)$ .... Ly38[3].

52. SPODUMEN. Triphane. Cleavage = T,M$\frac{1}{1}\frac{2}{6}$T.
• **1.** 5. P,T, M$\frac{1}{1}\frac{2}{6}$T ?

53. SCORODITE, Skorodit. Cleavage = m,t, m$\frac{1}{2}\frac{7}{0}$t.
4. 2. M,T, P_xM$\frac{1}{2}\frac{7}{0}$T, ...Cornwall............Ly71[2].
4. 5. M,T, m$\frac{1}{2}\frac{7}{0}$t. P_xM$\frac{1}{2}\frac{7}{0}$T, ...Peru. Saxony......Ly71[3].
4. 5. M,T, m$\frac{1}{2}\frac{7}{0}$t. p_+m, P_xM$\frac{1}{2}\frac{7}{0}$T, ...Saxony ....Ly71[4].
4. 5. M,T, m$\frac{1}{2}\frac{7}{0}$t, m$\frac{2}{1}\frac{0}{7}$t. P_xM$\frac{1}{2}\frac{7}{0}$T, P_M$\frac{1}{2}\frac{7}{0}$T, p_+mt_, ...Brazil...Ly71[5].

54. PREHNITE. Cleavage = P,m$\frac{5}{6}$t.
1. 3. P,M$\frac{5}{6}$T, ... Ratschinges. Tyrol...L471[1]. M ii 218[1]. Ly v 1. H75[187].
1. 3. P$\sqsubset$,M$\frac{5}{6}$T,...................(Koupholite)......J[70]. A[88]. P23[1]. H ii 606[4].
1. 5. P$\sqsubset$,T, M$\frac{5}{6}$T, ... Dumbarton......L471[2]. J[72]. Ly v 2. H75[189].
1. 5. P$\sqsubset$,m,T, m$\frac{5}{6}$t, ...L471[3]. J[71]. M ii 218[2]. H75[191].
3. 5. P,M$\frac{5}{6}$T. P_xM, ... Fahlun ......L471[5].
3. 5. P_,M$\frac{5}{6}$T. P$\frac{1}{2}\frac{1}{4}$T, ... Fahlun......L471[4]. H75[188].
3. 5. P_+,M$\frac{5}{6}$T. P$\frac{\psi}{3}$T, ... Fahlun......L471[4]. H75[190].
3. 5. P,M$\frac{5}{6}$T. P_xM, P_xT,.............................. Fahlun ......L471[6].
3. 5. P,T, M$\frac{5}{6}$T, P$\frac{\psi}{3}$T,...................................S[339]. M ii [13]. D286.
3. 5. P,m,T, M$\frac{5}{6}$T. p_+m, p_xm, p_xm,t_x,........... Ratschinges ......Ly37[2].
5. 5. m,t,M$\frac{5}{6}$T. P$\frac{1}{4}\frac{1}{3}$M, 2p_+m, p_+t, ...S[340]. P23[2].

55. PYROPHYLLITE. Cleavage = P,M_,T?

56. HARMOTOME. Cross Stone. Kreuzstein. Two varieties:—
1. POTASH HARMOTOME. Kalikreuzstein.
2. BARYTES HARMOTOME. Barytkreuzstein.

Cleavage = M,T. p$\frac{7}{10}$m.
3. 2. P_+,M_,T. P$\frac{7}{10}$M,p_xm,t_x,...M ii [22].
4. 2. M_,T. $P_x$$M_y$$T_x$, ... Oberstein......S[345]. P44[6]. H83[271]. Ly43[2].
4. 2. (M_,T. $P_x$$M_y$$T_x$) × 2, ... Andreasberg...J[84]. P44[5]. H83[272]. Ly43[3].
4. 5. m_,t. $P_x$$M_y$$T_x$, ...P44[7].
5. 2. M_,T. P$\frac{7}{10}$M, ...P44[2].
5. 2. M_,T. p$\frac{7}{10}$m, $P_x$$M_y$$T_x$, ...P44[4].
5. 2. M_,T. P$\frac{7}{10}$M, p_xm,t_x, ...Oberstein......P44[3]. Ly43[4].
5. 2. (M_,T. P$\frac{7}{10}$M, p_xm,t_x) × 2, ... Andreasberg......Ly43[5].
5. 2. (M_,T. P$\frac{7}{10}$M, p$\frac{\psi}{7}$m, p_xm,t_x) × 2, ... Strontian...M ii [40]. Ly43[6].
5. 5. M_,T. P$\frac{7}{10}$M, $P_x$$M_y$$T_x$, ... Strontian......H iii 146[2]. Ly43[7].

57. THOMSONITE. Cleavage = M,T.
3. 5. P_+,m, M$\frac{1}{1}\frac{0}{0}\frac{0}{1}$T. PM, ... Ly45[2].
3. 5. P_+,m,T, M$\frac{1}{1}\frac{0}{0}\frac{0}{1}$T. PM, ... Ly45[2].
3. 5. P_+,M,T, M$\frac{1}{1}\frac{0}{0}\frac{0}{1}$T. pm, ... Brooke, Ann. Phil. xvi. 104. T i 314.
3. 5. P_+,M,T, M$\frac{1}{1}\frac{0}{0}\frac{0}{1}$T, m_t, m_+t. pm, pt, pmt, ... L208. P125.

58. DESMINE. Stilbite. Radiated Zeolite. C Si^3+3 A Si^3+6 Aq.
Cleavage = m,T.
1. 2. P$\frac{2}{3}$, M$\frac{3}{5}$,T,..H84[278].

3. 2. p_{+},M,T. $P\frac{4}{7}M\frac{6}{7}T$,......Sim. Model 43. D[368]. Ly v 3. H84[280]. M ii [10].
3. 5. P_{+},M,T, $m\frac{19}{20}t$. $P\frac{4}{7}M\frac{6}{7}T$,..............................P24[3].
4. 2. M,T. $P\frac{4}{7}M\frac{6}{7}T$, ... Ly v 1. H84[279]. M ii 239[1]. A[96]. Rose [96].
4. 5. M, T, $m\frac{13}{14}t$. $P\frac{4}{7}M\frac{6}{7}T$, ...Ly43[2].

59. Epistilbite. Cleavage = T.
5. 3. $M\frac{7}{17}T$. P_xM, P_xT, $p_xm_yt_z$, ... Ly44[2].
5. 5. t_{+}, $M\frac{7}{17}T$. $P\frac{12}{17}M$, $P\frac{7}{24}T$, ...P129.

60. Polyhallite. Polyhalit.
1. 5. P,T_{-}, $M\frac{7}{11}T$, ...P204.

61. Caledonite. Cupreous Sulphato-carbonate of Lead.
Cleavage = T, $m\frac{11}{12}t$.
3. 5. P_{-},T, $M\frac{11}{12}T$. P_3M, $p_{-}m$, $p\frac{3}{4}t$, $3p_xm\frac{11}{12}t$,.........Lead Hills...P360[2]. S[123].

62. White Tellurium. Weisstellurerz. Cleavage = $m\frac{10}{13}t$.
3. 5. P_{-},m,t, $m\frac{10}{13}t$. P_3M, $P\frac{3}{4}T$, $p_xm_yt_z$, ...L688. P342.

63. Schilfglaserz. Sulphuret of Silver and Antimony.
5. 3. $M\frac{5}{6}t$, $3m_{-}t$. p_xm, $3P_xT$, ...Cleavage = $m\frac{5}{6}t$................ P299.

64. Fluellite.
5. 3. p_{+}. $P_{+}M_{-}T$,..............................M iii 101. P76.

65. Polymignite.
4. 5. M,T_{+}, $3m_xt$. $P_{-}MT_{+}$. $p_{+}^{a}m_{-}^{a}t^{a}$, ...D369. P262.

66. Brookite. Cleavage = t.
4. 5. M_{-}, $M\frac{5}{6}T$. $p_{+}t$, $P_xM_yT_z$, ... P256.
5. 5. M_{-}, t, $M\frac{5}{6}T$. $2p_xm$, $p_{+}t$, $3p_xm_yt_z$,..............................L725. M ii [190].

67. Lenticular Copper Ore. Linsenerz. Cuivre arséniaté octaédral.
Cleavage = $m\frac{7}{12}t$. $p\frac{11}{15}m$.
5. 3. $M\frac{7}{12}T$. $P\frac{11}{15}M$, ... Ly65[2]. P329[1].
5. 3. $M\frac{7}{12}T$. $p\frac{11}{15}m$, $2p_xm_yt_z$, ... P329[2].

68. Lazulite. Azurite. Cleavage = p.
5. 5. m, $M\frac{28}{29}T$. $p\frac{14}{23}m$, $P\frac{7}{4}T$, $p\frac{4}{7}t$, $P\frac{7}{4}M\frac{28}{29}T$, $p\frac{4}{7}m\frac{28}{29}t$, ... P159[2]. Ly38[2].

69. Childrenite.
4. 5. M, $M\frac{19}{20}T$. $P_{-}MT_{+}$, $p_{+}m_{-}t$,..............................Ly80[2].
5. 3. P. $p\frac{12}{23}m$, $p_{-}t$, $P\frac{3}{2}M\frac{19}{20}T$, ...P158. D188.

70. Forsterite.
3. 5. P,T,$m\frac{1}{2}t$. $P_2M\frac{1}{2}T$,..............................A[128]. P87.

71. SILLIMANITE.
1. 3. $P,M\frac{7}{8}T$, ... D320.
1. 3. $P,M\frac{3}{4}T$, ... P73.

72. MENGITE. Mengit. Monazite?
1. 3. P,M_T? .... R175.

73. KOENIGITE. Königine. Cleavage = P.
1. 3. $P,M\frac{10}{13}T$, ... Levy, Ann. Phil. xxvii. 194[1].
1. 5. $P,M,M\frac{10}{13}T$, ... idem. fig.[2].
3. 5. $P,M,M\frac{10}{13}T.\ P_{+}T$, ... idem. fig.[3].

74. MONTICELLITE.
4. 5. $T,\ M\_T.\ P_xT,\ P_xM_yT_z$, ... P403.

75. HERDERITE. Cleavage = $m\frac{9}{16}t$.
3. 5. $P\_,\ M\frac{4}{16}T.\ P\frac{5}{8}T,\ p\frac{13}{16}m\frac{9}{16}t$, ... P172.

76. HOPEITE. Cleavage = M.
3. 5. $P_{+},\ M,\ T\_,\ M\frac{1}{2}\frac{9}{2}T.\ p\frac{9}{1}t,\ p\frac{9}{1}m\_t$, ... P377.

CLASS V.—MINERALS BELONGING TO THE OBLIQUE PRISMATIC SYSTEM OF CRYSTALLISATION.

The AXES of all Combinations belonging to this Class are $= p^a_x m^a_y t^a_z$.

The constituent FORMS of the Combinations of this Class are as follow:—

Zones.	Homohedral Forms.	Hemihedral Forms.
Prismatic,	$M,\ M\_T,\ M_{+}T,\ T$,	$\frac{1}{2}M\_T,\ \frac{1}{2}M_{+}T$.
North,	$P,\ P\_M,\ P_{+}M,\ M$,	$\frac{1}{2}P\_M,\ \frac{1}{2}P_{+}M$.
East,	$P,\ P\_T,\ P_{+}T,\ T$,	$\frac{1}{2}P\_T,\ \frac{1}{2}P_{+}T$.
Octahedral,	$P_xM_yT_z$,	$\frac{1}{2}P_xM_yT_z$.

The homohedral Forms of the prismatic zone, $M, M\_T, M_{+}T, T$, or some of them, occur upon almost every Combination in this Class.

The homohedral Form P occurs very seldom.

The hemihedral prismatic Forms $\frac{1}{2}M\_T, \frac{1}{2}M_{+}T$, occur more frequently than would appear from the following list, for many Forms indicated as homohedral, consist of two pair of planes of unequal size.

The homohedral Form $P_xM_yT_z$ occurs seldom, perhaps not at all. It is a Form that properly belongs to the Prismatic System, so that the examples of $P_xM_yT_z$ given in the following Table, should probably be rendered $\frac{1}{2}P_xM_yT_z$, $\frac{1}{2}p_xm_yt_z$.

The homohedral Forms $P\_M, P_{+}M, P\_T, P_{+}T$, are also of rare, perhaps of doubtful, occurrence.

The greater part of the Combinations of this Class fall into two divisions:—

a.) Those which have M_xT, $\frac{1}{2}P_xM$ Zn and $\frac{1}{2}P_xM_yT_z$ Zne Znw.

b.) Those which have M_xT, $\frac{1}{2}P_xT$ Zw and $\frac{1}{2}P_xM_yT_z$ Znw Zsw.

These Combinations are commonly called *oblique prisms*, and it is said of those of the first division, that the *terminal plane*, namely, the Form $\frac{1}{2}P_xM$ Zn, is set on the *obtuse lateral edge of the prism;* and of the combinations of the second division it is said, that the *terminal plane*, namely, the Form $\frac{1}{2}P_xT$ Zw, is set on the *acute lateral edge of the prism.*

The hemihedral Forms $\frac{1}{2}M_-T$, $\frac{1}{2}M_+T$, $\frac{1}{2}P_-M$, $\frac{1}{2}P_+M$, $\frac{1}{2}P_-T$, $\frac{1}{2}P_+T$, always consist of a pair of parallel planes.—The hemihedral Form $\frac{1}{2}P_xM_yT_z$, consists of two pair of parallel planes. Hence, half the planes of any given hemihedral Form are placed on different sides of any given meridian.

1. REALGAR. Arsenic sulfuré rouge. Ruby Sulphur.
Cleavage = m,t, $M\frac{2}{3}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn.
5. 5. m,T, $M\frac{2}{3}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P_+M_-T$ Zn^2w Zn^2e, ... $Ly74^2$.
5. 5. M,T, $M\frac{2}{3}T$, M_+T, $M\ddagger T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p_+m$ Zs, $3(\frac{1}{2}p_xm_+t_-)$ Znw^2 Zne^2, $\frac{1}{2}P_+M_-T$ Zn^2w Zn^2e, ... $Ly74^6$.
5. 5. M,T, $M\frac{2}{3}T$, M_+T, $M\ddagger T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $2(\frac{1}{2}p_+m)$ Zs, $3(\frac{1}{2}p_xm_+t_-)$ Znw^2 Zne^2, $\frac{1}{2}P_+M_-T$ Zn^2w Zn^2e, ... $Ly74^7$.
5. 5. t, $M\frac{2}{3}T$, m_+t, $m\ddagger t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p_xm_+t_-$ Znw^2 Zne^2, ... $Ly74^3$
5. 5. T, $M\frac{2}{3}T$, M_+T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p_+m$ Zs, $\frac{1}{2}P_xM_+T_-$ Znw^2 Zne^2, ... $Ly74^4$.
5. 5. T, $M\frac{2}{3}T$, m_+t, $m\ddagger t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p_+m$ Zs, $\frac{1}{2}P_xM_+T_-$ Znw^2 Zne^2, $Ly74^5$.

2. PLAGIONITE. Cleavage = $m\frac{4}{7}t$.
5. 3. $M\frac{4}{7}T$. $\frac{1}{2}P_-M$ Zn, $\frac{1}{2}p_+m$ Zs, $p_xm_yt_z$, ... P346. T i 580.

3. MYARGYRITE. Hemi-prismatic Ruby-Blende. Ag Sb_2 S_4.
Cleavage = t. $\frac{1}{2}p_xm_yt_z$ Zn^2w.
5. 3. $M\frac{1}{1}\frac{4}{5}T$. $\frac{1}{2}P\frac{1}{3}T$ Zw, $\frac{1}{2}p\frac{1}{1}\frac{1}{2}t$ Ze, ... D^{97}. M ii [103].

4. RED ANTIMONY. Rothantimonerz. Antimoine oxidé sulfuré. Antimonblende. Rothspiesglaserz.
Cleavage = m,T.
5. 5. T, M_-T. $\frac{1}{2}p_xm_yt_z$ Zn^2e Zs^2e, $\frac{1}{2}P\frac{1}{3}T$ Zw, $\frac{1}{2}p\frac{4}{7}t$ Ze. Axes: $p\ddagger m^a_- t^z$.
L608.

5. TUNGSTATE OF IRON. Wolfram. Scheelin ferruginé. Tungstène.
Cleavage = m,T.
5. 2. M^6_7,T. $\frac{1}{2}P\frac{1}{2}M$ Zn, ... $H118^{324}$? $Ly79^2$
5. 2. M^6_7,T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{2}{1}M$ Zs, ... $Ly79^3$.
5. 2. M_-,T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $p_xm_yt_z$? .... $H118^{326}$.
5. 3. M^6_7T. P^6_7T, $p_xm_yt_z$? .... $H118^{325}$.
5. 5. M_-, M^6_7T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{2}m$ Zs, p^6_7t, ... D373. M ii [49]. P326.
5. 5. M_-,T, m^3_7t. $\frac{1}{2}P\frac{1}{2}M$ Zn, p_+m, $p_xm_yt_z$? .... $H118^{327}$.
5. 5. M_-, M^3_7T, M^6_7T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{2}m$ Zs, P^6_7T, $\frac{1}{2}p_xm_yt_z$ Zne Znw, ... $Ly79^6$.

5. 5. M_, $M\frac{3}{7}T$, $m\frac{6}{7}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{2}m$ Zs, $P\frac{6}{7}T$. $\frac{1}{2}p_x m_y t_z$ Zne Znw,
$\frac{1}{2}p_x m_y t_z$ Zne^2 Znw^2,............Ly79[7].

5. 5. M_,t, $M\frac{3}{7}T$, $m\frac{6}{7}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $P\frac{6}{7}T$. $\frac{1}{2}p_x m_y t_z$ Zne Znw,...............Ly79[4].

5. 5. M_,t, $M\frac{3}{7}T$, $m\frac{6}{7}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{2}m$ Zs, $P\frac{6}{7}T$, $\frac{1}{2}p_x m_y t_z$ Zne Znw, ...Ly79[5].

5. 5. M_,t, $M\frac{3}{7}T$, $m\frac{6}{7}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}$P=M Z^2s, $\frac{1}{2}p\frac{1}{2}m$ Zs^2, $P\frac{6}{7}T$, $\frac{1}{2}p_x m_y t_z$ Zne Znw,
$\frac{1}{2}p_x m_y t_z$ Zne^2 Znw^2,.................Ly79[8].

[Measurements gathered from Phillips and Mohs.]

6. CHROMATE OF LEAD. Rothbleierz. Red Lead Ore. Plomb chromaté.

The prism M T has an obtuse angle of 93°, Haüy = $M\frac{1}{1}\frac{8}{9}T$; of 93° 30′, Levy, Phillips, Brooke = $M\frac{1}{1}\frac{6}{7}T$; of 93° 40′, Mohs = $M\frac{1}{1}\frac{5}{6}T$.

Cleavage = m,t, $M\frac{1}{1}\frac{6}{7}t$.

4. 3. $M\frac{16}{17}T$. $\frac{1}{2}p_x m$ Zn, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Znw Zne, $\frac{1}{2}$p_$m\frac{16}{17}t$ Z^2sw Z^2se,
$\frac{1}{2}P_+M\_T$ Zs^2w Zs^2e,.........D233[1].

5. 3. $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}P_+M$ Zs, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw, ... Ly53[3].

5. 3. $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}p_+m$ Zn, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw,........................Ly53[4].

5. 3. $M\frac{17}{16}T$, $m\frac{17}{10}t$. $\frac{1}{2}p\frac{4}{17}m$ Zn, $\frac{1}{2}P_+M$ Zs, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw,............Ly53[5].

5. 3. $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}P_+M$ Zn, $\frac{1}{2}P_+M$ Zs, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw, ... Ly53[6].

5. 3. $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}P_+M$ Zs, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw, $\frac{1}{2}p_+m\frac{16}{17}t$ Zne Znw, Ly53[8].

5. 3. $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}P_+M$ Zs, $p_x t$, p_+t, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw,............Ly53[14].

5. 3. $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}p\frac{4}{17}m$ Zn, $\frac{1}{2}p_+m$ Zs, $p_x t$, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw,
$\frac{1}{2}p_x m\frac{16}{17}t$ Zse Zsw,...............Ly53[16].

5. 3. m_t, $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}p\frac{4}{17}m$ Zn, $\frac{1}{2}P_+M$ Zs, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw,
$\frac{1}{2}p_+m\frac{16}{17}t$ Zse Zsw,...............Ly54[17].

5. 3. $\frac{1}{2}M\frac{16}{17}T$, $\frac{1}{2}M\frac{16}{17}T$, $\frac{1}{2}M\frac{17}{10}T$, $\frac{1}{2}m\frac{17}{10}t$. $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw, ... H91[48].

5. 3. $\frac{1}{2}M\frac{16}{17}T$, $\frac{1}{2}m\frac{16}{17}t$, $\frac{1}{2}M\frac{17}{10}T$, $\frac{1}{2}m\frac{17}{10}t$. $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw, $\frac{1}{2}P_+M\frac{16}{17}T$ ZseZsw, H91[49].

5. 3. M_T. $\frac{1}{2}$P_M Zn,...Ly53[2].

5. 5. $m\frac{4}{17}t$, $m\frac{16}{17}t$, $M\frac{17}{10}T$. $\frac{1}{2}P_+M$ Zn, $\frac{1}{2}P_+M$ Zs,..............................Ly53[7].

5. 5. m, $M\frac{16}{17}T$. $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw, $\frac{1}{2}P_+M\frac{16}{17}T$ Zse Zsw,..................H91[50].

5. 5. m, $M\frac{16}{17}T$. $\frac{1}{2}p\frac{4}{17}m$ Zn, $\frac{1}{2}P_+M$ Zs, $\frac{1}{2}p\mp m$ Zs^2, $\frac{1}{2}P_x M_y T_z$ Zne Znw,
$\frac{1}{2}p_+mt\_$ Zne^2 Znw^2,............P368[2].

5. 5. m, $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}$p_m Zs, $\frac{1}{2}p_+m$ Zs, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Zne Znw,
$\frac{1}{2}p_+m\frac{16}{17}t$ Zne Znw,..............Ly53[15].

5. 5. t, M_T. $\frac{1}{2}$P_M Zn, $\frac{1}{2}p_x m_y t_z$ Zne Znw, $\frac{1}{2}p_+m_y t_z$ Zne Znw, ... Ly53[9].

5. 5. t, M_T. $\frac{1}{2}$P_M Zn, $\frac{1}{2}p_+m$ Zs, $\frac{1}{2}p_x m_y t_z$ Zne Znw, $\frac{1}{2}p_+m_y t_z$ Zne Znw,
Ly53[10].

5. 5. m, t, $M\frac{16}{17}T$, $m\frac{17}{10}t$. $\frac{1}{2}p_x m$ Zs, $p_x t$, P_+T, $\frac{1}{2}P\frac{4}{17}M\frac{16}{17}T$ Znw Zne,
$\frac{1}{2}$P_$M\frac{16}{17}T$ Z^2sw Z^2se, $\frac{1}{2}p_+m\_t$ Zs^2w Zs^2e,.........M ii [170]. D233[2].

7. GADOLINITE. Cleavage = ?

5. 3. $M\frac{7}{11}T$. $\frac{1}{2}P\frac{1}{7}M$ Zn, $p_+m\frac{7}{11}t$,...Ly32[2].

5. 3. $M\frac{7}{11}T$. $\frac{1}{2}p\frac{1}{7}m$ Zn, $\frac{1}{2}$P_M_+T Zne^2 Znw^2, $p_+m\frac{7}{11}t$, ... Ly32[3].

5. 5. m, $M\frac{7}{11}T$. $\frac{1}{2}P\frac{1}{7}M$ Zn, $\frac{1}{2}$P_M_+T, $\frac{1}{2}P_+M\frac{7}{11}T$, ...P101[2].

5. 5. T, $M\frac{7}{10}T$, $m\frac{10}{7}t$. $\frac{1}{2}p\frac{16}{7}m$ Zn, $\frac{1}{2}$p_m t Zne Znw, ... H69[123].

8. TABULAR SPAR. Tafelspath. Wollastonite. Schaalstein.
Cleavage = $\frac{1}{2}M\frac{98}{100}T$, $\frac{1}{2}M\frac{85}{100}T$.
5. 5. m, $\frac{1}{2}M\frac{98}{100}T$ nw, $\frac{1}{2}M\frac{85}{100}T$ ne. $\frac{1}{4}p\_m\frac{98}{100}t$ Z^2nw, $\frac{1}{4}p_{+}m\frac{98}{100}t$ Znw^2, $\frac{1}{4}p_{x}m\frac{98}{100}t$ Z^2se,..................P47^2.

9. AUGITE. Pyroxène. *An Isomorphous Group,* comprehending these varieties:—
1. Diopside. Mussite. Alalite.
2. Sahlite. Pyrgom. Malakolith. Fassaite.
3. Hedenbergite. Pyroxène noir.
4. Rhodonite. Manganhaltiger Augite.
5. Basaltic Augite. Gemeiner Augit.
6. Hypersthene. Labradorische Hornblende. Paulite.
7. Diallage. Smaragdite. Bronzite?

M_T on M_T at the north pole is 92° 18′ Haüy, Mohs = $M\frac{21}{25}T$;
is 92° 55′ Levy, Phillips = $M\frac{19}{20}T$.
$\frac{1}{2}$P_T on the vertical edge at the west pole is 106° 6′ Haüy = nearly
$P\frac{2}{7}T = P\frac{6}{21}T$.

I have adopted $M\frac{20}{21}T$ because the measures of the pyramids point out that number.

Cleavage = $M\frac{20}{21}T$, constant; m,t, $\frac{1}{2}p\frac{6}{21}t$, occasional.

1. 3. P,$M\frac{20}{21}T$,...A right-rhombic prism? (Hypersthene)...L516^1. H69^{121}.
1. 5. P,M,T, $M\frac{20}{21}T$, ...A right prism, ...Haüy's Ambigu,...L500^4. H66^{94}.
3. 5. P,M,t,$M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw, $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw, ...Ly30^2.
3. 5. p_{+},M,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw, ...Md. 53. L500^6. J^{115}. H68^{107}.
3. 5. P,M,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw, $\frac{1}{2}p\_mt_{+}$ Zne Zse,........................H68^{110}.
3. 5. PZ, M,T, $M\frac{20}{21}T$. $\frac{1}{2}p\_mt_{+}$ Z, $\frac{1}{2}P\frac{6}{21}T$ N,.....................Ly v^5. H68^{110a}.
3. 5. p_{+},M,T, $m\frac{20}{21}t$. $\frac{1}{2}p\frac{6}{21}t$ Zw, $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Z^2nw Z^2sw, $\frac{1}{2}p\frac{18}{21}m\frac{15}{21}t$ Znw^2 Zsw^2,
$\frac{1}{2}p_{+}m_{x}t$ Zn^2e Zs^2e,......(Hedenbergite)......L500^{14}. H69^{117}.
3. 5. (p,M,T, $m\frac{20}{21}t$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw) × 2, and crossed,......J^{118}. H69^{119}.
4. 5. T,$M\frac{10}{21}T$. $P\frac{12}{21}M\frac{10}{21}T$, ... (Fassaite)......L501^{21}. H67^{101}.
5. 2. M_, T. $\frac{1}{2}P\frac{6}{21}T$ Zw,.....................(Sahlite)......J^{113}. M ii^{146}. H66^{91}.
5. 3. $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw, ...Haüy's Primitive...(Mussite)...Model 87.
Ly30^1. L500^1. M ii^{44}. H66^{86}.
5. 3. $M\frac{20}{21}T$. $\frac{1}{2}P\frac{18}{21}M\frac{15}{21}T$ Z^2nw Z^2sw, $\frac{1}{2}p\ddagger m\frac{15}{21}t$ Zn^2w^2 Zs^2w^2,......(Fassaite)......
L500^{17}. H66^{88}.
5. 5. T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw,..................(Augite)............L500^2. H66^{87}.
5. 5. T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw, ...(Augite)...J^{113}. L500^6. H66^{90}.
5. 5. T, $M\frac{20}{21}T$. $\frac{1}{2}p\frac{6}{21}t$ Zw, $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw, ...(Augite) ...L500^7. H66^{92}.
5. 5. t. $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw, $\frac{1}{2}p\frac{6}{21}t$ Ze, $\frac{1}{2}p_{+}m_{x}t$ Zn^2w Zs^2w, $\frac{1}{2}p\frac{18}{21}m\frac{15}{21}t$ Z^2ne Z^2se,
$\frac{1}{2}p\ddagger m\frac{15}{21}t$ Zn^2e^2 Zs^2e^2,......(Fassaite)......L501^{18}. H68^{113}.
5. 5. t,$M\frac{20}{21}T$. $\frac{1}{2}p\frac{6}{21}t$ Zw, $\frac{1}{2}P\frac{6}{21}T$ Ze, $\frac{1}{2}p\frac{6}{21}m\frac{10}{21}t$ Z^2nwZ^2sw, $\frac{1}{2}P\frac{18}{21}M\frac{15}{21}T$ Zn^2w^2 Zs^2w^2,
$\frac{1}{2}P\frac{18}{21}M\frac{30}{21}T$ Zne Zse,.........M ii^{73}. D306^3.
5. 5. M,T,$M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}t$ Zw, $\frac{1}{2}P\frac{6}{21}T$ Ze, $\frac{1}{2}p\frac{6}{21}m\frac{10}{21}t$ Z^2nw Z^2sw,
$\frac{1}{2}P\frac{18}{21}M\frac{15}{21}T$ Zn^2w^2 Zs^2w^2, $\frac{1}{2}P\frac{18}{21}M\frac{15}{21}T$ Zn^2e^2 Zs^2e^2,............H68^{116}.

5. 5. M,T, $m\frac{20}{21}t$. $\frac{1}{2}p\frac{6}{21}t$ Zw, $\frac{1}{2}p\frac{6}{21}T$ Ze, $\frac{1}{2}p\frac{6}{21}m\frac{10}{21}t$ Z^2nw Z^2sw,
$\frac{1}{2}P\frac{18}{21}M\frac{15}{21}T$ Zn^2w^2 Zs^2w^2, $\frac{1}{2}P\frac{18}{21}M\frac{20}{21}T$ Zne Zse,...D306[2]. R[104, 104a]. M ii[176, 177].

5. 5. M,T, $M\frac{20}{21}T$. $P\frac{6}{21}T$, ... (Hypersthene)............L516[8]. H69[125].

5. 5. m,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw,.........(Sahlite) .... L500[8]. H66[93].

5. 5. M,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw, Axes: $p\underline{+}m^2t\underline{\ }$,...........Model 98.
H67[95]. Ly v[1]. J[114]. M ii[73]. L500[9]. D306[1].

5. 5. (M,T,$M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw) × 2,......Model 99. Ly v[2]. H67[96].

5. 5. m,T,$M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Z, $\frac{1}{4}P\frac{6}{21}M\frac{10}{21}T$ Nne Nse, ...H67[99]. J[116].

5. 5. m,T,$M\frac{20}{21}T$. $\frac{1}{2}p\frac{6}{21}t$ Zw, $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw, ... R[103]. Ly v[3]. H67[106].

5. 5. m,T,$m\frac{20}{21}t$, $m\frac{7}{21}t$. $\frac{1}{2}P\frac{6}{21}T$ Zw, ... (Sahlite)............L500[5]. H68[106].

5. 5. m,T,$m\frac{20}{21}t$, $m\frac{7}{21}t$. $\frac{1}{2}P\frac{6}{21}T$ Zw, $\frac{1}{2}p\frac{6}{21}m\frac{10}{21}t$ ZnwZsw...(Diopside)......H68[108].

5. 5. m,T,$M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw, $\frac{1}{2}p\frac{18}{21}m\frac{15}{21}t$ Znw Zsw,
J[116]. Ly v[8]. L501[20]. H68[109].

5. 5. M,T,$m\frac{20}{21}t$. $\frac{1}{2}p\frac{6}{21}T$ Ze, $\frac{1}{2}p_x M_y T_z$ Znw^2 Zsw^2, $\frac{1}{2}p_x M_y T_z$ Zn^2w Zs^2w,
L500[16]. H68[111].

5. 5. M,T,$M\frac{20}{21}T$. $\frac{1}{2}p\frac{6}{21}T$ Zw, $\frac{1}{2}p\frac{6}{21}t$ Ze, $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw, $\frac{1}{2}p\frac{18}{21}m\frac{20}{21}t$ Zne Zse,
L500[18]. Ly v[16]. H68[112].

5. 5. M,T,$M\frac{20}{21}T$. $\frac{1}{2}p\frac{6}{21}t$Zw, $\frac{1}{2}p\frac{6}{21}T$Ze, $\frac{1}{2}p\frac{6}{21}m\frac{10}{21}t$ Z^2nwZ^2sw, $\frac{1}{2}P\frac{18}{21}M\frac{15}{21}T$ $Zn^2w^2Zs^2w^2$,
$\frac{1}{2}p\frac{18}{21}M\frac{20}{21}T$ Zne Zse..........L500[10]. H68[114, 115].

5. 5. M,T,$M\frac{20}{21}T$. $\frac{1}{2}p\frac{6}{21}t$ Zw, $\frac{1}{2}p\frac{6}{21}T$ Ze, $\frac{1}{2}P\frac{18}{21}M\frac{15}{21}T$ Zn^2w^2 Zs^2w^2, $\frac{1}{2}p\frac{6}{21}m\frac{10}{21}t$ Z^2nwZ^2sw,
$\frac{1}{2}p\frac{18}{21}m\frac{20}{21}t$ Z^2ne Z^2se, $\frac{1}{2}p\ddagger m_+ t$ Zne^2 Zse^2 ...(Alalite)...L500[11]. H69[118].

5. 5. m,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw,... Axes: $p\underline{\ }m^2t\ddagger$, ...Ly v[1]. H67[96].

5. 5. M,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{5}{21}M\frac{10}{21}T$ Znw Zsw, ... L500[15]. H67[100].

5. 5. M,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw, $\frac{1}{2}p\frac{6}{21}T$ Ze,..........................Ly v[6]. H67[103].

5. 5. M,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw, $\frac{1}{2}P\ddagger M\frac{15}{21}T$ Zne Zse...(Baïkalite) L501[19]. H67[104].

5. 5. m,t, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{7}{21}T$, ...(Diallage).............................H70[128].

Levy gives figures of many other varieties of Augite, and some of them of very complicated combinations. It is, however, impossible to represent them in symbols, owing to the want of measurements. They do not differ in character from the combinations described above.—Haüy's measurements are very numerous, but many of them are useless for my purpose, while several angles necessary for completing the above symbols are unfortunately omitted.

10. WAGNERITE. Fluophosphate of Magnesia.

Cleavage = t, $M\frac{10}{11}T$.

5. 3. $M\frac{10}{11}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, ... (assumed primitive)....................P186.

5. 5. m, $m\frac{3}{10}t$, $M\frac{7}{10}T$, $m\frac{18}{10}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\_M_xT$ Z^2ne Z^2nw, $\frac{1}{2}p_+m_xt$
Zne^2 Znw^2, $\frac{1}{2}p_xm_yt_z$ Z^2se Z^2sw, $\frac{1}{2}p_xM_yT_z$ Zs^2e Zs^2w, $2(\frac{1}{2}p_+m_xt)$
Zse^2 Zsw^2,..M ii[19].

11. LITHIA MICA. Lithionglimmer. Lepidolite.

5. 5. t, $m\frac{10}{17}t$. $\frac{1}{2}P\frac{3}{17}M$ Zn,... Axes: $p\underline{\ }m\ddagger t^2$, ...?

12. MALACHITE. Green Carbonate of Copper. Fibrous Malachite.
Cuivre carbonaté verte.

Cleavage = $m\frac{4}{3}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn.

5. 5. m, $M\frac{4}{3}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, ...L155[1].

5. 5. (m, $M\frac{4}{3}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn) × 2, ... Ly62[2]. M ii[78].

5. 5. m, M$\frac{4}{3}$T. $\frac{1}{2}$P$\frac{1}{2}$M Zn, $\frac{1}{2}$p$_x$m Zs, ... L155[2].
5. 5. m, M$\frac{4}{3}$T. $\frac{1}{2}$P$\frac{1}{2}$M Zn, $\frac{1}{2}$p_m$_x$t Zne^2 Znw^2, ... L155[3].

13. CARBONATE OF SODA. Soda.
Cleavage = T, m$\frac{4}{3}$t.
5. 5. M, M$\frac{4}{3}$T. $\frac{1}{2}$P$\frac{4}{3}$M$\frac{4}{3}$T Znw Zsw,...p$_{+}^{a}$m$_{-}^{a}$t^{a}, ...H54[182].
5. 5. M, t, M$\frac{4}{3}$T. $\frac{1}{2}$p$\frac{4}{3}$t Zw, $\frac{1}{2}$P$\frac{4}{3}$M$\frac{4}{3}$T Znw Zsw,..............Artificial crystal.

14. TRONA. Sesqui-carbonate of Soda. Urao.
Cleavage = $\frac{1}{2}$M$\frac{4}{3}$T.
5. 3. $\frac{1}{2}$M$\frac{4}{3}$T nw, $\frac{1}{2}$M$\frac{4}{3}$T ne. $\frac{1}{2}$P_T Zw, $\frac{1}{2}$P$_+$T Ze,...p$_{+}^{a}$m$_{-}^{a}$t^{a},......P197. A[26].

15. PHOSPHATE OF COPPER. Phosphorsaures Kupferoxyd von Rheinbreitenbach. Hydrous Phosphate of Copper.
Cleavage = t, $\frac{1}{2}$p_t.
5. 3. M$\frac{6}{17}$T. $\frac{1}{2}$P_T Z^{2}w, $\frac{1}{2}$p$_+$t Zw^2, ...Ly62[2].
5. 5. t, M$\frac{6}{17}$T. $\frac{1}{2}$P_T Z^{2}w, $\frac{1}{2}$p$_+$t Zw^2, ...Ly62[3].
5. 5. t, M$\frac{6}{17}$T. $\frac{1}{2}$P_T Z^{2}w, $\frac{1}{2}$P$_+$M_T Znw^2 Zsw^2, ...Ly62[5].
5. 5. t, M$\frac{6}{17}$T. $\frac{1}{2}$P_T Z^{2}w, $\frac{1}{2}$ P$_+$T Zw^2, $\frac{1}{2}$p$_+$m_t Znw^2 Zsw^2, ...Ly62[6].
5. 5. m, t, M$\frac{6}{17}$T. $\frac{1}{2}$P_T Z^{2}w, $\frac{1}{2}$P$_+$M_T Znw^2 Zsw^2, ...Ly62[4].
[P$_+$M_T Znw^2 on Zsw^2 = 117° 50′, and on t = 123° 30′. Levy.]

16. OBLIQUE PRISMATIC ARSENIATE OF COPPER.
Cleavage = $\frac{1}{2}$P$\frac{2}{11}$T.
5. 3. M$\frac{8}{15}$T. $\frac{1}{2}$P$\frac{2}{11}$T Zw, ... (assumed primitive) ....P331[1]. Ly65[1].
5. 3. M$\frac{8}{15}$T. $\frac{1}{2}$P$\frac{2}{11}$T Z^{2}w, $\frac{1}{2}$P$\frac{11}{4}$T Ze, ...A[83].
5. 5. M$\frac{8}{15}$T. $\frac{1}{2}$P$\frac{2}{11}$T Z^{2}w, $\frac{1}{2}$p$\frac{5}{11}$t Zw^2, $\frac{1}{2}$P$\frac{11}{4}$T Ze, ... Ly65[2].

17. *An Isomorphous Group*, 1, 2:—

1. VIVIANITE. Phosphate of Iron. Fer phosphaté. Blue Iron Ore.
Cleavage = T.
5. 5. m, M$\frac{8}{11}$T. $\frac{1}{2}$P$_+$M Zn, $\frac{1}{2}$P$_+$M Zs,...................................Ly70[2].
5. 5. m, T, M$\frac{8}{11}$T. $\frac{1}{2}$P$\frac{4}{11}$M Zn, $\frac{1}{2}$p$_{x}^{4}$m$\frac{1}{x}$t Zne^2 Znw^2, ...Ly70[3].
5. 5. M, T, M$\frac{8}{11}$T, m$_+$t. $\frac{1}{2}$P_M Zn, ...Ly70[4].
5. 5. M, T, M$\frac{8}{11}$T, m$_+$t. $\frac{1}{2}$P_M Zn, $\frac{1}{2}$P_M$_x$T Zne^2 Znw^2, ... Ly70[5].
5. 5. M, T, M$\frac{8}{11}$T, m$_+$t. $\frac{1}{2}$P_M Zn, $\frac{1}{2}$p_m$_x$t Zne^2 Znw^2, $\frac{1}{2}$P_M$_x$T Zse Zsw,
Ly70[6].
5. 5. m, t, M$\frac{8}{11}$T. $\frac{1}{2}$p_m Zn, $\frac{1}{2}$P_M$_x$T Z^{2}ne Z^{2}nw, $\frac{1}{2}$p$_+$m$_x$t Zn^{2}e Zn^{2}w,
$\frac{1}{2}$P_M$_x$T Z^{2}se Z^{2}sw, $\frac{1}{2}$p$_+$m$_x$t Zs^{2}e Zs^{2}w,...........Ly70[7].

2. COBALT BLOOM. Kobaltblüthe. Cobalt arseniaté. Arseniate of Cobalt. Red Cobalt.
Cleavage = M, t.
5. 2. M, T. $\frac{1}{2}$P$\frac{2}{3}$T Zw, ...L161[1].
5. 5. M, T, m$\frac{1}{2}$t. $\frac{1}{2}$P$\frac{2}{3}$T Zw, ...L162[2].
5. 5. M, t, M$\frac{1}{2}$T, m$_+$t. $\frac{1}{2}$P$\frac{2}{3}$T Zw,...Axes: p$_{+}^{a}$m$_{-}^{a}$t^{a},...L162[3]. P289. Ly73[2].

18. HURAULITE. Phosphate of Iron and Manganese.

5. 3. $M\frac{3}{5}T.\frac{1}{2}P_{+}M_{x}T$ Zne Znw, ... Cleavage = 0, ... T i 518. P247.

19. HETEROSIDERITE?

20. PHARMACOLITE. Chaux arseniatée. Arseniate of Lime.
Cleavage = T.

5. 5. $M, T, m\frac{7}{20}t.\ \frac{1}{2}P\_M_{x}T$ Zne Znw, ... S^{334}.

5. 5. $M, T, m\frac{7}{20}t.\ \frac{1}{2}P\_M_{x}T\ Z^{2}ne\ Z^{2}nw,\ \frac{1}{2}p_{+}m_{x}t\ Zn^{2}e\ Zn^{2}w$, ...P181.

5. 5. $M, T, m\frac{7}{20}t.\ 3(\frac{1}{2}P_{x}M_{y}T_{z})$ Zne Znw, ... S^{336}.

21. STRAHLERZ?

22. TINCAL. Tinkal. Borax. Soude boratée.
Cleavage = $m, t, M\frac{16}{17}T$.

5. 5. $T, M\frac{16}{17}T.\ \frac{1}{2}P\frac{5}{17}T$ Zw, $\frac{1}{2}p\_m_{x}t\ Z^{2}nw\ Z^{2}sw,\ \frac{1}{2}p_{+}m_{x}t\ Zn^{2}w\ Zs^{2}w$,
$\frac{1}{2}P\_M_{x}T$ Zne Zse, ...$Ly25^{2}$.

5. 5. $m, T, M\frac{16}{17}T.\ \frac{1}{2}P\frac{5}{17}T$ Zw, $\frac{1}{2}p\_m_{x}t\ Z^{2}nw\ Z^{2}sw,\ \frac{1}{2}p_{+}m_{x}t\ Zn^{2}w\ Zs^{2}w$,
$\frac{1}{2}p\_m_{x}t$ Zne Zse, ... S^{72}. P199.

23. GLAUBER'S SALT. Glaubersalz. Soude sulfatée.
Cleavage = t (Leonhard), = $\frac{1}{2}P\frac{4}{13}M$ (Levy).

5. 3. $M\frac{11}{13}T.\ \frac{1}{2}P\frac{4}{13}M$, ... (primary)......................P198. Ly i 328.

5. 5. $m, T, m\frac{16}{17}t.\ 2(\frac{1}{2}p_{x}m)$ Zn, $2(\frac{1}{2}p_{x}m)$ Zs, $2(\frac{1}{2}p\ t)$ Zw, $2(\frac{1}{2}p_{x}t)$ Ze,
$2(\frac{1}{2}p_{x}m_{y}t_{z})$ Znw Zsw, $\frac{1}{2}p_{x}m_{y}t_{z}$ Zne Zse, ... M ii ^{56}. D173.

24. GYPSUM. Gyps. Selenite. Hydrous Sulphate of Lime. Chaux sulfatée. Fraueneis.
Cleavage = $m, T, \frac{1}{2}p\frac{3}{7}m$ Zn.

5. 2. $M\_, T.\ \frac{1}{2}P\frac{3}{7}M$ Zn, (Haüy's Primitive, but with p^{a} and t^{a} changed.)
Wolfach, ...Model 79. $L119^{1}$. $H29^{1}$.

5. 5. $M, t, M\frac{9}{13}T.\ \frac{1}{2}P\_M_{+}T$ Znw Zsw, $\frac{1}{2}PM_{+}T\_$ Zne Zse, ... $H30^{9}$.

5. 5. $m, T\_, m\frac{9}{13}t.\ \frac{1}{2}p\frac{5}{13}m\frac{9}{13}t$ Zne Znw, $\frac{1}{2}p\frac{6}{13}m\frac{10}{13}t$ Zse Zsw, ... $H30^{11}$.

5. 5. $T\_, M\frac{9}{13}T.\ \frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw, $\frac{1}{2}P\frac{6}{13}M\frac{10}{13}T$ Zse Zsw,
(Haüy's équivalente).........Model 75. $H30^{6}$. R^{99}. Ly v ^{3}.

5. 5. $T\_, M\frac{9}{13}T.\ \frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw, $p_{+}^{a}m^{a}t_{-}^{a}$, ...(trapezienne élargie)
Montmartre, ...Model 115......R^{100}. $Ly14^{2}$. $H29^{3}$.

5. 5. $T\_, M\frac{9}{13}T.\ \frac{1}{2}P\frac{5}{13}M\frac{9}{13}T\ Z^{2}ne\ Z^{2}nw,\ \frac{1}{2}p_{+}m_{x}t\ Zne^{2}\ Znw^{2}$, ...$H30^{5}$.

5. 5. $T\_, M\frac{9}{13}T.\ \frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ ZneZnw, $p_{-}^{a}m_{+}^{a}t^{a}$, (trapezienne) Lyv ^{1}. $H29^{2}$.

5. 5. $(T\_, M\frac{9}{13}T.\ \frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw$)\times 2$, ...(Hemitrope)...H i 535.

5. 5. $T\_, M\frac{18}{13}T.\ \frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$, Zne Znw, ...$p_{-}^{a}m_{+}^{a}t^{a}$, ...$H29^{4}$.

5. 5. $T\_, M\frac{18}{13}T.\ \frac{1}{2}p\frac{5}{13}m\frac{9}{13}t$ Zne Znw, $\frac{1}{2}p\frac{6}{13}m\frac{10}{13}t$ Zse Zsw, ...$H30^{7}$.

5. 5. $T\_, m\frac{9}{13}t, M\frac{18}{13}T.\ \frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw, ...$H30^{8}$.

5. 5. $T\_, M\frac{9}{13}T, m\frac{27}{13}t.\ \frac{1}{2}P_{x}M$ Zs, $\frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw, ... Bex...$H31^{12}$.

5. 5. $T\_, M\frac{9}{13}T, m\frac{18}{13}t, m\frac{27}{13}t.\ \frac{1}{2}P_{x}M$ Zs, $\frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ ZneZnw, Lyv^{5}. $H31^{13}$.

25. SULPHATE OF IRON. Eisenvitriol. Fer sulfaté. Green Vitriol.

Cleavage = $m\frac{6}{7}t.\ \frac{1}{2}p_{\frac{5}{20}}t,\ \frac{1}{2}p_{+}m_{x}t$ Znw Zsw.

5. 3. $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{5}{20}}T\ Z^2w,\ \frac{1}{2}P_{+\frac{5}{10}}T\ Zw^2,\ \frac{1}{2}P_{+\frac{8}{10}}T$ Ze,...$p_{+}^{a}m_{6}^{a}t_{7}^{a}$.

5. 3. $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{5}{20}}T\ Z^2w,\ \frac{1}{2}p_{x}t\ Zw,\ \frac{1}{2}p_{+\frac{5}{10}}t\ Zw^2,\ \frac{1}{2}p_{+\frac{8}{10}}t$ Ze,...$p_{+}^{a}m_{6}^{a}t_{7}^{a}$.

5. 3. $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{5}{20}}T\ Z^2w,\ \frac{1}{2}P_{+\frac{5}{10}}T\ Zw^2,\ \frac{1}{2}P_{+\frac{8}{10}}T$ Ze, $3(\frac{1}{4}p_{x}m_{y}^{1}t_{z})$ Zsw,
$\frac{1}{4}p_{x}m_{y}t_{z}$ Znw,......$p_{+}^{a}m_{6}^{a}t_{7}^{a}$.

5. 3. $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{5}{20}}T\ Z^2w,\ \frac{1}{2}P_{+\frac{5}{10}}T\ Zw^2,\ \frac{1}{2}P_{+\frac{8}{10}}T$ Ze, $3(\frac{1}{2}p_{x}m_{y}t_{z})$ Znw Zsw,
$p_{+}^{a}m_{6}^{a}t_{7}^{a}$.

5. 3. $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{5}{20}}T\ Z^2w,\ \frac{1}{2}P_{+\frac{5}{10}}T\ Zw^2,\ \frac{1}{2}P_{+\frac{8}{10}}T$ Ze, $3(\frac{1}{4}p_{x}m_{y}t_{z})$ Znw,
$3(\frac{1}{4}p_{x}m_{y}t_{z})$ Zsw,......$p_{+}^{a}m_{6}^{a}t_{7}^{a}$.

A great variety of factitious Crystals answer to the last two formulæ, the planes of the 3 Forms $\frac{1}{2}p_{x}\ m_{y}\ t_{z}$ varying considerably in size on the north and south sides of the crystal.

26. BARYTO-CALCITE. Barytocalcit.

Cleavage = $\frac{1}{2}PM$ Zs, $\frac{1}{2}P\frac{4}{6}M\frac{2}{6}T$ Zne Znw.

5. 3. $M\frac{1}{1}\frac{6}{7}T.\ \frac{1}{2}P\frac{5}{9}M$ Zn, ...M ii [46].

5. 3. $M\frac{1}{1}\frac{6}{7}T.\ \frac{1}{2}P\frac{4}{9}M$ Zn, $\frac{1}{2}pm$ Zs, $\frac{1}{2}p\frac{4}{6}m\frac{2}{6}t$ Zne Znw, ... D202. A[54]. P189.

5. 5. t, $M\frac{1}{1}\frac{6}{7}T,\ m\frac{1}{3}\frac{6}{7}t.\ \frac{1}{2}P\frac{5}{9}M$ Zn, $\frac{1}{2}p_{-}m$ Zn, $\frac{1}{2}pm$ Zs, $\frac{1}{2}P\frac{4}{6}M\frac{2}{6}T\ Z^2ne\ Z^2nw$,
$\frac{1}{2}p_{+}m_{x}t\ Zne^2\ Znw^2$,.........M ii [188]. S[33].

27. AZURE COPPER ORE. Kupferlasur. Cuivre carbonaté bleu.

Cleavage = m, T, $M\frac{6}{7}T.\ \frac{1}{2}p_{x}m_{y}t_{z}\ Zne^2\ Znw^2$.

5. 3. $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{3}{14}}M$ Zn, $\frac{1}{2}p_{+}m_{-}t$ Zne Znw, ... L152[1]. H101[140].

5. 3. $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{3}{14}}M$ Zn, $\frac{1}{2}p_{+}m_{-}t\ Zn^2e\ Zn^2w,\ \frac{1}{2}p_{-}m_{+}t\ Zne^2\ Znw^2$,
L152[2]. H101[141].

5. 3. $M\frac{6}{7}T.\frac{1}{2}P_{\frac{3}{14}}M$ Zn,$\frac{1}{2}p_{+}m_{-}t$ZneZnw, $\frac{1}{2}p_{+}m_{-}t$ZseZsw, H101[143]. L152[4].

5. 5. m, $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{3}{14}}M$ Zn, $\frac{1}{2}p_{+}m_{-}t$ Zne Znw, ... L152[5]. H101[144].

5. 5. m, $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{3}{14}}M$ Zn, $\frac{1}{2}P_{+}M_{-}T\ Zn^2e\ Zn^2w\ \frac{1}{2}p_{x}m_{y}t_{z}\ Zne^2\ Znw^2$,
Model 103. L152[6]. H101[145].

5. 5. M, $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{3}{14}}M\ Z^2n,\ \frac{1}{2}p_{\frac{8}{11}}m\ Zn^2,\ \frac{1}{2}p_{+}m_{-}t\ Zn^2e\ Zn^2w$,
$\frac{1}{2}p_{-}m_{+}t\ Zne^2\ Znw^2,\ \frac{1}{2}p_{+}m_{-}t$ Zse Zsw,.........H102[146].

5. 5. T, $M\frac{6}{7}T.\ \frac{1}{2}P_{\frac{3}{14}}M$ Zn, $\frac{1}{2}p_{-}m_{+}t\ Zne^2\ Znw^2$, ... L152[3]. H101[142].

28. TRIPHYLINE.

1. 3. $P,M\frac{4}{9}T$?.........Cleavage = $P,m\frac{4}{9}t$,..............................D219.

29. VAUQUELINITE. Chromate of Lead and Copper.

5. 5. m, $M_{-}T.\ \frac{1}{2}P\frac{4}{7}M$ Zn, ...Ly52[2].

5. 5. (m, $M_{-}T.\ \frac{1}{2}P\frac{4}{7}M$ Zn) $\times$ 2, ... P369. D234. Ly52[3].

30. TITANITE. Sphene. Titane calcareo-siliceux. Menakerz.

Cleavage = $M\frac{4}{3}T.\ \frac{1}{2}P\frac{15}{8}M$ Zn.

5. 3. $M\frac{4}{3}T.P\frac{4}{8}M$,..H118[231].

5. 3. $M\frac{4}{3}T.\ \frac{1}{2}P\frac{15}{8}M$ Zn, $\frac{1}{2}P\frac{4}{3}T$ Zs,...Sim. Model 81. M ii [1]. H118[230]. J[188].

5. 3. $M\frac{4}{3}T.\ P\frac{15}{8}M,\ p_{+}m_{x}t$, ... H118[232].

5. 3. $M\frac{4}{3}T.\ \frac{1}{2}P\frac{15}{8}M$ Zn, $\frac{1}{2}p\frac{5}{8}m$ Zs, $\frac{1}{2}p_{+}m_{x}t\ Zne^2\ Znw^2$, ...P259[3]. M ii [47].
A[134]. D360[2].

5. 3. M$\frac{3}{5}$T. $\frac{1}{2}$P$\frac{1}{8}$M Z'n, $\frac{1}{2}$p$_{+}$m Zn², $\frac{1}{2}$p$\frac{3}{8}$m Zs, ...R[101].
5. 5. M$\frac{3}{5}$T, $\frac{1}{2}$M$\frac{3}{5}$T ne sw. P$\frac{3}{8}$M, $\frac{1}{4}$P$_{+}$M$_x$T Zne Nsw, ...H118[323].

31. EPIDOTE, *comprehending four varieties:*

1. ZOISITE.
2. PISTACITE. Pistazit. Thallite.
3. MANGAN-EPIDOT. Epidote manganésifère.
4. BUCKLANDITE.

Cleavage = M, $\frac{1}{2}$P$\frac{1}{2}$M Zn.

5. 2. M_,T. $\frac{1}{2}$P$\frac{1}{2}$M Zn, ... Model 79[b]. (Haüy's primitive form, but in a different position,)......H74[172]. P29[1].
5. 5. M_. $\frac{1}{2}$P$\frac{1}{2}$M Zn, $\frac{1}{2}$p$\frac{1}{2}\frac{1}{4}$m Zs, $\frac{1}{2}$P$_{+}$MT_ Zne² Znw², ...Model 101. Ly36[2]. H74[174].
5. 5. M. $\frac{1}{2}$p$\frac{1}{2}$m Zn, $\frac{1}{2}$P$\frac{1}{2}\frac{1}{4}$M Zs, $\frac{1}{2}$P$_{+}$MT_ Zne Znw,............J[119]. H74[173].
5. 5. M, m$\frac{8}{11}$t. $\frac{1}{2}$P$\frac{1}{2}$M Zn², $\frac{1}{2}$P$\frac{1}{10}\frac{1}{4}$M Z²n, $\frac{1}{2}$P$\frac{1}{2}\frac{1}{4}$M Zs, 4($\frac{1}{2}$p$_x$m$_y$t$_z$) ZneZnw, 3($\frac{1}{2}$p$_x$m$_y$t$_z$) Zse Zsw,......M ii[77]. Mr[93].
5. 5. M_, t, M$\frac{1}{1}\frac{8}{1}$T. $\frac{1}{2}$P$\frac{1}{2}$M Zn, $\frac{1}{2}$p$\frac{3}{8}$m Zs, $\frac{1}{2}$P$_{+}$MT_ Zne² Znw²,...Mod. 101[a]. J[120]. Sim. H74[176].
5. 5. M,T, m$\frac{1}{1}\frac{8}{1}$t. $\frac{1}{2}$P$\frac{1}{2}$M Zn, $\frac{1}{2}$P$\frac{1}{2}\frac{1}{4}$M Zs, $\frac{1}{2}$P$_x$M$_y$T$_z$ Zne² Znw², $\frac{1}{2}$p$_x$m$_y$t$_z$ Zse² Zsw²,.........J[121]. H74[175].
5. 5. M,t, M$\frac{9}{11}$T. $\frac{1}{2}$P$\frac{1}{2}$M Z²n, $\frac{1}{2}$P$\frac{7}{4}$m Zn², $\frac{1}{2}$p$\frac{1}{2}\frac{1}{4}$m Zs, $\frac{1}{2}$p$_x$m$_y$t$_z$ Z²ne Z²nw, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zne² Znw², $\frac{1}{2}$p$_x$m$_y$t$_z$ Zse Zsw,......J[122]. H74[177].
5. 5. M,T, m$\frac{1}{1}\frac{8}{1}$t, m$\frac{9}{11}$t. $\frac{1}{2}$P$\frac{1}{2}\frac{1}{4}$M Z²n, $\frac{1}{2}$p$\frac{1}{16}$m Zn², $\frac{1}{2}$p$_x$m$_y$t$_z$ Z²ne Z²nw, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zne² Znw², $\frac{1}{2}$P$_x$M$_y$T$_z$ Zse Zsw,......J[123]. H74[178].

32. COUZERANITE. Cleavage = t.

5. 3. M$\frac{9}{10}$T. $\frac{1}{2}$P_M$_x$T Zne Znw,......................P122. L731[2]. S153.

33. EUCLASE. Euklas. Prismatic Emerald.

Cleavage = M,T. $\frac{1}{2}$p$\frac{6}{7}$m Zn.

5. 5. m, M$\frac{1}{2}\frac{6}{3}$T, m$\frac{7}{23}$t. 3($\frac{1}{2}$p$_x$m$_y$t$_z$) Zne Znw, $\frac{1}{2}$P$_x$M$_y$T$_z$ Zse Zsw, ...Ly33[2].
5. 5. m, M$\frac{1}{2}\frac{6}{3}$T, m$\frac{7}{23}$t. 4($\frac{1}{2}$p$_x$m$_y$t$_z$) Zne Znw, $\frac{1}{2}$P$_x$M$_y$T$_z$ Zse Zsw, ...Ly33[4].
5. 5. m, t, M$\frac{1}{2}\frac{6}{3}$T, m$\frac{7}{23}$t. 2($\frac{1}{2}$p$_x$m$_y$t$_z$) Zne Znw, $\frac{1}{2}$P$_x$M$_y$T$_z$ Zse Zsw, Ly33[3].
5. 5. m, t, 13m_t. $\frac{1}{2}$p$\frac{6}{7}$m Zn, $\frac{1}{2}$P^6_xM^7_y T Zne Znw, 5($\frac{1}{2}$p$_x$m$_y$t$_z$) Zse Zsw, P98.

[An imaginary combination, representing the planes of many Crystals.]

5. 5. T, m$\frac{7}{23}$t, M$\frac{1}{2}\frac{1}{3}$T, m$\frac{1}{2}\frac{6}{3}$t. $\frac{1}{2}$P$\frac{6}{7}$M Zn, $\frac{1}{2}$P^6_xM^7_y T Zne Znw, $\frac{1}{2}$P$_{+}$M_T Zn²e Zn²w, $\frac{1}{2}$P$_x$M$_y$T$_z$ Zse Zsw,......H72[152].
5. 5. T, m$\frac{7}{23}$t, m$\frac{1}{2}\frac{1}{3}$t, m$\frac{1}{2}\frac{6}{3}$t. 6($\frac{1}{2}$p$_x$m$_y$t$_z$) Zne Znw, 3($\frac{1}{2}$p$_x$m$_y$t$_z$) Zse Zsw, M ii[54]. H72[153].

34. TWO-AXED MICA. Zweiaxiger Glimmer. Bi-axial Mica.

Cleavage = $\frac{1}{2}$P$\frac{3}{17}$M Zn.

5. 5. T, M$\frac{4}{7}$T. $\frac{1}{2}$P$\frac{3}{17}$M Zn, ...Vesuvius. Greenland...........Ly42[2].
5. 5. T, M$\frac{4}{7}$T. $\frac{1}{2}$P$\frac{3}{17}$M Zn, $\frac{1}{2}$p_m$\frac{4}{7}$t Zn²e Zn²w, ...Siberia.........Ly42[3].
5. 5. T, M$\frac{4}{7}$T. $\frac{1}{2}$P$\frac{3}{17}$M Zn, $\frac{1}{2}$p$_{+}$m$\frac{4}{7}$t Zs²e Zs²w, ...Siberia.........Ly43[4].
5. 5. t, M$\frac{4}{7}$T. $\frac{1}{2}$P$\frac{3}{17}$M Zn, $\frac{1}{2}$p$_{+}$m$_x$t Zne² Znw², ...Siberia.........Ly43[5].

35. ACMITE. Akmit. Achmite.

Cleavage = m, T, m$\frac{1}{1}\frac{8}{9}$t.

5. 5. m,T, m$\frac{1}{1}\frac{8}{9}$t. P$\frac{16}{3}$M$\frac{4}{3}$T, ... T i 479.

5. 5. m,T, m$\frac{1}{1}\frac{8}{9}$t. P$\frac{16}{3}$M$\frac{4}{3}$T, $\frac{1}{2}$p$\frac{3}{10}$m$\frac{5}{10}$t Z^2nw Z^2sw, ... M ii [186]. P152. D315.

5. 5. m,T, M$\frac{1}{1}\frac{8}{9}$T. $\frac{1}{2}$P$\frac{3}{10}$T Zw, $\frac{1}{2}$p$\frac{3}{10}$m$\frac{5}{10}$t Znw Zsw, ... Ly32[2].

5. 5. m,T, m$\frac{1}{1}\frac{8}{9}$t. $\frac{1}{2}$P$\frac{16}{3}$M$\frac{4}{3}$T Zn^2w Zs^2w, $\frac{1}{2}$p$\frac{3}{10}$m$\frac{5}{10}$t Z^2nw Z^2sw,
$\frac{1}{2}$p$\frac{16}{3}$m$\frac{4}{3}$t Zne Zse,......Ly32[3].

36. HORNBLENDE. Amphibole, *comprehending five varieties:*

1. TREMOLITE. Grammatite. Amphibole blanc.
2. ACTYNOLITE. Strahlstein. Actinote.
3. ARFVEDSONITE.
4. BASALTIC HORNBLENDE. Amphibole noir.
5. ANTHOPHYLLITE.

Cleavage = m,t, M$\frac{1}{1}\frac{0}{9}$T.

5. 3. M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn,...(Haüy's primitive form)...Model 84.
H64[68]. P55[1]. Ly29[1].

5. 3. M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, 2($\frac{1}{2}$p$_x$m$_y$t$_z$) Zse Zsw, ... Ly29[2].

5. 3. M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P^4_xM$^{15}_y$T Zne Znw, ... D309[1]. S[243]. H64[71].

5. 5. m, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$p$\frac{4}{13}$m Zn, $\frac{1}{2}$P^4_xM$^{15}_y$T Zne Znw, ... S[245]. H64[73].

5. 5. M,T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P^4_xM$^{15}_y$T Zne Znw, ... S[247]. Ly v [2]. H65[80]. M ii [77].

5. 5. m,T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$p$\frac{4}{13}$m Zn, $\frac{1}{2}$P^4_xM$^{15}_y$T Zne Znw, ... D309[2]. H65[81].

5. 5. m,T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$p$\frac{6}{13}$m Zs, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zn^2e Zn^2w,
$\frac{1}{2}$p^4_xm$^{15}_y$t Zse Zsw,............H65[83].

5. 5. m.T, M$\frac{1}{1}\frac{0}{9}$T, m$\frac{3}{1}\frac{0}{9}$t. $\frac{1}{2}$p$\frac{4}{13}$m Z^2n, $\frac{1}{2}$p$_+$m Zs^2, 3($\frac{1}{2}$p$_x$m$_y$t$_z$) Zne Znw,
3 ($\frac{1}{2}$p$_x$m$_y$t$_z$) Zse Zsw,............M ii [76].

5. 5. T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P^4_xM$^{15}_y$T Zne Znw, ...S[244]. Ly v [1]. H64[72].

5. 5. T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$P^4_xM$^{15}_y$T Zse Zsw, ... Md. 112. H64[74, 79].
M ii [74]. P55[2]. S[246]. D309[3]. Ly v [3]. J[109].

5. 5. T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M N, $\frac{1}{2}$P^4_xM$^{15}_y$T Z, ... P55[3]. J[110]. H65[78].

5. 5. (T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$P^4_xM$^{15}_y$T Zse Zsw) × 2, Md. 113. H65[75].

5. 5. T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{3}{4}$P$\frac{4}{13}$M Zn Nn Ns, $\frac{1}{4}$P^4_xM$^{15}_y$T Zse Zsw, ...J[110, 3]. H65[77].

5. 5. t, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$p$_+$m$_x$t Zne^2 Znw^2, $\frac{1}{2}$P^4_xM$^{15}_y$T Z^2se Z^2sw,
$\frac{1}{2}$p$_+$m$_x$t Zse^2 Zsw^2,............S[248]. Ly v [7]. H65[82].

5. 5. T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zn^2e Zn^2w, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zne^2 Znw^2,
$\frac{1}{2}$P^4_xM$^{15}_y$T Z^2se Z^2sw, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zse^2 Zsw^2,......Ly v [8]. H65[84].

5. 5. (T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zn^2e Zn^2w, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zne^2 Znw^2,
$\frac{1}{2}$P^4_xM$^{15}_y$T Z^2se Z^2sw, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zse^2 Zsw^2) × 2,......Ly v [9]. H65[85].

5. 5. T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zne Znw, $\frac{1}{2}$p$_x$m$_y$t$_z$ Zse Zsw, Ly29[3].

5. 5. T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, 2($\frac{1}{2}$p$_x$m$_y$t$_z$) Zse Zsw, ... Ly29[4].

5. 5. T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$p$\frac{4}{13}$m Zn, 3($\frac{1}{2}$p$_x$m$_y$t$_z$) Zne Znw, 2($\frac{1}{2}$p$_x$m$_y$t$_z$) Zse Zsw,
D309[4].

37. *An Isomorphous Group*, 1, 2:—

1. FELSPAR. Feldspath. Potash-Felspar. Common Felspar. Adularia. Orthoklas.
2. RHYACOLITE. Rhyakolith. Glassy Felspar. Eisspath.

Cleavage = T, $\frac{1}{2}m^{15}_{26}t$ ne sw, $\frac{1}{2}P\frac{1}{2}M$ Zn Ns = (Model 105.)

5. 2. T.$\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{4}{3}m$ Zs,..........................L426[3]. J[98]. H79[230]. R[103].

5. 3. M^{15}_{26}T.$\frac{1}{2}P\frac{7}{15}M$ Zs, ... H79[232]. Ly39[2].

5. 3. M^{15}_{26}T.$\frac{1}{2}P\frac{1}{2}M$ Zn,..M ii[44]. Ly39[1].

5. 3. M^{15}_{26}T.$\frac{1}{2}p\frac{1}{2}m$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs, ...Ly39[3].

5. 3. M^{15}_{26}T.$\frac{1}{2}p\frac{1}{2}m$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs, $\frac{1}{2}p\frac{1}{2}m^{26}_{15}t$ Zne Znw, ...Ly39[4].

5. 3. M^{15}_{26}T.$\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs, ... Model 81. L426[5]. J[90]. M ii[1]. H79[234].

5. 3. $\frac{1}{2}M^{15}_{26}$T ne, $\frac{1}{2}M^{15}_{26}$T nw. $\frac{1}{2}P\frac{1}{2}M$ Zn, ... Model 81[a]. L426[2]. J[91]. H79[231].

5. 5. T, $\frac{1}{2}M^{15}_{26}$T ne, $\frac{1}{2}M^{15}_{26}$T nw. $\frac{1}{2}P\frac{1}{2}M$ Zn,...................L426[1]. J[92]. H79[233].

5. 5, T, M^{15}_{26}T.$\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{7}{15}m$ Z^{2}s, $\frac{1}{2}p\frac{4}{3}m$ Zs^2, $\frac{1}{2}p{-}m_x t$ Zse^2 Zsw^2,
L426[8]. J[96]. H80[243]. R[106, 106a].

5. 5. T, M^{15}_{26}T.$\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs, ... Model 109, with n and s reversed,
J[93]. Ly39[5]. L426[4]. M ii 252[3]. H80[237].

5. 5. T, M^{15}_{26}T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{4}{3}M$ Zs, ... M ii[61]. H80[236]. Ly39[6].

5. 5. T, M^{15}_{26}T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Z^{2}s, $\frac{1}{2}p\frac{4}{3}m$ Zs^2, ...L426[7]. J[95]. H80[238].

5. 5. T, M^{15}_{26}T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs, $\frac{1}{2}p{-}m_x t$ Zne^2 Znw^2, ...H80[241].

5. 5. (T, M^{15}_{26}T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{7}{15}m$ Z^{2}s, $\frac{1}{2}p\frac{4}{3}m$ Zs^2, $\frac{1}{2}p{-}m_x t$ Zse^2 Zsw^2) × 2,
Ly40[17].

5. 5. (T, M^{15}_{26}T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{4}{3}M$ Zs) × 2, ...M ii[80]. M ii[81].

5. 5. (T, m^{15}_{26}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{4}{3}M$ Zs, $\frac{1}{2}p{-}m_x t$ Zne^2 Znw^2) × 2, ...Ly40[19].

5. 5. T, $\frac{1}{2}M^{15}_{26}$T ne sw. $\frac{1}{2}P\frac{1}{2}M$ Zn Ns, ... *Position and planes of Haüy's primitive form, the parallélépipede obliquangle, Model* 105, *having its marked planes placed as follows:* T = $P\frac{1}{2}M$ Zn; M = M^{15}_{26}T ne; P = Te, ...Puy-de-Dôme,...P116[1]. H79[229].

5. 5. t, M^{15}_{26}T. $\frac{1}{2}P\frac{7}{15}M$ Zs,..H79[235].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs,...L426[6]. J[49]. A[102]. H80[242]. Ly40[8]. R[105].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{4}{3}M$ Zs, ... H80[240].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Z^{2}s, $\frac{1}{2}p\frac{4}{3}m$ Zs^2, ... Ly40[9].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{7}m$ Z^{2}s, $\frac{1}{2}P\frac{7}{15}M$ Zs^2, ...Ly40[10].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs, $\frac{1}{2}p{-}m_x t$ Zse^2 Zsw^2,...Ly40[11]. H81[245].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Z^{2}s, $\frac{1}{2}p\frac{4}{3}m$ Zs^2, $\frac{1}{2}p{-}m_x t$ Zne^2 Znw^2,
L426[9]. H81[246]. Ly40[12].

5. 5. T, M^{15}_{26}T. m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{4}{3}M$ Zs, $\frac{1}{2}p{-}m_x t$ Zne^2 Znw^2, ...H81[244].

5. 5. T, M^{15}_{26}T, M^{26}_{15}T. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{4}{3}M$ Zs, $\frac{1}{2}P{-}M_x T$ Zne^{2}Znw^2, $\frac{1}{2}p{-}m_x t$ Zse^{2}Zsw^2,
J[97]. H81[247]. Ly40[14].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Z^{2}s, $\frac{1}{2}p\frac{4}{3}m$ Zs^2, $\frac{1}{2}p{-}m_x t$ Zne^2 Znw^2,
$\frac{1}{2}p{-}m_x t$ Zse^2 Zsw^2,............Ly40[15].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs, $\frac{1}{2}p{-}m_x t$ Zne^2 Znw^2,
$\frac{1}{2}p{-}m_x t$ Zse^2 Zsw^2,............H81[249].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs^2, $\frac{1}{2}P\frac{1}{7}M$ Z^{2}s, $\frac{1}{2}p{+}m_x t$ Zs^{2}e Zs^{2}w,
$\frac{1}{2}p{-}m_x t$ Zse^2 Zsw^2,............Ly40[16].

5. 5. T, M^{15}_{26}T, m^{26}_{15}t. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Z^{2}s, $\frac{1}{2}p\frac{4}{3}m$ Zs^2, $\frac{1}{2}p{-}m_x t$ Zne^2 Znw^2,
$\frac{1}{2}p{-}m_x t$ Zse^2 Zsw^2,............H81[250].

FELSPAR, *Continued:*

5. 5. (T, $m^{15}_{26}t$, $m^{26}_{15}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{4}{3}M$ Zs,) × 2, ...Ly40[18].
5. 5. T, $M^{15}_{26}T$, $m^{26}_{15}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{7}m$ Z^2s, $\frac{1}{2}P\frac{7}{15}M$ Zs, $\frac{1}{2}p\frac{4}{3}m$ Zs^2,
$\frac{1}{2}p_{-}m_{x}t$ Zne^2 Znw^2, $\frac{1}{2}p_{-}m_{x}t$ Zse^2 Zsw^2,.........P116[2]. H81[224].
5. 5. T, $M^{15}_{26}T$, $m^{26}_{15}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{7}m$ Z^2s, $\frac{1}{2}P\frac{7}{15}M$ Zs, $\frac{1}{2}p\frac{4}{3}m$ Zs^2,
$\frac{1}{2}p_{-}m_{x}t$ Zse^2 Zsw^2,............M ii[62].
5. 5. (T, $M^{15}_{26}T$, $m^{26}_{15}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs^2, $\frac{1}{2}p\frac{1}{7}m$ Z^2s,) × 2, ... Ly40[9].
5. 5. (T, $M^{15}_{26}T$, $m^{26}_{15}t$, $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{4}{3}M$ Zs, $\frac{1}{2}p_{-}m_{x}t$ Zne^2 Znw^2,
$\frac{1}{2}p_{-}m_{x}t$ Zse^2 Zsw^2,) × 2,............Ly40[22].
5. 5. t, $M^{15}_{26}T$, $m^{26}_{15}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{7}m$ Z^2s, $\frac{1}{2}P\frac{4}{3}M$ Zs^2, $\frac{1}{2}p_{-}m_{x}t$ Zse^2 Zsw^2, H81[248].
5. 5. m, T, $m^{15}_{26}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs, ...Ly40[7].
5. 5. m, T, $M^{15}_{26}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Z^2s, $\frac{1}{2}p\frac{4}{3}m$ Zs^2,.......................H80[229].
5. 5. m, T, $m^{15}_{26}t$, $m^{26}_{15}t$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{7}m$ Z^2s, $\frac{1}{2}P\frac{7}{15}M$ Zs^2, ... Ly40[13].

38. GLAUBERITE. Sulphate of Lime and Soda. Brongniartine.
Cleavage = $m^{9}_{10}t$. $\frac{1}{2}P\frac{4}{10}T$ Zw.

5. 3. $M^{9}_{10}T$. $\frac{1}{2}P\frac{4}{10}T$ Zw, ... H55[183].
5. 3. $M^{9}_{10}t$. $\frac{1}{2}P\frac{4}{10}T$ Zw, $\frac{1}{2}P\frac{8}{10}M\frac{9}{10}T$ Znw Zsw,............M ii 55[2]. H55[186]. L270[1].
5. 5. t, $M^{9}_{10}T$. $\frac{1}{2}P\frac{4}{10}T$ Zw, $\frac{1}{2}P\frac{8}{10}M\frac{9}{10}T$ Znw Zsw, M ii 55[3]. Ti 138. P205. L270[2].
5. 5. t, $m^{9}_{10}t$. $\frac{1}{2}P\frac{4}{10}T$ Zw, $\frac{1}{2}p_{+}t$ Ze, $\frac{1}{2}P\frac{8}{10}M\frac{9}{10}T$ Znw Zsw, $\frac{1}{2}p\frac{8}{10}m\frac{9}{10}t$ Zn^2e Zs^2e,
$\frac{1}{2}p_{+}m_{x}t$ Zne^2 Zse^2,............M ii[60].
6. 5. $\frac{1}{2}P\frac{4}{10}T$ Zw, $\frac{1}{2}P\frac{8}{10}M\frac{9}{10}T$ Znw Zsw, ...L270[3]. M ii[59]. H55[185].

39. AZURE LEAD ORE. Bleilasur. Cupreous Sulphate of Lead.
Cleavage = T, $\frac{1}{2}P\frac{1}{3}T$ Zw.

5. 5. M_{-}, t, $M^{3}_{5}T$. $\frac{1}{2}P\frac{2}{9}T$ Zw, $\frac{1}{2}p_{-}t$ Z^2e, $\frac{1}{2}p_{+}t$ Ze^2, ... Ly56[3].

40. LEADHILLITE. Sulphato-tri-carbonate of Lead.
Cleavage = $\frac{1}{2}P1\frac{1}{18}T$ Zw.

5. 3. $M^{3}_{5}T$. $\frac{1}{2}P^{-}_{-}T$ Zw, $\frac{1}{2}p_{+}t$ Ze, ...Ly57[2].
5. 3. $M^{3}_{5}T$. $\frac{1}{2}P^{-}_{-}T$ Z^2w, $\frac{1}{2}p_{-}t$ Zw^2, $\frac{1}{2}p_{+}t$ Ze, ...Ly58[5].
5. 5. $\frac{1}{2}P^{-}_{-}T$ Zw, $\frac{1}{2}P_{+}T$ Ze, $\frac{1}{2}P_{-}M_{x}T$ Zn^2w Zs^2w, ...Ly57[4].
5. 5. $M^{3}_{5}T$. $\frac{1}{2}P^{-}_{-}T$ Z^2w, $\frac{1}{2}p_{-}t$ Zw^2, $\frac{1}{2}p_{-}m_{x}t$ Zn^2w Zs^2w,.................Ly57[3].
5. 5. T, $M^{4}_{7}T$. $\frac{1}{2}P1\frac{1}{18}T$ Zw, ... A[59].
5. 5. m,t, $M^{4}_{7}T$, $m^{16}_{7}t$. $\frac{1}{2}P^{-}_{-}T$ Zw, 3($\frac{1}{2}p_{x}t$) Zw, $\frac{1}{2}p_{x}t$ Ze, 9($\frac{1}{2}p_{x}m_{y}t_{z}$) Znw Zsw,
9 ($\frac{1}{2}p_{x}m_{y}t_{z}$) Zne Zsw,............M ii[171].
5. 5. t, $m^{4}_{7}t$. $\frac{1}{2}P1\frac{1}{18}T$ Z^2w, $\frac{1}{2}p_{+}t$ Zw^2, $\frac{1}{2}p_{+}t$ Ze, $\frac{1}{2}p_{+}m_{-}t$ Znw Zsw,
$\frac{1}{2}p_{+}m_{-}t$ Zne Zse,............P359.

41. LANARKITE. Sulphato-carbonate of Lead. Dyoxilite.
Cleavage = M.

5. 5. M, $M^{4}_{7}T$. $\frac{1}{2}P_{-}M$ Zn, $\frac{1}{2}p_{-}m$ Zs, $\frac{1}{2}p_{-}m_{x}t$ Zne Znw? .... P358.

42. GAY-LUSSITE. Carbonate of Lime and Soda.

5. 5. m, $M^{2}_{3}T$. $\frac{1}{2}P_{-}M$ Zn, ... Ti 139.

43. LAUMONITE. Efflorescent Zeolite. Lomonite.

Cleavage = m,t, $M\frac{15}{16}T$. $\frac{1}{2}p\frac{2}{5}t$ Zw.

5. 3. $M\frac{15}{16}T$. $\frac{1}{2}P\frac{2}{5}T$ Zw, ... Ly43[1].
5. 3. $M\frac{15}{16}T$. $\frac{1}{2}P\frac{5}{7}T$ Zw, ... M ii[44]. A[23]. Ly43[2].
5. 3. $M\frac{15}{16}T$. $\frac{1}{2}p\frac{2}{5}t$ Zw, $\frac{1}{2}P\frac{5}{7}T$ Ze, $\frac{1}{2}p+m_xt$ Znw[2] Zsw[2], ... S[272]. P[27].
5. 5. M, $M\frac{15}{16}T$. $\frac{1}{2}P\frac{2}{5}T$ Zw, $\frac{1}{2}P\frac{5}{7}T$ Ze, ... M ii 235[2]. Ly43[3].
5. 5. M, $M\frac{15}{16}T$. $\frac{1}{2}p\frac{2}{5}t$ Zw, $\frac{1}{2}P\frac{5}{7}T$ Ze, $\frac{1}{2}p+m_xt$ Znw[2] Zsw[2], ... Ly43[4].
5. 5. m, $M\frac{15}{16}T$. $\frac{1}{2}P\frac{2}{5}T$ Zw, $\frac{1}{2}P\frac{5}{7}T$ Ze, ... H84[276].
5. 5. m, t, $M\frac{15}{16}T$. $\frac{1}{2}P\frac{2}{5}T$ Zw, $\frac{1}{2}P\frac{5}{7}T$ Ze, ... H84[277].
5. 5. m, t, $M\frac{15}{16}T$. $\frac{1}{2}P\frac{5}{7}T$ Zw, ... M ii 234[2]. S3[2].

44. MESOTYPE. Needlestone, comprehending three varieties:—

1. NATROLITE. Mesotype. $N\,Si^3 + 3A\,Si + 2\,Aq.$
2. MESOLITE. Mesole. $N\,Si^3 + 2\,C\,Si^3 + 9\,A\,Si + 8\,Aq.$
3. SCOLEZITE. Needle Zeolite. $C\,Si^3 + 3\,A\,Si + 3\,Aq.$

Cleavage = $M\frac{4}{3}\frac{9}{0}T$.

5. 3. $M\frac{4}{3}\frac{9}{0}T$. 3p_m$_x$t, ... M ii[174]. D270. A[91].
5. 3. $M\frac{4}{3}\frac{9}{0}T$. $\frac{1}{2}$p_m$_x$t Zne Znw, ... S[292].
5. 3. $M\frac{4}{3}\frac{9}{0}T$. $\frac{1}{2}$P_M$_x$T Zne Znw, $\frac{1}{2}$P_M$_x$T Zse Zsw, ... Model 67.
L205[1]. D271. S[293]. J[73]. A[92]. R[98]. H85[291]. Ly45[2].
5. 3. $M\frac{4}{3}\frac{9}{0}T$. $\frac{1}{2}$p_m$_x$t Z[2]ne Z[2]nw, $\frac{1}{2}$P_M$_x$T Zne[2] Znw[2], $\frac{1}{2}$p_m$_x$t Z[2]se Z[2]sw,
$\frac{1}{2}$P_M$_x$T Zse[2] Zsw[2], ... Ly45[2].
5. 5. M, $M\frac{4}{3}\frac{9}{0}T$. $\frac{1}{2}$p_m$_x$t Z[2]ne Z[2]nw, $\frac{1}{2}$P_M$_x$T Zne[2] Znw[2],
$\frac{1}{2}$p_m$_x$t Z[2]se Z[2]sw, $\frac{1}{2}$P_M$_x$T Zse[2] Zsw[2], ... Ly45[3].
5. 5. M, $M\frac{4}{3}\frac{9}{0}T$. $\frac{1}{2}$P_M$_x$T Zne Znw, $\frac{1}{2}$P_M$_x$T Zse Zsw, ... P124. H85[292].
Ly v[2]. S[295].
5. 5. m, $M\frac{4}{3}\frac{9}{0}T$. 5p$_x$m$_y$t$_z$, ... S[294]. P123.
5. 5. T, $M\frac{4}{3}\frac{9}{0}T$. $\frac{1}{2}$P_M$_x$T Zne Znw, $\frac{1}{2}$P_M$_x$T Zse Zsw, ... L205[2]. Ly45[3].
5. 5. t, m+t, $M\frac{4}{3}\frac{9}{0}T$. p_m, P_M$_x$T, p+m$_x$t, ... S[296]. P126.
5. 5. m, t, $M\frac{4}{3}\frac{9}{0}T$. $\frac{1}{2}$P_M$_x$T Zne Znw, $\frac{1}{2}$P_M$_x$T Zse Zsw, ... J[74]. Fuchs.

45. STILBITE. Heulandite. Foliated Zeolite. Blätterzeolith.

$C\,Si^3 + 4\,A\,Si^3 + 6\,Aq.$

Cleavage = T.

5. 2. M_, T. $\frac{1}{2}P\frac{5}{6}T$ Zw, $\frac{1}{2}p\frac{1}{2}t$ Ze, $\frac{1}{2}$p$_x$m$_y$t$_z$ Znw Zsw, $\frac{1}{2}$p_m$_x$t Z[2]ne Z[2]se,
$\frac{1}{2}p\frac{1}{2}m_xt$ Zne[2] Zse[2], ... S[241]. Ly44[5]. P25[2].
5. 2. M_, T. $\frac{1}{2}P\frac{5}{6}T$ Zw, $\frac{1}{2}p\frac{1}{2}t$ Ze, $\frac{1}{2}$p$_x$m$_y$t$_z$ Znw Zsw, ... H84[281]. A[97]. Ly44[2].
5. 2. M_, T. $\frac{1}{2}P\frac{5}{6}T$ Zw, $\frac{1}{2}p\frac{1}{2}t$ Ze, $\frac{1}{2}$p$_x$m$_y$t$_z$ Znw Zsw, $\frac{1}{2}p\frac{1}{2}m_xt$ Zne[2] Zse[2],
Ly44[3].
5. 2. M_, T. $\frac{1}{2}P\frac{5}{6}T$ Zw, $\frac{1}{2}p\frac{1}{2}t$ Ze, $\frac{1}{2}$p$_x$m$_y$t$_z$ Znw Zsw, $\frac{1}{2}$p_m$_x$t Z[2]ne Z[2]se,
Ly44[4].

46. BREWSTERITE. $(N\dot{C})\,Si^3 + 4\,A\,Si^3 + 8\,Aq.$

Cleavage = T.

5. 5. m, $M\frac{2}{3}T$. $\frac{1}{2}$P_M$_x$T Zne Znw, ... T i 348[1].
5. 5. m, T, $M\frac{2}{3}T$. $\frac{1}{2}P\frac{1}{13}M$ Zn, $\frac{1}{2}$p_m$_x$t Zne Znw, ... A[99]. P45.
5. 5. m, T, $M\frac{2}{3}T$, m+t. $\frac{1}{2}$P_M$_x$T Zne Znw, ... Ly44[2].

5. 5. m, T, $M\frac{2}{3}T$, $m_{+}t$. $\frac{1}{2}P\frac{1}{13}M$ Zn, $\frac{1}{2}P\_M_xT$ Zne Znw, ...Ly44[3].
5. 5. m, T, $m_{\infty}t$, $m\frac{2}{3}t$, $m\frac{1}{2}t$, $m\frac{4}{7}t$. $\frac{1}{2}p\frac{1}{13}m$ Zn, $\frac{1}{2}P\_M_xT$ Zne Znw, ... S[77].

47. Datholite. Datolith. Humboldtite. Borate of Lime. Chaux boratée siliceuse. Borosilicate of Lime.

Cleavage = $m\frac{4}{3}t$, (but that of Humboldtite = t, Levy.)

3. 5. P, m, $M\frac{4}{3}T$. $p_{+}m$, $p_{+}m_xt$ Zn^2w,......................................Ly14[2].
3. 5. $P\sqsupset$, t, $m\frac{4}{3}t$, $m_{+}t$. $p_{+}m$, $p_{+}m_xt$ Zn^2w, ...Ly14[3].
3. 5. P, m, $M\frac{4}{3}T$. $p_{+}m$, p_m, $p_{+}m_xt$ Zn^2w, $p\_m\frac{1}{9}\frac{5}{9}t$ Znw^2,............Ly14[4].

According to Levy, the above right rhombic or prismatic combinations are Datholite, and the following oblique or hemi-prismatic combinations, Humboldtite. Other mineralogists state, that all the combinations of Datholite contain oblique forms.

5. 5. M, $M\frac{8}{9}T$. $\frac{1}{2}p\sqsupset m$ Z^2n, $\frac{1}{2}P\_M$ Zn^2, $4(\frac{1}{2}p_xm_yt_z)$ Zne^2 Znw^2, $3(\frac{1}{2}p_xm_yt_z)$ Zse^2 Zsw^2,.........Ly15[2].
5. 5. M, $M\frac{8}{9}T$. $\frac{1}{2}P\_M$ Zn, $3(\frac{1}{2}p_xm_yt_z)$ Zne^2 Znw^2, $3(\frac{1}{2}p_xm_yt_z)$ Zse^2 Zsw^2, Ly15[3].
5. 5. $M\frac{4}{3}T_j$ $M\frac{8}{9}T$. $P\frac{8}{9}M$ Z^2n, $p\frac{1}{1}\frac{5}{6}m$, $p\frac{4}{5}m$ Zn^2, $2(\frac{1}{2}p_xm_yt_z)$ Znw Zsw, $7(\frac{1}{2}p_xm_yt_z)$ Zse Zsw,.........M ii[71].
5. 5. $M\frac{4}{3}T$, $m\frac{8}{9}t$. $p\frac{8}{9}m$, $\frac{1}{2}P\frac{1}{34}T$ Z^2w, $\frac{1}{2}P\frac{4}{5}T$ Zw^2, $\frac{1}{2}P_xM_yT_z$ Znw Zsw, $\frac{1}{2}p_xm_yt_z$ Zne Zse,.........M ii[68].
5. 5. m, $M\frac{4}{3}T$, $M\frac{8}{9}T$. $p\frac{4}{5}m$ Zn^2, $P\frac{8}{9}M$ Z^2n, $\frac{1}{2}P\frac{1}{34}T$ Z^2w, $\frac{1}{2}P\frac{4}{5}T$ Zw^2, $2(\frac{1}{2}p_xm_yt_z)$ Znw Zsw, $3(\frac{1}{2}p_xm_yt_z)$ Zne Zse,.........M ii[69].
5. 5. t, $M\frac{4}{3}T$, $M\frac{8}{9}T$. $p\frac{4}{5}m$, $\frac{1}{2}P\frac{1}{34}T$ Z^2w, $\frac{1}{2}p\frac{4}{5}t$ Zw^2, $\frac{1}{2}p_xm_yt_z$ Znw Zsw, M ii[67]. S[156]. D284[1]. A[84].

48. Red Iron Vitriol. Rother Vitriol. Botryogen.

Cleavage = $M\frac{1}{2}\frac{4}{6}T$.

5. 3. $M\frac{1}{2}\frac{4}{6}T$, $M\frac{1}{1}\frac{4}{3}T$. $\frac{1}{2}P\_M$ Zn, $\frac{1}{2}p\_m_xt$ Zne Znw, ...P233. A[31].

49. Johannite. Uran Vitriol. Urane sulfaté.

5. 3. $M\frac{1}{2}T$. $\frac{1}{2}p\_m_xt$ Zne Znw,....................................P271. A[172].

50. Graphic Tellurium. Schrifterz. Schrift-Tellur.

Cleavage = p,m, $p\frac{11}{8}t$.

3. 5. p, M, T, $m\frac{4}{3}t$, $m\frac{2}{3}t$. $P\frac{11}{8}M$, $3p_xm_yt_z$, ...S[216]. D416. L690. P341.
3. 5. p, M, T, M_xT. P_T. $4p_xm_yt_z$,...M ii[35].

51. Flexible Sulphuret of Silver. Argent sulfuré flexible. Beigsamer Silberglanz.

Cleavage = T.

5. 5. M, T, $m\frac{1}{2}t$, mt. $\frac{1}{2}P\frac{7}{10}M$ Z^2n, $\frac{1}{2}p\frac{1.5}{5}m$ Zn^2, $3(\frac{1}{2}p_xm_yt_z)$ Zne Znw, L779[2]. P297.
5. 5. M, T, $m\frac{1}{2}t$, mt. $\frac{1}{2}P\frac{7}{10}M$ Z^2n, $\frac{1}{2}p_xm_yt_z$ Zne Znw,..................L779[1].

52. Humite. Cleavage = t.

3. 5. P, M, T, $m\frac{4}{21}t$, $m\frac{5}{21}t$, $m\frac{10}{21}t$, $M\frac{1}{2}\frac{2}{1}T$, $M\frac{8}{9}\frac{2}{1}T$, $m\frac{8}{9}\frac{2}{1}t$, $m\frac{8}{9}\frac{8}{1}t$. $P\frac{7}{13}M$, $P\frac{1}{2}T$, $p\frac{8}{3}t$, $11p\_m_xt$, $3p_{+}m_xt$,.........P89[2].

3. 5. P,M,T, 4m_xt. P½M, 2p_m_xt, ... Ly46[2].
3. 5. P,M,T, 5m_xt. P½M, 7p_m_xt, ... Ly46[3].

53. MONAZITE. Mengite. Lanthanite.
5. 3. M$\frac{1}{1}\frac{9}{1}$T. ½P$\frac{1}{1}\frac{4}{3}$M Zn, ... D448. T i 672.

54. TURNERITE. Pictite. Cleavage = m,T.
5. 5. t, m$\frac{9}{20}$t, M$\frac{9}{10}$T, m$\frac{2}{3}$t. ½P¼M Z^2n, ½P_M Zn^2, ½P_M Zs, ½P_M$\frac{9}{10}$T
Zn^2e Zn^2w, 5(½p_m_xt) Zne^2 Znw^2, 2P_xM$\frac{9}{10}$T Zs^2e Zs^2w,
½p$_+$m$\frac{2}{3}$t Zse^2 Zsw^2, ... P84[2].
5. 5. T, M$\frac{9}{20}$T, M$\frac{9}{10}$T, m$\frac{2}{3}$t. ½P¼M Z^2n, ½p_m Zs, 3p_m_xt Zne Znw,
p_m_xt Zse Zsw, ... Ly82[2].

CLASS VI.—MINERALS BELONGING TO THE DOUBLY OBLIQUE PRISMATIC SYSTEM OF CRYSTALLISATION.

The AXES of all Combinations belonging to this Class are = $p_x^a m_y^a t_z^a$.
The constituent FORMS of the Combinations of this Class are as follow :—

Homohedral Forms, ... M,T, M_xT.
Hemihedral Forms, ... ½M_xT.
Tetartohedral Forms, ... ¼$P_x M_y T_z$.

Every form consists of a pair of parallel planes, and none of the forms ever meet at a right angle. The combinations generally contain from 3 to 8 pair, but sometimes as many as 12 pair of planes, and always such as belong to the series—M,T, M_xT, ½M_xT. ¼$P_x M_y T_z$. Finally, there is, in every separate combination, at least two pair of planes of the prismatic zone, and one pair of scalene tetarto-octahedral planes.

1. BORACIC ACID. Sassolin. Acide boracique.
5. 5. t, ½m_t nw, ½m_t ne. ¼$P_x M_y T_z$ Znw.

2. DIASPORE. Diaspor. Cleavage = m$\frac{2}{3}$t nw.
5. 5. t, ½M$\frac{2}{3}$T nw, ½M$\frac{2}{3}$T ne. ¼P_M_xT Zsw, 3(¼$p_x m_y t_z$) Zne ?
Phillips, Annals Phil. July, 1822, p. 17.

3. CYANITE. Kyanite. Sappare. Disthène.
Cleavage = M, ½M$\frac{7}{2}$T nw. ¼p'_m_xt Znw.
M on ½M$\frac{7}{2}$T nw = 106° 6′. ¼P_M_xT Znw on M = 106° 55′, on ½M$\frac{7}{2}$T nw = 94° 38′. Haüy.
1. 5. P$_+$,M, ½m$\frac{7}{10}$t n^2w, ½M$\frac{7}{2}$T nw^2, ½m$\frac{2}{3}$t ne, ... (A right prism)...H63[60].
5. 5. M, ½M$\frac{7}{2}$T nw^2. ¼P_M_xT Z^2nw, ... M ii [38].
5. 5. M, ½M$\frac{7}{2}$T nw, ½M$\frac{7}{3}$T ne. ¼P_M_xT Z^2nw, ... L407[1]. Ly29[2].
5. 5. M, ½M$\frac{7}{2}$T nw, ½M$\frac{7}{3}$T ne. ¼P_M_xT Z^2sw, ... L407[1]. H63[67].
5. 5. M, ½m$\frac{7}{3}$t n^2w, ½M$\frac{17}{2}$T nw^2, ½m$\frac{8}{3}$t ne. ¼P_M_xT Z^2nw, ... P74.

5\. 5. M, $\frac{1}{2}m\frac{7}{10}t\ n^2w$, $\frac{1}{2}M\frac{7}{8}T nw^2$, $\frac{1}{2}m\frac{7}{8}t$ ne. $\frac{1}{4}P\_M_xT\ Z^2nw$,...Sim. Md. 107. Ly29[3]. H63[56]. S[206]. L407[2].

5\. 5. (M, $\frac{1}{2}m\frac{7}{10}t\ n^2w$, $\frac{1}{2}M\frac{7}{8}T\ nw^2$, $\frac{1}{2}m\frac{7}{8}t$ ne. $\frac{1}{4}P\_M_xT\ Z^2nw$) $\times$ 2, Ly29[6]. H63[59]. L407[4].

5\. 5. M, $\frac{1}{2}m\frac{7}{18}t\ n^2w$, $\frac{1}{2}m\frac{7}{10}t$ nw, $\frac{1}{2}M\frac{7}{8}T\ nw^2$, $\frac{1}{2}m\frac{7}{8}t$ ne. $\frac{1}{4}P\_M_xT\ Z^2nw$, Ly29[4]. H63[61].

5\. 5. M, $\frac{1}{2}m\frac{7}{10}t\ n^2w$, $\frac{1}{2}M\frac{7}{8}T\ nw^2$, $\frac{1}{2}m\frac{7}{8}t$ ne. $\frac{1}{4}P\_M_xT$ Zne, $\frac{1}{4}p\_m_xt$ Znw, $\frac{1}{4}P\_M_xT$ Zsw, $\frac{1}{4}p\_m_xt$ Zse,.........H63[60].

5\. 5. M, $\frac{1}{2}m\frac{7}{10}t\ n^2w$, $\frac{1}{2}M\frac{7}{8}T\ nw^2$, $\frac{1}{2}m\frac{7}{8}t$ ne. $\frac{1}{4}P\_M_xT\ Z^2nw$, $\frac{1}{4}P{+}M_xT\ Z^2ne$, Ly29[5].

4\. **Blue Vitriol.** Kupfervitriol. Cuivre sulfaté. Sulphate of Copper.

Cleavage $=$ $\frac{1}{2}m\frac{1}{2}t$ ne, $\frac{1}{2}m\frac{3}{5}t$ nw. $\frac{1}{4}p\_m_xt$ Znw.

$\frac{1}{4}P\_M_xT$ Znw on $\frac{1}{2}M\frac{1}{2}T$ ne $= 109° 32'$, on $\frac{1}{2}M\frac{3}{5}T$ nw $= 128° 37'$.

$\frac{1}{2}M\frac{1}{2}T$ ne on $\frac{1}{2}M\frac{3}{5}T$ nw $= 124° 2'$. Haüy.

5\. 3. $\frac{1}{2}M\frac{1}{2}T$ ne, $\frac{1}{2}M\frac{3}{5}T$ nw. $\frac{1}{4}P\_M_xT$ Znw,...(Haüy's Primitive) H102[153].

5\. 5. M, $\frac{1}{2}M\frac{1}{2}T$ ne, $\frac{1}{2}M\frac{3}{5}T$ nw. $\frac{1}{4}P\_M_xT$ Znw, ...H102[155].

5\. 5. M, $\frac{1}{2}M\frac{1}{2}T\ n^2e$, $\frac{1}{2}m_6t\ ne^2$, $\frac{1}{2}M\frac{3}{5}T$ nw. $\frac{1}{4}P\_M_xT$ Znw, ...H103[156].

5\. 5. M, $\frac{1}{2}M\frac{1}{2}Tn^2e$, $\frac{1}{2}m_6t\ ne^2$, $\frac{1}{2}M\frac{3}{5}Tn^2w$, $\frac{1}{2}m\frac{3}{5}t\ nw^2$. $\frac{1}{4}P\_M_xT$ Znw, H103[157].

5\. 5. M, $\frac{1}{2}M\frac{1}{2}T\ n^2e$, $\frac{1}{2}m_6t\ ne^2$, $\frac{1}{2}M\frac{3}{5}T$ nw. $\frac{1}{4}P\_M_xT$ Znw, $\frac{1}{4}p\_m_xt$ Zsw, H103[158].

5\. 5. M, $\frac{1}{2}M\frac{1}{2}T\ n^2e$, $\frac{1}{2}m_6t\ ne^2$, $\frac{1}{2}M\frac{3}{5}T$ nw. $\frac{1}{4}P\_M_xT\ Z^2nw$, $\frac{1}{4}p\_m_xt\ Znw^2$, $\frac{1}{4}P\_M_xT\ Zs^2w$, $\frac{1}{4}p{+}m_xt\ Zne^2$,.........H103[159].

5\. **Latrobite.** Diploite. Cleavage $=$ m, $\frac{1}{2}m_{16}t\ nw^2$. $\frac{1}{4}p\_m_xt\ Z^2nw$.

5\. 5. T, $\frac{1}{2}M\frac{1}{16}T$ nw. $\frac{1}{4}P\_M_xT\ Z^2nw$? .... P118.

6\. *An Isomorphous Group, comprehending four varieties:*

1. **Anorthite.** MS + 2 CS + 8 AS.
2. **Labradorite.** Labrador. Labrador Felspar.
3. **Oligoclase.** Oligoklas. Natronspodumen.
4. **Albite.** Soda-Felspar. Cleavelandite. Pericline.

Cleavage $=$ T, $\frac{1}{2}M\frac{4}{7}T$ ne sw. $\frac{1}{2}P_xM_yT_z\ Z^2s^2e\ N^3n^2w$.

[$P_x M_y T_z\ Z^3s^2e$ upon Te $= 93° 50'$.]

5\. 5. T, $M\frac{4}{7}T$. $\frac{1}{4}p_xm_yt_z\ Znw^2$, $\frac{1}{4}p_xm_yt_z\ Z^2ne$, $\frac{1}{4}P_xM_yT_z\ Z^2s^2e$, ... M ii[84].

5\. 5. T, $m\frac{4}{7}t$, m_+t. $\frac{1}{4}P_xM_yT_z\ Znw^2$, $\frac{1}{4}p_xm_yt_z\ Z^2nw$, $\frac{1}{4}P_xM_yT_z\ Z^2ne$, $\frac{1}{4}p_xm_yt_z\ Zn^2e$, $\frac{1}{4}P_xM_yT_z\ Z^3s^2e$, $\frac{1}{4}p_xm_yt_z\ Zsw^2$,......M ii[85].

5\. 5. T, $m\frac{4}{7}t$, m_+t. $\frac{1}{4}P_xM_yT_z\ Znw^2$, $\frac{1}{4}p_xm_yt_z\ Z^2ne$, $\frac{1}{4}p_xm_yt_z\ Zne^2$, $\frac{1}{4}P_xM_yT_z\ Z^2n^3e$, $\frac{1}{4}P_xM_yT_z\ Z^3n^2e$, $\frac{1}{4}p_xm_yt_z\ Zsw^2$,......M ii[86].

5\. 5. (T, $M\frac{4}{7}T$, M_+T. $\frac{1}{4}P_xM_yT_z\ Znw^2$, $\frac{1}{4}p_xm_yt_z\ Z^2ne$, $\frac{1}{4}P_xM_yT_z\ Z^3s^2e$) $\times$ 2, M ii[87].

5\. 5. (T, $M\frac{4}{7}T$. $\frac{1}{4}P_xM_yT_z\ Z^2ne$, $\frac{1}{4}P_xM_yT_z\ Z^3se$) $\times$ 2, ... M ii[88].

Examples of Albite, G. Rose, *Gilbert's Annalen der Physik*, 1823.

5\. 5. T, $\frac{1}{2}M\frac{4}{7}T$ nw, $\frac{1}{2}M\frac{4}{7}T$ ne. $\frac{1}{4}P_xM_yT_z$ Znw, $\frac{1}{4}p_xm_yt_z$ Zse, $\frac{1}{4}p_xm_yt_z$ Zsw, Rose 3[18, 19].

5. 5. $(T, M\frac{4}{7}T. \frac{1}{4}P_xM_yT_z$ Znw, $\frac{1}{4}p_xm_yt_z$ Zse, $\frac{1}{4}P_xM_yT_z$ Zsw) $\times$ 2, R 3[20, 21].

5. 5. $[T, M\frac{4}{7}T. \frac{1}{4}P_xM_yT_z$ Znw, $\frac{1}{4}p_xm_yt_z$ Zne, 3($\frac{1}{4}p_xm_yt_z$ Zse)] $\times$ 2, R3[22].

5. 5. $(T, M\frac{4}{7}T, m_+t. \frac{1}{4}p_xm_yt_z$ Znw, $\frac{1}{4}p_xm_yt_z$ Zne, $\frac{1}{4}p_xm_yt_z$ Z^2se,
$\frac{1}{4}P_xM_yT_z$ Zse^2) $\times$ 2,.........R3[24].

5. 5. $[T, M\frac{4}{7}T, m_+t. \frac{1}{4}P_xM_yT_z$ Znw, 2($\frac{1}{4}p_xm_yt_z$ Zne), 3($\frac{1}{4}p_xm_yt_z$ Zse)] $\times$ 2,
R3[25].

Examples of Anorthite, G. Rose, *Gilbert's Annalen der Physik*, 1823.

5. 3. T, $\frac{1}{2}M\frac{4}{8}T$nw, $\frac{1}{2}M\frac{5}{8}T$ne. $\frac{1}{4}p_xm_yt_z$ Znw^2, $\frac{1}{4}P_xM_yT_z$ Z^2ne, $\frac{1}{4}p_xm_yt_z$ Zn^{2}e,
$\frac{1}{4}p_xm_yt_z$ Zne^2, $\frac{1}{4}p_xm_yt_z$ Zs^{2}e^2, $\frac{1}{4}p_xm_yt_z$ Zs^{2}e, $\frac{1}{4}p_xm_yt_z$ Zs^{2}w^2, R3[30, 31].

5. 3. T, $\frac{1}{2}M\frac{4}{8}T$ nw, $\frac{1}{2}M\frac{5}{8}T$ ne. $\frac{1}{4}p_xm_yt_z$ Znw^2, $\frac{1}{4}P_xM_yT_z$ Z^2ne,
3($\frac{1}{4}p_xm_yt_z$ Zne), 4($\frac{1}{4}p_xm_yt_z$ Zse), 2($\frac{1}{4}p_xm_yt_z$ Zsw), ... R3[32, 33].

5. 5. T, $\frac{1}{2}M\frac{4}{8}T$ nw, $\frac{1}{2}M\frac{5}{8}T$ ne. $\frac{1}{4}p_xm_yt_z$ Znw^2, $\frac{1}{4}P_xM_yT_z$ Z^2ne,
$\frac{1}{4}p_xm_yt_z$ Zn^{2}e, $\frac{1}{4}p_xm_yt_z$ Zne^2, ...R3[28, 29].

5. 5. T, $\frac{1}{2}M\frac{4}{8}T$ n^{2}w, $\frac{1}{2}m_+t$ nw^2, $\frac{1}{2}M\frac{5}{8}T$ n^{2}e, $\frac{1}{2}m_+t$ ne^2, $\frac{1}{4}p_xm_yt_z$ Znw^2,
$\frac{1}{4}P_xM_yT_z$ Z^2ne, $\frac{1}{4}p_xm_yt_z$ Zne^2, 4 ($\frac{1}{4}p$ m_yt_z Zse), 3($\frac{1}{4}p_xm_yt_z$ Zsw),
R3[34].

Since the article on Felspar, page 87, was sent to press, I have been furnished, by the kindness of Dr. T. Thomson, with the volume of Gilbert's Annalen der Physik which contains G. Rose's account of the distinctions between the felspathic minerals. The measurements of Weiss, quoted in that article, give simpler formulæ for the felspar forms than do the measurements of Haüy and Phillips, which I had employed in the calculations. For example, the form $\frac{1}{2}P\frac{7}{13}M$ should be $\frac{1}{2}P\frac{1}{2}M$ (similar to the cleavage plane,) and, upon the same authority, the form $\frac{1}{2}P\frac{1}{7}M$ shoud be $\frac{1}{2}P\frac{1}{6}M$.

Distinctions between Felspar, Albite, and Anorthite.—1.) In the Prisms: Felspar $M\frac{1}{2}\frac{5}{6}T$ = 120° north angle, Albite $M\frac{4}{7}T$ = 121° north angle, Anorthite $\frac{1}{2}M\frac{4}{8}T$ nw, $\frac{1}{2}M\frac{5}{8}T$ ne = 121° north angle. The planes ne nw of Albite have the same inclination upon the two planes of T, but those of Anorthite have different inclinations. 2.) In the Pyramids: The chief terminal planes of Felspar are $\frac{1}{2}P_xM$ ZnNs, of Albite $\frac{1}{2}P_xM_yT_z$ Znw Nse, of Anorthite $\frac{1}{2}P_xM_yT_z$ Zne Nsw. 3.) In the Cleavage: The two principal cleavages of Felspar produce an edge that has an angle of 90°, but the edge produced by the principal cleavages of Albite has an angle of 93° 50′.

7. Petalite. Silicate of Alumina and Lithia.

Cleavage = m, $m\frac{3}{4}t$.

5. 3. $M\frac{3}{4}T. \frac{1}{4}P_xM_yT_z$ Znw ?

8. Axinite. Thumerstone.

Cleavage = $\frac{1}{2}m\frac{2}{3}t$ ne. $\frac{1}{2}p_-m_xt$ Z^3nw, $\frac{1}{2}p_-m_xt$ Z^3se.

5. 3. $M\frac{2}{3}T. \frac{1}{4}P_xM_yT_z$ Z^2nw, $\frac{1}{4}p_xm_yt_z$ Z^2ne^2, ...Ly34[2].

5. 3. $M\frac{2}{3}T. \frac{1}{4}P_xM_yT_z$ Z^2nw, $\frac{1}{4}P_xM_yT_z$ Zn^{2}e^2, $\frac{1}{4}p_xm_yt_z$ Z^2ne^2, ... Model 81[b],
with marked letters altered,......H73[167]. Ly34[3].

5. 3. $M\frac{2}{3}T. \frac{1}{4}P_xM_yT_z$ Z^2nw, $\frac{1}{4}p_xm_yt_z$ Zn^{2}w^2, $\frac{1}{4}p_xm_yt_z$ Zn^{2}e^2, H73[169]. Ly35[4].

5. 3. $M\frac{2}{3}T, \frac{1}{2}m_xt$ ne^2. $\frac{1}{4}P_xM_yT_z$ Z^2nw, $\frac{1}{4}p_xm_yt_z$ Zn^{2}w^2, $\frac{1}{4}p_xm_yt_z$ Zn^{2}e^2,Ly35[5].

AXINITE, *Continued:*

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ \frac{1}{4}p_xm_yt_z\ Z^2ne^2,$ M ii [80]. Ly35[6].

5\. 3. $M\frac{2}{3}T,\ \frac{1}{3}m_xt\ ne^2.\ \frac{1}{4}P_xM\ T_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ \frac{1}{4}p_xm_yt_z\ Z^2ne^2,$............Ly35[7].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}p_xm_yt_z\ Zn^2e^2,$ ....H73[166].

5\. 3. $M\frac{2}{3}T,\ \frac{1}{2}m_xt\ nw^2.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}p_xm_yt_z\ Zn^2w^2,$ .... H73[170].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ 2(\frac{1}{4}p_xm_yt_z\ Z^2ne^2)$............Ly35[8].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ 2(\frac{1}{4}p_xm_yt_z\ Z^2ne^2)$ ... H74[171].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ \frac{1}{4}p_xm_yt_z\ Z^2ne^2,\ \frac{1}{4}p_xm_yt_z\ Zse^2,$.........Ly35[9].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ \frac{1}{4}p_xm_yt_z\ Z^2ne^2,\ \frac{1}{4}p_xm_yt_z\ Zsw^2,$ ...... Ly35[10].

5\. 3. $M\frac{2}{3}T,\ \frac{1}{2}m_xt\ nw^2.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ 2(\frac{1}{4}p_xm_yt_z\ Z^2ne^2,)$.........Ly35[11].

5\. 3. $M\frac{2}{3}T,\ \frac{1}{3}m_xt\ ne^2.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ \frac{1}{4}p_xm_yt_z\ Z^2ne^2,\ \frac{1}{3}p_xm_yt_z\ Zsw^2,$.........Ly35[12].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ 2(\frac{1}{4}p_xm_yt_z\ Z^2ne^2),\ \frac{1}{4}p_xm_yt_z\ Zse,$.........Ly35[13].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ \frac{1}{4}p_xm_yt_z\ Z^2ne^2,\ \frac{1}{4}p_xm_yt_z\ Zse^2,\ \frac{1}{4}p_xm_yt_z\ Zsw^2,$.........Ly35[14].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}p_xm_yt_z\ Z^3nw,\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}p_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}p_xm_yt_z\ Zn^2e^2,\ \frac{1}{4}p_xm_yt_z\ Z^2ne^2,\ \frac{1}{4}p_xm_yt_z\ Z^3se,\ \frac{1}{4}p_xm_yt_z\ Zse^2,$ Ly35[15].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{4}p_xm_yt_z\ Z^3nw,\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ 2(\frac{1}{4}p_xm_yt_z\ Z^2ne^2),\ \frac{1}{4}p_xm_yt_z\ Zse^2,\ \frac{1}{4}p_xm_yt_z\ Zsw^2,$...Ly35[16].

5\. 3. $M\frac{2}{3}T,\ \frac{1}{3}m_xt\ ne^2.\ \frac{1}{4}p_xm_yt_z\ Z^3nw,\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ 2(\frac{1}{4}p_xm_yt_z\ Z^2ne^2),$.........Ly35[16].

5\. 3. $M\frac{2}{3}T,\ \frac{1}{3}m_xt\ nw^2.\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}p_xm_yt_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ \frac{1}{4}P_xM_yT_z\ Z^2ne^2,\ \frac{1}{4}p_xm_yt_z\ Zse^2,\ \frac{1}{4}p_xm_yt_z\ Zsw^2,$.........Ly35[17].

5\. 3. $M\frac{2}{3}T,\ \frac{1}{3}m_xt\ ne^2,\ \frac{1}{3}m_xt\ nw^2.\ \frac{1}{4}p_xm_yt_z\ Z^3nw,\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ \frac{1}{4}P_xM_yT_z\ Zn^2e^2,\ 2(\frac{1}{4}p_xm_yt_z\ Z^2ne^2),\ \frac{1}{4}p_xm_yt_z\ Zse^2,\ \frac{1}{4}p_xm_yt_z\ Zsw^2,$ .... Ly35[19].

5\. 3. $M\frac{2}{3}T.\ \frac{1}{3}m_xt\ ne^2,\ 2(\frac{1}{2}m_xt\ nw^2).\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ 3(\frac{1}{4}p_xm_yt_z\ Zne),\ \frac{1}{4}p_xm_yt_z\ Zse^2,\ \frac{1}{4}p_xm_yt_z\ Zsw^2,$ ......L36[20].

5\. 3. $M\frac{2}{3}T,\ 3(\frac{1}{3}m_xt).\ \frac{1}{4}p_xm_yt_z\ Z^3nw,\ \frac{1}{4}P_xM_yT_z\ Z^2nw,\ \frac{1}{4}P_xM_yT_z\ Zn^2w^2,\ 4(\frac{1}{4}p_xm_yt_z\ Zne),\ \frac{1}{4}p_xm_yt_z\ Zse^2,\ \frac{1}{4}p_xm_yt_z\ Zsw^2,\ \frac{1}{4}p_xm_yt_z\ Z^2sw,$ Ly36[22].

9\. BABINGTONITE.

Cleavage $=$ $m,\ \frac{1}{2}m\frac{12}{3}t\ nw^2.\ \frac{1}{4}P\_M_x\ T\ Z^2ne.$

5\. 5. $m,\ \frac{1}{2}m\frac{14}{13}t\ n^2w,\ \frac{1}{2}m\frac{12}{3}t\ nw^2,\ \frac{1}{2}M\frac{1}{10}T\ ne.\ \frac{1}{4}P\_M_x\ T\ Z^2ne,\ \frac{1}{4}p_+m_xt\ Zn^2e,$ Ly30[2]. P53.

Section III.

A SYSTEMATIC ARRANGEMENT OF THE CRYSTALS FOUND IN THE MINERAL KINGDOM, WITH A LIST OF THE MINERALS COMMON TO EACH CRYSTAL.

The Crystals comprised in this Catalogue are divided into Six Classes, and each Class into Five Orders, agreeably to the principles laid down in Part I., Section IV., page 71.

Classes:

1. Complete Prisms.
2. Complete Pyramids.
3. Complete Prisms combined with Incomplete Pyramids.
4. Incomplete Prisms combined with Complete Pyramids.
5. Incomplete Prisms combined with Incomplete Pyramids.
6. Incomplete Pyramids.

Orders common to every Class:

1. Square Equator.
2. Rectangular Equator.
3. Rhombic Equator.
4. Rhombo-Quadratic Equator.
5. Rhombo-Rectangular Equator.

The Orders are divided into Genera, and these into smaller groups, which differ in number according to the extent of each Genus.

Explanation of the Mineralogical Characters employed to discriminate the Minerals that Crystallise in the same Forms.

The characters employed are of four kinds, which are arranged in four separate columns after the names of the Minerals, and serve to denote,

1.) The Lustre and degree of Transparency.
2.) The Hardness.
3.) The Colour of the Streak.
4.) The Specific Gravity.

1) *Signs denoting the Lustre and Transparency:*

m signifies metallic lustre; opaque.
io — imperfect metallic lustre; opaque.
itl — imperfect metallic lustre; translucent.

When the lustre is non-metallic, it is not described, and the degree of transparency alone is then attended to, in which case,

o signifies opaque.
tp — transparent.
tl — translucent.

2.) *Signs denoting the Hardness:*

The scale of MOHS is adopted, and the hardness is expressed in numbers. When the hardness of a mineral is variable, the middle term is taken. Thus, 6½ means hardness varying from 6 to 6½. The sign which expresses hardness follows immediately that which expresses lustre or transparency.

MOHS'S SCALE OF HARDNESS.*

1. Talc.	6. Adularia.
2. Gypsum.	7. Rock Crystal.
3. Calc Spar.	8. Topas.
4. Fluor Spar.	9. Corundum.
5. Apatite.	10. Diamond.

" Numerous experiments of determining the degree of hardness, by the mere scratching of one substance with the other, have completely established, that this process alone is not sufficient, if we intend to make a more sure and extensive application of the characters that may be taken from hardness, than that which has hitherto been common in Mineralogy.

" But if we take several specimens of one and the same mineral, and pass them over a fine file, we shall find that an equal force will everywhere produce an equal effect, provided that the parts of the mineral in contact with the file be of a similar size, so that the one does not present to the file a very sharp corner, while the other is applied to it by a broad face. It is necessary, also, that *the force applied in this experiment, be always the least possible.*

" Every person, however little accustomed, will experience a very marked difference, if comparatively trying in this way any two subsequent members of the above scale, and thus the difference in their hardness will be easily perceived. A short practice is sufficient for rendering these perceptions more delicate and perfect, so that in a short time it is possible to determine differences in the hardness very much less than those between two subsequent members of the scale.

" Upon these observations is founded the application of the scale, the general principle of which consists in this, that the degree of hardness of the given mineral is compared with the degrees of hardness of the members of the scale, not immediately, by their mutual scratching, but mediately, *through the file*, and determined accordingly.

" The process of this determination is as follows:

" First, we try, with a corner of the given mineral, to scratch the members of the scale, beginning from above, in order that we may not waste unnecessarily the specimens representing lower members. After having thus arrived at the first, which is distinctly scratched by the given mineral, we have recourse to the file, and compare upon it the hardness of this degree, that of the next higher degree, and of the given mineral. Care must be taken to employ specimens of each of them nearly agreeing in form and size, and also as much as possible in the quality of their angles. From the resistance these bodies oppose to the file, and from the noise occasioned by their passing over it, we argue with perfect security upon their mutual relations in respect to hardness. The experiment is repeated with all the alterations thought necessary, till we may consider ourselves arrived at a fair estimate, which is at last expressed by the number of that degree with which it has been found to agree nearest, the decimals being likewise added, if required.

" The files answering best for the purpose are fine and very hard ones. Their absolute hardness is of no consequence; hence every file will be applicable whose hardness is in the necessary relation with that of the mineral. For it is not the hardness of the file with which we have to compare that of the minerals, but the hardness of another mineral by the medium of the file. From this observation it appears, that the application of the file widely differs from the methods of determining the hardness of minerals which have hitherto been in use, as scratching glass, striking fire with steel, cutting with a knife. scratching with the nail, &c."—*Mohs*, i. 305.

* Small cabinets containing suitable specimens of all these minerals, excepting the diamond, with a file for making the trials, are prepared for sale in Germany, and may be procured of R. GRIFFIN & Co. in Glasgow, Price 16s.

3.) *The colour of the streak.*

When the hardness of a mineral is determined by means of a file, a fresh surface is produced, which shows the colour of the mineral in powder while untarnished by contact with the air. This is the *streak.* The character is highly trustworthy, and is easily ascertained without injury to the specimens.

w	signifies	white.	br	signifies	brown.
gr	—	grey.	y	—	yellow.
bl	—	blue.	gn	—	green.
bk	—	black.	r	—	red.

Combinations of these letters indicate a mixed colour, or one colour passing into another. Thus, grw signifies greyish white; ry, reddish yellow; brbk, brownish black; blbk, blueish black, &c.

4.) *The specific gravity.*

This character is given in numbers, the standard or unity of which is water = 1.

Arrangement of the Signs.—The minerals which have a metallic lustre are placed first, those with an imperfect metallic lustre next, and those with a non-metallic lustre at the end of each list. Then the soft minerals are placed above the hard varieties, and the light minerals above the heavy varieties.

The fractions which, in many cases, are appended to the names of minerals, are the characteristics of the crystallographic symbols that are quoted in the descriptions prefixed to the groups.—See Class 1, Order 3, Genus 1.

CLASS I.—COMPLETE PRISMS.

Order 1. *Square Equator,*	Genus 1. Axes: $p^a\ m^a\ t^a$. Genus 2. Axes: $p^a_x\ m^a\ t^a$.
Order 2. *Rectangular Equator,*	Genus 1. Axes: $p^a_x\ m^a_y\ t^a_z$.
Order 3. *Rhombic Equator,*	Genus 1. Axes: $p^a_x\ m^a_y\ t^a_z$.
Order 4. *Rhombo-Quadratic Equator,*	Genus 1. Axes: $p^a_x\ m^a\ t^a$.
Order 5. *Rhombo-Rectangular Equator,* ...	Genus 1. Axes: $p^a_x m^a_{15} t^a_{13}$. Genus 2. Axes: $p^a_x m^a_{14} t^a_{13}$. Genus 3. Axes: $p^a_x m_y\ t^a_z$.

CLASS 1. ORDER 1. GENUS 1.

P,M,T. Model 1. The Cube.

Native Bismuth.........	m $2\frac{1}{4}$	w	$9\frac{3}{4}$
Sulphuret of Silver.....	m $2\frac{1}{4}$	gr	$7\frac{1}{5}$
Galena	m $2\frac{1}{2}$	gr	$7\frac{2}{3}$
Seleniuret of Lead.....	m $2\frac{1}{2}$	gr	$7\frac{2}{3}$
Do. Lead & Cobalt.....	m $2\frac{1}{2}$	gr	$7\frac{2}{3}$
Do. Lead & Mercury...	m $2\frac{1}{2}$	gr	$7\frac{2}{3}$
Do. Lead & Silver......	m $2\frac{1}{2}$	gr	$7\frac{2}{3}$

Genus 1 *Continued:*

Native Gold	m $2\frac{3}{4}$	y	15
Auriferous Silver	m $2\frac{3}{4}$	yw	12
Argentiferous Gold.....	m $2\frac{3}{4}$	wy	12
Native Silver............	m $2\frac{3}{4}$	w	$10\frac{1}{2}$
Native Copper	m $2\frac{3}{4}$	r	$8\frac{3}{5}$
Purple Copper	m 3	gr	5
Sulphuret of Tin	m 4	bk	$4\frac{1}{3}$
Platinum	m $4\frac{1}{2}$	gr	$17\frac{1}{3}$

Genus 1 Continued:

Sulpho-antimonite of Nickel?	m	5½	gr	6
Nickel Glance	m	5½	w	6
Tin White Cobalt	m	5½	gr	6½
Silver White Cobalt	m	5½	gr	6½
Tesseral Pyrites	m	5¾	w	6¾
Magnetic Iron Ore	m	6	bk	5
Iron Pyrites	m	6¼	brbk	5
Titanium	m	8	r	5⅓
Red Oxide of Copper	itl	3¾	r	6
Sulphur^t. of Manganese	io	3¾	gn	4
Chloride of Silver	tl	1¼	grbr	5⅔
Muriate of Ammonia	tl	1¾	w	1½
Chloride of Sodium	tp	2	w	2¼
Alum	tp	2¼	w	1¾
Arseniate of Iron	tl	2½	gn	3
Zinc Blende	tl	3¾	wbr	4
Fluorspar	tp	4	w	3¼
Analcime	tp	5½	w	2
Boracite	tl	7	w	3
Diamond	tp	10	w	3½

CLASS I. ORDER I. GENUS 2.

Divided into two groups:

a.) The right square-based prism. P_x, M, T. Models 2, 3.

b.) The right eight-sided prism. P_x, M, T, mt. Model 4.

Group a.

Black Tellurium	m	1½	grbk	7
Iron Pyrites,	m	6¼	bkbr	5
Chloride of Silver	o	1¼	grbr	5⅔
Muriate of Ammonia	tl	1¾	w	1½
Chloride of Sodium	tp	2	w	2¼
Uranite	tl	2¼	gn	3⅛
Cryolite	tl	2¾	w	3
Murio-Carbon. of Lead	tp	2¾	w	6
Molybdate of Lead	tl	3	w	6¾
Tungstate of Lead	tl	3	gr	8
Apophyllite	tp	4¾	w	2⅓
Gehlenite	o	5¾	w	3
Rutile	tl	6½	br	4¼
Idocrase	tl	6½	w	3⅜

Group b.

Black Tellurium	m	1½	grbk	7
Uranite	tl	2¼	gn	3⅛
Murio-Carbon. of Lead	tp	2¾	w	6
Molybdate of Lead	tl	3	w	6¾
Humboldtilite	tl	5	br	3
Wernerite	tp	5½	grw	2⅔
Mellilite	o	6	ry	3¼
Rutile	tl	6½	br	4¼
Idocrase	tl	6½	w	3⅜

CLASS I. ORDER 2. GENUS I.

Right Rectangular Prism.

P_+, M_-, T. Model 5.

Antimonial Silver	m	3½	w	9½
Iron Pyrites	m	6¼	bkbr	5
Anhydrite	tp	3¼	grw	3
Desmine	tp	3¾	w	2¼
Phospha. of Manganese	o	5½	ygr	3⅗
Chrysolite	tp	6¾	w	3⅓

CLASS I. ORDER 3. GENUS I.

Right Rhombic Prism.

P_x, M_-T. Model 6.

[The fraction appended to the name of the mineral is the characteristic of the rhombic prism, that is to say, the equivalent of the sign –. Thus, the prism of Jamesonite is P_x, $M\frac{5}{6}$T.]

Jamesonite $\frac{5}{6}$	m	2¼	gr	5¾
Sulphuret of Bismuth $\frac{7}{13}$	m	2¼	gr	6⅓
Arsenical Pyrites $\frac{2}{3}$	m	5¼	bk	6⅙
White Iron Pyrites $\frac{3}{4}$	m	6¼	bk	4⅗
Koenigite $\frac{10}{13}$	tp	2⅓	gn	3½
Muriate of Lead $\frac{4}{5}$	o	2¾	yw	7
Sulphate of Barytes $\frac{4}{5}$	tl	3¼	w	4⅖
Do. of Strontian $\frac{4}{5}$	tl	3¼	w	3⅗
Arragonite $\frac{5}{8}$	tl	3¾	grw	3
Red Oxide of Zinc $\frac{4}{5}$	tl	4¼	ry	5⅗
Mengite	o	5	w	5
Triphyline $\frac{5}{6}$	tl	5	grw	3⅗
Chiastolite $\frac{97}{100}$	tl	5¼	w	3
Hypersthene $\frac{89}{91}$	o	5½	wgr	3⅓
Fergusonite$_x$	o	5¾	br	5⅘

Genus 1 *Continued:*

Amblygonite $\frac{3}{4}$	tl	6	w	3
Sillimanite $\frac{7}{8}$, $\frac{3}{4}$	o	6¼	br	3⅗
Prehnite $\frac{5}{6}$	tl	6½	w	3
Staurolite $\frac{8}{17}$	o	7¼	w	3¾
Andalusite $\frac{49}{30}$	o	7½	w	3

P, M$\frac{47}{30}$T, m_t.

Pyrolusite,	m	2¾	brbk	4⅗

Class 1. Order 4. Genus 1.

Right square prism with the vertical edges bevelled.

P_+,M,T, m_t, m_+t.

Pinite,	o	2¼	w	2¾
Humboldtilite,	tl	5	br	3

P_+,M,T, MT, m_t, m_+t.

Idocrase,	tl	6½	w	3⅗

P_-, mt, m_t, m_+t.

Molybdate of Lead	tl	3	w	6¾
Rutile,	tl	6¼	br	4¼

Class 1. Order 5. Genus 1.

Regular six-sided prism.

P_x,T,M$\frac{15}{13}$$T_2$. = P_x,V. Model 7.

Sulphrt. of Molybdenm.	m	1¼	gr	4½
Graphite	m	1½	bk	2¼
Telluric Silver	m	2	gr	8½
Polybasite	m	2¼	bk	6¼
Vitreous Copper	m	2¾	gr	5¾
Sulphuret of Nickel	m	4	gr	6½
Magnetic Iron Pyrites	m	4	bk	4⅗
Antimonial Nickel	m	5	r	?
Osmium-Iridium	m	5	gr	19
Palladium	m	5	gr	12
Copper Nickel	m	5¼	br	7⅗
Ice	tp	1	w	1
Talc	tl	1¼	gnw	2¾
Chlorite	tl	1¼	gnw	2¾
Pinite	o	2¼	w	2¾
One-axed Mica	tp	2¼	wgr	3
Red Silver	o	2¼	r	5⅗
Cinnabar	tl	2¼	r	8
Sideroschisolite	o	2½	gn	3

Genus 1 *Continued:*

Cronstedtite	o	2½	gn	3⅓
Vanadiate of Lead	o	2¾	w	6⅗
Calcareous Spar	tp	3	grw	2¾
Phosphate of Lead	tl	3¾	yw	7
Arseniate of Lead	tl	3¾	yw	7
Fluoride of Cerium	o	4	y	4¾
Carbonate of Iron	tl	4	brw	3⅗
Pyrosmalite	o	4¼	br	3
Apatite	tl	5	w	3¼
Galmei	tl	5	w	4½
Nepheline	tl	6	w	2⅗
Dichroite	tl	7¼	w	2⅗
Beryl	tp	7¾	w	2⅗
Corundum	tp	9	w	4

Class 1. Order 5. Genus 2.

Regular twelve-sided prism.

P_x,m,T, m_2t$\frac{15}{13}$, M$\frac{15}{13}$$T_2$ = P_x,V, *v*.
Model 10.

The ratio of the axes m^a t^a *may* be exactly 14 to 13, and necessarily *must* be very near to that sum; since if t^a is taken = 13, then m^a can only fall between 13 and 15. See page 43. For practical purposes it is sufficient to know, that $p_x^a m_{14}^a t_{13}^a$ is used to indicate the ratio of the axes of such combinations as are represented by Models 10 and 52.

Vitreous Copper	m	2¾	gr	5¾
Magnetic Iron Pyrites	m	4	bk	4⅗
Pinite	o	2¼	w	2¾
Cronstedtite	o	2½	gn	3⅓
Calcareous Spar	tp	3	grw	2¾
Phosphate of Lead	tl	3¾	yw	7
Fluoride of Cerium	o	4	y	4¾
Apatite	tl	5	w	3¼
Nepheline	tl	6	w	2⅗
Dichroite	tl	7¼	w	2⅗
Beryl	tp	7¾	w	2⅗

Class 1. Order 5. Genus 3.

Rhombic prisms, with two or more of the vertical edges replaced.

Divided into three groups:

a.) containing P_x,M,M_T.
b.) containing P_x,T,M_T. Md. 8,9.
c.) containing P_x,M,T,M_T.

Group a.

Pyrolusite $\frac{47}{30}$ ....m $2\frac{3}{4}$ brbk $4\frac{3}{5}$
Uranite $\frac{1}{1}$..................tl $2\frac{1}{4}$ gn $3\frac{1}{8}$
Koenigite $\frac{10}{13}$ ....tp $2\frac{1}{2}$ gn $3\frac{1}{2}$
Sulphate of Barytes $\frac{4}{5}$ tl $3\frac{1}{4}$ w $4\frac{2}{5}$

P_+,M, $3(\frac{1}{2}m_x t)$.
Cyanite....................tl 6 w $2\frac{2}{3}$

Group b.

Sulph^t. of Antimony $\frac{24}{23}$ m 2 gr $4\frac{3}{5}$
Do. of Bismuth $\frac{7}{13}$......m $2\frac{1}{4}$ gr $6\frac{2}{3}$
Brittle Sulph^t. Silver $\frac{5}{8}$ m $2\frac{1}{4}$ bk $6\frac{1}{4}$
Antimonial Silver $\frac{7}{12}$...m $3\frac{1}{2}$ w $9\frac{1}{2}$
Sulphate of Potash $\frac{4}{7}$...tp $2\frac{3}{4}$ w $1\frac{3}{4}$
Do. of Barytes $\frac{4}{5}$ ....tl $3\frac{1}{4}$ w $4\frac{2}{5}$
Polyhallite $\frac{7}{11}$ ....o $3\frac{1}{2}$ rgr $2\frac{3}{4}$
Strontianite $\frac{2}{3}$ ....tl $3\frac{1}{2}$ w $3\frac{3}{5}$
Prehnite $\frac{5}{6}$ ....tl $6\frac{1}{2}$ w 3
Spodumen $\frac{15}{16}$............tl $6\frac{3}{4}$ w $3\frac{3}{5}$
Staurolite $\frac{8}{17}$ Md.9......o $7\frac{1}{4}$ w $3\frac{3}{4}$

Group c.

Sulph^t. of Antimony $\frac{24}{23}$ m 2 gr $4\frac{3}{5}$
Bournonite $\frac{15}{16}$ ....m $2\frac{3}{4}$ bk $5\frac{3}{4}$
Augite $\frac{20}{21}$ ....o $5\frac{1}{2}$ wgr $3\frac{1}{3}$
Prehnite $\frac{5}{6}$................tl $6\frac{1}{2}$ w 3

CLASS II.—COMPLETE PYRAMIDS.

Order 1. *Square Equator*,......................
- Genus 1. Axes: $p^a\ m^a\ t^a$.
- Genus 2. Axes: $p^a_x\ m^a\ t^a$.

Order 2. *Rectangular Equator*, ...
- Genus 1. Axes: $p^a_x\ m^a_y\ t^a_z$.

Order 3. *Rhombic Equator*, ...
- Genus 1. Axes: $p^a\ m^a\ t^a$.
- Genus 2. Axes: $p^a_x\ m^a\ t^a$.
- Genus 3. Axes: $p^a_x\ m^a_y\ t^a_z$.

Order 4. *Rhombo-Quadratic Equator*,......
- Genus 1. Axes: $p^a\ m^a\ t^a$.
- Genus 2. Axes: $p^a_x\ m^a\ t^a$.

Order 5. *Rhombo-Rectangular Equator*,...
- Genus 1. Axes: $p^a_x\ m^a_{15} t^a_{13}$.
- Genus 2. Axes: $p^a_x\ m^a_y\ t^a_z$.

Class 2. Order 1. Genus 1.

Divided into four groups:

a.) The Regular Octahedron.
PMT. Model 15.

b.) The Hemitrope or Twin Octahedron.
PMT × 2. Model 16.

c.) Various Triakisoctahedrons, as
$3P_2MT$. Md. 17.
$3P_2MT \times 2$.

d.) Combinations of the Regular Octahedron with the Triakisoctahedron, as PMT, $3p^3_2mt$.

Group a.

Native Lead ....m $1\frac{1}{2}$ gr $11\frac{1}{3}$
Native Bismuth.........m $2\frac{1}{4}$ w $9\frac{3}{4}$
Sulphuret of Silver.....m $2\frac{1}{4}$ gr $7\frac{1}{5}$
Galena ....m $2\frac{1}{2}$ gr $7\frac{2}{3}$
Native Copper ....m $2\frac{3}{4}$ r $8\frac{3}{5}$
Native Gold ....m $2\frac{3}{4}$ y 15
Native Silver............m $2\frac{3}{4}$ w $10\frac{1}{2}$
Purple Copper ....m 3 gr 5
Amalgam ....m $3\frac{1}{4}$ w $13\frac{3}{4}$
Copper Pyrites ....m $3\frac{3}{4}$ gnbk $4\frac{1}{6}$
Native Iron ....m $4\frac{1}{2}$ gr $7\frac{2}{3}$
Sulpho-antimonite of
Nickel.................m $5\frac{1}{2}$ gr 6

Group a Continued:

Tin White Cobalt......m $5\frac{1}{2}$ gr $6\frac{1}{3}$
Silver White Cobalt...m $5\frac{1}{2}$ gr $6\frac{1}{4}$
Sulphuret of Cobalt....m $5\frac{1}{2}$ gr $6\frac{1}{3}$
Magnetic Iron Ore.....m 6 bk 5
Iron Pyrites..............m $6\frac{1}{4}$ bkbr 5
Sulphut. of Manganese io $3\frac{3}{4}$ gn 4
Red Oxide of Copper itl $3\frac{3}{4}$ r 6
Chromate of Iron ......io $5\frac{1}{2}$ br $4\frac{1}{2}$
Chloride of Silver......o $1\frac{1}{4}$ grbr $5\frac{3}{5}$
Oxide of Arsenic.......o $1\frac{1}{2}$ w $3\frac{5}{8}$
Muriate of Ammonia...tl $1\frac{3}{4}$ w $1\frac{1}{2}$
Alum.......................tp $2\frac{1}{4}$ w $1\frac{3}{4}$
Zinc Blende ....tl $3\frac{3}{4}$ wbr 4
Fluorspar..................tp 4 w $3\frac{1}{7}$
Pyrochlore................tl 5 br $4\frac{1}{4}$
Leucite.....................tl $5\frac{1}{2}$ w $2\frac{1}{2}$
Nosian.....................tl $5\frac{1}{2}$ gr $2\frac{1}{4}$
Lapis Lazuli ....tl $5\frac{1}{2}$ bl 3
Yttrocerite................o 6 w $3\frac{1}{2}$
Haüyne ....tl 7 w $2\frac{2}{3}$
Pleonaste (bk)..........tl $7\frac{1}{2}$ w $3\frac{3}{5}$
Automalite (bl, gn) ....o 8 w $4\frac{1}{2}$
Spinel (r, bl)............tp 8 w $3\frac{1}{2}$
Diamond ....tp 10 w $3\frac{1}{2}$

Group b.

Native Silver.............m $2\frac{3}{4}$ w $10\frac{1}{2}$
Purple Copper ....m 3 gr 5
Magnetic Iron Ore.....m 6 bk 5
Alum.......................tp $2\frac{1}{4}$ w $1\frac{3}{4}$
Zinc Blende ....tl $3\frac{3}{4}$ wbr 4
Spinel (r, bl)............tp 8 w $3\frac{1}{2}$
Automalite (bl, gn).....o 8 w $4\frac{1}{2}$

Group c.

[The fraction is the characteristic of the triakisoctahedron.]

Galena $\frac{2}{1}$..................m $2\frac{1}{2}$ gr $7\frac{3}{5}$
Magnetic Iron Ore $\frac{3}{2}$...m 6 bk 5
Iron Pyrites $\frac{2}{1}$...........m $6\frac{1}{4}$ bkbr 5
Red Oxide of Copper $\frac{3}{1}$ itl $3\frac{3}{4}$ r 6
Fluorspar $\frac{3}{1}$...............tp 4 w $3\frac{1}{7}$
Diamond $\frac{3}{1}$...............tp 10 w $3\frac{1}{2}$

Group d.

Galena $\frac{2}{1}$..................m $2\frac{1}{2}$ gr $7\frac{3}{5}$
Magnetic Iron Ore $\frac{3}{2}$...m 6 bk 5

Group d Continued:

Red Oxide of Copper $\frac{3}{1}$ itl $3\frac{3}{4}$ r 6
Fluorspar $\frac{3}{1}$ ....tp 4 w $3\frac{1}{7}$
Spinel $\frac{3}{1}$ (r, bl) ....tp 8 w $3\frac{1}{2}$

CLASS 2. ORDER 1. GENUS 2.

Divided into three groups:

a.) Square-based Pyramids containing Forms of the North and East Zones. Example: P_xM, P_xT. Models 12, 13.

b.) Square-based Pyramids containing Forms of the Octahedral Zones. Example: P_xMT. Models 12, 13.

c.) Combinations of two or more square-based octahedrons of any Zones. Examples:
P_xM, P_xT, p_xmt.
P_xM, p_xm, P_xT, p_xt. Md. 14.
P_xM, P_xT, p_xmt, p_xmt.

The pyramids of groups *a* and *b* are not distinguishable by their external appearance. They may be discriminated by comparing their interfacial angles with the characteristics.

Group a.

Copper Pyrites $\frac{2}{1}$ ....m $3\frac{3}{4}$ gnbk $4\frac{1}{6}$
Chloride of Mercury $\frac{5}{8}$ tl $1\frac{1}{2}$ w $6\frac{1}{2}$
Molybdate of Lead $\frac{4}{3}$...tl 3 w $6\frac{3}{4}$
Tungstate of Lime $\frac{7}{8}$...tl $4\frac{1}{4}$ w 6
Hausmannite $\frac{5}{3}$ ....io $5\frac{1}{4}$ rbr $4\frac{3}{4}$
Anatase $\frac{5}{8}$ ....tl $5\frac{1}{2}$ w $3\frac{4}{5}$
Oxide of Tin $\frac{8}{9}$ ....tl $6\frac{1}{2}$ br 7

Group b.

Copper Pyrites $\frac{1}{1}$ ....m $3\frac{1}{2}$ gnbk $4\frac{1}{6}$
Mellite $\frac{3}{4}$ ....tp $2\frac{1}{4}$ w $1\frac{3}{5}$
Uranite ....tl $2\frac{1}{4}$ gn $3\frac{1}{8}$
Tungstate of Lead $\frac{5}{1}$...tl 3 gr 8
Tungstate of Lime $\frac{3}{2}$...tl $4\frac{1}{4}$ w 6
Apophyllite $\frac{5}{4}$............tp $4\frac{3}{4}$ w $2\frac{1}{3}$
Rutile $\frac{8}{9}$ ....tl $6\frac{1}{4}$ br $4\frac{1}{4}$
Zircon $\frac{8}{9}$...................tp $7\frac{1}{2}$ w $4\frac{1}{2}$

Group c.

Copper Pyrites ....m $3\frac{3}{4}$ gnbk $4\frac{1}{6}$
Hausmannite ....io $5\frac{1}{4}$ rbr $4\frac{3}{4}$
Braunite..................io $6\frac{1}{4}$ bk $4\frac{4}{5}$
Molybdate of Lead ....tl 3 w $6\frac{3}{4}$
Tungstate of Lead.....tl 3 gr 8
Tungstate of Lime.....tl $4\frac{1}{2}$ w 6
Anatase ....tl $5\frac{3}{4}$ w $3\frac{4}{5}$

Class 2. Order 2. Genus 1.

Models 82ᵃ, 82ᵇ; but these forms properly belong to Class 5, Ord. 3.

Class 2. Order 3. Genus 1.

Divided into two groups:

a.) Icositessarahedrons. Example: $3P\frac{1}{3}MT$. Model 22.

b.) Hexakisoctahedrons and Hemihexakisoctahedrons.

Group a.

$3P\frac{1}{3}MT$. Model 22.
Sulphuret of Silver.....m $2\frac{1}{4}$ gr $7\frac{1}{3}$
Amalgam ....m $3\frac{1}{4}$ w $13\frac{3}{4}$
Iron Pyrites ....m $6\frac{1}{4}$ bkbr5
Analcime ....tp $5\frac{1}{2}$ w 2
Leucite ....tl $5\frac{3}{4}$ w $2\frac{1}{2}$
Garnet ....tl 7 w 4

$3P\frac{2}{3}MT$. Similar to Model 22.
Native Gold ....m $2\frac{3}{4}$ y 15
Native Silver............m $2\frac{3}{4}$ w $10\frac{1}{2}$
Muriate of Ammonia...tl $1\frac{3}{4}$ w $1\frac{1}{2}$

$3P\frac{1}{3}MT \times 2$.
Native Gold ....m $2\frac{3}{4}$ y 15

$3P\frac{-}{-}MT$, $3P\frac{1}{3}MT$.
Analcime ....tp $5\frac{1}{2}$ w 2

pmt, $3P\frac{1}{3}MT$.
Native Gold ....m $2\frac{3}{4}$ y 15
Native Silver............m $2\frac{3}{4}$ w $10\frac{1}{2}$

Group b.

$6P\frac{1}{7}M\frac{1}{3}T$. Model 23.
Hexakisoctahedron.
Garnet ....tl $6\frac{1}{2}$ w $3\frac{3}{4}$

Group b Continued:

$6P\frac{1}{5}M\frac{1}{3}T$. Similar to Model 23. Hexakisoctahedron.
$\frac{1}{2}(6P\frac{1}{5}M\frac{1}{3}T.)$ Model 24. Hemihexakisoctahedron with inclined faces.
PMT, $6p\frac{1}{5}m\frac{1}{3}t$.
Diamond ....tp 10 w $3\frac{1}{2}$

$3P\frac{1}{2}M\frac{1}{3}T$. Model 25.
Hemihexakisoctahedron with parallel faces.
Iron Pyrites ....m $6\frac{1}{4}$ bkbr 5

Class 2. Order 3. Genus 2.

PMT, P_2M_3T, P_2MT_3.
Braunite..................io $6\frac{1}{4}$ bk $4\frac{4}{5}$

Class 2. Order 3. Genus 3.

Scalene Octahedrons, or octahedrons with a rhombic equator.

Example:

P_+M_-T, or $P\frac{19}{10}M\frac{8}{10}T$. Model 21.
Antimonial Silver ......m $3\frac{1}{2}$ w $9\frac{1}{2}$
Sulphur Md. 21.........tl 2 yw 2
Thenardite................tp 2 w $2\frac{3}{4}$
$P\frac{19}{10}M\frac{8}{10}T$, $p\frac{4}{10}m\frac{8}{10}t$.
Sulphur ....tl 2 yw 2

Class 2. Order 4. Genus 1.

The regular octahedron with the solid angles replaced by complex equiaxed scalene octahedrons.

PMT, $3p\frac{1}{2}mt$.
Sulphuret of Silver.....m $2\frac{1}{4}$ gr $7\frac{1}{3}$
Galena ....m $2\frac{1}{2}$ gr $7\frac{3}{5}$
Red Oxide of Copper itl $3\frac{3}{4}$ rs 6

PMT, $3p\frac{1}{3}mt$.
Native Gold ....m $2\frac{3}{4}$ y 15
Magnetic Iron Ore .....m 6 bk 5
Pleonaste, bk............tl $7\frac{1}{2}$ w $3\frac{4}{5}$

PMT, $3p\frac{1}{3}m\frac{1}{2}t$.
Iron Pyrites ....m $6\frac{1}{4}$ bkbr5

PMT, $6p_x m_y t_z$.
Red Oxide of Copper itl $3\frac{3}{4}$ rs 6

CLASS 2. ORDER 4. GENUS 2.

$P\frac{3}{2}M$, $P\frac{3}{2}T$, $p\frac{3}{2}mt$, $\frac{1}{2}(pm_-t_+, pm_+t_-)$. Also several similar square-based Combinations.

Tungstate of Lime .....tl 4¼ w 6

CLASS 2. ORDER 5. GENUS 1.

Divided into seven groups:

a.) Single Rhombohedrons. Exam.: $\frac{1}{2}P_xT$, $P_xM\frac{1}{1}\frac{6}{3}T_2$, or R_x.

R_1. Model 26^a. | $R\frac{5}{2}$. Model 26^c.
$R\frac{1}{2}$. Model 26^b. | R_2. Model 26^d.

b.) Combination of two equal rhombohedrons, forming a regular six-sided Pyramid. Example: P_xT, $P_xM\frac{1}{1}\frac{6}{3}T_2 = 2R_x$ Zw Ze. $2R\frac{1}{1}\frac{3}{3}$ Zw Ze. Model 26.

c.) Combination of two dissimilar Rhombohedrons. Example: $\frac{1}{2}P_-T$, $\frac{1}{2}p_+t$, $\frac{1}{2}P_-M\frac{1}{1}\frac{6}{3}T_2$, $\frac{1}{2}p_+m\frac{1}{1}\frac{6}{3}t_2$.

d.) Combination of three dissimilar Rhombohedrons. Example: $R\frac{1}{4}$ Zw, $R\frac{5}{8}$ Ze, $r\frac{1}{2}$ Ze. Model 26^e.

e.) Four Rhombohedrons combined.

f.) Rhombohedrons combined with Scalenohedrons. Example: $\frac{1}{2}p\frac{1}{2}t$, $\frac{1}{2}p\frac{1}{2}m\frac{1}{1}\frac{6}{3}t_2$, $\frac{1}{2}(3P_+M\mp T\pm)$.

g.) Scalenohedrons only. Example: $\frac{1}{2}(3P_1M\pm T\pm) = S_1$. Model 26^f.

Group a.

Tetradymite ½ ....m 1¾ gr 7½
Crichtonite $\frac{2}{1}$ ....m 4½ bk 4
Specular Iron ½ ....m 6 brr 5¼
Specular Iron ⅝ ....m 6 brr 5¼
Nitrate of Soda $\frac{1}{1}$ ......tp 1¾ w 2
Cinnabar $\frac{5}{2}$, Md. 26^c ...tl 2¼ r 8
Sulphato-tricarbonate of Lead, $\frac{5}{2}$ ....tp 2½ w 6¼

Calcar^s.Spar $\frac{1}{1}$ Md.26^a		tp 3	grw	2¾
Do. ½ Md.26^b				
Do. $\frac{2}{1}$ Md.26^d				
Do. $\frac{4}{1}$.........				
Do. $\frac{5}{1}$.........				
Do. $\frac{5}{2}$.........				

Group a Continued:

Plumbo-Calcite $\frac{1}{1}$ ......
Carbon^e. of Magnesia $\frac{1}{1}$ o 3½ grw 2⅔
Dreelite ¼ ....o 3½ w 3¼
Carbon^e.of Manganese½tl 3½ w 3⅗
Dolomite $\frac{1}{1}$..............tl 3¾ grw 3
Mesitinspar $\frac{1}{1}$...........

Carbonate of Iron $\frac{1}{1}$	tl 4	brw	3⅗
Do. ½			
Do. $\frac{5}{1}$			

Brown Spar $\frac{1}{1}$ ....tl 4¼ grw 3
Chabasite $\frac{5}{4}$ ....tp 4¼ w 2
Alunite $\frac{4}{1}$ ....tl 5 w 2¾

Galmei $\frac{1}{1}$...............	tl 5	w	4½
Do. $\frac{2}{1}$..............			
Do. ½..............			

Quartz $\frac{5}{4}$.................tp 7 w 2¾
Corundum $\frac{3}{2}$ ....tp 9 w 4

Group b.

Vitreous Copper ½...	m 2¾	gr	5¾
Do. $\frac{2}{1}$...			

Calcareous Spar $\frac{5}{1}$......tp 3 grw 2¾
Phosphate of Lead $\frac{1}{1}\frac{3}{3}$ tl 3¾ yw 7
Quartz $\frac{5}{4}$ ....tp 7 w 2¾

Corundum ⅝	tp 9	w	4
Do. $\frac{2}{3}$			

$2R\frac{1}{2}$, $2R\frac{2}{1}$.

Vitreous Copper ....m 2¾ gr 5¾

$2R\frac{16}{3}$, $2R\frac{2}{3}$.

Corundum ....tp 9 w 4

Group c.

Crichtonite ....m 4½ bk 4
Specular Iron............m 6 brr 5¼
Copper Mica ....tp 2 gn 2½
Calcareous Spar.........tp 3 grw 2¾
Dolomite ....tl 3¾ grw 3
Carbonate of Iron ......tl 4 brw 3⅗
Chabasite ....tp 4½ w 2
Galmei ....tl 5 w 4½
Quartz ....tp 7 w 2¾

Group d.

Calcareous Spar.........tp 3 grw 2¾
Chabasite Md. 26^e......tp 4¼ w 2

Group e.

Sulphato-tricarbonate of Lead............tp $2\frac{1}{2}$ w $6\frac{1}{4}$
Calcareous Spar.........tp 3 grw $2\frac{3}{4}$

Group f.

Specular Iron............m 6 brr $5\frac{1}{4}$
Red Silver...............o $2\frac{1}{4}$ r $5\frac{4}{5}$
Calcareous Spar.........tp 3 grw $2\frac{3}{4}$
Chabasite.................tp $4\frac{1}{4}$ w 2

Group g.

Red Silver...............o $2\frac{1}{4}$ r $5\frac{4}{5}$
Calcar^s. Spar Md. 26^f. tp 3 grw $2\frac{3}{4}$
Carbon^e. of Manganese tl $3\frac{1}{2}$ w $3\frac{3}{5}$

CLASS 2. ORDER 5. GENUS 2.

Irregular six-sided Pyramids.

$P_xT, P_xM_yT_z$.

Antimonial Silver......m $3\frac{1}{2}$ w $9\frac{1}{2}$
Sulphur...................tl 2 yw 2
Nitre......................tp 2 w 2
Sulphate of Potash.....tp $2\frac{3}{4}$ w $1\frac{3}{4}$
White Lead Ore........tl $3\frac{1}{4}$ w $6\frac{1}{2}$
Witherite.................tl $3\frac{1}{4}$ w $4\frac{1}{3}$

$p\frac{19}{10}t, P\frac{19}{10}M\frac{8}{10}T, p\frac{4}{10}m\frac{8}{10}t$.

Sulphur...................tl 2 yw 2

$P\frac{1}{3}T, p\frac{4}{3}t, P\frac{1}{5}M\frac{3}{5}T, p\frac{4}{5}m\frac{3}{5}t$.

Witherite.................tl $3\frac{1}{4}$ w $4\frac{1}{3}$

CLASS III.—COMPLETE PRISMS COMBINED WITH INCOMPLETE PYRAMIDS.

Order 1. *Square Equator,*....................	Genus 1. Axes: $p^a\ m^a\ t^a$. Genus 2. Axes: $p_x^a\ m^a\ t^a$.
Order 2. *Rectangular Equator,*..............	Genus 1. Axes: $p_x^a\ m_y^a\ t_z^a$.
Order 3. *Rhombic Equator,*..................	Genus 1. Axes: $p_x^a\ m_y^a\ t_z^a$.
Order 4. *Rhombo-Quadratic Equator,*......	Genus 1. Axes: $p^a\ m^a\ t^a$. Genus 2. Axes: $p_x^a\ m^a\ t^a$.
Order 5. *Rhombo-Rectangular Equator,*...	Genus 1. Axes: $p_x^a\ m_{15}^a\ t_{13}^a$. Genus 2. Axes: $p_x^a\ m_{14}^a\ t_{13}^a$. Genus 3. Axes: $p_x^a\ m_y^a\ t_z^a$.

CLASS 3. ORDER 1. GENUS 1.

This genus is divided into four groups:

a.) P,M,T predominant.—The cube with its edges and solid angles variously truncated. Models 27, 29, 31, 32, 35, 36, 38, 39, 40.

b.) MT.PM,PT predominant.—The rhombic dodecahedron with its edges and angles modified. Models 28, 34.

c.) PMT predominant.—The regular octahedron with its edges and angles modified. Models 30, 33.

d.) $\frac{1}{2}$PMT predominant.—The tetrahedron variously modified. Model 37.

Group a.

P,M,T, mt. pm, pt. Model 27.

Cube with edges replaced.

Sulphuret of Silver.....m $2\frac{1}{4}$ gr $7\frac{1}{5}$
Native Copper..........m $2\frac{3}{4}$ r $8\frac{3}{5}$
Native Silver............m $2\frac{3}{4}$ w $10\frac{1}{3}$
Native Gold.............m $2\frac{3}{4}$ y 15
Arsenical Nickel........m w
Chloride of Silver......o $1\frac{1}{4}$ grbr $5\frac{3}{5}$
Muriate of Ammonia...tp $1\frac{3}{4}$ w $1\frac{1}{3}$
Chloride of Sodium.....tp 2 w $2\frac{1}{4}$
Arseniate of Iron.......tl $2\frac{1}{2}$ gn 3
Fluorspar.................tp 4 w $3\frac{1}{7}$
Diamond..................tp 10 w $3\frac{1}{2}$

P,M,T. pmt. Similar to Model 29.

Cube with angles replaced.

Native Lead.............m $1\frac{1}{2}$ gr $11\frac{1}{3}$
Galena....................m $2\frac{1}{2}$ gr $7\frac{3}{5}$

Group a Continued:

Native Silver............m2½ w 10½
Native Copper..........m2½ r 8⅗
Purple Copper..........m3 gr 5
Sulphrt. of Manganese m3½ gn 4
Sulph8-antimonite of
Nickel...............m5½ gr 6
Tin White Cobalt......m5½ gr 6½
Bright White Cobalt...m5½ gr 6½
Sulphuret of Cobalt....m5½ gr 6⅓
Nickel Glance...........m5½ w 6
Magnetic Iron Ore.....m6 bk 5
Iron Pyrites.............m6¼ bkbr5
Arsenical Nickel........m w
Chloride of Silver......o 1½ grbr5⅗
Chloride of Sodium....tp2 w 2½
Alum.....................tp2½ w 1¾
Arseniate of Iron.......tl 2½ gn 3
Leucite...................tl 5¾ w 2½

P,M,T. PMT. Model 29.

Middle crystal between the cube and the octahedron.

Sulphuret of Silver.....m2¼ gr 7⅕
Galena...................m2½ gr 7⅗
Native Copper..........m2½ r 8⅗
Native Gold..............m2½ y 15
Bright White Cobalt...m5½ gr 6½
Zinc Blende.............tl 3¾ wbr 4
Fluorspar.................tp4 w 3⅐

P,M,T, mt. pm, pt, pmt. Md. 31.

Cube with edges and angles replaced.

Sulphuret of Silver.....m2¼ gr 7⅕
Galena....................m2½ gr 7⅗
Native Copper..........m2½ r 8⅗
Native Silver............m2½ w 10½
Native Gold.............m2½ y 15
Arsenical Grey Coppr. m4 rgr 4⅔
Arsenical Nickel........m w
Red Oxide of Copper itl3½ r 6
Arseniate of Iron......tl 2½ gn 3
Fluorspar................tp4 w 3⅐

P,M,T, mt. pm, pt, PMT. Md. 31.

Galena....................m2½ gr 7⅗
Tin White Cobalt......m5½ gr 6½
Sarcolite.................tl 5½ w 2

Group a Continued:

P,M,T. ½pmt. Model 38.

Native Gold.............m2½ y 15
Arseniate of Iron........tl 2½ gn 3

P,M,T, mt. pm, pt, ½pmt. Md. 36.

Arseniate of Iron.......tl 2½ gn 3
Boracite..................tl 7 w 3

P,M,T, MT. PM, PT, ½PMT, ½pmt.
Model 35.

Boracite..................tl 7 w 3

P,M,T. 3p½mt. Model 39.

Sulphuret of Silver.....m2¼ gr 7⅕
Iron Pyrites.............m6¼ bkbr5
Fluorspar................tp4 w 3⅐
Analcime................tp5½ w 2

P,M,T. 3p⅓mt. Sim. Md. 39.

Native Gold............m2¾ y 15
Fluorspar................tp4 w 3⅐

P,M,T. pmt, 3p½mt.
P,M,T. PMT, 3p₂mt, 3p₃mt.

Galena....................m2½ gr 7⅗

P,M,T. pmt, 3p₂mt.
P,M,T. ½pmt, ½(3p₂mt).

Arseniate of Iron......tl 2½ gn 3

P,M,T. pmt, 3p⅓m½t.

Iron Pyrites.............m6¼ brbk5

P,M,T. PMT, 6p⅓m½t.

Diamond.................tp10 w 3½

P,M,T,mt.pm,pt, 3p⅓mt.
P,M,T,MT.PM,PT,3p⅓mt, 6p¼m½t,
6p 1/11 m⅓t⅓.
P.M,T. 6p¼m½t. Sim. Mod. 40.

Fluorspar................tp 4 w 3⅐

P,M,T,MT.PM,PT,pmt, ½(3p½mt).
P,M,T, MT. PM, PT, ½PMT Znw,
½pmt Zne, ½(3p½mt) Z²ne,
½(6p⅓m⅓t) Z²n²w.

Boracite..................tl 7 w 3

Group b.

p,m,t, MT.PM, PT. Model 28.

Native Copper..........m2¾ r 8⅗
Amalgam................m3¼ w 13¾
Magnetic Iron Ore.....m6 bk 5

Group b Continued:

Red Oxide of Copper...itl3¾ r 6
Chloride of Silver ......o 1½ grbr5½
Zinc Blende..............tl 3¾ wbr 4
Nosian ....tl 5¾ gr 2¼
Lapis Lazuli ....tl 5¾ bl 3
Haüyne ....tl 7 w 2⅔
Garnet ....tl 7 w 3¾

p,m,t, MT. PM, PT, pmt. Md. 34.
Sulphuret of Silver .....m 2¼ gr 7¼
Arsenical Grey Coppʳ. m 4 rgr 4⅔

p,m,t, MT. PM, PT, pmt, 3p½mt.
Amalgam ....m 3½ w 13¾

p,m,t, MT. PM, PT, ½pmt.
p,m,t, MT. PM, PT, ½pmt Znw, ½ (3p½mt) Z²ne.
P,M,T, MT. PM,PT, ½PMT Znw, ½PMT Zne, ½(3p½mt) Z²nw.
P,M,T, MT. PM, PT ½PMT Znw, ½ (6p½m½t) Zne.
Boracite ....tl 7 w 3

p,m,t, MT. PM, PT, 3p½mt.
Amalgam ....m 3½ w 13¾
Sodalite ....tl 5¾ w 2¼
Nosian ....tl 5¾ gr 2¼
Lapis Lazuli ....tl 5¾ bl 3
Haüyne ....tl 7 w 2⅔

p,m,t, MT. PM, PT, pmt. 3p½mt.
Magnetic Iron Ore .....m 6 bk 5

P,M,T, MT. PM, PT, PMT, 3p½mt, 3p¾mt.
Arsenical Grey Copper m 4 rgr 4⅔

Group c.

p,m,t. PMT. Model 30.
Galena ....m 2½ gr 7⅗
Native Copper ....m 2¾ r 8⅗
Native Silver............m 2¾ w 10½
Native Gold ....m 2¾ y 15
Purple Copper ....m 3 gr 5
Copper Pyrites ....m 3¾ gnbk4⅙
Arsenical Grey Coppʳ. m 4 rgr 4⅔
Platin-Iridium ....m 4½ gr 17
Tin White Cobalt......m 5½ gr 6½
Sulphuret of Cobalt ...m 5½ gr 6⅓
Bright White Cobalt...m 5½ gr 6½

Group c Continued:

Magnetic Iron Ore.....m 6 bk 5
Iron Pyrites ....m 6½ brbk5
Red Oxide of Copper itl3¾ r 6
Alum ....tp 2¼ w 1¾
Chloride of Silver ......o 1½ grbr 5½
Zinc Blende ....tl 3¾ wbr 4
Fluorspar ....tp 4 w 3¼
Diamond ....tp 10 w 3½

p,m,t. mt. pm, pt, PMT. Sim. Md. 33.
Arsenical Grey Coppʳ. m 4 rgr 4⅔
Franklinite...............m 6½ br 5
Red Oxide of Copper itl3¾ r 6
Zinc Blende ....tl 3¾ wbr 4

P,M,T, mt. pm, pt, PMT. Md. 33.
Alum ....tp 2¼ w 1¾
Fluorspar ....tp 4 w 3¼

P,M,T, PMT, 3p,mt.
P,M,T,mt.pm,pt,PMT, 3p,mt.
P,M,T,mt.pm,pt,PMT, 3p½mt, 3p,mt.
Galena ....m 2½ gr 7⅗

p,m,t, MT.PM,PT, PMT, 3p½mt.
Tin White Cobalt......m 5½ gr 6½
Tesseral Pyrites ....m 5¾ w 6¾
Red Oxide of Copper itl3¾ r 6

p,m,t, MT. PM, PT, PMT, 3p½mt.
Magnetic Iron Ore.....m 6 bk 5
Zinc Blende ....tl 3¾ wbr 4
Fluorspar ....tp 4 w 3¼
Pleonaste (bk) ....tl 7½ w 3⅔

p,m,t, MT. PM, PT, PMT, ½(3p½mt).
Zinc Blende ....tl 3¾ wbr 4

Group d.

p,m,t. ½PMT.
Grey Copper ....m 3½ bk 5
Copper Pyrites ....m 3¾ gnbk4⅙
Boracite ....tl 7 w 3

p,m,t. ½PMT. ½pmt.
Zinc Blende ....tl 3¾ wbr 4

p,m,t, mt. pm, pt, ½PMT. Md. 37.
p,m,t, mt. pm, pt, ½PMT, ½pmt.
Grey Copper ....m 3 to 4 bk 5
Do. Antimonial ......m 3 to 4 bk 5
Boracite ....tl 7 w 3

CLASS 3. ORDER 1. GENUS 2.

This genus is divided into two groups:

a.) The prism has four vertical planes.—A right square prism with the terminal edges or the solid angles replaced. Exam.: $P\frac{5}{4}$,M,T. $p\frac{5}{4}$mt. Md. 41.

b.) The prism has eight vertical planes.—A right square prism with the lateral edges and the terminal edges or the solid angles replaced. Example: p_{+},M,T,MT.$P\frac{3}{4}$M,$P\frac{3}{4}$T. Md.42.

Group a.

Black Tellurium	m	$1\frac{1}{4}$	grbk	7
Galena	m	$2\frac{1}{2}$	gr	$7\frac{2}{3}$
Copper Pyrites	m	$3\frac{3}{4}$	gnbk	$4\frac{1}{6}$
Chloride of Mercury...	tl	$1\frac{1}{2}$	w	$6\frac{1}{2}$
Mellite	tp	$2\frac{1}{4}$	w	$1\frac{3}{5}$
Uranite	tl	$2\frac{1}{4}$	gn	$3\frac{1}{8}$
Muriocarbon. of Lead	tp	$2\frac{3}{4}$	w	6
Molybdate of Lead.....	tl	3	w	$6\frac{3}{4}$
Tungstate of Lead......	tl	3	gr	8
Apophyllite Md. 41	tp	$4\frac{3}{4}$	w	$2\frac{1}{3}$
Anatase	tl	$5\frac{3}{4}$	w	$3\frac{4}{5}$
Rutile	tl	$6\frac{1}{4}$	br	$4\frac{1}{4}$
Idocrase	tl	$6\frac{1}{2}$	w	$3\frac{2}{5}$

Group b.

Black Tellurium.........	m	$1\frac{1}{4}$	grbk	7
Uranite	tl	$2\frac{1}{4}$	gn	$3\frac{1}{8}$
Muriocarbon. of Lead	tp	$2\frac{3}{4}$	w	6
Apophyllite	tp	$4\frac{3}{4}$	w	$2\frac{1}{3}$
Oerstedtite	o	$5\frac{1}{2}$	br	$3\frac{3}{5}$
Anatase	tl	$5\frac{3}{4}$	w	$3\frac{4}{5}$
Rutile	tl	$6\frac{1}{4}$	br	$4\frac{1}{4}$
Oxide of Tin............	tl	$6\frac{1}{2}$	br	7
Idocrase Md. 42	tl	$6\frac{1}{2}$	w	$3\frac{2}{5}$

CLASS 3. ORDER 2. GENUS 1.

Rectangular prism with the terminal edges or the solid angles replaced. Example:

p_{+},M,T. $P\frac{4}{7}M\frac{6}{7}$T. Md. 43.

Bournonite...............	m	$2\frac{3}{4}$	bk	$5\frac{3}{4}$
Anhydrite	tp	$3\frac{1}{4}$	grw	3

Genus 1 *Continued:*

Desmine Md. 43	tp	$3\frac{3}{4}$	w	$2\frac{1}{5}$
Harmotome	tl	$4\frac{1}{2}$	w	$2\frac{2}{5}$
Olivine	tp	$6\frac{3}{4}$	w	$3\frac{2}{5}$

CLASS 3. ORDER 3. GENUS 1.

Rhombic prism, with the terminal edges or the solid angles, or both, replaced.

P_x,M_T. p_xm, p_xt, $p_x m_y t_z$.
Model 44 is P_x,M_T. p_xm.

[The characteristic added to the name relates to the prism, and is the substitute for the sign – in the above general formula.]

Arsenical Pyrites $\frac{2}{3}$	m	$5\frac{3}{4}$	bk	$6\frac{1}{8}$
White Iron Pyrites $\frac{3}{4}$	m	$6\frac{1}{4}$	bk	$4\frac{3}{5}$
Lievrite $\frac{6}{9}$, $\frac{8}{9}$	io	$5\frac{3}{4}$	gnbk	4
Thenardite $\frac{1}{2}$............	tp	2	w	$2\frac{3}{4}$
Celestine $\frac{4}{5}$	tp	$3\frac{1}{4}$	w	$3\frac{7}{8}$
Heavy Spar $\frac{4}{5}$, $\frac{3}{4}$	tp	$3\frac{1}{4}$	w	$4\frac{2}{5}$
White-Lead Ore $\frac{3}{5}$.....	tl	$3\frac{1}{4}$	w	$6\frac{1}{2}$
Arragonite $\frac{5}{8}$............	tl	$3\frac{3}{4}$	grw	3
Euchroite $\frac{3}{5}$	tl	$3\frac{3}{4}$	gn	$3\frac{2}{5}$
Libethenite $\frac{9}{10}$	tl	4	gn	$3\frac{3}{4}$
Fergusonite	o	$5\frac{3}{4}$	br	$5\frac{4}{5}$
Andalusite $\frac{49}{50}$	o	$7\frac{1}{2}$	w	3
Topas $\frac{19}{36}$, $\frac{19}{24}$, $\frac{19}{18}$, $\frac{19}{12}$,	tp	8	w	$3\frac{1}{2}$

CLASS 3. ORDER 4. GENUS 1.

This genus is divided into nine groups:

a.) P,M,T predominant.—The cube with its edges and angles replaced by equiaxed combinations of unequiaxed forms, of the kinds enumerated at page 15. Examples:

P,M,T, mt_2. pm_2, p_2t, $3p\frac{1}{3}m\frac{1}{2}t$.
P,M,T, mt_3; m_3t. pm_3, p_3m, pt_3, p_3t, Model 45.
P,M,T. PMT, $6p\frac{1}{3}m\frac{1}{2}t$.

b.) MT.PM,PT, the rhombic dodecahedron predominant.

Genus 1 *Continued:*

c.) $MT_+.PM_+,P_+T$, the pentagonal dodecahedron, predominant. Examples:
p,m,t,M½T.P½M,P⅔T. Md. 47.
p,m,t,M½T.P½M,P⅔T, PMT. Model 48.

d.) MT_+, M_+T. PM_+, P_+M, PT_+, P_+T, the tetrakishexahedron, predominant. Similar to Md. 45, but having p,m,t smaller.

e.) PMT, the regular octahedron, predominant. Example: P,M,T. PMT, 3p½mt.

f.) ½PMT, the tetrahedron, predom^t. p,m,t. ½PMT, ½(3p½mt).

g.) 3P_MT, the icositessarahedron, predominant. Example: p,m,t. 3P½MT.

h.) $3P\_MT_+$, the hemihexakisoctahedron, predominant. Exam.: p,m,t. 3P⅓M½T. Model 46.

i.) $6P\_MT_+$, the hexakisoctahedron, predominant. Exam.: p,m,t. 6P¼M½T.

Group a.

Galena	m 2½	gr	7⅗
Native Copper	m 2¾	r	8⅗
Bright White Cobalt	m 5½	gr	6½
Iron Pyrites	m 6½	brbk	5
Red Oxide of Copper	itl 3¾	r	6
Chloride of Sodium	tp 2	w	2¼
Fluorspar Md. 45	tp 4	w	3⅐
Diamond	tp 10	w	3½

Group b.

Amalgam	m 3½	w	13¾

Group c.

Bright White Cobalt	m 5½	gr	6½
Iron Pyrites Md. 47, 48	m 6½	brbk	5

Group d.

Fluorspar	tp 4	w	3⅐

Group e.

Galena	m 2½	gr	7⅗
Native Gold	m 2¾	y	15
Bright White Cobalt	m 5½	gr	6½

Group e Continued:

Iron Pyrites	m 6½	brbk	5
Red Oxide of Copper	itl 3¾	r	6

Group f.

Grey Copper	m 3½	bk	5

Group g.

Sulphuret of Silver	m 2¼	gr	7¼
Native Gold	m 2¾	y	15
Grey Copper	m 3½	bk	5
Iron Pyrites	m 6½	brbk	5
Analcime	tp 5½	w	2

Group h.

Iron Pyrites Md. 46	m 6½	brbk	5

Group i.

Fluorspar	tp 4	w	3⅐

CLASS 3. ORDER 4. GENUS 2.

Right square prism, with the vertical edges bevelled, and the terminal edges, or the solid angles, replaced by oblique planes. Ex.: P_+,M,T, mt, m½t, m⅔t. p⅔m, p⅔t.

Uranite	tl 2¼	gn	3⅛
Murio-Carbo^e. of Lead	tp 2¾	w	6
Apophyllite	tp 4¾	w	2¼
Wernerite	tp 5½	grw	2⅗
Somervillite	– 5½	y	–
Idocrase	tl 6½	w	3⅗

CLASS 3. ORDER 5. GENUS 1.

The regular six-sided prism, Model 7, with the terminal edges, or the solid angles, or both, replaced. Examples:

P,T,M⅓⅓T₂. pm, pm₂t⅓⅓. Md. 56.

P,T,M⅓⅓T₂. ½p⅔m Zn, ½p⅔m₂t⅓⅓. = P,V. r⅔ Zn. Model 57.

P,T, M⅓⅓T₂. p⅔t, p⅔m⅓⅓t₂. Md. 58.

P,T, M⅓⅓T₂. pm, p⅙⅙t, pm₂t⅓⅓, p½⅙m⅓⅓t₂. Md. 52 without the six rectangular vertical planes.

Genus 1 *Continued:*

Graphite	m $1\frac{1}{2}$	bk	$2\frac{1}{4}$
Black Tellurium	m $2\frac{1}{4}$	w	$6\frac{1}{3}$
Polybasite	m $2\frac{1}{2}$	bk	$6\frac{1}{4}$
Vitreous Copper	m $2\frac{3}{4}$	gr	$5\frac{3}{4}$
Magnetic Iron Pyrites	m 4	bk	$4\frac{2}{3}$
Osmium-Iridium	m 5	gr	19
Specular Iron	m 6	rbr	$5\frac{1}{4}$
One-axed Mica	tp $2\frac{1}{4}$	wgr	3
Red Silver	o $2\frac{1}{4}$	r	$5\frac{4}{5}$
Cinnabar	tl $2\frac{1}{4}$	r	8
Calcareous Spar	tp 3	grw	$2\frac{3}{4}$
Dolomite	tl $3\frac{3}{4}$	grw	3
Phosphate of Lead	tl $3\frac{3}{4}$	yw	7
Arseniate of Lead	tl $3\frac{3}{4}$	yw	7
Coquimbite	tl ?	w	?
Pyrosmalite	o $4\frac{1}{4}$	br	3
Apatite Md. 58	tl 5	w	$3\frac{1}{4}$
Eudialyte	o $5\frac{1}{4}$	w	3
Nepheline	tl 6	w	$2\frac{2}{3}$
Quartz	tp 7	w	$2\frac{3}{4}$
Beryl Md. 56	tp $7\frac{3}{4}$	w	$2\frac{3}{5}$
Corundum Md. 57	tl 9	w	4

CLASS 3. ORDER 5. GENUS 2.

Regular twelve-sided prism, Md. 10, P,m,T, $m_2t\frac{1}{1}\frac{5}{3}$, $M\frac{1}{1}\frac{5}{3}T_2$, or P,V, *v*, with the terminal edges or the solid angles, or both, replaced.

Example:

P,m,T, $m_2t\frac{1}{1}\frac{5}{3}$, $M\frac{1}{1}\frac{5}{3}T_2$. pm, $p\frac{1}{2}\frac{5}{6}t$, $pm_2t\frac{1}{1}\frac{5}{3}$, $p\frac{1}{2}\frac{5}{6}m\frac{1}{1}\frac{5}{3}t_2$. Md. 52.

Vitreous Copper	m $2\frac{3}{4}$	gr	$5\frac{3}{4}$
Magnetic Iron Pyrites	m 4	bk	$4\frac{2}{3}$
Pinite	o $2\frac{1}{4}$	w	$2\frac{3}{4}$
Calcareous Spar	tp 3	grw	$2\frac{3}{4}$
Phosphate of Lead	tl $3\frac{3}{4}$	yw	7
Apatite	tl 5	w	$3\frac{1}{4}$
Eudialyte	o $5\frac{1}{4}$	w	3
Nepheline	tl 6	w	$2\frac{2}{3}$
Dichroite	tl $7\frac{1}{4}$	w	$2\frac{3}{5}$
Beryl	tp $7\frac{3}{4}$	w	$2\frac{3}{5}$

CLASS 3. ORDER 5. GENUS 3.

Divided into five groups:

a.) The prism has 4 vertical planes. Example: P_-,$M\frac{4}{3}T$. $p\frac{4}{3}t$. The planes $p\frac{4}{3}t$ meet and form two edges on the equator. Hence the equator is six-sided, although the prism has only four vertical planes.

b.) The prism has 6 vertical planes. Example: P_+,T, $M\frac{8}{17}T$. $p\frac{7}{12}m$. Model 55.

c.) The prism has 8 vertical planes. Examples:
P_-,m,t_+, $M\frac{4}{3}T$. $p\frac{4}{3}m$, $p\frac{4}{3}t$. Md. 50.
p_+,M_-,T, $M\frac{8}{9}T$. $P\frac{5}{4}M$, $P\frac{10}{9}T$,
$p\frac{5}{9}m\frac{8}{9}t$, Model 51.
p_+,M,T, $M\frac{20}{21}T$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$
Znw Zsw. Model 53.

d.) The prism has 9 vertical planes. Exam. of the 9 vertical planes:
$\frac{1}{2}$M,t, $\frac{1}{2}M_+T$, m_-t.
$\frac{1}{2}$m,T, $\frac{1}{2}m_+t$, M_-T.

e.) The prism has above 9 vertical planes. Example:
p_+,M,T, $m\frac{10}{36}t$, $m\frac{10}{18}t$. $P\frac{28}{29}T$.

Group a.

Arsenical Pyrites	m $5\frac{3}{4}$	bk	$6\frac{1}{8}$
White Iron Pyrites	m $6\frac{1}{4}$	bk	$4\frac{2}{3}$
Sulphur	tl 2	yw	2
Heavy Spar	tl $3\frac{1}{4}$	w	$4\frac{2}{5}$
Herderite	tl 5	w	3
Prehnite	tl $6\frac{1}{2}$	w	3

Group b.

Sulphuret of Bismuth	m $2\frac{1}{4}$	gr	$6\frac{3}{5}$
Brittle Sulphr^t. of Silv^r.	m $2\frac{1}{4}$	bk	$6\frac{1}{4}$
Antimonial Silver	m $3\frac{1}{2}$	w	$9\frac{1}{2}$
Koenigite	tp $2\frac{1}{2}$	gn	$3\frac{1}{2}$
Caledonite	tl $2\frac{3}{4}$	gnw	$6\frac{2}{5}$
Sulphate of Lead	tl $2\frac{3}{4}$	w	$6\frac{1}{3}$
Celestine	tp $3\frac{1}{4}$	w	$3\frac{7}{8}$
Heavy Spar	tl $3\frac{1}{4}$	w	$4\frac{2}{5}$
Witherite	tl $3\frac{1}{4}$	w	$4\frac{1}{3}$
White Lead Ore	tl $3\frac{1}{4}$	w	$6\frac{1}{2}$

Group b Continued:

Strontianite	tl	3½	w	3⅗
Thomsonite	tp	5	w	2⅖
Siliceous Oxide of Zinc	o	5	w	3½
Prehnite	tl	6½	w	3
Forsterite	tl	7	w	—
Andalusite	o	7½	w	3
Staurolite Md. 55	o	7½	w	3¾
Topaz	tp	8	w	3½

Group c.

Graphic Tellurium	m	1½	gr	5¾
White Tellurium	m	2½	yw	10
Bournonite	m	2¾	bk	5¾
Pyrolusite	m	2½	brbk	4⅗
Nitre	tp	2	w	2
Hopeite	tl	2¾	w	2½
Heavy Spar Md. 50	tl	3¼	w	4⅖
Desmine	tp	3¾	w	2⅕
Thomsonite	tp	5	w	2⅖
Augite	o	5½	wgr	3⅓

Group c Continued:

Prehnite	tl	6½	w	3
Olivine Md. 51	tl	6¾	w	3⅖

Group d.

Pinite	o	2½	w	2¾
Tourmaline	tp	7¼	w	3
Tourmaline	o	7¼	bk	3

Group e.

Graphic Tellurium	m	1½	gr	5¾
Bournonite	m	2¾	bk	5¾
Antimonial Silver	m	3½	w	9½
Tantalite	io	6	brbk	6
Columbite	io	6	brbk	6
Celestine	tl	3¼	w	3⅞
Heavy Spar	tl	3¼	w	4⅖
White Lead Ore	tl	3¼	w	6½
Euchroite	tl	3¾	gn	3⅖
Thomsonite	tp	5	w	2⅖
Olivine	tp	6¾	w	3⅖
Humite	tp	6¾	y	
Topaz	tp	8	w	3½

CLASS IV.—INCOMPLETE PRISMS COMBINED WITH COMPLETE PYRAMIDS.

Order 1. *Square Equator,*	Genus 1. Axes: $p^a\ m^a\ t^a$. Genus 2. Axes: $p^a_x\ m^a\ t^a$.
Order 2. *Rectangular Equator,*	Genus 1. Axes: $p^a_x\ m^a_y\ t^a_z$.
Order 3. *Rhombic Equator,*	Genus 1. Axes: $p^a\ m^a\ t^a$. Genus 2. Axes: $p^a_x\ m^a\ t^a$. Genus 3. Axes: $p^a_x\ m^a_y\ t^a_z$.
Order 4. *Rhombo-Quadratic Equator,*	Genus 1. Axes: $p^a\ m^a\ t^a$. Genus 2. Axes: $p^a_x\ m^a\ t^a$.
Order 5. *Rhombo-Rectangular Equator,*	Genus 1. Axes: $p^a_x\ m^a_{15}\ t^a_{13}$. Genus 2. Axes: $p^a_x\ m^a_{14}\ t^a_{13}$. Genus 3. Axes: $p^a_x\ m^a_y\ t^a_z$.

CLASS 4. ORDER 1. GENUS 1.
Divided into three groups:

a.) The Rhombic Dodecahedron alone. MT.PM, PT. Md. 63.

b.) The Rhombic Dodecahedron predominant, but combined with other forms. Examples:
MT.PM, PT, pmt. Md. 65.
MT.PM, PT, ½pmt.
MT.PM, PT, 3p⅓mt.
(MT.PM, PT) × 2.

c.) The Regular Octahedron predominant. Examples:
mt.pm, pt, PMT. Model 64.
MT.PM, PT, PMT, 3p⅓mt.
mt.pm, pt, PMT, 3p+mt.

Group a.

Sulphuret of Silver	m	2¼	gr	7⅕
Bismuth	m	2¼	w	9¾
Copper	m	2¾	r	8⅗
Silver	m	2¾	w	10½
Gold	m	2¾	y	15

Group a Continued:

Amalgam	m 3¼	w	13¾
Grey Copper, Mixed	m 3½	bk	5
Do. Antimonial	m 3½	bk	5
Do. Arsenical	m 4	rgr	4⅖
Magnetic Iron Ore	m 6	bk	5
Red Oxide of Copper	itl 3¾	r	6
Muriate of Ammonia	tl 1¾	w	1½
Chloride of Silver	o 1¼	grbr	5⅗
Chloride of Sodium	tp 2	w	2¼
Zinc Blende	tl 3¾	wbr	4
Fluorspar	tl 4	w	3⅐
Sodalite	tl 5¾	w	2¼
Cancrinite	tl 5¾	w	2⅓
Nosian	tl 5¾	gr	2¼
Lapis-Lazuli	tl 5¾	bl	3
Haüyne	tl 7	w	2⅔
Garnet	tl 7	w	4
Rhodizite	tl 7	w	3
Uwarowite	tp 7	w	3¼
Pyrope	tl 7	w	3¾
Pleonaste, bk	tl 7½	w	3⅗
Spinel, r,bl	tp 8	w	3½
Diamond	tp 10	w	3½

Group b.

Bismuth	m 2¼	w	9¾
Copper	m 2¾	r	8⅗
Silver	m 2¾	w	10½
Amalgam	m 3¼	w	13¾
Grey Copper, Antim[l].	m 3½	bk	5
Magnetic Iron Ore	m 6	bk	5
Red Oxide of Copper	itl 3¾	r	6
Zinc Blende	tl 3¾	wbr	4
Nosian	tl 5¾	gr	2¼
Lapis Lazuli	tl 5¾	bl	3
Haüyne	tl 7	w	2⅔
Rhodizite	tl 7	w	3
Boracite	tl 7	w	3
Pleonaste, bk	tl 7½	w	3⅗
Diamond	tp 10	w	3½

Group c.

Bismuth	m 2¼	w	9¾
Galena	m 2½	gr	7⅔
Copper	m 2¾	r	8⅗
Gold	m 2¾	y	15
Amalgam	m 3¼	w	13¾

Group c Continued:

Zinc Blende	m 3¾	wbr	4
Arsen[l]. Grey Copper	m 4	rgr	4⅖
Magnetic Iron Ore	m 6	bk	5
Iron Pyrites	m 6¼	brbk	5
Franklinite	m 6¼	br	5
Red Oxide of Copper	itl 3¾	r	6
Chromate of Iron	io 5½	br	4½
Fluorspar	tl 4	w	3⅐
Rhodizite	tl 7	w	3
Pleonaste, bk	tl 7½	w	3⅗
Spinel, r bl	tp 8	w	3½
Diamond	tp 10	w	3½

CLASS 4. ORDER 1. GENUS 2.

Divided into two groups:

a.) The prism has four vertical planes. Examples:
MT. pm, pt.
MT. P⅔MT. Model 61.
(M,T. P⅔M, P⅔T) × 2. Md.62.
MT. p⅔m, p⅔t, P$\frac{3}{2}$MT.

b.) The prism has eight vertical planes. Examples:
M,T, MT. P⅔MT. Model 60.
m,t, MT. p_2mt, P⅔MT, p_2mt_2,
p_2m_3t.
M,T, MT. P⅗M, P⅗T. Md. 59.
M,T, MT. P⅗M, P⅗T, p⅔mt.

Group a.

Sulphuret of Silver	m 2¼	gn	7⅕
Magnetic Iron Ore	m 6	bk	5
Chloride of Mercury	tl 1½	w	6½
Muriate of Ammonia	tl 1¾	w	1½
Mellite	tp 2¼	w	1⅗
Murio-Carb[e]. of Lead	tp 2¾	w	6
Molybdate of Lead	tl 3	w	6¾
Tungstate of Lead	tl 3	gr	8
Apophyllite	tp 4¾	w	2⅓
Phosphate of Yttria	o 4¾	b	4¼
Rutile	tl 6¼	br	4¼
Oxide of Tin Md. 62	tl 6½	br	7
Garnet	tl 7	w	3
Zircon Md. 61	tp 7½	w	4½
Spinel	tp 8	w	3½

Group b.

Chloride of Mercury...tl $1\frac{1}{2}$ w $6\frac{1}{2}$
Murio-Carbn[e]. of Lead tp $2\frac{3}{4}$ w 6
Wernerite Md. 59......tp $5\frac{1}{4}$ grw $2\frac{3}{5}$
Oerstedtite ....o $5\frac{1}{2}$ br $3\frac{3}{5}$
Anatase ....tl $5\frac{3}{4}$ w $3\frac{4}{5}$
Rutile.....................tl $6\frac{1}{4}$ br $4\frac{1}{4}$
Oxide of Tin............tl $6\frac{1}{2}$ br 7
Zircon Md. 60...........tp $7\frac{1}{2}$ w $4\frac{1}{2}$

Class 4. Order 2. Genus 1.

Scalene Octahedron, with the lateral angles replaced by the planes of a right rectangular prism. $M_{-}, T. P_x M_y T_z$.

Desmine ....tp $3\frac{3}{4}$ w $2\frac{1}{5}$
Scorodite ....tl $3\frac{3}{4}$ w $3\frac{2}{5}$
Harmotome ....tl $4\frac{1}{2}$ w $2\frac{2}{5}$

Class 4. Order 3. Genus 1.

Combinations containing either the Tetrakishexahedron or the Pentagonal Dodecahedron. Exam.: $M\frac{1}{2}T, M\frac{2}{1}T. P\frac{1}{2}M, P\frac{2}{1}M, P\frac{1}{2}T, P\frac{2}{1}T$. Model 68.

Copper Md. 68 ....m $2\frac{3}{4}$ r $8\frac{3}{5}$
Gold Md. 68 ....m $2\frac{3}{4}$ y 15
Iron Pyrites ....m $6\frac{1}{4}$ brbk 5
Fluorspar ....tl 4 w $3\frac{1}{7}$

Class 4. Order 3. Genus 2.

$M\frac{1}{2}T, M\frac{2}{1}T. P\frac{1}{3}MT$.

Rutile.....................tl $6\frac{1}{4}$ br $4\frac{1}{4}$

Class 4. Order 3. Genus 3.

Rhombic prisms terminated by rectangular or scalene pyramids; or scalene octahedrons with the horizontal edges replaced by the vertical planes of rhombic prisms. Examples:
$M\frac{8}{10}T. P\frac{19}{10}M\frac{8}{10}T$. Model 66.
$M_{-}T. P_x M_y T_z, p_x m_y t_z$.
$M\frac{4}{5}T. p\frac{3}{4}m, p\frac{5}{4}t$.

[The characteristic relates to the prism.]

Sulph[t]. of Antimony $\frac{24}{25}$ m 2 gr $4\frac{3}{5}$
Antimonial Silver $\frac{7}{12}$...m $3\frac{1}{2}$ w $9\frac{1}{2}$
Manganite $\frac{5}{6}$ ....io 4 rbr $4\frac{1}{3}$
Lievrite $\frac{6}{9}$ ....io $5\frac{3}{4}$ gnbk 4
Sulphur $\frac{8}{10}$ Md. 66......tl 2 yw 2
Sulph[e]. of Magnesia $\frac{99}{100}$ tp $2\frac{1}{4}$ w $1\frac{3}{4}$
Sulphate of Zinc $\frac{99}{100}$ tp $2\frac{1}{4}$ w 2
Chromate of Lead $\frac{16}{17}$ o $2\frac{3}{4}$ ry 6
Celestine $\frac{4}{5}$..............tp $3\frac{1}{4}$ w $3\frac{7}{8}$
Heavy Spar $\frac{4}{5}, \frac{3}{4}$.........tl $3\frac{1}{4}$ w $4\frac{2}{5}$
Mesotype $\frac{49}{50}$ Md. 67...tp $5\frac{1}{4}$ w $2\frac{1}{4}$
Aeschynite $\frac{1}{2}$ ....o $5\frac{1}{2}$ bk $5\frac{1}{2}$
Topas $\frac{19}{36}, \frac{19}{18}$ ....tp 8 w $3\frac{1}{2}$

Class 4. Order 4. Genus 1.

Divided into three groups:

a.) MT.PM,PT predominant. Ex.:
MT. PM, PT, $3p\frac{1}{2}mt$.
MT. PM, PT, PMT, $3p\frac{1}{3}mt$.
MT. PM, PT, $3P\frac{1}{2}MT$, $6p\frac{1}{3}m\frac{1}{2}t$.

b.) PMT predominant. Examples:
mt. pm, pt, PMT, $3p\frac{1}{3}mt$.
$m\frac{1}{2}t, m\frac{2}{1}t. p\frac{1}{2}m, p\frac{2}{1}m, p\frac{1}{2}t, p\frac{2}{1}t$, PMT.

c.) $3P\frac{1}{2}MT$ predominant. Example:
mt. pm, pt, $3P\frac{1}{2}MT$. Md. 69.

Group a.

Amalgam ....m $3\frac{1}{4}$ w $13\frac{3}{4}$
Grey Copper, Arsen[l]. m 4 rgr $4\frac{2}{5}$
Magnetic Iron Ore.....m 6 bk 5
Red Oxide of Copper itl $3\frac{3}{4}$ r 6
Garnet ....tl 7 w 4
Pleonaste bk ....tl $7\frac{1}{2}$ w $3\frac{3}{5}$
Spinel r, bl...............tp 8 w $3\frac{1}{2}$

Group b.

Magnetic Iron Ore.....m 6 bk 5
Iron Pyrites ....m $6\frac{1}{4}$ brbk 5
Red Oxide of Copper itl $3\frac{3}{4}$ r 6
Fluorspar ....tl 4 w $3\frac{1}{7}$
Pleonaste bk ....tl $7\frac{1}{2}$ w $3\frac{3}{5}$

Group c.

Arsen[l]. Grey Copper...m 4 rgr $4\frac{3}{5}$
Garnet Md. 69 ....tl 7 w 4

CLASS 4. ORDER 4. GENUS 2.

Combination of the right square prism with bevelled edges (the 12-sided or 16-sided prism of the pyramidal system), with square-based pyramids. Ex.:

M, T, $m\frac{1}{2}t$, $m\frac{2}{1}t$. $P\frac{4}{4}MT$.

m, t, MT, $m\frac{1}{3}t$, $m\frac{3}{1}t$. $p\frac{3}{3}m$, $p\frac{3}{3}t$, $P\frac{3}{3}MT$.

Mellite	tp $2\frac{1}{4}$	w	$1\frac{2}{3}$
Apophyllite	tp $4\frac{3}{4}$	w	$2\frac{1}{3}$
Wernerite	tp $5\frac{1}{4}$	grw	$2\frac{2}{3}$
Rutile	tl $6\frac{1}{4}$	br	$4\frac{1}{4}$
Oxide of Tin	tl $6\frac{1}{2}$	br	7

CLASS 4. ORDER 5. GENUS 1.

Divided into four groups:

a.) Regular six-sided prism with a rhombohedral or three-faced pyramid at each end. Exam.:

T, $M\frac{1}{1}\frac{5}{3}T_2$. $\frac{1}{2}PM$ Zn, $\frac{1}{2}PM_2T\frac{1}{1}\frac{5}{3}$. or V. R_1 Zn. Model 71.

T, $M\frac{1}{1}\frac{5}{3}T_2$. $\frac{1}{2}PT$ Zw, $\frac{1}{2}PM\frac{1}{1}\frac{5}{3}T_2$. or V. R_1 Zw. Model 72.

b.) Regular six-sided prism with regular six-sided pyramid at each end. Examples:

T, $M\frac{1}{1}\frac{5}{3}T_2$. $P\frac{5}{4}T$, $P\frac{5}{4}M\frac{1}{1}\frac{5}{3}T_2$. or V. $2R\frac{5}{4}$. Model 73.

T, $M\frac{1}{1}\frac{5}{3}T_2$. $P\frac{1}{1}\frac{5}{3}T$, $P\frac{1}{1}\frac{5}{3}M\frac{1}{1}\frac{5}{3}T_2$. or V. $2R\frac{1}{1}\frac{5}{3}$. Model 74.

c.) Regular six-sided prism terminated by two or more dissimilar pyramids, but without scalenohedrons. Examples:

V. $R\frac{5}{4}$ Zw, $r\frac{5}{4}$ Ze.

T, $M\frac{1}{1}\frac{5}{3}T_2$. $p\frac{5}{2}m$, $P\frac{5}{4}T$, $p\frac{5}{2}m_2t\frac{1}{1}\frac{5}{3}$, $P\frac{5}{4}M\frac{1}{1}\frac{5}{3}T_2$.

d.) Regular six-sided prism terminated by complex pyramids with scalenohedrons. Examples:

T, $M\frac{1}{1}\frac{5}{3}T_2$. $\frac{1}{2}(3P_1M\pm T\mp)$.

V. $2R\frac{5}{4}$ ZwZe, $3s_+$.

Group a.

Red Silver	o $2\frac{1}{4}$	r	$5\frac{2}{3}$
Sulphato-tricarbonate of Lead,	tp $2\frac{1}{2}$	w	$6\frac{1}{4}$
Calcareous Spar	tp 3	grw	$2\frac{3}{4}$
Dioptase	tp 5	gn	$3\frac{1}{4}$
Phenakite	tp 6	w	3
Willelmine	o 6	rw	$4\frac{1}{4}$
Corundum	tp 9	w	4

Group b.

Sulph^t. of Molybdenum	m $1\frac{1}{4}$	gr	$4\frac{1}{2}$
Zinkenite	m $3\frac{1}{4}$	gr	$5\frac{1}{3}$
Magnetic Iron Pyrites	m 4	bk	$4\frac{2}{3}$
Calcareous Spar	tp 3	grw	$2\frac{3}{4}$
Phos^e. of Lead Md. 74	tl $3\frac{3}{4}$	yw	7
Arseniate of Lead	tl $3\frac{3}{4}$	yw	7
Apatite	tl 5	w	$3\frac{1}{4}$
Phenakite	tp 6	w	3
Quartz Md. 73	tp 7	w	$2\frac{3}{4}$
Corundum	tp 9	w	4

Group c.

Red Silver	o $2\frac{1}{4}$	r	$5\frac{2}{3}$
Calcareous Spar	tp 3	grw	$2\frac{3}{4}$
Dolomite	tl $3\frac{1}{2}$	grw	3
Arseniate of Lead	tl $3\frac{3}{4}$	yw	7
Carbonate of Iron	tl 4	brw	$3\frac{2}{3}$
Chabasite	tp $4\frac{1}{4}$	w	2
Phenakite	tp 6	w	3
Quartz	tp 7	w	$2\frac{3}{4}$
Beryl	tp $7\frac{3}{4}$	w	$2\frac{2}{3}$

Group d.

Red Silver	o $2\frac{1}{4}$	r	$5\frac{2}{3}$
Calcareous Spar	tp 3	grw	$2\frac{3}{4}$
Carbonate of Iron	tl 4	brw	$3\frac{2}{3}$
Chabasite	tp $4\frac{1}{4}$	w	2
Quartz	tp 7	w	$2\frac{3}{4}$

CLASS 4. ORDER 5. GENUS 2.

The vertical planes of the twelve-sided prism, m, T, $m_2t\frac{1}{1}\frac{5}{3}$, $M\frac{1}{1}\frac{5}{3}T_2$, Model 10, terminated by rhombohedral pyramids, or by rhombohedrons and scalenohedrons.

Vitreous Copper	m $2\frac{3}{4}$	gr	$5\frac{3}{4}$
Calcareous Spar	tp 3	grw	$2\frac{3}{4}$
Apatite	tl 5	w	$3\frac{1}{4}$

CLASS 4. ORDER 5. GENUS 3.

Divided into three groups:

a.) The prism has either three or nine vertical planes, and the pyramidal terminations are rhombohedral, and generally different at each end. Exam.:

Mn,$\frac{1}{3}$M,T$\frac{1}{13}$se sw.R$\frac{1}{3}$Zn,$\frac{1}{3}$r,Nn.
mn,T,$\frac{1}{3}$m,t$\frac{1}{13}$se sw, M$\frac{1}{13}$T,. $\frac{1}{2}$r$\frac{1}{2}$
Zn, $\frac{1}{3}$r,Zn, $\frac{1}{3}$R,Zs, $\frac{1}{3}$R$\frac{1}{3}$Ns.

b.) The prism has six vertical planes (= T,M_T), and the terminations are scalene octahedrons. Examples:

T_,M$\frac{3}{4}$T. p$\frac{4}{3}$m, p,t, p$\frac{3}{5}$m$\frac{3}{4}$t.
T_,M$\frac{9}{13}$T. $\frac{1}{3}$P$\frac{5}{13}$M$\frac{9}{13}$T ZneZnw,
$\frac{1}{3}$P$\frac{6}{13}$M$\frac{10}{13}$T Zse Zsw. Md. 75.

c.) The prism has any other number of vertical planes than 3, 6, or 9; and the terminations are scalene pyramids. Exam.:

m_,P$\frac{19}{10}$M$\frac{8}{10}$T. Model 70.

Group a.

Tourmaline	tp	7$\frac{1}{4}$	w	3
Tourmaline	o	7$\frac{1}{4}$	w	3

Group b.

[The characteristic relates to the form M_T.]

Sulph. of Antimony $\frac{2}{2}\frac{4}{3}$	m	2	gr	4$\frac{3}{5}$
Antimonial Silver $\frac{7}{12}$	m	3$\frac{1}{2}$	w	9$\frac{1}{2}$

Group b Continued:

Nitre $\frac{11}{19}$	tp	2	w	2
Gypsum $\frac{9}{13}$ Md. 75	tp	2	w	2$\frac{1}{4}$
Sulphur	tl	2	yw	2
Sulphate of Zinc $\frac{99}{100}$	tp	2$\frac{1}{4}$	w	2
Heavy Spar $\frac{3}{4}$	tl	3$\frac{1}{4}$	w	4$\frac{2}{5}$
Witherite $\frac{3}{5}$	tl	3$\frac{1}{4}$	w	4$\frac{1}{3}$
White Lead Ore $\frac{3}{5}$	tl	3$\frac{1}{4}$	w	6$\frac{1}{2}$
Strontianite $\frac{3}{5}$	tl	3$\frac{1}{2}$	w	3$\frac{3}{5}$
Arragonite $\frac{5}{8}$	tl	3$\frac{3}{4}$	grw	3
Childrenite $\frac{19}{20}$	tl	4$\frac{3}{4}$	w	?
Prismatic Iron Ore $\frac{23}{23}$	tl	5$\frac{1}{4}$	ybr	4
Monticellite	tp	5$\frac{1}{2}$	y	?
Augite $\frac{20}{21}$	o	5$\frac{1}{2}$	wgr	3$\frac{1}{4}$
Brookite $\frac{5}{6}$	tl	5$\frac{3}{4}$	yw	?

Group c.

Sulphuret of Antimony	m	2	gr	4$\frac{3}{5}$
White Iron Pyrites	m	6$\frac{1}{4}$	bk	4$\frac{3}{5}$
Tantalite	io	6	brbk	6
Columbite	io	6	brbk	6
Polymignite	io	6$\frac{1}{2}$	br	4$\frac{4}{5}$
Sulphur	tl	2	yw	2
Sulphate of Magnesia	tp	2$\frac{1}{4}$	w	1$\frac{3}{4}$
Desmine	tp	3$\frac{3}{4}$	w	2$\frac{1}{5}$
Scorodite	tl	3$\frac{3}{4}$	w	3$\frac{2}{5}$
Harmotome	tl	4$\frac{1}{2}$	w	2$\frac{2}{5}$
Prismatic Iron Ore	tl	5$\frac{1}{4}$	ybr	4
Chrysoberyl	tp	8$\frac{1}{2}$	w	3$\frac{3}{4}$

CLASS V.—INCOMPLETE PRISMS COMBINED WITH INCOMPLETE PYRAMIDS.

Order 1. *Square Equator,*	Genus 1.	Axes: $p^x\ m^x\ t^x$.
	Genus 2.	Axes: $p^x_x\ m^x\ t^x$.
Order 2. *Rectangular Equator,*	Genus 1.	Axes: $p^x_x\ m^x_y\ t^x_z$.
Order 3. *Rhombic Equator,*	Genus 1.	Axes: $p^x_x\ m^x_y\ t^x_z$.
Order 4. *Rhombo-Quadratic Equator,*	Genus 1.	Axes: $p^x\ m^x\ t^x$.
	Genus 2.	Axes: $p^x_x\ m^x\ t^x$.
Order 5. *Rhombo-Rectangular Equator,*	Genus 1.	Axes: $p^x_x\ m^x_{18}\ t^x_{18}$.
	Genus 2.	Axes: $p^x_x\ m^x_y\ t^x_z$.

CLASS 5. ORDER 1. GENUS 1.

Combinations in which the tetrahedron is predominant. Exam.:

mt. pm, pt, $\frac{1}{2}$PMT. Md. 78.

Bismuth	m	2$\frac{1}{4}$	w	9$\frac{4}{5}$
Grey Copper	m	3$\frac{1}{2}$	bk	5
Helvine	tl	6$\frac{1}{4}$	w	3

Class 5. Order 1. Genus 2.

Divided into two groups:

a.) Quadratic pyramids combined with the horizontal planes P.

Examples:

$P\_ . P\frac{4}{3}M, P\frac{4}{3}T$. Model 76.

p. pm, pt, PMT. Model 77.

b.) Pyramidal forms combined with vertical prismatic forms.

Group a.

Black Tellurium.........	m	$1\frac{1}{4}$	grbk	7
Copper Pyrites Md. 77	m	$3\frac{3}{4}$	gnbk	$4\frac{1}{6}$
Braunite..................	io	$6\frac{1}{4}$	bk	$4\frac{4}{5}$
Mellite	tp	$2\frac{1}{4}$	w	$1\frac{3}{5}$
Uranite	tl	$2\frac{1}{4}$	gn	$3\frac{1}{8}$
Tungstate of Lead......	tl	3	gr	8
Molyb^{e}. of Lead Md. 76	tl	3	w	$6\frac{3}{4}$
Tungstate of Lime	tl	$4\frac{1}{4}$	w	6
Apophyllite	tp	$4\frac{3}{4}$	w	$2\frac{1}{3}$
Anatase	tl	$5\frac{3}{4}$	w	$3\frac{4}{5}$

Group b.

Edingtonite	tl	$4\frac{1}{4}$	w	$2\frac{3}{4}$
Rutile......................	tl	$6\frac{1}{4}$	br	$4\frac{1}{4}$

Class 5. Order 2. Genus 1.

All with vertical prismatic planes, except Bournonite. Examples:

$M_x, T. P\frac{4}{7}M$. Model 79^a.

$M\_, T. \frac{1}{2}P\frac{3}{7}M$ Zn. Model 79.

$M\_, T. \frac{1}{2}P\frac{1}{2}M$ Zn. Model 79^b.

Bournonite...............	m	$2\frac{3}{4}$	bk	$5\frac{3}{4}$
Pyrophyllite?............	tp	$1\frac{1}{2}$	gnw	$2\frac{4}{5}$
Gypsum Md. 79.........	tp	2	w	$2\frac{1}{3}$
Cobalt Bloom	tl	$2\frac{1}{4}$	r	3
Heulandite................	tl	$3\frac{3}{4}$	w	$2\frac{1}{5}$
Harmotome	tl	$4\frac{1}{2}$	w	$2\frac{2}{5}$
Tungstate of Iron	o	$5\frac{1}{4}$	rbr	$7\frac{1}{5}$
Augite	o	$5\frac{1}{2}$	wgr	$3\frac{1}{3}$
Felspar	tl	6	grw	$2\frac{3}{5}$
Epidote Md. 79^b.........	tl	$6\frac{1}{2}$	grw	$3\frac{1}{3}$
Olivine	tp	$6\frac{3}{4}$	w	$3\frac{2}{5}$
Chrysoberyl Md. 79^a...	tp	$8\frac{1}{2}$	w	$3\frac{3}{4}$

Class 5. Order 3. Genus 1.

Divided into nine groups:

a.) Scalene octahedrons with the apex replaced by the horizontal planes P. Example:

$p_{+} . P\frac{19}{10}M\frac{8}{10}T$. Model 80.

The planes P_xM, P_xT may also be present.

b.) Type: $M\_T. P_xM$. A right rhombic prism with homohedral oblique terminations belonging to the north zone. $P_xM_yT_z$ may also be present, but no form belonging to the east zone. Example:

$M\frac{4}{5}T. P\frac{10}{13}M$.

c.) Type: $M\_T. \frac{1}{2}P_xM$ Zn. A right rhombic prism, terminated by hemihedral oblique forms belonging to the north zone. It may also have $\frac{1}{2}p_xm_yt_z$ Zne Znw, or $\frac{1}{2}p_xm_yt_z$ Zse Zsw, but not $\frac{1}{2}p_xm_yt_z$ Znw Zsw, nor $\frac{1}{2}p_xm_yt_z$ Zne Zsw, nor any form belonging to the east zone. Exam.:

$M\frac{15}{16}T. \frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs. Model 81.

$M\frac{10}{19}T. \frac{1}{2}P\frac{4}{15}M$ Zn. Md. 84.

d.) Type: $M\_T. P_xT$. A right rhombic prism, terminated by homohedral oblique forms belonging to the east zone. It may also have the form $P_xM_yT_z$ but no form belonging to the north zone. Examples:

$M\frac{4}{5}T. P\frac{5}{4}T$. Axes: $p^{a}_{12}m^{a}_{4}t^{a}_{5}$. Model 82.

$M\frac{4}{5}T. P\frac{5}{4}T$. Axes: $p^{a}_{25}m^{a}_{16}t^{a}_{20}$. Model 82^b.

$M\frac{6}{10}T. P\frac{7}{10}T$. Axes $p^{a}_{7}m^{a}_{6}t^{a}_{10}$. Model 82^a.

$M\frac{10}{36}T, m\frac{10}{18}t. P\frac{28}{29}T, p\frac{35}{36}m\frac{10}{36}t$.

e.) Type: M_T. $\frac{1}{2}P_xT$ Zw. A right rhombic prism terminated by hemihedral oblique forms belonging to the east zone. It may also have $\frac{1}{2}p_xm_yt_z$ Znw Zsw, or $\frac{1}{2}p_xm_yt_z$ Zne Zse, but not $\frac{1}{2}p_xm_yt_z$ Zne Znw, nor $\frac{1}{2}p_xm_yt_z$ Zse Zsw, nor any form belonging to the north zone. Exam.: $M\frac{20}{21}T$. $\frac{1}{2}P_{\frac{6}{21}}T$ Zw. Model 87.

f.) Type: M_T.P_xM,P_xT. A right rhombic prism terminated by a combination of homohedral oblique forms belonging both to the north and east zones. It may also have the form $P_xM_yT_z$. Example: $M\frac{2}{3}T$. $p\frac{4}{7}m$, $P\frac{3}{10}T$.

g.) Type: M_T. $\frac{1}{2}P_xM_yT_z$ Zne Znw Nse Nsw. A right rhombic prism terminated by a hemihedral scalene octahedron having the positions Zne Znw Nse Nsw. It must have no forms belonging to the north and east zones, but it may have other hemioctahedrons in the positions Zse Zsw Nne Nnw. Ex.: $M\frac{3}{5}T$. $\frac{1}{2}P_+M_xT$ Zne Znw.

h.) Type: M_T. $\frac{1}{2}P_xM_yT_z$ Znw Zsw Nne Nse. It may also have $\frac{1}{2}p_xm_yt_z$ Zne Zse Nnw Nsw. It must have no forms belonging to the north or east zones.

i.) Type: M_T. $\frac{1}{4}P_xM_yT_z$ Znw Nse. A doubly oblique combination, containing three pair of parallel planes, of which two pair must belong to the prismatic and one pair to the octahedral zone, and none of which must meet at a right angle. Several other varieties of $\frac{1}{4}p_xm_yt_z$ may be present. The prism M_T is frequently $\frac{1}{2}$M_T, $\frac{1}{2}$m_t.

Example:

$M\frac{2}{5}T$. $\frac{1}{4}P_xM_yT_zZ^2w$, $\frac{1}{4}p_xm_yt_zZn^2e^2$, $\frac{1}{4}p_xm_yt_zZ^2ne$.

Group a.

Sulphur Md. 80 ....tl 2 yw 2
Childrenite...............tl $4\frac{3}{4}$ w ?
Fluellite ....tp ? w ?

Group b.

[The characteristic relates to M_T.]

Arsenical Iron $\frac{5}{9}$ ....m $5\frac{1}{2}$ bk $7\frac{1}{4}$
Lievrite $\frac{6}{9}$ ....io $5\frac{3}{4}$ gnbk 4
Lenticular Cop^r.Ore $\frac{7}{12}$ tl $2\frac{1}{2}$ blgn 3
Celestine $\frac{4}{5}$..................tp $3\frac{1}{4}$ w $3\frac{7}{8}$
Heavy Spar $\frac{3}{4}$ ....tl $3\frac{1}{2}$ w $4\frac{2}{5}$
Wavellite $\frac{5}{9}$ ....tl $3\frac{3}{4}$ w $2\frac{1}{3}$
Felspar $\frac{15}{26}$, Md. 81 .....tl 6 grw $2\frac{3}{5}$

Group c.

[The characteristic relates to M_T.]

Plagionite $\frac{4}{7}$ ....m $2\frac{1}{2}$ bkgr $5\frac{2}{5}$
Glauber's Salt $\frac{11}{13}$ ....tp $1\frac{3}{4}$ w $1\frac{1}{2}$
Red Iron Vitriol $\frac{15}{26}$ ...tl $2\frac{3}{4}$ y 2
Chromate of Lead $\frac{16}{17}$ o $2\frac{3}{4}$ ry 6
Azure Copper Ore $\frac{6}{7}$...tp $3\frac{1}{2}$ bl $3\frac{4}{5}$
Baryto-Calcite $\frac{10}{11}$ ......tp 4 w $3\frac{2}{3}$
Wagnerite $\frac{10}{11}$.............tl $5\frac{1}{2}$ w $3\frac{1}{8}$
Titanite $\frac{2}{3}$ ....tl $5\frac{1}{4}$ w $3\frac{1}{2}$
Hornblende $\frac{10}{19}$, Md.84 o $5\frac{1}{2}$ gr 3
Felspar $\frac{15}{26}$, Md.81, 81^a tl 6 grw $2\frac{3}{5}$
Monazite $\frac{10}{11}$ ....tl 6 rw $4\frac{8}{9}$
Rutile $\frac{1}{2}$ ....tl $6\frac{1}{2}$ br $4\frac{1}{4}$
Gadolinite $\frac{7}{11}$.............o $6\frac{3}{4}$ gngr $4\frac{1}{4}$

Group d.

[The characteristic relates to M_T.]

Galena $\frac{7}{10}$ ....m $2\frac{1}{2}$ gr $7\frac{2}{3}$
Silver $\frac{7}{10}$.....................m $2\frac{3}{4}$ w $10\frac{1}{2}$
Amalgam $\frac{7}{10}$ ....m $3\frac{1}{2}$ w $13\frac{3}{4}$
Sulphr^t. of Cobalt $\frac{7}{10}$...m $5\frac{1}{2}$ gr $6\frac{1}{3}$
Tin White Cobalt $\frac{7}{10}$...m $5\frac{1}{2}$ gr $6\frac{1}{2}$
Arsenical Pyrites $\frac{2}{3}$ ....m $5\frac{3}{4}$ bk $6\frac{1}{8}$
Magnetic Iron Ore $\frac{7}{10}$ m 6 bk 5
White Iron Pyrites $\frac{3}{4}$...m $6\frac{1}{2}$ bk $4\frac{3}{5}$
Red Ox. of Copper $\frac{7}{10}$ itl $3\frac{1}{2}$ r 6
Lievrite $\frac{6}{9}$ ....io $5\frac{3}{4}$ gnbk 4
Oxide of Arsenic $\frac{7}{10}$...o $1\frac{1}{2}$ w $3\frac{2}{3}$
Nitre $\frac{11}{19}$......................tp 2 w 2
Sulph^e. of Lead $\frac{4}{5}$, Md.82^b tl $2\frac{3}{4}$ w $6\frac{1}{3}$
Celestine $\frac{4}{5}$..................tp $3\frac{1}{2}$ w $3\frac{7}{8}$
Heavy Spar $\frac{4}{5}$, $\frac{3}{4}$, Md.82 tl $3\frac{1}{2}$ w $4\frac{2}{5}$

Group d Continued:

White Lead Ore $\frac{6}{10}$,
Model 82[a] ....tl $3\frac{1}{4}$ w $6\frac{1}{2}$
Muriate of Copper $\frac{2}{3}$...tl $3\frac{1}{4}$ gn $4\frac{2}{5}$
Olivenite $\frac{9}{10}$ ....tl $3\frac{1}{4}$ gnbr $4\frac{1}{2}$
Junkerite $\frac{5}{7}$...............o $3\frac{1}{2}$ y $3\frac{4}{5}$
Arragonite $\frac{5}{8}$ ....tl $3\frac{3}{4}$ grw 3
Libethenite $\frac{9}{10}$ ....tl 4 gn $3\frac{3}{4}$
Topas $\frac{19}{36}$ Md. 90 ......tp 8 w $3\frac{1}{2}$
Spinel $\frac{7}{10}$, r, bl ....tp 8 w $3\frac{1}{2}$

Group e.

[The characteristic relates to M_T.]

Magnetic Iron Ore $\frac{7}{10}$ m 6 bk 5
Myargyrite $\frac{14}{15}$ ....io $2\frac{1}{4}$ r $5\frac{1}{4}$
Red Ox. of Copper $\frac{7}{10}$ itl $3\frac{3}{4}$ r 6
Sulphate of Iron $\frac{6}{7}$......tp 2 w $1\frac{4}{5}$
Leadhillite $\frac{3}{5}$..............tl $2\frac{1}{2}$ w $6\frac{1}{3}$
Glauberite $\frac{9}{10}$...........tl $2\frac{3}{4}$ w $2\frac{1}{5}$
Trona $\frac{4}{5}$ ....tl $2\frac{3}{4}$ w $2\frac{1}{8}$
Oblique prismatic Ar-
seniate of Cop[r]. $\frac{8}{15}$...tp $2\frac{3}{4}$ gn $4\frac{1}{4}$
Laumonite $\frac{15}{16}$............tl 3 w $2\frac{1}{5}$
Phosphate of Copper $\frac{6}{17}$ tl $4\frac{3}{4}$ gn $4\frac{1}{5}$
Tungstate of Iron $\frac{6}{7}$....o $5\frac{1}{4}$ rbr $7\frac{1}{5}$
Augite $\frac{20}{21}$, Md. 87......o $5\frac{1}{2}$ wgr $3\frac{1}{3}$

Group f.

[The characteristic relates to M_T.]

Schilfglaserz $\frac{5}{6}$ ....m $2\frac{1}{4}$ w $5\frac{1}{2}$
Needle Ore $\frac{1}{2}$ ....m $2\frac{1}{4}$ gr 6
Arsenical Pyrites $\frac{2}{3}$.....m $5\frac{3}{4}$ bk $6\frac{1}{8}$
Manganite $\frac{5}{6}$ ....io 4 rbr $4\frac{1}{3}$
Lievrite $\frac{6}{9}$ ....io $5\frac{3}{4}$ gnbk 4
Celestine $\frac{4}{5}$...............tp $3\frac{1}{4}$ w $3\frac{7}{8}$
Heavy Spar $\frac{3}{4}$............tl $3\frac{1}{4}$ w $4\frac{2}{5}$
Epistilbite $\frac{7}{17}$............tp $4\frac{1}{4}$ w $2\frac{1}{4}$
Topas $\frac{19}{36}$ ....tp 8 w $3\frac{1}{2}$

Group g.

[The characteristic relates to M_T.]

Johannite $\frac{1}{2}$...............tl $2\frac{1}{4}$ gn $3\frac{1}{5}$
Chromate of Lead $\frac{16}{17}$...o $2\frac{3}{4}$ ry 6
Huraulite $\frac{3}{5}$ ....tl $3\frac{1}{2}$ yw $2\frac{1}{4}$
Couzeranite $\frac{9}{10}$ ....o 5 gr $2\frac{2}{3}$
Mesotype $\frac{48}{50}$ ....tp $5\frac{1}{4}$ w $2\frac{1}{4}$
Hornblende $\frac{18}{19}$ ....o $5\frac{1}{2}$ gr 3

Group h.

Augite $\frac{20}{21}$ ....o $5\frac{1}{2}$ wgr $3\frac{1}{3}$

Group i.

Blue Vitriol ....tl $2\frac{1}{2}$ w $2\frac{1}{4}$
Petalite ....tl $6\frac{1}{4}$ w $2\frac{1}{2}$
Axinite Md. 81[b].........tl $6\frac{3}{4}$ w $3\frac{1}{4}$

CLASS 5. ORDER 4. GENUS 1.

Divided into four groups:

a.) MT.PM,PT, the rhombic dodecahedron, predominant. Exam.: MT. PM, PT, $\frac{1}{2}$pmt Znw, $\frac{1}{2}$(3p$\frac{1}{3}$mt)Z^2ne. Model 95.

b.) M$\frac{1}{2}$T.P$\frac{1}{2}$M, P$\frac{2}{1}$T, the pentagonal dodecahedron, either alone or predominant. Examples:
M$\frac{1}{2}$T. P$\frac{1}{2}$M, P$\frac{2}{1}$T. Model 91.
M$\frac{1}{2}$T.P$\frac{1}{2}$M,P$\frac{2}{1}$T,PMT. Md.92.

c.) PMT, the regular octahedron, predominant. Example:
m$\frac{1}{2}$t.p$\frac{1}{2}$m, p$\frac{2}{1}$t, PMT. Md. 93.

d.) $\frac{1}{2}$PMT, the tetrahedron, predominant. Example:
mt. pm. pt, $\frac{1}{2}$PMT, $\frac{1}{2}$(3p$\frac{1}{2}$mt) Z^2nw. Model 94.

Group a.

Zinc Blende Md. 95...tl $3\frac{3}{4}$ wbr 4

Group b.

Bright White Cobalt...m $5\frac{1}{2}$ gr $6\frac{1}{4}$
Iron Pyrites ....m $6\frac{1}{4}$ brbk 5

Group c.

Bright White Cobalt...m $5\frac{1}{2}$ gr $6\frac{1}{4}$
Iron Pyrites ....m $6\frac{1}{4}$ brbk 5

Group d.

Grey Copper*..........m $3\frac{1}{2}$ bk 5

CLASS 5. ORDER 4. GENUS 2.

(MT, m$\frac{1}{2}$t. $\frac{1}{2}$P$\frac{3}{2}$M) × 2.
(M,T, MT, mt, m_3t. $\frac{1}{2}$P$\frac{3}{2}$M) × 2.
Rutile.....................tl $6\frac{1}{4}$ br $4\frac{1}{4}$

CLASS 5. ORDER 5. GENUS 1.

Divided into four groups:

a.) A rhombohedron combined with P. Examples:

P. R_1. Model 114ᵃ.
P. $R\frac{3}{2}$. Model 114.

b.) A regular six-sided pyramid combined with P. Example:

$P_-. 2R\frac{1}{3}$ Zw Ze. Model 96.

c) Two or more rhombohedrons or regular six-sided pyramids, or both, combined with P. Ex.:

$P.\frac{1}{2}pt,\frac{1}{2}P_4T,\frac{1}{2}pm\frac{1}{3}t_2,\frac{1}{2}P_4M\frac{1}{3}T_2$

d.) Scalenohedrons combined with P, and commonly also with rhombohedrons or with regular six-sided pyramids. Exam.:

$p. P\frac{3}{2}T, P\frac{3}{2}M\frac{1}{3}T_2, \frac{1}{2}(3p_x m\pm t\mp)$.
$P_+. \frac{1}{2}(3P\ddagger M\pm T\mp) = P_+. S\ddagger$.

Group a.

[The characteristic relates to $\frac{1}{2}P_xT$.]

Antimony $\frac{1}{2}$	m 3½	w	6½
Arsenic $\frac{1}{3}$	m 3½	w	3¾
Crichtonite $\frac{8}{1}$, $\frac{1}{3}$	m 4½	bk	4
Specular Iron $\frac{3}{2}$	m 6	brr	5¼
Copper Mica $\frac{7}{1}$	tp 2	gn	2½
Calcareous Spar $\frac{1}{1}$, $\frac{1}{2}$, $\frac{8}{1}$, $\frac{4}{1}$, $\frac{3}{2}$, Md. 114ᵃ......	tp 3	grw	2¾
Carbᵉ. of Manganese $\frac{1}{1}$	tl 3½	w	3⅔
Dolomite $\frac{1}{1}$, $\frac{4}{1}$	tl 3¾	grw	3
Carbonate of Iron $\frac{1}{1}$, $\frac{5}{1}$	tl 4	brw	3⅔
Alunite $\frac{1}{3}$	tl 5	w	2¾
Corundum $\frac{3}{2}$ Md. 114	tp 9	w	4

Group b.

[The characteristic relates to P_xT.]

Vitreous Copper $\frac{2}{1}$, $\frac{1}{1}$...	m 2¾	gr	5¾
Magnetic Iron Pyrites ½	m 4	bk	4⅔
Specular Iron $\frac{7}{4}$, $\frac{5}{1}$, $\frac{3}{2}$...	m 6	brr	5¼
Phosphate of Lead $\frac{13}{13}$ Model 96	tl 3¾	yw	7
Arseniate of Lead $\frac{5}{6}$ ...	tl 3½	yw	7
Corundum $\frac{2}{3}$, $\frac{16}{3}$	tp 9	w	4

Group c.

Osmium-Iridium	m 5	gr	19
Specular Iron............	m 6	brr	5¼
Copper Mica	tp 2	gn	2½
Red Silver...............	o 2¼	r	5⅔
Cinnabar	tl 2¼	r	8
Calcareous Spar.........	tp 3	grw	2¾
Dolomite	tl 3¾	grw	3
Levyne	tl 4	w	2
Carbonate of Iron	tl 4	brw	3⅔
Alunite	tl 5	w	2¾
Corundum	tp 9	w	4

Group d.

Specular Iron............	m 6	brr	5¼
Titanitic Iron Ore.......	io 5½	bk	4⅔
Calcareous Spar.........	tp 3	grw	2¾

CLASS 5. ORDER 5. GENUS 2.

Divided into twenty-six groups:

a.) Scalene octahedron truncated by the planes P, and having additional planes that cut the equator. Examples:

$p_+, m_-. P\frac{10}{10}M\frac{8}{10}T$.
$p_+. p\frac{8}{10}t, P\frac{10}{10}M\frac{8}{10}T$.

b.) Type: $M\_T. P_xT$. A rhombic prism with dihedral terminations which cut the equator and render it six-sided. Ex.:

Md. 82 held in such a position that the axes become $p^a_- m_+^c t^a$.

The following symbols give the type of each group:

c.) M, $M\_T. P_xM$.
d.) M, $M\_T. P_xT$.
e.) M, $M\_T. P_xM$, P_xT.
f.) T, $M\_T. P_xM$.
g.) T, $M\_T. P_xT$.
h.) T, $M\_T. P_xM$, P_xT.

Genus 2 *Continued:*

i.) $M,T,\ M\_T.\ P_xM.$

j.) $M,T,\ M\_T.\ P_xT.$

k.) $M,T,\ M\_T.\ P_xM,\ P_xT.$

l.) $M.\ \frac{1}{2}P_xM,\ \frac{1}{2}P_xM_yT_z$ Zne Znw.

m.) $M,\ M\_T.\ \frac{1}{2}P_xM$ Zn.

n.) $M,\ M\_T.\ \frac{1}{2}P_xT$ Zw.

o.) $M,\ M\_T.\ \frac{1}{2}P_xM_yT_z$ Zne Znw.

p.) $M,\ M\_T.\ \frac{1}{2}P_xM_yT_z$ Znw Zsw.

q.) $T,\ M\_T.\ \frac{1}{2}P_xM$ Zn.

r.) $T,\ M\_T.\ \frac{1}{2}P_xT$ Zw.

s.) $T,\ M\_T.\ \frac{1}{2}P_xM_yT_z$ Zne Znw.

t.) $T,\ M\_T.\ \frac{1}{2}P_xM_yT_z$ Znw Zsw.

u.) $M,T,\ M\_T.\ \frac{1}{2}P_xM$ Zn.

v.) $M,T,\ M\_T.\ \frac{1}{2}P_xT$ Zw.

w.) $M,T,\ M\_T.\ \frac{1}{2}P_xM$ Zn, $\frac{1}{2}P_xT$ Zw.

x.) $M,T,\ M\_T.\ \frac{1}{2}P_xM_yT_z$ Zne Znw.

y.) $M,T,\ M\_T.\ \frac{1}{2}P_xM_yT_z$ Znw Zsw.

z. $\left\{\begin{array}{l} M,\ \frac{1}{2}M_xT.\ \frac{1}{4}P_xM_yT_z \\ T,\ \frac{1}{2}M_xT.\ \frac{1}{4}P_xM_yT_z \end{array}\right\}$ doubly oblique combinations.

Compare these groups with those of Class 5. Order 3. Genus 1. The same additional Forms are admissible here that are stated there to be admissible, and the same Forms are excluded here that are said there to be excluded, from the groups of particular types. Thus, *group c* must have no Forms belonging to the east zone, and *group g* must have none belonging to the north zone. Groups containing planes of either of these zones may have any number of Forms belonging to the same zone. When $\frac{1}{2}P_xM_yT_z$ is part of the type, the combination must have neither $\frac{1}{2}P_xM$ nor $\frac{1}{2}P_xT$. When $\frac{1}{2}P_xM_yT_z$ is not part of the type, it may be present on the combinations that contain $\frac{1}{2}P_xM$ or $\frac{1}{2}P_xT$ in any number of varieties. The combination must contain all the Forms that are named in the type.

Examples:

		Group.
$M\_,M^4_5T.P^4_5M$ .... Model	100	*c*
$M,M^4_5T.P^5_4T$	104	*d*
$\text{T},M^5_8T.P^7_{10}T$	111	*g*
$T'\_,M^6_{10}T.p^7_4m,P^{10}_7T$	110	*h*
$\text{M},\text{T},M^5_8T.P^7_{10}T$	97	*j*
$M\_.\frac{1}{2}P^1_2M$ Zn, $\frac{1}{2}p^{11}_{24}m$ Zs, ... $\frac{1}{2}P_+MT\_$ Zne² Znw²	101	*l*
$m,M^6_7T.\frac{1}{2}P^5_{14}M$ Zn, $\frac{1}{2}P_+M\_T$ Zn²e Zn²w, $\frac{1}{2}p_xm_yt_z$ Zne² Znw²	103	*m*
$T,\frac{1}{2}M^{15}_{26}T$ ne sw. $\frac{1}{2}P^1_2M$ Zn Ns,	105	*q*
$T,M^{15}_{26}T.\frac{1}{2}P^1_2M$ Zn, $\frac{1}{2}P^7_{15}M$ Zs,	109	*q*
$T,M^{10}_{19}T.\frac{1}{2}P^4_{15}M$ Zn, $\frac{1}{2}P^4_xM^{15}_yT$ Zse Zsw	112	*q*
$T\_,M^9_{13}T.\frac{1}{2}P^5_{13}M^9_{13}T$ Z²ne Z²nw	115	*s*
$M\_,t,m^{18}_{11}t.\frac{1}{2}P^1_2M$ Zn, $\frac{1}{2}p^3_2m$ Zs $\frac{1}{2}\text{P}_+\text{MT}\_$ Zne² Znw²	101ª	*u*
$\text{M},T,\text{M}^{20}_{21}\text{T}.\frac{1}{2}P^6_{21}M^{10}_{21}T$ Znw Zsw	98	*y*
$(\text{M},T,\text{M}^{20}_{21}\text{T}.\frac{1}{2}P^6_{12}M^{10}_{21}T$ Znw Zsw$) \times 2$	99	*y*
$M,3(\frac{1}{2}m_xt).\frac{1}{4}P\_M_+T$ Z²ne ...	107	*z*

Group a.

Sternbergite	m	$1\frac{1}{4}$	bk	$4\frac{1}{4}$
Bournonite	m	$2\frac{3}{4}$	bk	$5\frac{3}{4}$
Antimonial Silver	m	$3\frac{1}{2}$	w	$9\frac{1}{2}$
Sulphur	tl	2	yw	2

Group b.

Arsenical Pyrites $\frac{2}{3}$	m	$5\frac{1}{2}$	bk	$6\frac{1}{8}$
White Iron Pyrites $\frac{3}{4}$ ...	m	$6\frac{1}{2}$	bk	$4\frac{3}{5}$
White Lead Ore $\frac{5}{3}$	tl	$3\frac{1}{4}$	w	$6\frac{1}{2}$
Andalusite $\frac{49}{50}$	o	$7\frac{1}{2}$	w	3

Group c.

Allanite	io	6	gr	4
Heavy Spar $\frac{4}{5}$ Md. 100	tl	$3\frac{1}{2}$	w	$4\frac{2}{3}$

Group d.

Sulphᵉ. of Lead $\frac{4}{5}$ Md. 104	tl	$2\frac{3}{4}$	w	$6\frac{1}{3}$
Celestine $\frac{4}{5}$	tp	$3\frac{1}{2}$	w	$3\frac{7}{8}$
Heavy Spar $\frac{1}{3}$, $\frac{4}{5}$	tl	$3\frac{1}{2}$	w	$4\frac{2}{5}$
Libethenite $\frac{9}{10}$	tl	4	gn	$3\frac{3}{4}$
Topas $\frac{19}{12}$	tp	8	w	$3\frac{1}{2}$

Group e.

Sulphate of Lead $\frac{4}{5}$	tl	$2\frac{3}{4}$	w	$6\frac{1}{3}$
Heavy Spar $\frac{4}{5}$	tl	$3\frac{1}{4}$	w	$4\frac{2}{5}$
Brochantite $\frac{3}{5}$	tp	$3\frac{3}{4}$	gn	$3\frac{3}{4}$
Lazulite $\frac{28}{29}$	tl	$5\frac{1}{2}$	bl	3
Andalusite $\frac{49}{50}$	o	$7\frac{1}{2}$	w	3

Group f.

Celestine $\frac{4}{5}$	tp	$3\frac{1}{4}$	w	$3\frac{7}{8}$
Heavy Spar $\frac{3}{4}$	tl	$3\frac{1}{4}$	w	$4\frac{2}{5}$
Wavellite $\frac{5}{9}$	tl	$3\frac{3}{4}$	w	$2\frac{1}{3}$
Sil^c^. Ox. of Zinc $\frac{4}{5}$	o	5	w	$3\frac{1}{2}$

Group g.

Arsenical Pyrites $\frac{2}{3}$	m	$5\frac{3}{4}$	bk	$6\frac{1}{8}$
Nitre $\frac{11}{19}$	tp	2	w	2
White Antimony $\frac{2}{3}$	tl	$2\frac{3}{4}$	w	$5\frac{3}{5}$
Sulphate of Lead $\frac{4}{5}$	tl	$2\frac{3}{4}$	w	$6\frac{1}{3}$
Witherite $\frac{3}{5}$	tl	$3\frac{1}{4}$	w	$4\frac{1}{3}$
Heavy Spar $\frac{4}{5}$	tl	$3\frac{1}{4}$	w	$4\frac{2}{5}$
White Lead Ore $\frac{3}{5}$	tl	$3\frac{1}{4}$	w	$6\frac{1}{2}$
Muriate of Copper $\frac{2}{3}$	tl	$3\frac{1}{4}$	gn	$4\frac{2}{5}$
Arragonite $\frac{5}{8}$ Md. 111	tl	$3\frac{3}{4}$	grw	3
Libethenite $\frac{9}{10}$	tl	4	gn	$3\frac{3}{4}$
Sil^c^. Ox. of Zinc $\frac{4}{5}$	o	5	w	$3\frac{1}{2}$
Prismatic Iron Ore $\frac{23}{25}$	tl	$5\frac{1}{4}$	ybr	4

Group h.

Needle Ore $\frac{1}{2}$	m	$2\frac{1}{4}$	gr	6
Mascagnine	tl	1	y	?
Celestine $\frac{4}{5}$	tp	$3\frac{1}{4}$	w	$3\frac{7}{8}$
Heavy Spar $\frac{3}{4}$	tl	$3\frac{1}{4}$	w	$4\frac{2}{5}$
White Lead Ore $\frac{3}{5}$ Model 110	tl	$3\frac{1}{4}$	w	$6\frac{1}{2}$
Epistilbite $\frac{7}{17}$	tp	$4\frac{1}{4}$	w	$2\frac{1}{4}$
Sil^c^. Ox. of Zinc $\frac{4}{5}$	o	5	w	$3\frac{1}{2}$
Topas $\frac{19}{36}$, $\frac{10}{18}$	tp	8	w	$3\frac{1}{2}$

Group i.

Picrosmine $\frac{1}{2}$	tl	$2\frac{3}{4}$	w	$2\frac{3}{5}$
Chrysoberyl $\frac{7}{10}$	tp	$8\frac{1}{2}$	w	$3\frac{3}{4}$

Group j.

Orpiment $\frac{4}{5}$, $\frac{3}{2}$	tl	$1\frac{3}{4}$	y	$3\frac{1}{2}$
Anhydrite $\frac{9}{10}$	tp	$3\frac{1}{4}$	grw	3
White Lead Ore $\frac{3}{5}$	tl	$3\frac{1}{4}$	w	$6\frac{1}{2}$
Muriate of Copper $\frac{2}{3}$	tl	$3\frac{1}{4}$	gn	$4\frac{2}{5}$
Arragonite $\frac{5}{8}$ Md. 97	tl	$3\frac{3}{4}$	grw	3
Hypersthene $\frac{20}{21}$	o	$5\frac{1}{2}$	wgr	$3\frac{1}{3}$
Chrysoberyl $\frac{7}{10}$	tp	$8\frac{1}{2}$	w	$3\frac{3}{4}$

Group k.

Glauber's Salt $\frac{16}{17}$	tp	$1\frac{3}{4}$	w	$1\frac{1}{2}$
Haidingerite $\frac{5}{6}$	tp	$2\frac{1}{4}$	w	$2\frac{3}{4}$
Heavy Spar $\frac{4}{5}$	tl	$3\frac{1}{4}$	w	$4\frac{2}{5}$
Olivenite $\frac{9}{10}$	tl	$3\frac{1}{4}$	gnbr	$4\frac{1}{3}$
Brochantite $\frac{3}{5}$	tp	$3\frac{3}{4}$	gn	$3\frac{3}{4}$
Brookite $\frac{5}{6}$	tl	$5\frac{3}{4}$	yw	
Prehnite $\frac{5}{6}$	tl	$6\frac{1}{2}$	w	3
Olivine $\frac{4}{9}$	tp	$6\frac{3}{4}$	w	$3\frac{2}{5}$
Chrysoberyl $\frac{7}{10}$	tp	$8\frac{1}{2}$	w	$3\frac{3}{4}$

Group l.

Epidote Md. 101	tl	$6\frac{1}{2}$	grw	$3\frac{1}{3}$

Group m.

Vivianite $\frac{8}{11}$	tl	$1\frac{3}{4}$	bl	$2\frac{3}{5}$
Gay-Lussite $\frac{2}{3}$	tp	$2\frac{1}{2}$	gr	2
Lanarkite $\frac{4}{7}$	tl	$2\frac{1}{2}$	w	7
Vauquelinite	tl	$2\frac{3}{4}$	gn	$5\frac{3}{4}$
Chromate of Lead $\frac{16}{17}$	o	$2\frac{3}{4}$	ry	6
Azure Copper Ore $\frac{6}{7}$ Model 103	tp	$3\frac{3}{4}$	bl	$3\frac{3}{5}$
Malachite $\frac{4}{5}$	tl	$3\frac{3}{4}$	gn	4
Wagnerite $\frac{7}{10}$	tl	$5\frac{1}{4}$	w	$3\frac{1}{8}$
Tungstate of Iron $\frac{6}{7}$, $\frac{3}{7}$	o	$5\frac{1}{4}$	rbr	$7\frac{1}{3}$
Hornblende $\frac{10}{19}$	o	$5\frac{1}{2}$	gr	3
Datolite $\frac{5}{8}$	tl	$5\frac{1}{2}$	w	3
Epidote $\frac{8}{11}$	tl	$6\frac{1}{2}$	grw	$3\frac{1}{3}$
Gadolinite $\frac{7}{11}$	o	$6\frac{3}{4}$	gngr	$4\frac{1}{4}$

Group n.

Laumonite $\frac{15}{16}$	tl	4	w	$2\frac{1}{3}$

Group o.

Chromate of Lead	o	$2\frac{3}{4}$	ry	6
Mesotype $\frac{49}{50}$	tp	$5\frac{1}{4}$	w	$2\frac{1}{4}$
Brewsterite $\frac{2}{3}$	tp	$5\frac{1}{4}$	yw	$2\frac{1}{5}$
Euclase $\frac{16}{25}$, $\frac{7}{25}$	tp	$7\frac{1}{2}$	w	3

Group p.

Carbonate of Soda $\frac{4}{5}$	tp	$1\frac{1}{4}$	w	$1\frac{2}{5}$

Group q.

Realgar $\frac{2}{3}$	tl	$1\frac{3}{4}$	yr	$3\frac{3}{5}$
Gypsum $\frac{9}{13}$	tp	2	w	$2\frac{1}{3}$
Lepidolite	tl	$2\frac{1}{4}$	rw	$2\frac{4}{5}$
Two-axed Mica $\frac{4}{7}$	tp	$2\frac{1}{4}$	wgr	3
Chromate of Lead	o	$2\frac{3}{4}$	ry	6
Azure Copper Ore $\frac{6}{7}$	tp	$3\frac{3}{4}$	bl	$3\frac{3}{5}$
Baryto-Calcite $\frac{10}{11}$	tp	4	w	$3\frac{3}{5}$
Turnerite $\frac{9}{10}$	tl	5	grw	?

Group q Continued:

Mesotype $\frac{40}{50}$ ....tp $5\frac{1}{4}$ w $2\frac{1}{4}$
Hornblende $\frac{10}{19}$ Md.112 o $5\frac{1}{2}$ gr 3
Felspar $\frac{15}{26}$ Md.109,105 tl 6 grw $2\frac{3}{5}$
Gadolinite ....o $6\frac{3}{4}$ gngr $4\frac{1}{4}$
Euclase ....tp $7\frac{1}{2}$ w 3

Group r.

Red Antimony ....o $1\frac{1}{2}$ rbr $4\frac{1}{2}$
Tincal $\frac{16}{17}$ ....tl $2\frac{1}{4}$ w $1\frac{2}{3}$
Leadhillite $\frac{4}{7}$ ....tl $2\frac{1}{2}$ w $6\frac{1}{3}$
Glauberite $\frac{9}{10}$............tl $2\frac{3}{4}$ w $2\frac{4}{5}$
Phosph^e^. of Copper $\frac{6}{17}$ tl $4\frac{1}{2}$ gn $4\frac{1}{5}$
Augite $\frac{20}{21}$................o $5\frac{1}{2}$ wgr $3\frac{1}{3}$

Group s.

Gypsum $\frac{9}{13}$ Md.75,115 tp 2 w $2\frac{1}{3}$
Mesotype $\frac{40}{50}$ ....tp $5\frac{1}{4}$ w $2\frac{1}{4}$
Hornblende $\frac{10}{19}$..........o $5\frac{1}{2}$ gr 3
Euclase $\frac{16}{25}$, $\frac{7}{25}$ ....tp $7\frac{1}{2}$ w 3

Group t.

Augite $\frac{20}{21}$ ....o $5\frac{1}{2}$ wgr $3\frac{1}{3}$

Group u.

Flexible Sulphuret of
  Silver $\frac{1}{2}$ ....m 5 yr ?
Vivianite $\frac{8}{11}$ ....tl $1\frac{1}{2}$ bl $2\frac{2}{3}$
Realgar $\frac{2}{3}$ ....tl $1\frac{3}{4}$ yr $3\frac{3}{5}$
Chromate of Lead $\frac{16}{17}$ o $2\frac{3}{4}$ ry 6
Brewsterite $\frac{2}{5}$............tp $5\frac{1}{4}$ yw $2\frac{1}{5}$
Tungstate of Iron $\frac{5}{7}$, $\frac{6}{7}$ o $5\frac{1}{4}$ rbr $7\frac{1}{5}$
Hornblende $\frac{10}{19}$ ....o $5\frac{1}{2}$ gr 3
Felspar $\frac{15}{26}$ ....tl 6 grw $2\frac{3}{5}$
Epidote $\frac{9}{11}$, $\frac{10}{11}$, Md.101^a^ tl $6\frac{1}{2}$ grw $3\frac{1}{3}$
Euclase $\frac{16}{25}$...............tp $7\frac{1}{2}$ w 3

Group v.

Carbonate of Soda $\frac{4}{5}$...tp $1\frac{1}{4}$ w $1\frac{2}{3}$
Cobalt Bloom $\frac{1}{2}$.........tl $2\frac{1}{4}$ r 3
Tincal $\frac{16}{17}$.................tl $2\frac{1}{4}$ w $1\frac{2}{3}$

Group v Continued:

Leadhillite $\frac{4}{7}$ ....tl $2\frac{1}{2}$ w $6\frac{1}{3}$
Azure Lead Ore $\frac{3}{5}$......o $2\frac{3}{4}$ bl $5\frac{1}{3}$
Laumonite $\frac{15}{16}$............tl 4 w $2\frac{1}{3}$
Phosphate of Cop^r^. $\frac{6}{17}$ tl $4\frac{1}{2}$ gn $4\frac{1}{5}$
Augite $\frac{20}{21}$ ....o $5\frac{1}{2}$ wgr $3\frac{1}{3}$
Sahlite $\frac{20}{21}$ ....o $5\frac{1}{2}$ wgr $3\frac{1}{3}$

Group w.

Glauber's Salt $\frac{16}{17}$.......tp $1\frac{1}{2}$ w $1\frac{1}{2}$

Group x.

Gypsum $\frac{9}{13}$................tp 2 w $2\frac{1}{3}$
Pharmacolite $\frac{7}{20}$.........o $2\frac{1}{4}$ w $2\frac{3}{4}$
Mesotype $\frac{40}{50}$ ....tp $5\frac{1}{4}$ w $2\frac{1}{4}$
Hornblende $\frac{10}{19}$ ....o $5\frac{1}{2}$ gr 3
Euclase $\frac{16}{25}$...............tp $7\frac{1}{2}$ w 3

Group y.

Gypsum $\frac{9}{13}$ ....tp 2 w $2\frac{1}{3}$
Augite $\frac{20}{21}$ Md. 98, 99 o $5\frac{1}{2}$ wgr $3\frac{1}{3}$
Acmite $\frac{18}{19}$ ....o $6\frac{1}{4}$ ygr $3\frac{1}{4}$

Group z.

[Including several complex combinations likely to be mistaken, when held in certain positions, for doubly oblique combinations.]

Allanite ....io 6 gr 4
Boracic Acid............o $1\frac{1}{2}$ w $1\frac{1}{2}$
Blue Vitriol..............tl $2\frac{1}{2}$ w $2\frac{1}{4}$
Tabular Spar............tl $4\frac{1}{2}$ w $2\frac{4}{5}$
Titanite ....tl $5\frac{1}{4}$ w $3\frac{1}{2}$
Babingtonite ....o $5\frac{3}{4}$ bk $3\frac{1}{2}$
Felspar Md. 105 ....tl 6 grw $2\frac{3}{5}$
Latrobite ....o 6 rw $2\frac{3}{4}$
Diaspore ....o 6 gr $2\frac{2}{5}$
Cyanite Md. 107 ....tl 6 w $3\frac{2}{3}$
Albite.......................tl 6 w $2\frac{3}{5}$
Petalite ....tl $6\frac{1}{4}$ w $2\frac{1}{2}$
Axinite Md. 81^b^.........tl $6\frac{3}{4}$ w $3\frac{1}{4}$

CLASS VI.—INCOMPLETE PYRAMIDS.

Order 1. *Square Equator,*.................... } Genus 1. Axes: p^2 m^2 t^2.
Genus 2. Axes: p^2_x m^2 t^2.

Order 2. *Rectangular Equator,* .... Genus 1. Axes: ?

Order 3. *Rhombic Equator,* .... Genus 1. Axes: p^2 m^2 t^2.

Order 4. *Rhombo-Quadratic Equator,*...... Genus 1. Axes: p^2 m^2 t^2.

Order 5. *Rhombo-Rectangular Equator,* ... Genus 1. Axes: p^2_x m^2_y t^2_z.

Class 6. Order 1. Genus 1.

Divided into two groups:

a.) The regular tetrahedron. Ex.: $\frac{1}{2}$PMT. Model 117.

b.) The right and left tetrahedron. Ex.: $\frac{1}{2}$PMT, $\frac{1}{2}$pmt. Md. 118.

Group a.

Silver.....................m $2\frac{3}{4}$ w $10\frac{1}{2}$
Gold........................m $2\frac{3}{4}$ y 15
Grey Copper............m $3\frac{1}{2}$ bk 5
Copper Pyrites.........m $3\frac{3}{4}$ gnbk $4\frac{1}{6}$
Bismuth Blende.........tl $3\frac{3}{4}$ ygr 6
Zinc Blende.............tl $3\frac{3}{4}$ wbr 4
Helvine...................tl $6\frac{1}{4}$ w 3
Spinel, r, bl..............tp 8 w $3\frac{1}{2}$
Automalite...............o 8 w $4\frac{1}{2}$

Group b.

Bismuth..................m $2\frac{1}{4}$ w $9\frac{3}{4}$
Grey Copper............m $3\frac{1}{2}$ bk 5
Copper Pyrites.........m $3\frac{3}{4}$ gnbk $4\frac{1}{6}$
Magnetic Iron Ore.....m 6 bk 5
Zinc Blende.............tl $3\frac{3}{4}$ wbr 4
Helvine...................tl $6\frac{1}{4}$ w 3
Spinel, r, bl..............tp 8 w $3\frac{1}{2}$
Automalite...............o 8 w $4\frac{1}{2}$

Class 6. Order 1. Genus 2.

$\frac{1}{2}$PMT or $\frac{1}{2}$P$\frac{100}{101}$MT predominant.

Copper Pyrites.........m $3\frac{3}{4}$ gnbk $4\frac{1}{6}$

Class 6. Order 3. Genus 1.

$\frac{1}{2}$(3P_MT). Example:
$\frac{1}{2}$(3P$\frac{1}{3}$MT). Model 119.

Grey Copper............m $3\frac{1}{2}$ bk 5
Copper Pyrites.........m $3\frac{3}{4}$ gnbk $4\frac{1}{6}$
Bismuth Blende.........tl $3\frac{3}{4}$ ygr 6

Class 6. Order 4. Genus 1.

Divided into two groups:

a.) $\frac{1}{2}$PMT predominant. Example: $\frac{1}{2}$PMT, $\frac{1}{2}$(3p$\frac{1}{2}$mt).

b.) $\frac{1}{2}$(3P$\frac{1}{2}$MT) predominant. Ex.: $\frac{1}{2}$PMT Znw, $\frac{1}{2}$(3P$\frac{1}{2}$MT) Z^2nw.

Group a.

Grey Copper............m $3\frac{1}{2}$ bk 5

Group b.

Grey Copper............m $3\frac{1}{2}$ bk 5
Arseniate of Iron......tl $2\frac{1}{2}$ gn 3
Bismuth Blende.........tl $3\frac{3}{4}$ ygr 6

Class 6. Order 5. Genus 1.

p$\frac{12}{10}$t, P$\frac{12}{10}$M$\frac{8}{10}$T. Model 120.

Sulphur..................tl 2 yw 2

Section IV.

A DESCRIPTIVE CATALOGUE OF THE MODELS OF CRYSTALS EMPLOYED TO ILLUSTRATE THIS SYSTEM OF CRYSTALLOGRAPHY.

The Models are one hundred and twenty in number, and, with two or three exceptions, they all represent crystals that have been found among Minerals. They are arranged in this catalogue according to the classification of crystals adopted in Part II. Section III.

I have added to the description of every model, the name of a mineral which it particularly characterises; but as the same crystal often indicates a variety of minerals, I have prefixed to each model the number of the Class, Order, and Genus, in which an account is given of all the minerals that have been found crystallised in the shape of that model.

The positions of the axes p^a m^a t^a in every model, are indicated by the letters P, M, T, stamped upon each, as already explained at page 2, Part I.

Since the models were stamped, I have changed my views regarding the positions in which some of them should be held for examination and description. In such cases, new numbers are given to the models, and the alterations are fully described in the catalogue. To render the positions of the equator and of the north and east meridians perfectly evident, I have marked portions of them on many of the models with coloured inks. The reader will be so good as to remark that

a *brown* line indicates the equator,
a *blue* line indicates the north meridian,
a *purple* line indicates the east meridian.

The axis p^a of every combination is situated in the intersection of the north meridian with the east meridian; the axis m^a, in the intersection of the north meridian with the equator; and the axis t^a, in the intersection of the east meridian with the equator. Consequently, the meeting of a blue line with a purple line upon a model shows the position of a pole of the axis p^a, the meeting of a blue line with a brown line, shows the position of a pole of the axis m^a, and the meeting of a purple line with a brown line, shows the position of a pole of the axis t^a. In general, the three poles thus indicated by the coloured lines on the models, are p^a_z, m^a_n, t^a_w.

It is also necessary to remark, that since the brown line is on the prismatic zone, all planes that cross it evenly belong to the prismatic series, and are expressed by the symbols M, M_xT, or T; that since the blue line is on the north zone, all planes that cross it evenly are expressed by the symbols P, P_xM, or M; and that since the purple line is on the east zone, all planes that cross it evenly are expressed by the symbols

P, P_xT, or T. There being no lines drawn to indicate the octahedral zones, the planes belonging to those zones are to be sought for in the octants or open spaces enclosed by the three coloured lines.

The angles of inclination of the planes of the different forms upon one another, are given in the "Table of Angles," in PART I.

It may be considered desirable in some instances to extend the coloured lines completely round the models, or to make other marks with the same colours. I think it, therefore, not improper to state in what manner the coloured inks are prepared.

Blue Ink.—This is prepared by dissolving pounded indigo in concentrated sulphuric acid, and diluting the solution with thick gum-arabic water.

Brown Ink.—Boil catechu in water, in such proportion as will make a pretty strong decoction. To this, when cold, add a solution of Bichromate of Potash. The weight of the catechu should be about four times that of the Bichromate of Potash. It is unnecessary to add gum arabic to this ink, because the decoction of catechu is of itself sufficiently thick.

Purple Ink.—Boil logwood in water in such quantity as will make a strong decoction. To this, when perfectly cold, add a solution of Protomuriate of Tin. Thicken with strong gum-arabic water.

Write with a clean quill pen, having a fine point.

Black lines may be written on the models with japan writing ink, or with common writing ink thickened with gum, and these can be effaced by diluted muriatic acid. The black lead pencil may also be employed to write with, and its traces yield to the action of caoutchouc. If durable marks are desired, it is best to mix amber varnish with turpentine, and colour it with lamp black or vermillion.

Those who wish to remove the coloured lines, need only wash the brown lines with diluted muriatic acid, or the blue lines and purple lines with a solution of bleaching powder, to effect what they desire. The models can be cleaned from dust by caoutchouc, and from grease by soap and water. The coloured marks can also be removed by a solution of caustic potash.

COMPLETE PRISMS.

CLASS. ORDER. GENUS. NO. OF MODEL.

1. 1. 1. **1**. P,M,T. The cube .... *Galena.*

1. 1. 2. **2**. $P\frac{1}{3}$,M,T. A short right square prism.................. *Rutile.*
[According to the letters stamped upon this model, it is $P_{\frac{5}{11}}$,M,T.]

1. 1. 2. **3**. $P\frac{5}{4}$,M,T. A long right square prism .... *Apophyllite.*

1. 1. 2. **4**. P_+,M,T, mt. A right square prism, M,T, with the lateral edges replaced by another right square prism, mt. *Egeran.*

1. 2. 1. **5**. P_+,M_-,T: or $P_{+\frac{9}{10}}$,$M_{\frac{9}{10}}$,T. A right rectangular prism...... *Anhydrite.*

1. 3. 1. **6**. P_-,$M\frac{4}{5}$T. A right rhombic prism .... *Heavy Spar.*

1. 5. 1. **7**. P_x,T,$M_{+\frac{5}{3}}T_2$: or P_x,V. The regular six-sided prism. *Apatite.*

1. 5. 3. **8**. P_x,T,M_-T. A right rhombic prism, M_-T, with the acute lateral edges replaced by the form T. Axes: $p_x^a m_+ t^a$. Altered to No. 79[a] .... *Chrysoberyl.*

1. 5. 3. **9**. (P_+,T,$M_{\frac{8}{17}}$T) × 2. A twin crystal of a right rhombic prism, $M_{\frac{8}{17}}$T, with the acute lateral edges replaced by the form T. Axes: $p_x^a m_-^a t^a$........................ *Staurolite.*

C. O. G. NO.

1\. 5. 2. **10.** $P^{\cdot}_{x}, m, T, m_2t\frac{1\,5}{1\,3}, M\frac{1\,5}{1\,3}T_2$: or P_x, V, v. The regular twelve-sided prism of the rhombohedral system .... *Apatite.*

11. A right prism with a rhomboidal base. Altered to No. 79[b].

COMPLETE PYRAMIDS.

2\. 1. 2. **12.** $P\frac{2}{3}MT$. An obtuse quadratic octahedron, consisting of a form of the octahedral zones, ... *Zircon.*

2\. 1. 2. **13.** $P\frac{5}{2}M, P\frac{5}{2}T$. An acute quadratic octahedron, consisting of a form of the north zone, $P\frac{5}{2}M$, combined with a similar and equal form of the east zone, $P\frac{5}{2}T$ .... *Anatase.*

2\. 1. 2. **14.** $P\frac{5}{2}M, p\frac{1}{2}m, P\frac{5}{2}T, p\frac{1}{2}t$. An obtuse quadratic octahedron, $p\frac{1}{2}m, p\frac{1}{2}t$, combined with an acute quadratic octahedron, $P\frac{5}{2}M, P\frac{5}{2}T$, ... *Anatase.*

2\. 1. 1. **15.** PMT. The regular octahedron......... *Magnetic Iron Ore.*

2\. 1. 1. **16.** PMT $\times$ 2. The hemitrope or twin regular octahedron. The plane of junction of the two crystals is $=$ PMT Nse. *Magnetic Iron Ore.*

2\. 1. 1. **17.** $3P_2MT$: or P_2MT, PM_2T, PMT_2. A triakisoctahedron ... *Diamond.*

18. $\frac{1}{2}(3P\frac{3}{2}MT)$: or $\frac{1}{2}(P\frac{3}{2}MT, PM\frac{3}{2}T, PMT\frac{3}{2})$. A hemitriakis-octahedron, a form which only occurs in combination, and not in a separate state.

2\. 2. 1. **19.** $P\frac{5}{4}M, P\frac{4}{3}T$. Axes: $p^{a}_{20}m^{a}_{16}t^{a}_{15}$. A rectangular octahedron. Altered to No. 82[b] .... *Sulphate of Lead.*

2\. 2. 1. **20.** $P\frac{10}{6}M, P\frac{10}{7}T$. Axes: $p^{a}_{10}m^{a}_{6}t^{a}_{7}$. A rectangular octahedron. Altered to No. 82[a] .... *Carbonate of Lead.*

2\. 3. 3. **21.** $P\frac{19}{10}M\frac{8}{10}T$. Axes: $p^{a}_{19}m^{a}_{8}t^{a}_{10}$. An acute scalene octahedron, or octahedron with a rhombic base .... *Sulphur.*

2\. 3. 1. **22.** $3P\frac{1}{2}MT$: or $P\frac{1}{2}MT, PM\frac{1}{2}T, PMT\frac{1}{2}$. An icositessarahedron. [Compare this model with No. 119] .... *Leucite.*

2\. 3. 1. **23.** $6P\frac{1}{7}M\frac{1}{3}T$: or $P\frac{1}{7}M\frac{1}{3}T, PM\frac{1}{7}T\frac{1}{3}, P\frac{1}{3}MT\frac{1}{7}, P\frac{1}{7}MT\frac{1}{3}, P\frac{1}{3}M\frac{1}{7}T, PM\frac{1}{3}T\frac{1}{7}$. A hexakisoctahedron .... *Garnet.*

2\. 3. 1. **24.** $\frac{1}{2}(6P\frac{1}{3}M\frac{1}{2}T)$: or $\frac{1}{2}(P\frac{1}{3}M\frac{1}{2}T, PM\frac{1}{3}T\frac{1}{2}, P\frac{1}{2}MT\frac{1}{3}, P\frac{1}{3}MT\frac{1}{2}, P\frac{1}{2}M\frac{1}{3}T, PM\frac{1}{2}T\frac{1}{3})$. A hemihexakisoctahedron with inclined faces .... *Diamond.*

2\. 3. 1. **25.** $3P\frac{1}{3}M\frac{1}{2}T$: or $P\frac{1}{3}M\frac{1}{2}T, PM\frac{1}{3}T\frac{1}{2}, P\frac{1}{2}MT\frac{1}{3}$. A right hemihexakisoctahedron with parallel faces .... *Iron Pyrites.*

2\. 5. 1. **26.** $P\frac{1\,3}{1\,3}T, P\frac{1\,3}{1\,3}M\frac{1\,5}{1\,3}T_2$: or $2R\frac{1\,3}{1\,3}$ Zw Ze. An obtuse regular six-sided pyramid, or combination of two equal rhombohedrons in contrary positions .... *Phosphate of Lead.*

2\. 5. 1. **26[a].** $\frac{1}{2}PT, \frac{1}{2}PM\frac{1\,5}{1\,3}T_2$: or R_1. An obtuse rhombodedron .... Haüy's *primitive form* of C. S. .... *Calcareous Spar.*

2\. 5. 1. **26[b].** $\frac{1}{2}P\frac{1}{2}T, \frac{1}{2}P\frac{1}{2}M\frac{1\,5}{1\,3}T_2$: or $R\frac{1}{2}$. An obtuse rhombohedron ...... Haüy's *equiaxe* .... *Calcareous Spar.*

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2. 5. 1. **26**^{c}. $\frac{1}{2}P\frac{8}{3}T$, $\frac{1}{2}P\frac{8}{3}M^{1\,5}_{1\,3}T_2$: or $R\frac{8}{3}$. An acute rhombohedron........ Haüy's *primitive form of* C ....*Cinnabar.*

2. 5. 1. **26**^{d}. $\frac{1}{2}P_2T$, $\frac{1}{2}P_2M^{1\,5}_{1\,3}T_2$: or R_2. An acute rhombohedron........... Haüy's *inverse*..............................*Calcareous Spar.*

2. 5. 1. **26**^{e}. $R\frac{5}{4}$ Zw, R$\frac{5}{8}$ Ze, $r\frac{5}{2}$ Ze. Combination of three rhombohedrons, all belonging to the east zone ....*Chabasite.*

2. 5. 1. **26**^{f}. $\frac{1}{2}PM^{1\,5}_{1\,3}T\frac{2}{3}$, $\frac{1}{2}PM^{1\,5}_{2\,6}T\frac{1}{2}$, $\frac{1}{2}PM^{\,5}_{1\,3}T_2$: or $\frac{1}{2}(3P_1M^{\pm}_{\pm}T^{\mp}_{+})$. A scalenohedron: or scalene six-sided pyramid. Haüy's *Metastatique*..............................*Calcareous Spar.*

COMPLETE PRISMS COMBINED WITH INCOMPLETE PYRAMIDS.

3. 1. 1. **27**. P,M,T, MT. PM, PT. The cube, No. 1, with its edges replaced by the planes of the rhombic dodecahedron, No. 63 ....*Fluorspar.*

3. 1. 1. **28**. p,m,t, MT. PM, PT. The rhombic dodecahedron, No. 63, with its acute angles replaced by the planes of the cube, No. 1 ....*Nosian.*

3. 1. 1. **29**. P,M,T. PMT. Middle crystal between the cube, No. 1, and the regular octahedron, No. 15 .... *Galena.*

3. 1. 1. **30**. p,m,t. PMT. The regular octahedron, No. 15, with its angles replaced by the planes of the cube, No. 1. *Fluorspar.*

3. 1. 1. **31**. P,M,T, mt. pm, pt, PMT. The cube, No. 1, with its edges replaced by the planes of the rhombic dodecahedron, No. 63, and its angles by the planes of the regular octahedron, No. 15 ....*Galena.*

3. 1. 1. **32**. The same combination, lettered, to show the number and positions of the planes of each form.

3. 1. 1. **33**. P,M,T, mt. pm, pt, PMT. The regular octahedron, No. 15, with its solid angles replaced by the planes of the cube, No. 1, and its edges by the planes of the rhombic dodecahedron, No. 63...................................*Fluorspar.*

3. 1. 1. **34**. p,m,t, MT. PM, PT, pmt. The rhombic dodecahedron, No. 63, with its four-faced angles replaced by the planes of the cube, No. 1, and its three-faced angles by the planes of the regular octahedron, No. 15 ... *Sulphuret of Silver.*

3. 1. 1. **35**. P,M,T, MT. PM, PT, $\frac{1}{2}$PMT, $\frac{1}{2}$pmt. The cube, No. 1, with its edges replaced by the planes of the rhombic dodecahedron, No. 63, and its angles by the planes of the right and left tetrahedron, No. 118...................... *Boracite.*

3. 1. 1. **36**. P,M,T, mt. pm, pt, $\frac{1}{2}$pmt. The cube, No. 1, with its edges replaced by the planes of the rhombic dodecahedron, No. 63, and its alternate solid angles by the planes of the tetrahedron, No. 117 ....*Boracite.*

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3. 1. 1. **37**. p,m,t, mt. pm,pt, $\frac{1}{2}$PMT. The tetrahedron, No. 117, with its edges replaced by the planes of the cube, No. 1, and its angles by the planes of the rhombic dodecahedron, No. 63 .... *Boracite.*

[Nos. 36 and 37, so very unlike in external appearance, are composed of precisely similar forms, and differ only in the relative magnitude of the same forms on the different combinations.]

3. 1. 1. **38**. P,M,T. $\frac{1}{2}$pmt. The cube, No. 1, with its alternate solid angles replaced by the tetrahedron, No. 117, the planes of which occupy the positions Znw Zse Nne Nsw......... *Arseniate of Iron.*

3. 1. 1. **39**. P,M,T. 3p$\frac{1}{2}$mt. The cube, No. 1, with its solid angles replaced by the planes of the icositessarahedron, No. 22, situated three in the place of each angle of the cube, and the planes inclining on the planes of the cube. *Analcime.*

3. 1. 1. **40**. P,M,T. 6p$\frac{1}{4}$m$\frac{1}{2}$t. The cube, No. 1, with its solid angles replaced by the planes of a hexakisoctahedron, similar to No. 23, situated six in the place of each solid angle of the cube .... *Fluorspar.*

3. 1. 2. **41**. P$\frac{5}{4}$,M,T. p$\frac{5}{4}$mt. A right square prism, No. 3, with the solid angles replaced by the planes of an acute quadratic octahedron belonging to the octahedral zones .... *Apophyllite.*

3. 1. 2. **42**. p$_+$,M,T. MT. P$\frac{1}{4}$M, P$\frac{1}{4}$T. A right square prism, with the lateral edges replaced by the planes of another right square prism, similar to No. 4, and the terminal edges replaced by the planes of an obtuse quadratic octahedron, composed of forms belonging to the north and east zones .... *Idocrase.*

3. 2. 1. **43**. p$_+$,M,T. P$\frac{4}{7}$M$\frac{6}{7}$T. A right rectangular prism, similar to No. 5, with the angles replaced by the planes of an obtuse scalene octahedron.................................. *Desmine.*

3. 3. 1. **44**. P$_-$,M$\frac{4}{5}$T, p$\frac{4}{5}$m. A right rhombic prism, P$_-$,M$\frac{4}{5}$T, No. 6, with its obtuse solid angles replaced by a form of the east zone, p$\frac{4}{5}$m, the planes of which occupy the positions Zn Zs Nn Ns.. *Heavy Spar.*

3. 4. 1. **45**. P,M,T, m$\frac{1}{3}$t, m$\frac{3}{1}$t. p$\frac{1}{3}$m, p$\frac{3}{1}$m, p$\frac{1}{3}$t, p$\frac{3}{1}$t. The cube, No. 1, with its edges bevelled by the planes of the tetrakishexahedron, No. 68 .... *Fluorspar.*

3. 4. 1. **46**. p,m,t. P$\frac{1}{3}$M$\frac{1}{2}$T. A right hemihexakisoctahedron with parallel faces, No. 25, with its acute solid angles replaced by the planes of the cube, No. 1 .... *Iron Pyrites.*

3. 4. 1. **47**. p,m,t, M$\frac{1}{2}$T. P$\frac{1}{2}$M, P$\frac{2}{1}$T. A pentagonal dodecahedron, No. 91, with its six longest edges replaced by the planes of the cube, No. 1 .... *Iron Pyrites.*

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3\. 4. 1. **48**. p,m,t, $M\frac{1}{2}T$. $P\frac{1}{2}M$, $P\frac{2}{1}T$, PMT. The middle crystal between the regular octahedron, No. 15, and the pentagonal dodecahedron modified by the planes of the cube, No. 47, and therefore containing the planes of Models 1, 15, and 91 .... *Iron Pyrites.*

3\. 4. 1. **49**. The same combination, lettered, to show the number and positions of the planes of each form.

3\. 5. 3. **50**. P_-,m,t_+, $M\frac{4}{3}T$. $p\frac{4}{3}m$, $p\frac{4}{3}t$. A right rhombic prism (P_-,$M\frac{4}{3}T$, No. 6), with its obtuse vertical edges replaced by the prismatic form m, its acute vertical edges by the prismatic form t, its obtuse solid angles by a form of the north zone, $p\frac{4}{3}m$, and its acute solid angles by a form of the east zone, $p\frac{4}{3}t$ .... *Heavy Spar.*

3\. 5. 3. **51**. p_+,M_-,T, $\textsc{m}\frac{4}{9}\textsc{t}$. $P\frac{5}{4}M$, $P\frac{10}{9}T$, $p\frac{5}{9}m\frac{4}{9}t$. A right rectangular prism, No. 5, with its vertical edges replaced by the rhombic form $\textsc{m}\frac{4}{9}\textsc{t}$, its long terminal edges by the form $P\frac{5}{4}M$, its short terminal edges by the form $P\frac{10}{9}T$, and its solid angles by the scalene octahedron $p\frac{5}{9}m\frac{4}{9}t$ ..... *Olivine.*

3\. 5. 2, **52**. P,m,T, $m_2t\frac{1}{1}\frac{5}{3}$, $M\frac{1}{1}\frac{5}{3}T_2$. pm, $p\frac{1}{2}\frac{5}{6}t$, $pm_2t\frac{1}{1}\frac{5}{3}$, $p\frac{1}{2}\frac{5}{6}m\frac{1}{1}\frac{5}{3}t_2$: or P,V, *v*. $2r_1$ Zn Zs, $2r\frac{1}{2}\frac{5}{6}$ Zw Ze. The regular twelve-sided prism, No. 10, with its terminal edges replaced by a regular six-sided pyramid belonging to the east zone, $2r\frac{1}{2}\frac{5}{6}$ Zw Ze, similar to No. 26, and its solid angles replaced by the six-sided pyramid $2r_1$ Zn Zs, also similar to No. 26, but belonging to the north zone .... *Beryl.*

3\. 5. 3. **53**. p_+,$\textsc{m}$,T, $\textsc{m}\frac{20}{21}\textsc{t}$. $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ Znw Zsw. A right rhombic prism, $\textsc{p}_+$,$\textsc{m}\frac{20}{21}\textsc{t}$, with its obtuse vertical edges replaced by the form $\textsc{m}$, its acute vertical edges by the form T, and its four terminal edges Znw Zsw Nne Nse by the obtuse scalene hemioctahedron $\frac{1}{2}P\frac{6}{21}M\frac{10}{21}T$ .... *Augite.*

54. Altered to No. 101^a.

3\. 5. 3. **55**. P_+,T,$M\frac{8}{17}T$. $p\frac{1}{1}\frac{7}{2}m$. A right rhombic prism, P_+,$M\frac{8}{17}T$, with its acute vertical edges replaced by the form T (constituting the individuals of No. 9), and its obtuse solid angles replaced by the form $p\frac{1}{1}\frac{7}{2}m$, which occupies the positions Zn Zs Nn Ns .... *Staurolite.*

3\. 5. 1. **56**. P,T,$M\frac{1}{1}\frac{5}{3}T_2$. pm, $pm_2t\frac{1}{1}\frac{5}{3}$: or P,V. $2r_1$ Zn Zs. The regular six-sided prism, No. 7, with its solid angles replaced by the planes of a regular six-sided pyramid, similar to No. 26, but in a different position .... *Beryl.*

3\. 5. 1. **57**. P,T, $M\frac{1}{1}\frac{5}{3}T_2$. $\frac{1}{2}p\frac{3}{2}m$ Zn, $\frac{1}{2}p\frac{3}{2}m_2t\frac{1}{1}\frac{5}{3}$: or P,V. $r\frac{3}{2}$ Zn. The regular six-sided prism, No. 7, with its alternate solid angles replaced by the planes of an acute rhombohedron, whose zenith planes have the positions Zn Zse Zsw .... *Corundum.*

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3\. 5. 1. **58.** $P,T,M\frac{1}{1}\frac{5}{3}T_2.\ p\frac{4}{3}t,\ p\frac{4}{3}m\frac{1}{1}\frac{5}{3}t_2$: or $P,V.\ 2r\frac{4}{3}$ Zw Ze. The regular six-sided prism, No. 7, with its terminal edges replaced by the planes of an obtuse regular six-sided pyramid, similar to No. 26 .... *Apatite.*

INCOMPLETE PRISMS COMBINED WITH COMPLETE PYRAMIDS.

4\. 1. 2. **59.** M,T, mt. $P\frac{2}{3}M$, $P\frac{2}{3}T$. A right square prism, No. 3, with its lateral edges replaced by the planes of another right square prism, No. 4, and its terminal edges and terminal planes replaced by the planes of an obtuse quadratic octahedron similar to No. 12, but composed of forms belonging to the north and east zones .... *Wernerite.*

4\. 1. 2. **60.** M,T, mt. $P\frac{2}{3}MT$. A right square prism, No. 3, with its lateral edges replaced by another right square prism, No. 4, and its solid angles and terminal planes replaced by the planes of the obtuse quadratic octahedron, $P\frac{2}{3}MT$, No. 12 .... *Zircon.*

4\. 1. 2. **61.** MT. $P\frac{2}{3}MT$. The obtuse quadratic octahedron, $P\frac{2}{3}MT$, No. 12, with its horizontal edges replaced by the vertical planes of a right square prism, MT................. *Zircon.*

4\. 1. 2. **62.** (M,T. $P\frac{2}{3}M$, $P\frac{2}{3}T$) × 2. A hemitrope or twin crystal, each individual of which is an obtuse quadratic octahedron similar to No. 12, having its horizontal edges replaced by the vertical planes of a right square prism, M,T, forming a combination similar to No. 61. The plane of junction is P_MT Nse .... *Oxide of Tin.*

4\. 1. 1. **63.** MT. PM, PT. The rhombic dodecahedron .... *Garnet.*

4\. 1. 1. **64.** mt. pm, pt, PMT. The regular octahedron, No. 15, with its edges replaced by the planes of the rhombic dodecahedron, No. 63 .... *Red Oxide of Copper.*

4\. 1. 1. **65.** MT. PM, PT, pmt. The rhombic dodecahedron, No. 63, with its obtuse three-faced angles replaced by the planes of the regular octahedron, No. 15 .... *Amalgam.*

4\. 3. 3. **66.** $M\frac{8}{10}T.\ P\frac{1}{1}\frac{9}{10}M\frac{8}{10}T$. The acute scalene octahedron, No. 21, with its equatorial edges replaced by the vertical planes of a rhombic prism, $M\frac{8}{10}T$ .... *Sulphur.*

4\. 3. 3. **67.** $M\frac{4}{3}\frac{9}{0}T.\frac{1}{2}P\_M\frac{4}{3}\frac{9}{0}T$ ZneZnw, $\frac{1}{2}P\_M\frac{4}{3}\frac{9}{0}T$ ZseZsw. A rhombic prism nearly square-based, with its terminal edges and terminal planes replaced by the planes of an obtuse scalene octahedron, or by the planes of two obtuse scalene hemioctahedrons differing a little in size......... *Mesotype.*

4\. 3. 1. **68.** $M\frac{1}{2}T$, $M\frac{2}{1}T.\ P\frac{1}{2}M$, $P\frac{2}{1}M$, $P\frac{1}{2}T$, $P\frac{2}{1}T$. A tetrakishexahedron. *Native Copper.*

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4\. 4. 1. **69.** mt. pm, pt, $3P\frac{1}{2}MT$. The icositessarahedron, No. 22, combined with the rhombic dodecahedron, No. 63. The planes of the latter are distinguished by their rhombic shape, whereas the modified planes of the icositessarahedron are hexagonal ....*Garnet.*

4\. 5. 3. **70.** m_. $P\frac{19}{10}M\frac{8}{10}T$. The scalene octahedron, No. 21, with its obtuse solid angles replaced by the planes of the prismatic form m ....*Sulphur.*

4\. 5. 1. **71.** $T, M\frac{15}{13}T_2$. $\frac{1}{2}PM$ Zn, $\frac{1}{2}PM_2T\frac{15}{13}$: or V. R_1 Zn. The regular six-sided prism, No. 7, terminated by the obtuse rhombohedron, No. 26[a], but having the latter situated on the north instead of the east zone ....*Calcareous Spar.*

4\. 5. 1. **72.** $T, M\frac{15}{13}T_2$. $\frac{1}{2}PT$ Zw, $\frac{1}{2}PM\frac{15}{13}T_2$: or V. R_1 Zw. The regular six-sided prism, No. 7, terminated by the obtuse rhombohedron, No. 26[a], the planes of both forms retaining their usual positions. Or, we may describe No. 72, as the obtuse rhombohedron, No. 26[a], with its acute lateral angles replaced by the planes of the regular six-sided prism, No. 7.................................*Calcareous Spar.*

4\. 5. 1. **73.** $T, M\frac{15}{13}T_2$. $P\frac{5}{4}T$, $P\frac{5}{4}M\frac{15}{13}T_2$: or V. $2R\frac{5}{4}$ Zw Ze. The regular six-sided prism, No. 7, terminated by a regular six-sided pyramid similar to No. 26 .... *Quartz.*

4\. 5. 1. **74.** $T, M\frac{15}{13}T_2$. $P\frac{13}{13}T$. $P\frac{13}{13}M\frac{15}{13}T_2$: or V. $2R\frac{13}{13}$ Zw Ze. The obtuse regular six-sided pyramid, No. 26, with its equatorial edges replaced by the planes of the regular six-sided prism, No. 7 ....*Phosphate of Lead.*

4\. 5. 3. **75.** T_, $M\frac{9}{13}T$. $\frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$ Zne Znw, $\frac{1}{2}P\frac{6}{13}M\frac{10}{13}T$ Zse Zsw. A right rhombic prism, $M\frac{9}{13}T$, with its acute lateral edge deeply replaced by the planes of the form T, four of its terminal edges, Zne Znw Nse Nsw, replaced by the planes of the obtuse scalene hemioctahedron $\frac{1}{2}P\frac{5}{13}M\frac{9}{13}T$, and its other four terminal edges, Zse Zsw Nne Nnw, by the planes of the obtuse scalene hemioctahedron, $\frac{1}{2}P\frac{6}{13}M\frac{10}{13}T$. *Gypsum.*

INCOMPLETE PRISMS COMBINED WITH INCOMPLETE PYRAMIDS.

5\. 1. 2. **76.** P_. $P\frac{4}{5}M$, $P\frac{4}{5}T$. An obtuse quadratic octahedron, similar to No. 12, but belonging to the north and east zones, with its two obtuse solid angles replaced by the planes of the form P ....*Molybdate of Lead.*

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5. 1. 2. **77.** p. pm, pt, PMT. A square-based octahedron, very nearly the same as the regular octahedron, with its summits replaced by the planes of the form p, and its oblique edges by the planes of the form pm belonging to the north zone, and pt belonging to the east zone; all of which forms, except p, are nearly if not absolutely equiaxed. See part II. page 36................*Copper Pyrites.*

5. 1. 1. **78.** mt. pm, pt, $\frac{1}{2}$PMT. The regular tetrahedon, No. 117, with its solid angles replaced by the planes of the rhombic dodecahedron, No. 63 .... *Mixed Grey Copper.*

5. 2. 1. **79.** M_{-}, T. $\frac{1}{2}$ P$\frac{3}{7}$M Zn. A right rectangular prism, M_{-},T, terminated by the planes of a hemihedral form of the north zone, $\frac{1}{2}$P$\frac{3}{7}$M Zn Ns ....*Gypsum.*

5. 2. 1. **79**[a]. M_x,T. P$\frac{4}{7}$M. A right rectangular prism, M_x,T, with dihedral terminations belonging to the north zone, and consisting of the form P$\frac{4}{7}$M ....*Chrysoberyl.*

This model, held in the position indicated by the stamped letters, shows a very common variety of the regular six-sided prism, in which the form T predominates. Example: Phosphate of Lead, Vanadiate of Lead, Arseniate of Lead, &c. Symbol P_x, T_{-}, $M\frac{1}{1}\frac{5}{3}T_2$.

5. 2. 1. **79**[b]. M_{-},T. $\frac{1}{2}$P$\frac{1}{2}$M Zn. A right rectangular prism, M_{-},T, terminated by the planes of the hemihedral form $\frac{1}{2}$P$\frac{1}{2}$M Zn Ns ....*Epidote.*

This model, held in the position indicated by the stamped letters, represents *a right prism with a rhomboidal base,* which form was adopted by Haüy as the primitive form of Epidote.

Compare No. 79[b] with Nos. 101 and 101[a].

5. 3. 1. **80.** p_{+}. P$\frac{19}{10}$M$\frac{8}{10}$T. The scalene octahedron, No. 21, with the summits replaced by the horizontal form P.......*Sulphur.*

5. 3. 1. **81.** M$\frac{1}{2}\frac{5}{6}$T. $\frac{1}{2}$P$\frac{1}{2}$MZn, $\frac{1}{2}$ P$\frac{7}{13}$M Zs. A right rhombic prism, M$\frac{1}{2}\frac{5}{6}$T, terminated by two hemihedral forms of the north zone; the obtuse angles Zn Ns being replaced by the form $\frac{1}{2}$P$\frac{1}{2}$M, and the obtuse angles Zs Nn, by the form $\frac{1}{2}$P$\frac{7}{13}$M. But according to some crystallographers, (Weiss and Rose), these oblique forms are $\frac{1}{2}$P$\frac{1}{2}$M Zn, $\frac{1}{2}$p$\frac{1}{2}$m Zs, or forms of the same kind but of different magnitude.........*Felspar.*

5. 3. 1. **81**[a]. $\frac{1}{2}$M$\frac{1}{2}\frac{5}{6}$T ne, $\frac{1}{2}$m$\frac{1}{2}\frac{5}{6}$t nw. $\frac{1}{2}$P$\frac{1}{2}$M Zn. A right rhombic prism similar to M$\frac{1}{2}\frac{5}{6}$T in No. 81, but consisting of two pair of planes of unequal size. It is terminated by a hemihedral form of the north zone, the planes of which replace the Zn Ns obtuse angles of the rhombic prism. The planes $\frac{1}{2}$ P$\frac{7}{13}$M, which in No. 81 replace the Zs Nn obtuse angles of the rhombic prism, are absent from this combination ....*Felspar.*

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5. 3. 1. **81**ᵃ. *Continued:*

Compare No. 81ᵃ with No. 105, which contains the same forms, excepting that $\frac{1}{2}M\frac{15}{26}T$ nw is replaced by T.

Model 81ᵃ also nearly represents some of the forms of Felspar that are described by the symbols $M\frac{15}{26}T$. $\frac{1}{2}P\frac{1}{3}M$ Zn. Axes: $p^a_+ m^a_- t^a$. However, the resemblance of this model to either of the forms described is only approximate, the interfacial angles being different from those of Felspar.

5. 3. 1. **81**ᵇ. $M\frac{2}{3}T$. $\frac{1}{4}P_x M_y T_z$ Z^2nw, $\frac{1}{4}P_x M_y T_z$ Zu^2e^2, $\frac{1}{4}p_x m_y t_z$ Z^2ne^2. A right rhombic prism, terminated by three tetarto-octahedral forms having the positions indicated in the symbols.

This is one of the doubly oblique combinations, which all consist of one homohedral or two or more hemihedral vertical prismatic forms, with one or more tetarto-octahedral forms, forming three or more pair of planes situated obliquely to one another........................*Axinite.*

5. 3. 1. **82**. $M\frac{4}{3}T$. $P\frac{5}{4}T$. Axes: $p^a_{12} m^a_4 t^a_5$. A right rhombic prism, $M\frac{4}{3}T$, terminated by an oblique form of the east zone, $P\frac{5}{4}T$, the planes of which are placed on the Zw Ze Ne Nw acute angles of the prism. The eidogen of the east zone is much larger than that of the prismatic zone, so that the combination has an acicular or columnar appearance......
Columnar Heavy Spar.

5. 3. 1. **82**ᵃ. $M\frac{6}{10}T$. $P\frac{7}{10}T$. Axes: $p^a_7 m^a_6 t^a_{10}$. A right rhombic prism, $M\frac{6}{10}T$, terminated by an oblique form of the east zone, $P\frac{7}{10}T$, the planes of which have the positions Zw Ze Ne Nw..................................*Carbonate of Lead.*

This form is commonly considered to be a rectangular octahedron $=$ $P\frac{10}{6}M$, $P\frac{10}{7}T$. See note at page 66.

5. 3. 1. **82**ᵇ. $M\frac{4}{3}T$. $P\frac{5}{4}T$. Axes: $p^a_{25} m^a_{16} t^a_{20}$. A right rhombic prism, $M\frac{4}{3}T$, terminated by an oblique form of the east zone, $P\frac{5}{4}T$, the planes of which have the positions Zw Ze Ne Nw....
Sulphate of Lead.

This model, held in the position indicated by the letters stamped upon it, is a rectangular octahedron. Nos. 82ᵃ and 82ᵇ are combinations of the same kind, only differing in dimensions. Compare No. 82ᵇ with No. 104.

83. M_T. $\frac{1}{2}P\_M$ Zn. Altered to No. 26ᵃ.

5. 3. 1. **84**. $M\frac{10}{19}T$. $\frac{1}{2}P\frac{4}{13}M$ Zn. Axes: $p^a_4 m^a_{10} t^a_{19}$. A right rhombic prism, $M\frac{10}{19}T$, terminated by an oblique hemihedral form of the north zone, $\frac{1}{2}P\frac{4}{13}M$, whose planes are situated Zn Ns...*Hornblende.*

85. M_T. $\frac{1}{2}P\_M$ Zn. Altered to No. 26ᵇ.

86. M_T. $\frac{1}{2}P\_M$ Z^2n, $\frac{1}{2}p_+m_.$ Zn^2. Altered to No. 114ᵃ.

C. O. G. NO.

5. 3. 1. **87**. $M\frac{2}{2}\frac{0}{1}T$. $\frac{1}{2}P\frac{6}{21}T$ Zw. Axes: $p^a_6 m^a_{10} t^a_{21}$. A right rhombic prism, $M\frac{2}{2}\frac{0}{1}T$, terminated by an oblique hemihedral form of the east zone, $\frac{1}{2}P\frac{6}{21}T$, the planes of which occupy the positions Zw Ne .... *Augite.* (*Mussite*).

88. M_T. $\frac{1}{2}$P_T Zw. Altered to No. 26[c].

89. M_T. $\frac{1}{2}$P_T Zw. Altered to No. 26[d].

5. 3. 1. **90**. $M\frac{1}{3}\frac{0}{6}T$, $m\frac{1}{1}\frac{0}{8}t$. $P\frac{2}{2}\frac{8}{9}T$, $p\frac{3}{3}\frac{3}{6}m\frac{1}{3}\frac{0}{6}t$. Axes: $p^a_+ m^a_- t^a$. A right rhombic prism, $M\frac{1}{3}\frac{0}{6}T$, which has its acute vertical edges bevelled by another right rhombic prism, $m\frac{1}{1}\frac{0}{8}t$, its acute solid angles replaced by an oblique form of the east zone, $P\frac{2}{2}\frac{8}{9}T$, the planes of which are situated Zw Ze Ne Nw, and its obtuse solid angles bevelled by the planes of the obtuse scalene octahedron, $p\frac{3}{3}\frac{3}{6}m\frac{1}{3}\frac{0}{6}t$ .... *Topas.*

5. 4. 1. **91**. $M\frac{1}{2}T$. $P\frac{1}{2}M$, $P\frac{2}{1}T$. A pentagonal dodecahedron, the planes of which consist of three equal and similar rhombic forms, the first belonging to the prismatic zone, the second to the north zone, and the third to the east zone. See Part I., page 35 .... *Iron Pyrites from Elba.*

5. 4. 1. **92**. $M\frac{1}{2}T$. $P\frac{1}{2}M$, $P\frac{2}{1}T$, PMT. The middle crystal between the pentagonal dodecahedron, No. 91, and the regular octahedron, No. 15 .... *Iron Pyrites from Elba.*

5. 4. 1. **93**. $m\frac{1}{2}t$. $p\frac{1}{2}m$, $p\frac{2}{1}t$, PMT. The regular octahedron, No. 15, with its solid angles bevelled by the planes of the pentagonal dodecahedron, No. 91; a combination resembling No. 92 in the number and positions of its planes, but differing in respect to their relative magnitude ....

Bright White Cobalt from Tunaberg.

5. 4. 1. **94**. mt. pm, pt, $\frac{1}{2}$PMT, $\frac{1}{2}(3p\frac{1}{2}mt)$ Z^2nw. Axes: $p^a m^a t^a$. The regular tetrahedron, No. 117, with its solid angles replaced by the planes of the rhombic dodecahedron, No. 63, and its edges bevelled by the planes of the triakistetrahedron (or hemiicositessarahedron), No. 119.........

Grey Copper.

5. 4. 1. **95**. MT. PM, PT, $\frac{1}{2}$pmt Znw, $\frac{1}{2}(3p\frac{1}{3}mt)$ Z^2ne. The rhombic dodecahedron, No. 63, with four of its obtuse three-faced angles replaced by the planes of the right tetrahedron, No. 117, whose positions are Znw Zse Nne Nsw, and with its six acute four-faced angles replaced by the planes of a left triakistetrahedron, a form similar in figure to No. 119, but in different positions, having its two upper planes situated Z^2ne Z^2sw, instead of Z^2nw Z^2se, as marked upon the model .... *Zinc Blende from Kapnik.*

5. 5. 1. **96**. p_. $P\frac{1}{1}\frac{3}{5}T$, $P\frac{1}{1}\frac{3}{3}M\frac{1}{1}\frac{3}{3}T_2$: or p_. $2R\frac{1}{1}\frac{3}{3}$ Zw Ze. The obtuse regular six-sided pyramid, No. 26, with the summits replaced by the planes of the form p...........................

Phosphate of Lead from Johann-Georgenstadt.

C. O. G. NO.

5. 5. 2. **97.** M,T, $M\frac{5}{8}T$. $P\frac{7}{10}T$. A right rhombic prism, $M\frac{5}{8}T$, with its obtuse vertical edges replaced by the form M, its acute vertical edges by the form T, and its acute solid angles by an oblique form of the east zone, $P\frac{7}{10}T$, the planes of which are situated Zw Ze Ne Nw. Compare this Model with No. 111.........*Arragonite from Piedmont.*

5. 5. 2. **98.** M, T, $M\frac{2}{3}\frac{9}{1}T$. $\frac{1}{2}P\frac{6}{21}M\frac{1}{3}\frac{9}{1}T$ Znw Zsw. Axes: $p\frac{1}{+}m^{a}t^{a}$. A combination similar to No. 53, excepting that it is without the horizontal planes P...................*Augite.*

5. 5. 2. **99.** (M,T, $M\frac{2}{3}\frac{9}{1}T$. $\frac{1}{2}P\frac{6}{21}M\frac{1}{3}\frac{9}{1}T$ Znw Zsw) $\times$ 2. A hemitrope or twin crystal of the combination represented by No. 98. If the latter were divided into two pieces by a section passing through the north meridian, and the halves were joined together by the same plane, after inverting one of them, the result would be the same as No. 99. *Augite.*

5. 5. 2. **100.** M_, $M\frac{4}{5}T$. $P\frac{4}{3}M$. Axes: $p^{a}m^{a}_{-}t\frac{1}{+}$. A right rhombic prism, $M\frac{4}{5}T$, No. 6, with its obtuse vertical edges replaced by the planes of the prismatic form M, and its obtuse solid angles by an oblique form of the north zone, $P\frac{4}{3}M$, the planes of which are situated Zn Zs Ns Nn...*Heavy Spar.*

5. 5. 2. **101.** M_. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{1}{2}\frac{1}{4}m$ Zs, $\frac{1}{2}P_{+}MT_{-}$ Zne² Znw². Axes: $p^{a}m^{a}_{-}t\frac{1}{+}$. A right rectangular prism, M_,T, No. 5, with its Zn Ns edges replaced by the planes of an oblique hemihedral form of the north zone, $\frac{1}{2}P\frac{1}{2}M$ Zn Ns, No. 79ᵇ, its Zs Nn edges replaced by another oblique hemihedral form of the north zone, $\frac{1}{2}p\frac{1}{2}\frac{1}{4}m$ Zs Nn, and its east and west vertical planes replaced by the scalene hemioctahedron, $\frac{1}{2}P_{+}MT_{-}$, whose planes are situated Zne² Znw² Nse² Nsw², and meet at the east and west poles ....*Epidote.*

5. 5. 2. **101ᵃ.** M_, t, $M\frac{1}{1}\frac{8}{1}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}p\frac{3}{2}m$ Zs, $\frac{1}{2}P_{+}MT_{-}$ Zne² Znw². Axes: $p^{a}m^{a}_{-}t_{+}$. A right rectangular prism, M_,T, No. 5, with its vertical edges replaced by the planes of a right rhombic prism, $M\frac{1}{1}\frac{8}{1}T$, its Zn Ns edges by the planes of an oblique hemihedral form of the north zone, $\frac{1}{2}P\frac{1}{2}M$; its Zs Nn edges by the planes of an oblique hemihedral form of the north zone, $\frac{1}{2}p\frac{3}{2}m$; and its Ze Zw Nw Ne edges by the planes of a scalene hemioctahedron, $\frac{1}{2}P_{+}MT_{-}$ the positions of which are Zne² Znw² Nse² Nsw², ...*Epidote.*

102. m, M_T. $\frac{1}{2}P\_M$ Z²n, $\frac{1}{2}p_{+}m$ Zs², $\frac{1}{2}P_{x}M_{y}T_{z}$ Zn²e Zn²w, $\frac{1}{2}p_{x}m_{y}t_{z}$ Zne² Znw². Altered to No. 26ᵉ.

5. 5. 2. **103.** m, $M\frac{6}{7}T$. $\frac{1}{2}P\frac{3}{14}M$ Zn, $\frac{1}{2}P_{+}M\_T$ Zn²e Zn²w, $\frac{1}{2}p_{x}m_{y}t_{z}$ Zne² Znw². A right rhombic prism, $M\frac{6}{7}T$, with its obtuse vertical edges replaced by the form m, its obtuse solid angles Zn Ns by an oblique hemihedral form of the

. O. G. NO.

. 5. 2. **103.** *Continued:*
north zone, $\frac{1}{2}P\frac{3}{14}M$, its Zne Znw Nse Nsw terminal edges by the planes of a scalene hemioctahedron, $\frac{1}{2}P_{+s}M_{-}T$ Zn^2e Zn^2w Ns^2e Ns^2w, and its acute solid angles by the planes of a scalene hemioctahedron, $\frac{1}{2}p_x m_y t_z$ Zne^2 Znw^2 Nse^2 Nsw^2.........*Azure Copper Ore.*

A comparison of this model with No. 102 (26^c) will show how nearly the oblique prismatic combinations are related to the rhombohedral combinations.

. 5. 2. **104.** M, $M\frac{4}{5}T$. $P\frac{5}{4}T$. A right rhombic prism, $M\frac{4}{5}T$, with its obtuse vertical edges replaced by the form M, and its acute angles replaced by an oblique form of the east zones, $P\frac{5}{4}T$, the planes of which, situated Zw Ze Ne Nw, constitute dihedral terminations to the prism..............

Sulphate of Lead.

This combination is the same as No. 82^b, excepting that it has the form M additional.

. 5. 2. **105.** T, $\frac{1}{2}M\frac{1}{2}\frac{5}{6}T$ ne sw. $\frac{1}{2}P\frac{1}{2}M$ Zn Ns. This model represents the cleavage form of Felspar, and is *Haüy's* assumed primitive form of that mineral. It consists of the form T, half the planes of the form $M\frac{1}{2}\frac{5}{6}T$, the usual rhombic prism of felspar, and this half holding the positions ne sw, being a parallel pair of planes, and finally, the oblique hemihedral form of the north zone, $\frac{1}{2}P\frac{1}{2}M$ Zn Ns, the form so characteristic of the felspar crystals, where T upon $\frac{1}{2}P\frac{1}{2}M$ measures 90°.

The relation of this combination to the ordinary rhombic crystals of felspar, is distinctly shown by the figure drawn on the north east quadrant of No. 109. On holding the west plane of No. 105 against the east plane of No. 109, and bringing the two Zn planes to the same level, the agreement of the planes $M\frac{1}{2}\frac{5}{6}T$ and $\frac{1}{2}P\frac{1}{2}M$, on the two models, may be both seen and felt.

Felspar from Puy-de-Dôme.

106. $M_{-}T$. $\frac{1}{2}P_x M$ Zn. Altered to 81^a.

. 5. 2. **107.** M, $\frac{1}{2}m\frac{7}{10}t$ n^2w, $\frac{1}{2}M\frac{7}{2}T$ nw^2, $\frac{1}{2}m\frac{7}{8}t$ ne. $\frac{1}{4}P_{-}M_{+}T$ Z^2ne. A doubly oblique combination, consisting of the form M, three hemi-rhombic vertical forms, $\frac{1}{2}m_x t$, each of them being a parallel pair of planes, and one scalene tetarto-octahedron, occupying the positions Z^2ne N^2sw. *Cyanite.*

108. Altered to No. 81^b.

. 5. 2. **109.** T, $M\frac{1}{2}\frac{5}{6}T$. $\frac{1}{2}P\frac{1}{2}M$ Zn, $\frac{1}{2}P\frac{7}{15}M$ Zs. This combination is the same as No. 81, with the addition of the form T

Felspar.

The figure drawn upon the north east quadrant of the model, is explained in the description of No. 105.

C. O. G. NO.

5. 5. 2. **110**. T$_-$, M$\frac{6}{10}$T. p$\frac{7}{4}$m, P$\frac{10}{7}$T. Axes: p$\frac{2}{+}$m^{2}t$\underline{\ }$. A right rhombic* prism, M$\frac{6}{10}$T, similar to the vertical form of the combination, No. 82^a, with its acute vertical edges deeply replaced by the form T, its obtuse solid angles replaced by an oblique form of the north zone, p$\frac{7}{4}$m, whose planes occupy the positions Zn Zs Nn Ns; and its acute solid angles, by an oblique form of the east zone, P$\frac{10}{7}$T, the planes of which are situated Ze Zw Ne Nw *White Lead Ore.*

5. 5. 2. **111**. T$_+$, M$\frac{5}{8}$T. P$\frac{7}{10}$T. This combination is the same as No. 97, excepting that it is without the form M. *Arragonite from Piedmont.*

5. 5. 2. **112**. T,M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn,$\frac{1}{2}$P^4_xM$^{15}_7$T Zse Zsw. A right rhombic prism, M$\frac{1}{1}\frac{0}{9}$T, with its acute vertical edges replaced by the form T, its Zn Ns obtuse solid angles replaced by an oblique hemihedral form of the north zone, $\frac{1}{2}$P$\frac{4}{13}$M, and its Zse Zsw Nne Nnw terminal edges replaced by the planes of the scalene hemioctahedron, $\frac{1}{2}$P^4_xM$^{15}_7$T. *Hornblende.*

5. 5. 2. **113**. (T, M$\frac{1}{1}\frac{0}{9}$T. $\frac{1}{2}$P$\frac{4}{13}$M Zn, $\frac{1}{2}$P^4_xM$^{15}_7$T Zse Zsw) $\times$ 2. A hemitrope or twin crystal of the combination represented by No. 112. If we suppose No. 112 to be divided by the east meridian, one of the halves to be turned upside down, and the two to be joined together by the same planes as before, the result would be similar to No. 113. *Hornblende.*

5. 5. 1. **114**. P$_-$ $\frac{1}{2}$P$\frac{3}{2}$T, $\frac{1}{2}$P$\frac{3}{2}$M$\frac{1}{1}\frac{5}{3}$T$_2$: or P$_-$. R$\frac{3}{2}$. An acute rhombohedron nearly similar to No. 26^d, with its acute solid angles deeply truncated by the horizontal form P...*Corundum.*

According to the letters stamped upon this model, the description of it would be, M$_-$T. $\frac{1}{2}$P$_-$T Ze, $\frac{1}{2}$P$_+$T Zw.

5. 5. 1. **114^a**. P$_-$. $\frac{1}{2}$PT, $\frac{1}{2}$PM$\frac{1}{1}\frac{5}{3}$T$_2$: or P$_-$. R$_1$. An obtuse rhombohedron, R$_1$, No. 26^a, with its obtuse solid angles truncated by the horizontal planes P...............*Calcareous Spar.*

According to the letters stamped upon this model, it should be described as follows: M$_-$T.$\frac{1}{2}$P$_-$M Z^{2}n,$\frac{1}{2}$p$_+$m Zn^2.

5. 5. 2. **115**. T$_-$, M$\frac{9}{13}$T. $\frac{1}{2}$P$\frac{5}{13}$M$\frac{9}{13}$T Z^{2}ne Z^{2}nw. This combination is the same as that represented by No. 75, excepting that one of the obtuse scalene hemioctahedrons, namely, $\frac{1}{2}$P$\frac{6}{13}$M$\frac{1}{1}\frac{5}{3}$T Zse Zsw Nne Nnw, which occurs upon No. 75, does not occur upon this model...........................
Gypsum from Montmartre.

116. M$_-$T. P$_+$T. $\frac{1}{2}$P$_x$M$_y$T$_z$ Z^{2}ne Z^{2}nw. Altered to No 26^f.

INCOMPLETE PYRAMIDS.

C. G. O. NO.

6. 1. 1. **117.** $\frac{1}{2}$PMT. The regular tetrahedron, or hemioctahedron, or right tetrahedron; the four planes of which occupy the positions Znw Zse Nne Nsw .... *Grey Copper.*

6. 1. 1. **118.** $\frac{1}{2}$PMT, $\frac{1}{2}$pmt. The right tetrahedron, with its solid angles replaced by the planes of the left tetrahedron, the latter occupying the positions Zne Zsw Nse Nnw. *Grey Copper.*

6. 3. 1. **119.** $\frac{1}{2}$(3P$\frac{1}{2}$MT). A triakistetrahedron, or hemiicositessarahedron; the hemihedral form of the combination represented by No. 22 .... *Grey Copper.*

6. 5. 1. **120.** p$\frac{19}{10}$t, P$\frac{19}{10}$M$\frac{8}{10}$T. An acute scalene octahedron, No. 21, with its acute terminal edges replaced by an oblique form of the east zone, p$\frac{19}{10}$t, which possesses the same relations to the axes p^a and t^a as do the planes of the octahedron itself .... *Sulphur.*

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